

## Discursive Path-Dependencies

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**Abstract.** Discursive path-dependence is the phenomenon that decisions or arguments over multiple interconnected propositions may depend on the order in which the propositions are considered. I develop a formal model of sequential decision or argumentation processes over multiple propositions, focussing on so-called *modus ponens* processes. Drawing on examples of path-dependencies at the levels of both individual argumentation and collective decisions, I prove three main results. (1) Path-dependence occurs if and only if an individual's or a group's initial dispositions on a set of propositions violate deductive closure. (2) If we impose the conditions of universal domain, anonymity and decisiveness on a (collective) modus ponens decision process, path-dependencies are in principle unavoidable. (3) Path-dependence makes sequential decision or argumentation processes vulnerable to two types of strategic manipulation: manipulation by changes of the decision-path, and manipulation by expression of untruthful views on the propositions. Finally, I discuss escape-routes from the problem of path-dependence.

### 1. Introduction

Decisions over multiple propositions may depend on the order in which the propositions are considered. The conclusions of an argument may depend on the order in which the individual steps of the argument are presented. This is the phenomenon of *discursive path-dependence*. A decision process, or an argument, over multiple propositions is said to be *path-dependent* if the overall outcome is not invariant under changes in the order in which the propositions are considered.

Path-dependencies may occur at both individual and collective levels. At an individual level, many of us have probably made the puzzling observation that there can be two alternative arguments with the following properties: both begin with premises that we are immediately inclined to accept, both use only instances of valid logical inferences, and yet the two arguments lead to opposite conclusions. At a collective level, the order of decisions can also matter. Suppose a government decides first to commit itself to keeping taxes low. If it honours this prior commitment, it will then have to reject subsequent

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proposals in favour of increased expenditure, although it would have accepted the very same proposals if only it had considered them *before* making a decision on taxation.

When, and why, can path-dependencies occur in decisions or arguments over multiple propositions? The present paper addresses this question. At the centre of the discussion is a new set of social-choice-theoretic results. The results show that, in a large class of decision problems, path-dependencies are *in principle* unavoidable. In particular, path-dependencies can occur even when the dispositions of the decision-maker(s) to say ‘yes’ or ‘no’ on each proposition are held constant. The results further identify a simple necessary and sufficient condition for the occurrence of certain path-dependencies at collective and individual levels, as well as sufficient conditions for their avoidance.

Why should we care about path-dependencies? Path-dependence is responsible for the possibility of two types of strategic manipulation in decisions or arguments over multiple propositions: manipulation by agenda setting, and manipulation by expression of untruthful views. As we will see, in situations of path-dependence, an agenda-setter – someone who determines the order in which the propositions are considered – can considerably influence the outcome of a decision process. But suppose that the order in which the propositions are considered is held completely fixed (for whatever reason). Why should we *then* care about path-dependence? As we will also see, the mere *existence* of an alternative decision-path which *would* change the outcome of a decision process (even if that alternative decision-path is never adopted) may create incentives for individuals to strategically express untruthful views in a decision or argumentation process. Thus the *possibility* of path-dependence may have adverse effects even if we always stick to the same decision-path. Identifying conditions for avoiding path-dependencies is therefore also relevant from the perspective of avoiding these two types of strategic manipulation.

In section 2, I present a simple example from the context in which path-dependencies have been studied most extensively, namely the context of voting over alternatives. In section 3, I introduce decisions over multiple propositions, and give an example of path-dependence in that context. In section 4, I develop a general model of sequential decisions or arguments over multiple propositions. In section 5, I state a theorem showing that under certain conditions path-dependencies are in principle unavoidable. In section 6, I give an example of path-dependencies at the level of individual argumentation. The example illustrates a necessary and sufficient condition for the occurrence of path-dependencies, which is then discussed in greater detail in section 7. In section 8, I discuss the two types of strategic manipulation that path-dependence

may give rise to. And in section 9, I explore three escape-routes from path-dependencies at the collective level. Section 10, finally, contains some concluding remarks. Formal proofs are given in an appendix.

Although the model abstracts from many details and complications of real world decision and argumentation processes, the aim of the model is to highlight, in a particularly distilled form, some of those central features of such processes that give rise to path-dependencies. If successful, the model can illuminate our understanding of when, and why, such path-dependencies occur and what their implications are.

