Reconciling Theoretical and Empirical

Human Capital Earnings Functions

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Abstract

To interpret estimates of empirical earnings functions, and to resolve sample selection problems such as "tenure bias", the wage determination process must be specified. This paper shows that an earnings function can be interpreted as a wage offer in a labour market auction in which the worker accepts the highest offer received. The elasticity of the wage is one with respect to general human capital, and less than one for specific capital. There is a sample selection problem because only accepted offers are observed. This causes underestimation of the return to specific capital in a cross-sectional earnings regression.

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1. Introduction

Estimation of empirical earnings functions, in which the logarithm of the wage is regressed on measures of human capital for a cross-section of individuals, has been a popular technique in labour market research since the work of Mincer (1974), yet the earnings function has surprising little theoretical foundation. The original Mincerian earnings function, containing measures of general human capital such as years of schooling and experience, could be interpreted on the basis of competitive theory as a technological relationship – the effect of human capital on productivity. But the introduction of measures of specific human capital such as job tenure (Mincer and Jovanovic, 1981) and, in more recent studies, of firm and industry variables as well as individual variables, precludes the possibility of perfectly competitive wage determination, and raises the question: what are we estimating?

The answer must be, as Topel (1991) assumes in his discussion of the potential bias in the estimation of the returns to experience and tenure, that it is some kind of wage determination function. But without some assumptions about the nature of the wage determination process it is not possible to resolve the potential sample selection problem in the estimation of earnings functions, nor is it clear how to interpret the results.

The sample selection problem arises because of labour turnover. A worker who is observed working for a particular firm does so as a result of choices made by the worker and the firm: employment determination occurs jointly with wage determination. To fully understand wages, we would like to know for each worker what he would have earned if randomly assigned to any firm, but we only observe his wage in the chosen firm.

The effect of sample selection on estimation of the return to tenure has been particularly contentious. It has been argued that there is a positive bias, because tenure may be correlated with unobserved match quality, or worker quality. In apparent support of this argument, Abraham and Farber (1987) and Altonji and Shakotko (1987) found the effect of tenure on wages to be almost insignificant after controlling for unobserved heterogeneity. But Topel (1991) argued that a matching model of turnover could be expected to cause a negative bias. Further, he criticised the methods used in the earlier empirical papers to control for heterogeneity, and found a strong positive tenure effect in his own analysis of the same data[1].

These arguments have been conducted at an informal level. Most empirical discussions of such problems make the implicit assumption of some kind of matching process to determine employment, combined with an unspecified wage determination process. The purpose of this paper is to present a formal model of wage and employment determination in a labour market characterised by match heterogeneity, which is consistent with the existing theory of general and specific human capital, and which provides a precise interpretation of the empirical earnings function.

It is shown that the empirical earnings function can be regarded as a wage offer function in a labour market auction in which employers make wage offers after privately observing match quality, and workers accept the highest available offer. In estimation of this function a sample selection problem arises because we observe only those offers that are accepted. Sample selection bias leads to under-estimation of the return to specific human capital in a least-squares wage regression.

2. Wage Determination with General and Specific Human Capital

2.1 Theoretical Models

Classical human capital theory suggests that wages respond fully to general human capital but only partially to specific human capital (Becker, 1962; Oi, 1962). The intuition that the return to specific human capital will be shared between worker and firm, in the form of a wage higher than the worker's opportunity wage but lower than productivity, was formalised by Hashimoto (1981). In Hashimoto's model, the value of the worker's specific human capital is subject to shocks. Its ex-post value within the firm (within-firm productivity) is observed only by the firm, and its ex-post value elsewhere (the opportunity wage) is observed only by the worker. The asymmetry of information may result in inefficient separation and loss of specific capital; to minimise this loss it is desirable for the worker and firm to predetermine the wage. Hashimoto derives the optimal predetermined wage; he does not prove "sharing" in the sense that the wage lies strictly between the expected alternative wage and expected productivity, but it can be shown that if the two shocks are identically distributed[2] then it will do so. In this model the external labour market is assumed to be perfectly competitive, so that the return to general human capital accrues to the worker.

Hashimoto's analysis was important because it showed that the wage of a worker possessing specific human capital is determinate only under the assumption that wage and employment decisions are affected by asymmetric information, but the model begs some awkward questions. If specific human capital can have an ex-post value in other firms, is it really specific? How can the firm be unaware of the worker's alternative wage in a competitive labour market? These problems can be avoided as in Carmichael's (1983) model by assuming instead that the worker experiences a taste shock which affects his utility of remaining in the current firm. But with this interpretation Hashimoto's model does not necessarily predict "sharing" unless we make the rather arbitrary assumptions that the taste shock is proportional to the size of the investment in specific human capital, and distributed independently but identically to the productivity shock.

More importantly for the purpose of the present paper, it is clear that if empirical earnings functions represent *predetermined wage functions* then there cannot be a sample selection problem arising from employment choices, precisely because the wage is predetermined, before the shocks which determine employment decisions are realised. These shocks contribute some unobserved quality to the match; they affect the separation decision and hence whether or not the wage is observed, but have no effect on the wage itself. The predetermined wage reflects only the worker's expected human capital, and observed wages are a random sample of predetermined wages.

Thus, if we believe that we are estimating predetermined wage functions there is no need to be concerned about sample selection bias in the estimation of returns to human capital variables. But as noted by Gibbons and Waldman (1999), post-training wages are not typically specified in a contract. Moreover, the predetermined wage is not the only type of employment contract possible for workers with specific human capital. Wages may be determined ex-post by bargaining; when asymmetry of information makes bargaining infeasible or costly, Hall and Lazear (1983) identified other simple second-best contracts, such as allowing worker or firm to set the wage unilaterally. There is no reason to suppose such contracts are less efficient than the predetermined wage; Stevens (1994) shows that it may be preferable for the firm to set the wage. Having chosen a mechanism for wage determination, a worker and firm wishing to invest in specific human capital can determine their respective shares of the expected match surplus under that contract, and bargain over the sharing of costs.

