The Hedge Fund Game

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January 2, 2008

Abstract. This paper examines theoretical properties of incentive contracts in the hedge fund industry. We show that it is very difficult to structure incentive payments that distinguish between unskilled managers, who cannot generate excess market returns, and skilled managers who can deliver such returns. Under any incentive scheme that does not levy penalties for underperformance, managers with no investment skill can game the system so as to earn (in expectation) the same amount per dollar of funds under management as the most skilled managers. We consider various ways of eliminating this "piggy-back effect," such as forcing the manager to hold an equity stake or levying penalties for underperformance. The nature of the derivatives market means that none of these remedies can correct the problem entirely.

Acknowledgments. The authors thank Jon Parker, Andrew Patton, Kislaya Prasad, Matt Raifman, Tarun Ramadorai, and Neil Shephard for helpful comments.

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1. Background

Hedge funds are largely unregulated investment pools that have become increasingly important in the marketplace. There are currently more than 8000 hedge funds with well over one trillion dollars under management [Ibbotson and Chen, 2006]. The typical fee structure is a two-part pricing scheme in which the manager takes a fixed annual percentage of funds under management (the *management fee*), plus another percentage on that portion of returns that exceed some pre-established benchmark (the *incentive*). A fairly common arrangement is a management fee of 1-2%, an incentive fee of about 20%, and a benchmark in the region of 5-10% [Ackermann, McEnally, and Ravenscraft, 1999]

The purpose of such incentive schemes is to reward exceptional performance and to align the interests of investors and managers as closely as possible. Two-part incentive schemes certainly reward performance, but they are not very satisfactory on the second count. One reason is that the convexity of the fee structure encourages mangers to employ strategies with high variance, which is not always in the best interests of the investors, particularly those who are risk averse. A second problem is that the pay-as-you-go feature encourages managers to push high returns forward in time, because poor returns later are not used to offset the amounts earned from high returns early on. The impact of the incentive structure on dynamic portfolio choice has been examined in a number of papers, including Starks (1987), Carpenter (1998), and Ackermann, McEnally, and Ravenscraft (1999).[1]

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[1] For statistical analyses of recent hedge fund performance see Malkiel and Saha (2005) and Ibbotson and Chen (2006).

A third problem with these schemes is that they are susceptible to manipulation by managers who can mimic exceptional performance records with high probability (and earn large fees), without delivering exceptional performance *in expectation*. The purpose of this paper is to show that this 'piggy-back effect' is potentially very large and very difficult to correct. In particular, we shall show that it is virtually impossible to set up an incentive structure that rewards skilled hedge fund managers without at the same time rewarding unskilled managers and outright con artists.[2] In fact, any incentive scheme that does not directly penalize underperformance can be gamed by the manager so that he earns (in expectation) the same amount per dollar of funds under management as can the most skilled managers.

This rather surprising result, which is established in theorem 1 below, stems from the unusual flexibility of the derivatives market. It also has a disturbing corollary: since the cost of entry is low relative to the potentially enormous fees, the industry may be swamped by managers who are gaming the system rather than delivering high returns, which could ultimately lead to a collapse in investor confidence. The problem can be attenuated by insisting on greater transparency in the strategies that funds employ, but designing measures of risk exposure that cannot be circumvented turns out to be an extremely challenging problem.

[2] A "con artist" is a fund manager who *knows* that he cannot generate excess returns and tries to fool his investors into thinking that he can, whereas an "unskilled manager" is one who *imagines* that he can generate excess returns even though he cannot. It is rational for both types of managers to use strategies that maximize returns, which (given the fee structure) means exposing their investors to large losses with a small but non-negligible probability. Both types of managers produce bad outcomes for the investors irrespective of their intentions.