## 2. An Example: The Killer Amendment Problem

Suppose a committee has to make a decision between the status-quo ( $SQ$ ), a proposal ( $P$ ) and the same proposal with an amendment ( $PA$ ). For simplicity, there are three individual decision-makers, with preferences as shown in table 1:

	1 <sup>st</sup> preference	2 <sup>nd</sup> preference	3 <sup>rd</sup> preference
Individual 1	$PA$	$P$	$SQ$
Individual 2	$P$	$SQ$	$PA$
Individual 3	$SQ$	$PA$	$P$

**Table 1**

**Case 1.** Suppose the committee decides first whether to adopt the proposal ( $P$ ) or to keep the status-quo ( $SQ$ ). There is a majority of two out of three (individuals 1 and 2) in favour of the proposal. Suppose the committee decides next whether to amend the proposal ( $PA$ ) or to keep it without the amendment ( $P$ ). There is a majority of two out of three (individuals 1 and 3) in favour of amending the proposal. The resulting majority preference ordering is  $PA > P > SQ$  and the amended proposal is adopted.

**Case 2.** Suppose the committee decides first whether to amend the proposal ( $PA$ ) or to keep it without the amendment ( $P$ ). There is a majority of two out of three (individuals 1 and 3) in favour of amending the proposal. Suppose the committee decides next whether to adopt the amended proposal ( $PA$ ) or to keep the status-quo ( $SQ$ ). There is a majority of two out of three (individuals 2 and 3) in favour of keeping the status-quo. The resulting majority preference ordering is  $SQ > PA > P$  and the status-quo is kept.

The two cases lead to very different outcomes: adopting the amended proposal in case 1, and keeping the status-quo in case 2. Moreover, this happens in spite of the fact that the preferences of the individual decision-makers are exactly the same in both cases. The only difference between the two cases is the *decision-path*, i.e. the order in which the decisions are taken. Thus the decision process is path-dependent.

The label *killer amendment problem* refers to the strategic potential of this path-dependency. A clever manipulator, who opposes the proposal, might introduce the amendment at an early stage in the decision process in the expectation that the status-quo will be preferred to the amended proposal, although the proposal without the amendment would be preferred to the status-quo.

Path-dependencies in the context of voting over alternatives have been studied extensively in social choice theory. For example, the recognition of path-dependencies like the killer amendment problem is a central aspect of Riker's influential account of political manipulation and his critique of populist democracy (Riker 1982). The problem of decision making over multiple propositions, by contrast, has only recently received attention, and it is this problem that we now address.

### 3. Decisions over Multiple Interconnected Propositions

In decisions or arguments over multiple interconnected propositions, views on some propositions constrain views on others.<sup>2</sup> A formal model is introduced in section 4 below. Examples of systems of multiple interconnected propositions are political programmes, legal doctrines, ideologies, or scientific theories. The difference between decisions over such systems and classical problems of voting over alternatives is the following. In decisions over multiple propositions, unlike in classical voting problems, the outcome of the decision is *not* a single winning alternative (for example, one of  $PA$ ,  $P$ ,  $SQ$ ) or a ranking of the alternatives (for example,  $SQ > PA > P$ ), but rather an entire *set* of propositions, which may stand in various logical relations to each other. To model a legal doctrine adequately, for instance, it is more helpful to view the legal doctrine as a *set* consisting of multiple propositions than as a single 'entity' or alternative.

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<sup>2</sup> Decision problems over multiple interconnected propositions are the subject of a growing body of literature in the fields of law, economics and philosophy. In particular, a new paradox of aggregation affecting such decision problems, the so-called "doctrinal paradox" or "discursive dilemma", has received a lot of attention. See, for instance, Kornhauser and Sager 1986, 1993; Kornhauser 1992; Chapman 1998, 2002; Brennan 2001; Pettit 2001a,b; List and Pettit 2002; Bovens and Rabinowicz 2001.