In this paper we develop an alternative to the predetermined wage model of individual wage determination with specific and general human capital. The labour market is characterised by match heterogeneity, to be consistent with the assumptions underlying many empirical analyses. Like Hashimoto and Hall and Lazear we assume that information asymmetries rule out bilateral bargaining; when the worker's productivity varies across firms it is plausible to suppose that firms are not fully aware of the alternative opportunities available to their employees, and that employees are uncertain about their own value to their employer. In contrast to these two papers, the external labour market is not perfectly competitive: this follows from the assumptions that not only the worker's existing match but also his potential matches can vary in quality, and that match productivity is the private information of the employer. We then assume that the wage and employment of an individual worker are determined according to a private-value auction mechanism: firms make simultaneous wage offers and the worker accepts the highest offer. We will show that the labour market auction results in sharing of the return to specific human capital[3], but that in contrast to the predetermined wage mechanism, there is a sample selection problem in the estimation of the worker's share of the return.

2.2 Empirical Issues: Specific Capital, Match Quality and Tenure

The focus in recent empirical work has been on how to estimate the "true" return to tenure. But the original question that Mincer and Jovanovic (1981) hoped to address by including tenure in a cross-sectional earnings function was whether specific human capital was an important determinant of wages. We need to know whether employment relationships are characterised by a significant amount of specific capital if, for example, we want to assess the gains and losses associated with labour mobility and displacement.

There are two problems with the attempt to address these more fundamental issues by estimating the return to tenure. First, if specific capital is normally acquired during the first months of employment, a tenure variable measured in years may not capture the pattern of accumulation very well. But also, it is argued (for example by Abraham and Farber, 1987) that to estimate the "true" return to tenure it is necessary to eliminate the effect on wages of an unobservable component of match quality, constant throughout the duration of a match, and correlated with tenure because high quality matches tend to last longer. But this kind of match quality is, by definition, specific human capital: it is a permanent component of match productivity that would be lost if the worker were to change employers. By removing it we underestimate the extent of the worker's loss if match is destroyed. It might be preferable simply to regard tenure as a rather poor proxy for the specific value of a match, picking up both accumulating specific capital and permanent match quality.

Do these arguments mean that, whatever measure of specific human capital is used in an earnings function, we should treat unobservable match quality as unobservable specific capital? If so, we have a measurement error problem, not a sample selection problem. The answer depends on what we want to estimate.

The approach we take in this paper is to use a static theoretical model of wage determination, which could form the basis for a static empirical model. The productivity of a match is determined by the worker's general and specific human capital, which is assumed to be observable by all agents, and also affected by match quality shocks of expected value zero, observed only by agents within the match. *Observable specific human capital* is the amount by which the worker's productivity in his present job is expected by all agents, ex-ante (before match qualities are realised), to exceed his productivity elsewhere. It also represents the expectation by agents in the external labour market of the worker's specific value, at the time when wages are determined. We will suppose that the econometrician wishes to estimate the worker's return to observable specific capital.

We also assume for simplicity that the econometrician has a perfect measure of observable specific capital. It is straightforward to adapt the analysis in the usual way if he observes it with error. We will return to discuss the implications of tenure acting as a noisy signal of specific human capital in section 5 below.

3. An Auction Model of Wage Determination

Consider a worker attached to firm 0, who has general human capital $g \geq 1$, and additional human capital $k \geq 1$ specific to firm 0. His productivity if he remains in firm 0 is v_0, where:

$$\ln v_0 = \ln g + \ln k + \ln \varepsilon_0 \tag{1}$$

There are n alternative employers, and if he moves to an alternative firm i, his productivity is v_i:

$$\ln v_i = \ln g + \ln \varepsilon_i \tag{2}$$

$\varepsilon_i > 0$ ($i \geq 0$) represents the quality of his match with firm i, and is observable only by firm i; g and k are common knowledge.

(1) and (2) capture the technological relationship between human capital and productivity. Wages are determined through a first-price private-value auction: each of the n+1 firms privately observes its own match quality, and makes a wage offer w_i; the worker accepts the highest offer.

3.1 Distributional Assumptions

Match qualities $\varepsilon_0, \varepsilon_1, \ldots\ldots \varepsilon_n$ are independent and identically distributed with density and distribution functions $f(\varepsilon)$ and $F(\varepsilon)$. In order to apply results from auction theory we make the following distributional assumptions:

D1: Finite support: $\varepsilon_i \in [\underline{\varepsilon}, \bar{\varepsilon}]$

D2: $f(\underline{\varepsilon}) > 0$

We also require:

D3: The density of $\ln\varepsilon_i$ is strictly log-concave.

A log-concave function is a function whose logarithm is concave. The class of log-concave densities is a wide one, and includes the uniform distribution and the truncated normal and log-normal distributions (Caplin and Nalebuff, 1991).