2. A 'whimsical' example

The nature of the incentive problem can be illustrated by a somewhat whimsical example. An enterprising man named Oz sets up a new hedge fund with the aim of earning 10% in excess of some benchmark rate of return, say 5%. The fund will run for five years, and investors can cash out at the end of each year if they wish. The fee is 'two and twenty': 2% annually for funds under management, and a 20% incentive fee for annual investment returns that exceed the 5% benchmark.

Step 1. Oz raises $100 million and starts operations on the first day of the year. He invests the $100 million in Treasury bills yielding 4% and deposits them in escrow at a bank. Then he writes an event-driven option that works as follows: if at the end of the year the fourth digit in the S&P 500's closing value is a '2', Oz pays the option holder $100 million; otherwise the option holder pays Oz $10 million. This is a 9:1 bet in which the bad outcome is very bad but has low probability, and the good outcome is pretty good and has high probability. Moreover the option is worth money to any buyer that is large and highly diversified, because its expected payoff is $1 million (100 x 10% - 10 x 90% = 1).

Step 2. Oz sells the option for its fair market value ($1 million). He then rents some computer terminals, hires a few bright young 'quants' (in case his investors want to eyeball the operation), and takes a long vacation.

Step 3. At the end of the year the closing value of the S&P 500 is reported. If the fourth digit is not a '2', Oz gets his $10 million. Together with the interest ($4 million) and the sale of the option ($1 million) this makes for a stellar return of

15% -- right on target! The investors are pleased and Oz collects $4 million: $2 million in management fees plus a $2 million performance bonus. If the 'unlucky 2' does occur, Oz closes the fund early.

The chances are very good, however, that this will not happen. Oz can then repeat the gambit next year with even greater profits because the fund has now grown by 11%, net of fees. Indeed, the chances are over 59% that, by the end of the fifth year an 'unlucky 2' will *never* have occurred. In this case the fund has returned 15% per year and Oz looks very skilled indeed.[3]

A little calculation shows that Oz grosses nearly $25 million if the fund does not go bust in the first five years, and over $10 million (in expectation) if it does. Overall, Oz's expected gross earnings come to about $19 million in spite of the fact that he has no special investment skill.

We shall call Oz's strategy a *piggy-back strategy*. While it is doubtful that hedge fund managers use strategies that are this transparent, more sophisticated versions of it probably are in use. Lo (2001) gives the following concrete example: take short positions in S&P 500 put options that mature in 1-3 months and are about 7% out-of-the-money (using the investors' funds as collateral). The chances are high that such options will expire worthless, in which case the

[3] In practice, incentive schemes often specify that the manager's next incentive bonus will not be paid until the fund meets its previous highest value (*high water mark*). This does not change Oz's payments, however, because he keeps the fund growing until it crashes.

manager makes money -- indeed quite a lot of money, as Lo shows by putting the strategy through its paces using historical data.[4]

Of course, these gains come at a price, namely, there is a small probability that the market will decline sharply, the puts will be in-the-money, and the fund will lose a great deal. But this event is rare, and before it happens the manager will earn very large fees for delivering apparently above-average performance, when the *expected* returns are at best average (but this fact is concealed from the investors).

The same logic is at work in Oz's strategy, and in many other strategies one can imagine. While Oz's strategy is extremely transparent, however, our purpose here is not to concoct the cleverest way to deceive investors using this approach. Rather, we exploit the transparency to establish two general theoretical points: i) it is extremely difficult to detect, from a fund's track record, whether a manager is actually able to deliver excess returns, is merely lucky, or is an outright con artist; ii) it is virtually impossible to design the incentive structure so that it rewards skilled managers without also rewarding the unskilled (and unscrupulous) ones also.

[4] Lo claims that strategies of this general type are not merely theoretical possibilities, but constitute a "well-known artifice employed by unscrupulous hedge-fund managers to build an impressive track record quickly..." (Lo, 2001, p.23). He also shows how a manager can conceal such a strategy by taking a series of complicated positions that effectively create a *synthetic* short position.