Often decisions over multiple interconnected propositions are not made simultaneously, but sequentially. And *arguments* over multiple propositions are by their nature always sequential. Some propositions may be regarded as more fundamental than others, and may therefore be considered earlier in a sequential decision or argumentation process. Or a system of multiple interconnected propositions may be built up only over time. In a sequential decision or argumentation process, the different propositions typically constrain each other in an asymmetrical way. Earlier decisions may influence or constrain later ones. If, at any point, there is a conflict between an individual's or a group's initial disposition (to say 'Yes' or 'No') on a new proposition and previously accepted propositions, then prior decisions or commitments are often harder to overrule than the initial disposition on the new proposition. The method of resolving such conflict in favour of these prior decisions or commitments will be called *modus ponens* decision making, and will be defined more formally below. Other methods are conceivable, particularly *modus tollens* decision making, i.e. the method of resolving conflict in favour of the disposition on a new proposition while revising previous decisions.

A simple example illustrates that the decision-path in decisions over multiple propositions may matter. The example is a version of an example given by Pettit (2001b), which he describes as a "discursive dilemma". Suppose a multi-member government has to make decisions on the following propositions.

$P$  : Spending on education shall be increased.

$Q$  : Spending on health care shall be increased.

$R$  : Spending on defence shall be increased.

$S$  : Taxes shall be increased.

It is unanimously agreed that these propositions are interconnected in the following way:  $P$ ,  $Q$  and  $R$  can be accepted simultaneously *only if*  $S$  is also accepted, i.e.  $((P \wedge Q \wedge R) \rightarrow S)$ . The argument for  $((P \wedge Q \wedge R) \rightarrow S)$  is that increasing spending on education, health care and defence necessitates a tax increase. By contrast, increasing spending on two or fewer of these items is possible without a tax increase.

Decisions on the propositions are made sequentially, possibly over an extended period of time. For simplicity, there are three individual decision-makers, with views on the propositions as shown in table 2. Note that the set of views of each individual is consistent.

	$P$ “increase spending on education”	$Q$ “increase spending on health care”	$R$ “increase spending on defence”	$S$ “increase taxes”	$((P \wedge Q \wedge R) \rightarrow S)$
Individual 1	yes	yes	no	no	yes
Individual 2	yes	no	yes	no	yes
Individual 3	no	yes	yes	no	yes

Table 2

**Case 1.** The propositions are considered in the following order:  $S$  at time 1 (e.g. January),  $P$  at time 2 (e.g. February),  $Q$  at time 3 (e.g. March), and  $R$  at time 4 (e.g. April). At time 1,  $S$  is unanimously rejected. The government is therefore committed to not increasing taxes. At time 2,  $P$  is accepted by a majority of two out of three. At time 3,  $Q$  is accepted by a majority of two out of three. At time 4, the government faces a dilemma. A majority of two out of three supports  $R$ . However, the government is already committed to increasing spending on two items (by the acceptance of  $P$  and  $Q$ ) and to not increasing taxes (by the rejection of  $S$ ). Increasing spending on an additional third item would necessitate a tax increase (by the acceptance of  $((P \wedge Q \wedge R) \rightarrow S)$ ). An acceptance of  $R$  is therefore inconsistent with the government’s prior commitments. If the government does not honour its prior commitments, it may of course revise some of them (and thereby risk losing credibility and reelection). If, however, the government honours its prior commitments, then it must reject  $R$ , overruling the positive majority verdict on  $R$  for the sake of consistency. The outcome of the decision process is  $P$ ,  $Q$ ,  $\neg R$  (the negation of  $R$ ) and  $\neg S$  (the negation of  $S$ ).

**Case 2.** The propositions are considered in the following order:  $P$  at time 1,  $Q$  at time 2,  $R$  at time 3, and  $S$  at time 4. At times 1, 2 and 3, respectively,  $P$ ,  $Q$  and  $R$  are each accepted by majorities of two out of three. At time 4, the government faces a dilemma.  $S$  is unanimously rejected. However, the government is already committed to increasing spending on three items (by the acceptance of  $P$ ,  $Q$  and  $R$ ). And increasing spending on these three items necessitates a tax increase (by the acceptance of  $((P \wedge Q \wedge R) \rightarrow S)$ ). A rejection of  $S$  is therefore inconsistent with the government’s prior commitments. Again, if the government does not honour its prior commitments, it may revise them. But if it does honour them, it must accept  $S$ , overruling the negative majority verdict on  $S$  for the sake of consistency. The outcome of the decision process is  $P$ ,  $Q$ ,  $R$  and  $S$ .