Strictly log-concave densities have the property that the log of the distribution function and the log of the integral of the distribution function are also strictly concave functions. So the assumption that $\ln\varepsilon_i$ is strictly log-concave implies, for $f(\varepsilon)$ and $F(\varepsilon)$, that the elasticity of $F(\varepsilon)$ is strictly decreasing:

$$\frac{de}{d\varepsilon} < 0 \text{ where } e(\varepsilon) \equiv \frac{\varepsilon f(\varepsilon)}{F(\varepsilon)} \tag{3}$$

A fortiori, $f(\varepsilon)$ and $F(\varepsilon)$ are log-concave; so also are $f_n(\varepsilon)$ and $F_n(\varepsilon) \equiv F^n(\varepsilon)$, the density and distribution of $\max_{i=1,..,n} \varepsilon_i$. For the subsequent analysis, it is useful to note that, if we define:

$$H_n(\varepsilon) \equiv \int_{\underline{\varepsilon}}^{\varepsilon} F_n(x)dx \tag{4}$$

then not only is F_n / H_n a decreasing function (since H_n is log-concave), but also:

$$\frac{d}{d\varepsilon}\left(\frac{\varepsilon F_n(\varepsilon)}{H_n(\varepsilon)}\right) < 0 \tag{5}$$

The proof of (5) is given in the appendix.

3.2 Analysis of the Auction

The effect of specific human capital is to introduce an asymmetry between the players in the wage determination game. We will look for an equilibrium in which firm 0 offers $w_0=w_0(\varepsilon_0,g,k)$ and each of the n alternative firms uses the same strategy $w_i=w_a(\varepsilon_i,g,k)$. Firms choose their wage offer w to maximise payoffs:

for firm 0: $\Pi_0(\varepsilon_0, w) = (v_0 - w)\Pr[w_a(\varepsilon_j) \le w \; \forall j > 0]$

and for firm i>0: $\Pi_a(\varepsilon_i, w) = (v_i - w)\Pr[w_a(\varepsilon_j) \le w \; \forall j > 0, j \ne i, w_0(\varepsilon_0) \le w]$

Since general human capital g multiplies productivity for all firms, it is immediately clear that the elasticity of wage offers with respect to general human capital is unity:

LEMMA 1: If $W_0(\varepsilon, k)\equiv w_0(\varepsilon,1,k)$ and $W_a(\varepsilon, k)\equiv w_a(\varepsilon,1,k)$ are equilibrium wage offers for the case g=1, then $w_0 = gW_0(\varepsilon, k)$ and $w_a = gW_a(\varepsilon, k)$ are equilibrium wage offers for the general case $g\geq1$.

Hence we need only analyse the case g=1. Maskin and Riley (1996a,b) have proved the existence and uniqueness of equilibrium in auctions of this form. The equilibrium strategies are monotonic in ε, so we can define inverse wage offer functions: $\phi_0(w,k) \equiv W_0^{-1}(w,k)$ and $\phi_a(w,k) \equiv W_a^{-1}(w,k)$. Following the standard analysis, the first order conditions for maximisation of the payoff functions above yield a pair of differential equations for the inverse offer functions ϕ_0 and ϕ_a:

$$n\frac{f(\phi_a)}{F(\phi_a)}\frac{\partial\phi_a}{\partial w} = \frac{1}{k\phi_0 - w} \tag{6}$$

$$\frac{f(\phi_0)}{F(\phi_0)}\frac{\partial\phi_0}{\partial w} + (n-1)\frac{f(\phi_a)}{F(\phi_a)}\frac{\partial\phi_a}{\partial w} = \frac{1}{\phi_a - w} \tag{7}$$

Let $\underline{w}(k)$ and $\overline{w}(k)$ be the lowest and highest wage offers, respectively, of firm 0: that is: $\phi_0(\underline{w}) = \underline{\varepsilon}$ and $\phi_0(\overline{w}) = \bar{\varepsilon}$. The following results are a direct application of the results for asymmetric first-price auctions given by Maskin and Riley (1996a,b):

LEMMA 2:

(i) If k=1, $\underline{w}$=$\underline{\varepsilon}$; otherwise $\underline{w} = \arg\max F(w)^n (k\underline{\varepsilon} - w)$ and $\underline{\varepsilon}$<$\underline{w}$<$k\underline{\varepsilon}$.

(ii) $\phi_a(\underline{w}) = \underline{w}$ and $\phi_a(\overline{w}) = \bar{\varepsilon}$.

(iii) The differential equations (6) and (7) have a unique monotonic solution ϕ_0,ϕ_a satisfying $\phi_0(\underline{w}) = \underline{\varepsilon}$, $\phi_a(\underline{w}) = \underline{w}$, $\phi_0(\overline{w}) = \phi_a(\overline{w}) = \bar{\varepsilon}$.

Thus all firms make the same highest offer $\overline{w}$, and the same *effective* lowest offer $\underline{w}$. The alternative firms may have realised productivity lower than $\underline{w}$, in which case they cannot obtain the worker. Note also that whenever the productivity of the worker in the potential match is greater than the lowest wage offer $\underline{w}$, firms obtain a positive expected surplus:

$$\phi_a(\varepsilon) > \varepsilon \ \forall \varepsilon > \underline{w} \text{ and } \phi_0(k\varepsilon) > \varepsilon \ \forall \varepsilon \geq \underline{\varepsilon} \tag{8}$$

3.2.1 The Symmetric Case, *k*=1

When there is no specific capital, so that all bidders are the same, the system reduces with $\phi_0(w,1)$=$\phi_a(w,1)$≡$\phi(w)$ to a single differential equation:

$$n\frac{f(\phi)}{F(\phi)}\frac{\partial\phi}{\partial w} = \frac{1}{\phi - w} \tag{9}$$

which can be solved directly by standard techniques to yield:

$$W_0(\varepsilon,1) = \varepsilon - \frac{H_n(\varepsilon)}{F_n(\varepsilon)} \tag{10}$$

Using (5) it can be seen that the proportional “mark-down” of the wage offer below productivity, $(\varepsilon - W_0)/\varepsilon$, increases with unobserved match quality ε.