3. The piggy-back theorem

Suppose that a fund starts with size 1 at the beginning of year 1 and runs for T years. Let the *ordinary rate of return* be $r \geq 0$. We can think of this as the rate of return achievable in a given asset class by an 'ordinary' investment manager who has no special investment skill. After t years a fund run by an ordinary manager will be of size $(1+r)^t$. A skilled investment manager, by contrast, is able to generate excess returns, by which we mean a rate of return greater than r.

Let us express the total return in year t by the random variable $(1+r)X_t$, where $X_t \geq 0$. There are *excess returns* if $X_t > 1$, *deficient returns* if $X_t < 1$, and *ordinary returns* if if $X_t = 1$. We shall only be interested in the stochastic sequence of returns that a trading strategy generates; we need not concern ourselves with the *mechanism* that leads to these returns, such as the specific ways in which managers exploit arbitrage opportunities in the market.

It will be notationally convenient to deflate the dollar amounts in each period t by $(1+r)^t$. Thus the sequence of random variables $\{X_1, X_2, ..., X_T\}$ represents the series of returns achieved relative to the ordinary rate r, which will no longer appear in the notation. To avoid infinite returns, we shall assume from now on that the realized values of the X_t are bounded above by some number $u > 0$, that is,

$$0 \leq X_t \leq u. \qquad (1)$$

The object of investors is to maximize final wealth, namely, $W_T = \prod_{1 \leq t \leq T} X_t$.

A specific realization of the stochastic process will be denoted by $(x_1, x_2, ..., x_T) \in R_+^T$. Any such realization generates a series of payments to the manager. A *contract* is a function ϕ that maps each realized sequence $(x_1, x_2, ..., x_T)$ of any finite length to a total payment $\phi(x_1, x_2, ..., x_T)$. The payments may be doled out over time or paid in one lump sum. (If doled out over time we assume that they accumulate at the ordinary rate of return, so that the total payment is just the sum of payments along the way.) In this section we shall assume that $\phi(x_1, x_2, ..., x_T)$ is *continuous*, *nonnegative*, and *monotone non-decreasing* in each of its arguments. Later we shall consider the effect of negative payments, that is, penalties that the manager must pay out of his own pocket if he underperforms.

A person with *no skill* is someone like Oz who can generate ordinary returns every period but not more, that is, $E[X_t : x_1, x_2, ...x_{t-1}] \leq 1$. A series of returns $(x_1, x_2, ..., x_T)$ is *consistently good* if $x_t \geq 1$ for $1 \leq t \leq T$. This is the best that a manager with no skill can deliver in expectation.

Piggy-Back Theorem. Under any continuous, non-negative, monotonic contract ϕ, *a manager with no skill can earn, per dollar of initial investment, at least*

$$\max\{\phi(x_1, x_2, ..., x_T) / \prod_{1 \leq t \leq T} x_t : \text{ all } x_t \geq 1\}. \tag{2}$$

The maximum value will be called the *optimal reward ratio* for the contract ϕ.

Proof. Without loss of generality assume that the initial size of the fund is $x_0 = 1$. Choose a particular sequence $(x_1, x_2, ..., x_T)$ that maximizes (2), where by assumption all $x_t \geq 1$. For each period $t \leq T$ define a binomial process X_t such that the manager is paid $x_t - 1 \geq 0$ with probability $1/x_t$ and the manager pays -1 with probability $1-1/x_t$. Note that the expected value of X_t is zero. the manager can cover the negative payment with the fund's initial value. At the beginning of each period t the manager writes an option with distribution $(x_0 x_1 x_2 \cdots x_{t-1})X_t$. As in the Oz example the event that triggers the option is external and has the required probability distribution, namely, $(1/x_t, 1-1/x_t)$. The option is *covered* because the manager's obligation (in case of a bad draw) is $-(x_0 x_1 x_2 \cdots x_{t-1})$, which is the current value of the fund. (The manager can guarantee this because (being unskilled) he is simply going to put the fund into a vehicle that *assures* an ordinary rate of return.)