In each case, the decision process has two characteristics. First, the decision process is *aggregation-driven*: the decision-makers’ initial disposition to collectively accept or

reject each proposition is determined by aggregation over their individual views, specifically by majority voting. Second, the decision process is *consistency-driven*: the decision-makers are consistency-minded in the sense that they are prepared to overrule their majority verdict on some propositions for the sake of ensuring consistency with their verdicts on others (in the present example, prior ones).

Given these two characteristics, we have seen that cases 1 and 2 lead to different outcomes:  $P, Q, \neg R$  and  $\neg S$  in case 1, and  $P, Q, R$  and  $S$  in case 2. As in the killer amendment example, the only difference between the two cases is the decision-path. The views of the individual decision-makers are exactly the same in both cases, as is the modus ponens method of decision making.

This suggests that a decision process which is both aggregation-driven and consistency-driven may be path-dependent. It also raises the question of whether this path-dependency is just an artefact of the specific example, or whether it is an instance of a more general problem. We now turn to this question.

#### 4. A Simple Model

To define a *sequential decision or argumentation process*, a few preliminary definitions are due. Let  $X$  be the set of those propositions on which decisions are to be made, informally the set of propositions ‘on the table’. In the example of the previous section,  $X$  includes the atomic propositions  $P, Q, R, S$  and the compound proposition  $((P \wedge Q \wedge R) \rightarrow S)$ .<sup>3</sup> A *decision-path* is an ordering of all the propositions contained in  $X$ , which determines which proposition is to be considered first, which second, which third, and so on. There are (at least) two possible interpretations of a decision-path. A decision-path might be interpreted as the *temporal order* in which the propositions are considered in a decision or argumentation process over multiple interconnected propositions, i.e. earlier propositions in the decision-path come up earlier in time than later ones. Alternatively, a decision-path might be interpreted as the *order of epistemic or logical priority* that is assigned to the propositions, i.e. earlier propositions in the decision-path are regarded as epistemically or logically “weightier than”, or “prior to”, later ones.

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<sup>3</sup> As well as the negations of these propositions (for technical reasons). To ensure a nontrivial decision problem, it is assumed that  $X$  contains at least two distinct atomic propositions, their conjunction and the negation of their conjunction.

Now suppose the propositions are considered, one by one, in the order determined by a given decision-path. Then we need to specify the criteria of acceptance or rejection of each proposition in that sequence. This specification has two components.

**Initial dispositions.** The individual or group making the decision or argument may have an *initial disposition* to accept or reject a given proposition. This initial disposition can be interpreted as the verdict the individual or group would give on that proposition if the proposition were to be considered in isolation, with no reference to other propositions (particularly previously accepted ones). These initial dispositions can be represented by an acceptance/rejection function  $\delta$  which maps each proposition to either 1 or 0. For any proposition  $\phi$ ,  $\delta(\phi) = 1$  means that the individual or group has an initial disposition to accept  $\phi$ , and  $\delta(\phi) = 0$  means that the individual or group has an initial disposition to reject  $\phi$ .

To illustrate, in the case of a group, the initial disposition on each proposition might be the group's majority verdict on that proposition. In the case of an individual, the initial disposition on each proposition might be a function of the *prima facie* plausibility the individual attributes to that proposition.

**A conflict resolution rule.** As mentioned above, there may sometimes be a conflict between an individual's or a group's initial disposition on a new proposition and previously accepted propositions. In the example of the multi-member government in the previous section, such a conflict arises when the initial disposition towards increasing spending on a third item (at time 4 in case 1) is inconsistent with the government's prior commitment to keeping taxes low. A *conflict resolution rule* is a method of resolving such conflict.

The *modus ponens* rule resolves such conflict by accepting the logical implications of previously accepted propositions and overruling the initial disposition on the new proposition. The *modus tollens* rule resolves such conflict by accepting the initial disposition on the new proposition and revising previously accepted propositions.