3.2.2 Results for the Asymmetric Auction

In the asymmetric case, we first compare the strategy of the current employer with that of alternative employers. Examining the differential equations (6) and (7), we have:

LEMMA 3: If $k>1$: $W_a(\varepsilon) < W_0(\varepsilon)$ for all $\varepsilon \in (\underline{\varepsilon}, \bar{\varepsilon})$

and $W_0(\varepsilon) < W_a(k\varepsilon)$ for all $k\varepsilon \in (\underline{\varepsilon}, \bar{\varepsilon})$.

PROOF: See Appendix.

The second inequality in Lemma 3 tells us that firm 0, where the worker has specific value, offers a lower wage than an alternative firm would offer with the same total productivity. Thus the current employer bids less aggressively than alternative firms (resulting, as usual in asymmetric first-price auctions, in inefficient allocation of the worker).

However, the first inequality says that firm 0 offers a higher wage than an alternative firm would offer with the same unobserved match quality. This means that, as we would expect, the presence of specific human capital tends to reduce the turnover probability. For the worker stays if and only if $W_0(\varepsilon_0, k) > \max_{i=1,..,n} W_a(\varepsilon_i, k)$. So the probability $P(\varepsilon_0, k)$ that the worker will stay with the current employer is given by:

$$P(\varepsilon_0, k) = F_n(\theta) \qquad \text{where } \theta(\varepsilon, k) \equiv \phi_a(W_0(\varepsilon, k), k) \tag{11}$$

and Lemma 3 implies that $\theta(\varepsilon, k) > \varepsilon = \theta(\varepsilon, 1)$, so:

$$P(\varepsilon_0, k) > P(\varepsilon_0, 1) \tag{12}$$

Next, we examine the impact of specific human capital on the strategy of the current employer by comparing the asymmetric solution $W_0(\varepsilon,k)$ of (6) and (7) with the symmetric solution $W_0(\varepsilon,1)$ of (9):

LEMMA 4 (Sharing of specific human capital): If k>1, $W_0(\varepsilon,1) < W_0(\varepsilon,k) < kW_0(\varepsilon,1)$ for all $\varepsilon \geq \underline{\varepsilon}$.

PROOF: See Appendix.

Lemma 4 states that specific human capital raises the wage, but by less than the increase in productivity. Combined with the result from Lemma 1, $w_0 = gW_0(\varepsilon, k)$, this implies that the wage determination process has the "classical" properties first proposed by Becker (1962): the return to general human capital accrues to the worker, but the return to specific human capital is shared between worker and firm.

4. Derivation of the Earnings Function

Now consider a first-order approximation to $\ln(w_0)$, the (log) wage offer of the current employer, valid for k close to 1 (that is, when specific human capital is small relative to general human capital):

$$\ln w_0 \approx \ln g + \alpha(\varepsilon_0)\ln k + \ln W_0(\varepsilon_0,1) \tag{13}$$

$$\text{where } \alpha(\varepsilon) \equiv \frac{1}{W_0(\varepsilon,1)}\left.\frac{\partial W_0}{\partial k}\right|_{k=1}$$

From Lemma 4, we have $0 < \dfrac{W_0(\varepsilon,k) - W_0(\varepsilon,1)}{(k-1)W_0(\varepsilon,1)} < 1$ for k>1 and $\varepsilon \geq \underline{\varepsilon}$. Taking the limit as k tends to 1 gives:

$$0<\alpha(\varepsilon)<1 \;\forall\, \varepsilon \in [\underline{\varepsilon},\bar{\varepsilon}] \tag{14}$$

Equation (13) is reminiscent of an empirical earnings function, except that the coefficient on specific human capital is not constant, but varies with the error term. Intuitively, we might expect this coefficient to decrease as unobserved match quality ε_0 increases. For we know (section 3.2.1) that in the absence of specific capital firms mark-down the wage more when match quality is high, so it seems plausible that high match quality obviates the need to reward observable specific human capital. Lemma 5 confirms this intuition for the normal and uniform distributions:

LEMMA 5: When unobserved match quality, lnε, has a uniform distribution, or a symmetric truncated normal distribution, the elasticity of the wage with respect to observable specific human capital declines with match quality: $\frac{d\alpha}{d\varepsilon} < 0$.

PROOF: See Appendix.

Taking expectations of (13) with respect to ε_0 gives:

$$E[\ln w_0] \approx \ln g + \bar{\alpha} \ln k + c \tag{15}$$

where $c = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \ln W_0(\varepsilon,1) dF(\varepsilon)$, and $\bar{\alpha} = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \alpha(\varepsilon) dF(\varepsilon) \in (0,1)$. Hence:

$$\ln w_0 \approx \ln g + \bar{\alpha} \ln k + c + \eta \tag{16}$$

where the error term η has expectation zero and is given by:

$$\eta \equiv (\ln W_0(\varepsilon_0,1) - c) + (\alpha(\varepsilon_0) - \bar{\alpha}) \ln k \tag{17}$$

Thus we have established:

PROPOSITION 1: A human capital earnings function can be interpreted as a log-linear approximation to the wage offer function in a labour market auction, valid when specific human capital is small relative to general human capital. The elasticity of the wage with respect to general human capital is 1, and the elasticity with respect to specific human capital lies strictly between 0 and 1.

4.1 Sample Selection Bias

The sample selection problem in estimation of (16) is that a wage offer w_0 is observed only if it is accepted. From (12) we know that (for k close to 1) specific human capital k increases the probability of acceptance $P(\varepsilon_0, k)$. Unobserved match quality ε_0 also increases this probability, which suggests a negative sample selection bias in estimation of the coefficient on specific human capital: amongst workers who accept the wage offer of their current firm, those with higher *k* will have, on average, lower unobserved match quality than those with low *k*, and this will obscure the effect of *k* on the wage.