At the end of T periods he will have generated the sequence $(x_1, x_2, ..., x_T)$ with probability $1/w_T$ where $w_T = \prod_{1 \leq t \leq T} x_t$ is the final value of the fund. With probability $1-1/w_T$ the fund crashes at or before period T and the final value is 0. Since the contract carries no penalties for underperformance (ϕ is non-negative), the manager's expected earnings equal $\phi(x_1, x_2, ..., x_T) / \prod_{1 \leq t \leq T} x_t$, which is the reward ratio. This is one feasible strategy of the unskilled manager, and others may yield even higher earnings, but all we needed to show is that the unskilled manager can earn at least this amount. Q.E.D.

A *financial wizard* is someone who can consistently generate excess returns over an extended period of time. Specifically, an ε*-wizard* is a person who can consistently generate excess returns of size $\varepsilon > 0$. Suppose the fund runs for T periods. Under the contract ϕ, she will be paid, per dollar initially invested, the amount $\phi(1+\varepsilon, 1+\varepsilon, ..., 1+\varepsilon)$. Moreover the final value of her fund will be $w_T = (1+\varepsilon)^T$ with certainty. Hence her expected earnings *per final dollar in the fund* equal $\phi(1+\varepsilon, 1+\varepsilon, ..., 1+\varepsilon)/(1+\varepsilon)^T$.

Corollary. A person with no skill can earn, in expectation, at least as much per dollar of final value in his fund, as can an ε-wizard per dollar of final value in her fund.

The essential idea of the theorem is that an unskilled manager can piggy-back on any series of returns that a skilled manager might generate. The manager does this by creating a series of covered options that allow him to *mimic* the returns of more skilled managers with nonzero probability, hence he gets the skilled manager's rewards with this same probability.[5]

It is important to realize that the unskilled manager does not need to mimic the *distribution* of returns that a skilled manager would generate. The piggy-back theorem shows that much less is demanded of an unskilled manager: all he needs to do is mimic a series of returns that *could have been generated* by a skilled manager. Therefore, he might as well mimic a series of returns that gives a big bang for the buck, namely, a series that solves the maximum problem in (2).

To appreciate the magnitude of the piggy-back effect, suppose that a skilled manager can generate excess returns of 1 + ε = 1.03 and does so for ten years in a

[5] Of course, skilled managers can exaggerate the level of their skill using the same general approach. Unskilled managers can mimic these exaggerated returns if they choose, but their risk of exposure is higher.

row. Let R be the total reward to such a manager. The theorem shows that an unskilled manager with the same size fund initially can get *at least* .74R in expectation ($(1.03)^{-10} \approx .74$). Note also that this is a lower bound: there could be other series of returns that yield even higher expected rewards to the unskilled manager.

More generally, suppose that R is the expected reward to an ε*-wizard* over T periods. An unskilled manager who employs an piggy-back strategy will not be exposed in T periods with probability $(1+\varepsilon)^{-T}$. Hence in expectation such a manager earns at least $R(1+\varepsilon)^{-T}$. Table 1 shows how the *probability of not being exposed* varies with ε and T.

ε	$T=5$	$T=10$	$T=20$
.01	95%	91%	82%
.03	86	74	55
.05	78	61	38
.10	62	39	15
.20	40	16	3

Table 1. Probability that a piggy-back strategy is not exposed for various values of ε and T. (All probabilities rounded to the nearest whole percent.)

The *length* of a reporting period is not material to the results in table 1. Suppose, for example, that managers were required to report their returns at the end of each day instead of at the end of each year. A manager who can generate excess returns of ε per year can, on average, generate excess returns of *per day* equal to $(1+\varepsilon)^{1/365} - 1 \approx \varepsilon / 365$. An unskilled manager can mimic each of these daily returns with probability $(1+\varepsilon)^{-1/365}$. Hence, over T years, the unskilled manager can mimic this series with probability $(1+\varepsilon)^{-T}$, just as before.