For simplicity, we here restrict our consideration to decision or argumentation processes that use the *modus ponens* rule for resolving conflict, but the model can in principle be generalized. The use of the *modus ponens* rule (as opposed to the *modus tollens* rule or some combination of the two) may seem to be most plausible when we

adopt the epistemic or logical interpretation of a decision-path (as opposed to the temporal interpretation); or when the temporal order in which the propositions occur naturally coincides with their epistemic or logical order; or when we are dealing with a legal or political decision process in which a temporal asymmetry is determined, respectively, by the law or by external demands such as political credibility or feasibility. But the model to be developed is *formally* neutral with regard to different such interpretations and does not hinge on any specific one of them. (The reference to “time  $t$ ” in a decision-path is not intended to presuppose a temporal interpretation, and can easily be replaced with a reference to “step  $t$ ”.)

We define a *modus ponens decision process* in terms of a formal procedure.

- Consider the propositions, one by one, in the order determined by a given decision-path, say proposition  $\phi_1$  at time 1, proposition  $\phi_2$  at time 2, and so on.
- At each time  $t$ , when proposition  $\phi_t$  is being considered, determine whether or not previously accepted propositions in the sequence have a logical implication for the acceptance or rejection of  $\phi_t$ .

If yes:

Ignore the initial disposition ( $\delta(\phi_t)$ ) on  $\phi_t$ .

Accept  $\phi_t$  if previously accepted propositions imply  $\phi_t$ .

Reject  $\phi_t$  if previously accepted propositions imply the negation of  $\phi_t$ .

If no:

Accept  $\phi_t$  if the initial disposition on  $\phi_t$  is positive, i.e. if  $\delta(\phi) = 1$ .

Reject  $\phi_t$  if the initial disposition on  $\phi_t$  is negative, i.e. if  $\delta(\phi) = 0$ .

The example of the multi-member government in the previous section is a straightforward application of a modus ponens decision process. As noted there, a modus ponens decision process is both aggregation-driven and consistency-driven. Further, note that, as the process never allows the acceptance of a new proposition that conflicts with previously accepted propositions, it guarantees consistent outcomes.

## 5. Path Dependencies at a Collective Level – A General Theorem

The example of the multi-member government has shown that, if a group’s initial dispositions on each proposition are defined by majority voting over individual views on that proposition, then the resulting modus ponens decision process may be path-

dependent. This raises the question of whether there are other ways of defining the group's initial dispositions such that path-dependencies can be avoided.

To state a general theorem in answer to this question, suppose there are  $n$  individuals, labelled 1, 2, ...,  $n$ . Following the definition of initial dispositions above, the views of each individual,  $i$ , over the propositions will be represented by an acceptance/rejection function  $\delta_i$ . The subscript  $i$  indicates that the function corresponds to individual  $i$ . The label  $\delta$  without a subscript refers to the acceptance/rejection function of the group as a whole.

Suppose further, as a best-case scenario, that the views of each individual satisfy some strong rationality conditions: *completeness*, *consistency* and *deductive closure*.

**Completeness.** An individual's views are *complete* if, for any proposition  $\phi$ , the individual accepts the proposition  $\phi$  or its negation  $\neg\phi$  (formally,  $\delta_i(\phi)=1$  or  $\delta_i(\neg\phi)=1$ ).

**Consistency.** An individual's views are *consistent* if the individual never accepts a proposition  $\phi$  and its negation  $\neg\phi$  simultaneously (formally, *not both*  $\delta_i(\phi)=1$  and  $\delta_i(\neg\phi)=1$ ).<sup>4</sup>

**Deductive Closure.** An individual's views are *deductively closed* if, whenever the individual accepts a set of propositions  $\Psi$  and  $\Psi$  implies another proposition  $\phi$ , then the individual also accepts  $\phi$  (formally, *if* [ $\delta_i(\psi)=1$  for every  $\psi$  in  $\Psi$ ] *and* [ $\Psi$  implies  $\phi$ ], *then*  $\delta_i(\phi)=1$ ).

The set of views *across all individuals* is called a *profile of individual views over the propositions*.

The views of the  $n$  individuals on each proposition  $\phi$  can be represented by a vector of 0s and 1s (corresponding to individuals accepting and rejecting  $\phi$ , respectively). For example, if there are 5 individuals, the vector  $\langle 0, 0, 1, 0, 1 \rangle$  means that individuals 3 and 5 accept  $\phi$ , and individuals 1, 2 and 4 reject  $\phi$ .