To verify this conjecture, we need to examine the expectation of the error term (17), given acceptance of the wage offer. The density of ε_0 conditional on the wage having been accepted is given by:

$$\hat{f}(\varepsilon,k) = \frac{f(\varepsilon)F_n(\theta)}{\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_n(\theta)dF(x)} \tag{18}$$

and:

$$E[\eta|\text{acceptance}] = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (\ln W_0(\varepsilon,1)-c)\hat{f}(\varepsilon,k)d\varepsilon + \ln k \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (\alpha(\varepsilon)-\bar{\alpha})\hat{f}(\varepsilon,k)d\varepsilon \tag{19}$$

The direction of the sample selection bias in the estimation of $\bar{\alpha}$ is the sign of the derivative of this expected error with respect to *k*. Since we are assuming that *k* is close to 1, this is given by the sign of:

$$\left.\frac{\partial}{\partial k} E[\eta|\text{acceptance}]\right|_{k=1} = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (\ln W_0(\varepsilon,1)-c)\left.\frac{\partial \hat{f}}{\partial k}\right|_{k=1} d\varepsilon + \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (\alpha(\varepsilon)-\bar{\alpha})\hat{f}(\varepsilon,1)d\varepsilon \tag{20}$$

Thus there are two separate effects leading to sample selection bias. The first term is shown in Lemma 6(i) below to be unambiguously negative. It represents the standard sample selection effect discussed above: observed workers with higher specific human capital have, on average, lower unobserved match quality. The second term arises because the wage return to specific capital, α, varies with match quality. If, as was demonstrated in Lemma 5 for the normal and uniform distributions, α declines as match quality ε increases, then conditioning on acceptance gives more weight to observations of workers with high ε and low α. So the average reward for specific human capital amongst observed workers is lower than the unconditional mean and the second term is also negative. Thus we have:

LEMMA 6: (i) $\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}(\ln W_0(\varepsilon,1)-c)\left.\frac{\partial\hat{f}}{\partial k}\right|_{k=1} d\varepsilon < 0$

(ii) If $\alpha(\varepsilon)$ is decreasing in ε, $\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}(\alpha(\varepsilon)-\overline{\alpha})\,\hat{f}(\varepsilon,1)d\varepsilon < 0$

PROOF: See Appendix.

Now, using Lemmas 5 and 6, we have proved:

PROPOSITION 2: When the distribution of unobserved match quality, $\ln\varepsilon$, is normal (with symmetric truncation) or uniform, estimation of the effect on the expected wage offer of specific human capital, $\overline{\alpha}$, is negatively biased due to sample selection.

5. Conclusions

Without specifying the wage determination process we cannot interpret empirical earnings functions nor resolve the associated problem of sample selection bias. If wages are predetermined as in the standard model of specific human capital there is no such bias, but if the earnings function represents a wage offer, we have demonstrated that there will be a downward bias in the estimated coefficient of specific human capital in a cross-sectional regression.

The analysis has shown that a wage offer function is an attractive interpretation of the earnings function, in that it is consistent with earnings functions as typically estimated, and has the "correct" theoretical properties - in particular the elasticities with respect to general and specific human capital are as conventionally assumed.

What does the model tell us about “tenure bias”? If there is a deterministic functional relationship between tenure and specific human capital then we can apply the results of the model directly, and conclude that if we are estimating a wage offer function, tenure bias is likely to be negative. Workers with high observable specific human capital (and tenure) receive a wage premium which makes them more likely to stay even when unobserved match quality is low, so these workers have, on average, lower unobserved match quality. This reduces the apparent return to tenure in cross-sectional data.

If, on the other hand, unobservable shocks to the value of a match can have a permanent component, it was argued in section 2.2 that this is conceptually equivalent to specific capital and should be treated as such. Even if the worker and firm did not intentionally invest in this capital, the problem of finding an appropriate wage to protect it from loss, once they and competing employers are aware of its existence, is the same. A period of discovery of permanent match quality might, anyway, be interpreted as an investment period. In this case, there cannot be a deterministic relationship between tenure and specific capital, but it is plausible to suppose that competing employers may use tenure as a signal of its existence.

Since the model is static, is does not allow us to investigate this issue further. Nevertheless, we can conclude that one of the standard arguments for a positive bias on the return to tenure – that high tenure merely reflects high match quality – is misleading. If match quality is permanent, and known to the market, it is effectively specific human capital; then, if it is not observed by the econometrician we should expect a negative bias due to measurement error. Temporary, or currently unobserved, match quality introduces a negative bias.

The other argument for a positive bias is that workers may have an unobservable characteristic - perhaps "reliability" - that is associated both with higher productivity and with a tendency to remain in the same job. This is not addressed by the model presented here. We should also note that there are reasons for the inclusion of tenure that are not related to the existence of specific capital, such as Lazear's (1979) shirking story. Again, the model has nothing to say about wages determined according to incentive considerations.

Finally, we have not considered the inclusion of firm characteristics in the earnings function. A possible extension of the model would be to allow for an observable firm specific component of productivity. For the current employer, firm 0, this would affect wage determination in exactly the same way as specific human capital – with individual wage determination it does not matter whether this additional component applies to one worker only or to all employees of the firm. Thus the same result applies: sample selection will lead to a negative bias. The analysis of the auction would be considerably more complex, however, if every competing firm had a different firm specific characteristic. This would introduce asymmetries between all the players in the wage determination game, and general results would be hard to obtain – the outcome would depend on the whole vector of firm characteristics.