One lesson to draw from this table is that unskilled managers can get high rewards relative to skilled ones unless the latter are very skilled for long periods of time. For example, it is fairly easy to fake 5% excess returns over 5 years, whereas it is hard to fake 20% excess returns over 10 years without getting caught. Still, 5% excess returns is quite an enticing prospect. Thus an unskilled manager might prefer to mimic a series of moderately high returns rather than a series of very high returns, because he can get away with the former with much higher probability than the latter.

Another implication is that investors will have difficulty discriminating between managers who are truly talented and those who have no talent (or are con artists) based solely on their "track records." Suppose, for example, that an investor wants to be 95% confident that a given history of returns $(x_1, x_2, ..., x_T)$ was not generated by a scam of the type described in section 2. The preceding argument shows that this will be the case if and only if $1/(\prod_{1\le t\le T} x_t) \le .05$. In other words, only if a fund has grown at least *twenty-fold* compared to an alternative safe investment can the investor be 95% certain that it was not generated by a scam.

4. Restructuring the incentives

In this section we consider whether the problem can be remedied by restructuring the incentive schemes in the hedge fund industry. There are two separate problems that a properly designed incentive scheme needs to address.

The first is how to align the interests of the manager and the investors more closely. The second is how to distinguish between skilled and unskilled managers. The former is the *alignment problem* whereas the second is the *separation problem*.

The alignment problem is a standard one in the theory of contracts, and can be addressed in several ways: a) by rewarding the manager only on the basis of final total returns; b) by forcing the manager to hold a sizable equity stake; c) by levying penalties for underperformance. We shall consider each of these remedies in turn. A basic conclusion is that, while they may partially alleviate the alignment problem, they do not solve the separation problem: under almost any arrangement, unskilled managers will be able to piggy-back on the rewards of skilled ones to some extent.

a) Payments based on final returns

One way to align the objectives of the manager and the investors is to make the manager's payments depend only on the final value of the fund $w_T = \prod_{1 \le t \le T} x_t$. In other words, the contract should take the form $\phi(x_1, x_2, ..., x_T) = g(\prod_{1 \le t \le T} x_t)$ for some nonnegative, monotone increasing function $g(w_T)$. To separate the skilled from the unskilled, we need the payments to be zero whenever $\prod_{1 \le t \le T} x_t \le 1$. Unfortunately this means that the payment function is convex in a neighborhood of 1, which encourages risk-taking by the manager. If investors are risk-averse the interests of manager and investors will therefore not be fully aligned.

Moreover, even if the function $g(w_T)$ is zero for $w_T \le 1$, the separation problem remains acute. This follows at once from the piggy-back theorem: an unskilled manager can make at least $\max_w g(w)/w$. Therefore *under a final-wealth payment scheme, an unskilled manager can make at least as much per dollar of final wealth as can the most skilled managers*.

b) *Require the manager to hold an equity stake in the fund*

Suppose that the fund manager is required to hold an equity stake in the fund. Let $\theta \in (0,1)$ be the proportion of the fund's value that he is required to hold during the fund's lifetime T. We begin by noting that this requirement is easy to undermine, because the manager can always take positions in the derivatives market that effectively offset the gains and losses generated by his share of the fund. However, even if such offsetting positions can be prohibited, the requirement does not do much to solve the piggyback problem.