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Appendix

PROOF of (5):

$$\frac{d}{d\varepsilon}\left(\frac{\varepsilon F_n}{H_n}\right) \overset{\text{sgn}}{=} \frac{\int F^n}{F^n} - \frac{\int G}{G} \quad \text{where } G(\varepsilon) \equiv \frac{d}{d\varepsilon}\left(\varepsilon F^n(\varepsilon)\right) = F^n(\varepsilon)\left(1 + ne(\varepsilon)\right)$$

$$= \int_{\underline{\varepsilon}}^{\varepsilon} \left(\frac{F^n(x)}{F^n(\varepsilon)} - \frac{G(x)}{G(\varepsilon)} \right) dx$$

$$= \int_{\underline{\varepsilon}}^{\varepsilon} \frac{nF^n(x)}{G(\varepsilon)} \left(e(\varepsilon) - e(x) \right) dx < 0 \quad \text{since by (3) } e \text{ is decreasing.} \quad \square$$

PROOF of LEMMA 3: Subtracting (6) from (7), and writing $e(\varepsilon) \equiv \dfrac{\varepsilon f(\varepsilon)}{F(\varepsilon)}$:

$$\frac{e(\phi_0)}{\phi_0}\frac{\partial \phi_0}{\partial w} - \frac{e(\phi_a)}{\phi_a}\frac{\partial \phi_a}{\partial w} = \frac{1}{\phi_a - w} - \frac{1}{k\phi_0 - w} \qquad \text{(A1)}$$

First consider $y(w) \equiv \phi_0 - \phi_a$. If y=0 at $w_0 \in (\underline{w}, \overline{w}]$, (A1)⇒ $\dfrac{dy}{dw} > 0$. But y=0 at $\overline{w}$.

Hence there can be no other such w_0, and $y(w) < 0 \;\forall\; w < \overline{w}$.

Now let $z(w) \equiv \phi_a - k\phi_0$. If z=0 at $w_0 \in (\underline{w}, \overline{w})$, (A1)⇒ $e(\phi_0)\dfrac{\partial \phi_0}{\partial w} - e(k\phi_0)\dfrac{1}{k}\dfrac{\partial \phi_a}{\partial w} = 0$.

And from (3), $e(k\phi_0) < e(\phi_0)$, so at w_0, $\dfrac{dz}{\partial w} > 0$. But $z < 0$ at $\overline{w}$. Hence there can be no such w_0, and $z(w) < 0 \;\forall\; w \leq \overline{w}$.

Thus we have established that $\phi_0(w) < \phi_a(w) < k\phi_0(w)$ for all $w \in (\underline{w}, \overline{w})$; inverting gives the required inequalities for the wage offer functions. ∎

PROOF of LEMMA 4: Subtracting (9) from (6), and using Lemma 3:

$$\frac{e(\phi_a)}{\phi_a}\frac{\partial\phi_a}{\partial w} - \frac{e(\phi)}{\phi}\frac{\partial\phi}{\partial w} - = \frac{1}{k\phi_0 - w} - \frac{1}{\phi - w} < \frac{1}{\phi_a - w} - \frac{1}{\phi - w} \qquad \text{(A2)}$$

We will prove the inverse inequality: $\phi_0(w) < \phi(w) < \phi_0(kw)$ for all $w \geq \underline{w}$.

For the lower inequality, let $y(w) \equiv \phi_a - \phi$. At $\underline{w}(k)$ ($>\underline{\varepsilon}$), from (8), $y<0$. If $y=0$ at $w_0 \in (\underline{w}, \overline{w}]$, (A2)$\Rightarrow \frac{dy}{dw} < 0$. Hence there is no such w_0, and $\phi_a < \phi \;\; \forall \; w < \overline{w}$. From Lemma 3, $\phi_0 < \phi \;\; \forall \; w < \overline{w}$.

For the upper inequality, let $\psi \equiv \phi(w/k)$. (9) can be written: $\frac{e(\psi)}{\psi}\frac{\partial\psi}{\partial w} = \frac{1}{n}\frac{1}{k\psi - w}$.

Also, from (6) and (7): $\frac{e(\phi_0)}{\phi_0}\frac{\partial\phi_0}{\partial w} = \frac{1}{\phi_a - w} - \frac{n-1}{n}\frac{1}{k\phi_0 - w}$. Subtracting:

$$\frac{e(\phi_0)}{\phi_0}\frac{\partial\phi_0}{\partial w} - \frac{e(\psi)}{\psi}\frac{\partial\psi}{\partial w} = \left(\frac{1}{\phi_a - w} - \frac{1}{k\phi_0 - w}\right) + \frac{1}{n}\left(\frac{1}{k\phi_0 - w} - \frac{1}{k\psi - w}\right) \qquad \text{(A3)}$$

Consider $z(w) \equiv \phi_0 - \psi$. When $w=k\underline{\varepsilon}$, from (8), $z>0$.

If $z=0$ at $w_0 > k\underline{\varepsilon}$, (A3)$\Rightarrow \frac{dz}{dw} > 0$. Hence there is no such w_0, and $z>0 \;\forall\; w > k\underline{\varepsilon}$, or equivalently $\phi(w) < \phi_0(kw)$ for all $w \geq \underline{\varepsilon}$. This establishes the inequalities for ϕ_0 and ϕ; inverting gives the inequalities for the wage offer functions. ∎

PROOF of LEMMA 5: Transform equations (6) and (7), putting $w = W_0(\varepsilon, k)$ and $\theta(\varepsilon, k) \equiv \phi_a(W_0(\varepsilon, k), k)$ as in (11):

$$n\frac{e(\theta)}{\theta}\frac{\partial\theta}{\partial\varepsilon}(k\varepsilon - W_0) = \frac{\partial W_0}{\partial\varepsilon} \qquad \text{(6')}$$

$$\left[(n-1)\frac{e(\theta)}{\theta}\frac{\partial\theta}{\partial\varepsilon}+\frac{e(\varepsilon)}{\varepsilon}\right](\theta-W_0)=\frac{\partial W_0}{\partial\varepsilon} \tag{7'}$$

Differentiating (6') and (7') with respect to *k*, setting *k*=1, and rearranging:

$$\frac{\partial^2 W_0}{\partial\varepsilon\partial k}=n(\varepsilon-W_0)\frac{\partial}{\partial\varepsilon}\left(\frac{e(\varepsilon)}{\varepsilon}\frac{\partial\theta}{\partial k}\right)+n\frac{e(\varepsilon)}{\varepsilon}\left(\varepsilon-\frac{\partial W_0}{\partial k}\right)$$

$$0=(\varepsilon-W_0)\frac{\partial}{\partial\varepsilon}\left(\frac{e(\varepsilon)}{\varepsilon}\frac{\partial\theta}{\partial k}\right)+n\frac{e(\varepsilon)}{\varepsilon}\left(\varepsilon-\frac{\partial W_0}{\partial k}\right)$$

Now, when *k*=1, $\varepsilon-W_0=H_n(\varepsilon)/F_n(\varepsilon)$ (equation 6), and $\alpha(\varepsilon)\equiv\frac{1}{W_0(\varepsilon,1)}\frac{\partial W_0}{\partial k}\bigg|_{k=1}$.

Define similarly $\beta(\varepsilon)\equiv\frac{1}{\varepsilon}\frac{\partial\theta}{\partial k}\bigg|_{k=1}$. We can transform these equations to obtain a pair of differential equations in α and β:

$$\beta'+\frac{e'}{e}\beta+\frac{nF_n}{H_n}(1-\beta)=0 \tag{A4}$$

$$\alpha'=\left(\frac{ne}{\varepsilon-H_n/F_n}\right)\left(1-\alpha-n(1-\beta)\right) \tag{A5}$$

We already know that 0<α<1. From Lemma 3 $\varepsilon<\theta<k\varepsilon$, so 0<$\beta$<1.

At $\underline{\varepsilon}$, $\alpha=\beta=\frac{1}{\underline{\varepsilon}}\frac{d\underline{w}}{dk}\bigg|_{k=1}$. From Lemma 2(i) $\underline{w}$ satisfies $n\frac{f(\underline{w})}{F(\underline{w})}(k\underline{\varepsilon}-\underline{w})=1$, from which we obtain:

$$\frac{1}{\underline{\varepsilon}}\frac{d\underline{w}}{dk}=\frac{nf^2(\underline{w})}{(n+1)f^2(\underline{w})-F(\underline{w})f'(\underline{w})}$$

When *k*=1, $\underline{w}=\underline{\varepsilon}$; hence $\alpha(\underline{\varepsilon})=\beta(\underline{\varepsilon})=\frac{n}{n+1}$.

Consider equation (A4). This can be written:

$$\beta'=\left(-\frac{1}{\varepsilon}-\frac{f'}{f}+\frac{f}{F}+\frac{nF_n}{H_n}\right)\beta-\frac{nF_n}{H_n}$$

The bracketed term is positive (by (3)); so when $\beta \le \frac{n}{n+1}$:

$$\beta' \le \frac{n}{n+1}\left(-\frac{1}{\varepsilon}-\frac{f'}{f}+\frac{f}{F}-\frac{F_n}{H_n}\right) < \frac{n}{n+1}\left(-\frac{1}{\varepsilon}-\frac{f'}{f}-(n-1)\frac{f}{F}\right)$$

(using the log-concavity of H_n).

When lnε has a uniform distribution, $\frac{1}{\varepsilon}+\frac{f'}{f}=0$, and β' is negative whenever

$\beta \le \frac{n}{n+1}$. Since $\beta(\underline{\varepsilon}) = \frac{n}{n+1}$, this means that β' is negative for all ε.

When lnε is truncated normal[4], $\frac{1}{\varepsilon}+\frac{f'}{f} = -\frac{\ln\varepsilon}{\varepsilon}$, so by the same argument β' is

negative for all $\varepsilon \le 1$. Now suppose $\beta' = 0$ at some $1 < \varepsilon_0 < \bar{\varepsilon}$.

Differentiating (A4) $\Rightarrow \beta'' \overset{\text{sgn}}{=} \frac{\partial}{\partial\varepsilon}\left[\left(-\frac{\varepsilon e'}{e}\frac{H_n(\varepsilon)}{\varepsilon F_n(\varepsilon)}\right)\right]$ at ε_0.

$H_n/\varepsilon F_n$ is positive and increasing, and, by Lemma A1, so is $\left(-\frac{\varepsilon e'}{e}\right)$ for the normal

distribution when $\varepsilon \ge 1$. So $\beta'' > 0$ at ε_0. But this is impossible because at

$\bar{\varepsilon}$, $\theta(\varepsilon) = \varepsilon\ \forall k$, so β=0, and $\beta' < 0$. Hence $\beta' < 0\ \forall\varepsilon$.

Now consider equation (A5).

At $\underline{\varepsilon}$, $\alpha = \beta \Rightarrow \alpha' < 0$. Differentiating shows that if $\alpha' = 0$ at some $\varepsilon_0 > \underline{\varepsilon}$,

$\alpha'' \overset{\text{sgn}}{=} \beta' < 0$. This is impossible, so $\alpha' < 0\ \forall\varepsilon$. ∎

LEMMA A1: When lnε has a symmetric truncated normal distribution,

$\frac{d}{d\varepsilon}\left(-\frac{\varepsilon}{e}\frac{de}{d\varepsilon}\right) > 0$ for all ε>1.