To see why, let $(x_1, x_2, ...x_T)$ be some series of returns that a skilled manager can generate, and let $\phi = \phi(x_1, x_2, ...x_T)$ be the corresponding payment. At the end of the series his wealth is

$$\theta \prod_{1 \le t \le T} x_t + (1-\theta)\phi . \quad (3)$$

The piggy-back theorem shows that an unskilled manager can generate this same series with probability $1/\prod_{1 \le t \le T} x_t$. His expected wealth at the end of the period is composed of two parts: the expected value of his own stake before fees, which is exactly θ (because he makes bets with zero expected gains); and the expected fees from the investors, which amount to $(1-\theta)\phi / \prod_{1\le t\le T} x_t$. Hence the unskilled manager's end-wealth is, in expectation,

$$\theta + (1-\theta)\phi / \prod_{1\le t\le T} x_t \,. \tag{4}$$

It follows from (3) and (4) that the *ratio* of the unskilled to the skilled manager's end-wealth is $1/\prod_{1\le t\le T} x_t$, which is the same as the ratio of their earnings in the unconstrained case.

c) Assess penalties for underperformance

Theoretically this is the most satisfactory approach, but it still does not solve the separation problem. Consider a contract ϕ that calls for negative payments for some sequences of sub-par returns. We do not need to specify which returns trigger negative payments, but we will assume that they are bounded below by some number $-\delta$. The payments must be enforceable, so the manager must put δ in escrow (earning normal rates of return) until the end of period T.

Consider some series of returns $(x_1, x_2, \ldots . x_T)$ that can be generated by a skilled manager. Let $w = \prod_{1\le t\le T} x_t$. Conditional on this realization of returns, the skilled manager's end-wealth is $\delta + \phi(w)$. He realizes, however, that if he had not opened the fund to investors, but simply applied his skills to the amount held in escrow (δ), he would have had δw. Therefore his participation constraint is

$$\delta(w-1) \le \phi(w). \tag{5}$$

Now consider an unskilled manager who piggy-backs on the sequence $(x_1, x_2, ...x_T)$. His expected end-wealth is

$$\phi(w)/w - \delta(1-1/w). \tag{6}$$

Assuming that the skilled manager participates, the unskilled manager's end-wealth divided by the skilled manager's end-wealth is

$$(1/w) - \delta/(\delta + \phi(w)). \tag{7}$$

Under the other regulatory regimes this ratio was $1/w$, so in the current regime the penalties are in fact penalizing the unskilled manager. Nevertheless they are not sufficient to keep out the unskilled (and the con artists). Indeed, suppose that the penalties are high enough that the unskilled manager's earnings are negative. Then from (6) we obtain $\phi(w)/w - \delta(1-1/w) < 0$, which implies that $\phi(w) < \delta(w-1)$. But then the participation constraint (5) of the skilled manager is not satisfied. It follows that *any contract with penalties that keeps out the unskilled managers keeps out all the skilled managers as well.*

5. Discussion

In this paper we have shown how easy it is to mimic a series of excess returns without being able to generate such returns in expectation. It suffices to place a series of bets, each of which generates a modest excess return with high probability and a large loss with low probability. As long as the excess returns are not too excessive, and the series not too long, the probability of being exposed is low. Furthermore, it is essentially impossible to design an incentive scheme that keeps out people who are pursuing such strategies (either unwittingly or by design), without keeping out everybody.

We draw two conclusions. First, investors cannot distinguish between true financial wizards and those who are mediocre (or duplicitous) without knowing their track records over long periods of time, and even then there is no sure way to discriminate between them. Second, because it is easy to fake excess returns and earn a lot of money in the process, mediocre managers and con artists will be attracted to the market. The situation is analogous to an automobile 'lemons' market with the added feature that 'lemons' can be manufactured at will (Akerlof, 1970). Indeed, it is analogous to a car market with the following characteristics: i) every car is one of a kind; ii) the car's engine is locked in a black box and no one can see how it works (it's not protected under patent law); iii) anyone can cobble together a car that delivers apparently superior performance for a period of time and then breaks down completely. In such a case one would expect the price of cars -- both good and bad – to collapse, because buyers cannot tell the difference between them. A similar fate may await the hedge fund industry unless ways are found to make their functioning more transparent.

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