PROOF: $e(\varepsilon) \equiv \dfrac{\varepsilon f(\varepsilon)}{F(\varepsilon)} = \dfrac{g(\ln\varepsilon)}{G(\ln\varepsilon)}$ where $g(x) = \dfrac{c}{\sqrt{2\pi}}\exp(-x^2/2)$ for some constant c and G is the corresponding distribution function.

It is sufficient to prove that $y(x) \equiv \dfrac{d^2 \ln}{dx^2}\left(\dfrac{g(x)}{G(x)}\right) < 0$ for $x>0$.

Evaluating this: $y = \dfrac{g}{G}\left(x + \dfrac{g}{G}\right) - 1$. When $x=0$, symmetric truncation $\Rightarrow G=½$, and it can be verified that $y<0$. Then:

$$\frac{dy}{dx} = -(y+1)\left(x + \frac{g}{G}\right) - \frac{g}{G}y$$

so if $y=0$ at any $x_0>0$, $dy/dx<0$. Hence there can be such x_0, and $y<0$ $\forall x>0$. □

PROOF of LEMMA 6: (i) $\dfrac{\partial}{\partial k}\left(F_n(\theta)\right)\Big|_{k=1} = f_n(\varepsilon)\dfrac{\partial\theta}{\partial k}\Big|_{k=1} = nF_n(\varepsilon)e(\varepsilon)\beta(\varepsilon)$ where β is defined as in the proof of Lemma 5. Hence:

$$\frac{\partial\hat{f}}{\partial k}\bigg|_{k=1} = n\frac{\left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_n(x)dF(x)\right) f(\varepsilon)F_n(\varepsilon)e(\varepsilon)\beta(\varepsilon) - \left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_n(x)e(x)\beta(x)dF(x)\right) f(\varepsilon)F_n(\varepsilon)}{\left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_n(\varepsilon)dF(x)\right)^2}$$

and it is required to prove that:

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_n(\varepsilon)dF(\varepsilon)\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (\ln W_0(\varepsilon,1) - c)F_n(\varepsilon)e(\varepsilon)\beta(\varepsilon)dF(\varepsilon)$$
$$< \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_n(\varepsilon)e(\varepsilon)\beta(\varepsilon)dF(\varepsilon)\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (\ln W_0(\varepsilon,1) - c)F_n(\varepsilon)dF(\varepsilon)$$

or equivalently, since $F_n(\varepsilon)f(\varepsilon) = f_{n+1}(\varepsilon)/(n+1)$:

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}(\ln W_0(\varepsilon,1) - c)e(\varepsilon)\beta(\varepsilon)dF_{n+1}(\varepsilon) < \int_{\underline{\varepsilon}}^{\bar{\varepsilon}}e(\varepsilon)\beta(\varepsilon)dF_{n+1}(\varepsilon)\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}(\ln W_0(\varepsilon,1) - c)dF_{n+1}(\varepsilon)$$

We know that $\ln W_0$ is an increasing function of ε, and from equation (A4) $e\beta$ is a decreasing function of ε. Hence by Lemma A2 below this inequality holds.

$$\hat{f}(\varepsilon,k) = \frac{f(\varepsilon)F_n(\theta)}{\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}F_n(\theta)dF(x)}$$

(ii) $\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}(\alpha(\varepsilon)-\bar{\alpha})\hat{f}(\varepsilon,1)d\varepsilon \overset{\text{sgn}}{=} \int_{\underline{\varepsilon}}^{\bar{\varepsilon}}(\alpha(\varepsilon)-\bar{\alpha})F_n(\varepsilon)dF(\varepsilon)$

$< \int_{\underline{\varepsilon}}^{\bar{\varepsilon}}(\alpha(\varepsilon)-\bar{\alpha})dF(\varepsilon)\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}F_n(\varepsilon)dF(\varepsilon)$ by Lemma A2

$= 0$ by definition of $\bar{\alpha}$. ∎

LEMMA A2: If $K(\varepsilon)$ is a distribution function with support $[\underline{\varepsilon},\bar{\varepsilon}]$, and $y(\varepsilon)$ is increasing and $z(\varepsilon)$ is decreasing on $[\underline{\varepsilon},\bar{\varepsilon}]$, then

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} y(\varepsilon)z(\varepsilon)dK(\varepsilon) < \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} y(\varepsilon)dK(\varepsilon)\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} z(\varepsilon)dK(\varepsilon)$$

PROOF:

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} y(\varepsilon)z(\varepsilon)dK(\varepsilon) - \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} y(\varepsilon)dK(\varepsilon)\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} z(\varepsilon)dK(\varepsilon) = \frac{1}{2}\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}(y(\varepsilon)-y(x))(z(\varepsilon)-z(x))dK(\varepsilon)dK(x)$$

and the integrand on the right-hand side is everywhere negative. ∎

Footnotes

[1] Barth (1997), using a Norwegian dataset that enabled him to control for firm heterogeneity, also found a significant positive tenure effect.

[2]More precisely, the shock to internal productivity must be distributed identically to the negative of the shock to the opportunity wage.

[3]This is a model of wage determination for a worker already possessing some specific and some general human capital; we do not model the investment process, and the sharing of costs. However, it would be straightforward to do so: the incentives for the worker and firm to invest can be determined by calculating their expected returns.

[4] This proof is for the standard normal with mean zero and unit variance; it extends to the general case in the obvious way.