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<sup>4</sup> Note that this is a narrow (syntactic) notion of consistency. It requires only that no proposition and its negation be simultaneously accepted, but not that there exists a semantic model (an assignment of truth-values to all propositions) that would make all the accepted propositions simultaneously true. On the given narrow (syntactic) notion of consistency, the set  $\{P, (P \rightarrow Q), \neg Q\}$ , for instance, is consistent, since no proposition and its negation are both contained in it. On a broader (semantic) notion of consistency, by contrast, the set is inconsistent, since there exists no semantic model that would make all the propositions in this set simultaneously true. However, in conjunction with deductive closure, the narrow notion of consistency entails the broader one (in the propositional calculus).

To determine the group's initial disposition on  $\phi$ , we require a way of *aggregating* the  $n$  individual views on  $\phi$  – represented by a vector of 0s and 1s – into a single overall disposition in the form of 1 (acceptance) or 0 (rejection). For example, the method of majority voting defines the group's initial disposition on  $\phi$  by determining whether there are more 0s or more 1s among the individual views on  $\phi$ . If a group's initial dispositions are defined by *aggregation* over the individual views, we will call the group's acceptance/rejection function  $\delta$  an *aggregation function*.

We can now restate the observation as well as the question raised at the beginning of this section. First, the observation: Majority voting is an aggregation function which may lead to path-dependencies in modus ponens decision processes. And, second, the question: Are there any alternative aggregation functions under which there will be no such path-dependencies? As we will now see, if we impose some undemanding minimal conditions on an aggregation function, then the answer to this question is negative. Under these minimal conditions, path-dependencies would, then, be *in principle* unavoidable.

The conditions are the following:

**Universal Domain.** The domain of admissible individual views over the propositions includes all logically possible profiles of individual views satisfying completeness, consistency and deductive closure.

**Anonymity.** An aggregation function is invariant under permutations of the individuals. (Informally, this requires that all individuals have equal weight in determining the group's initial disposition on any proposition.)

**Decisiveness.** The modus ponens decision process produces a determinate decision on every proposition (acceptance of the proposition or acceptance of its negation).

**Theorem 1.** There exists no aggregation function (satisfying universal domain and anonymity) for which a modus ponens decision process (satisfying decisiveness) is invariant under changes of the decision-path.

The formal proof of the theorem is given in the appendix, but a brief informal explanation of the mechanism underlying theorem 1 will be given in section 7. It is important to be clear about what exactly theorem 1 states. The theorem is premised on three conditions. First, individuals are free to form any logically possible combination of views on the

propositions, provided their views satisfy the three rationality conditions of completeness, consistency and deductive closure. Second, an aggregation function should give equal weight to all individuals in determining the group's initial disposition on any proposition; specifically, this requires that no individual or subset of individuals be dictatorial. Third, a modus ponens decision process should be decisive, producing a determinate decision on every proposition. The theorem states that, under these conditions, it is impossible to find an aggregation function for which a modus ponens decision process is always invariant under changes of the decision-path. This does *not* mean that the outcome will be path-dependent for *every* profile of individual views. Rather, it means that no aggregation function can *guarantee* the avoidance of path-dependencies; under *any* aggregation function, there will exist *some* profiles of individual views for which the outcome of a modus ponens decision process is path-dependent.

To explain the mechanism underlying theorem 1, it is necessary to go one step further. Before we can state a theorem identifying a necessary and sufficient condition for the occurrence of path-dependencies it is helpful to consider an example – albeit a somewhat contrived one – of path-dependencies in arguments at an individual level.

## 6. Path-Dependencies at an Individual Level – An Example

Suppose an individual has an *initial disposition* to *accept* each of the following propositions. That is, if the individual were to consider each proposition *in isolation*, he or she would find the proposition sufficiently plausible to accept.

$P$  : Young people are to be free to decide their own life plans after school.

$Q$  : Compulsory national service reduces the number of crimes committed by young people.

$R$  : Compulsory national service is justifiable.

$(P \rightarrow \neg R)$  : If young people are to be free to decide their own life plans after school, then compulsory national service is not justifiable.

$(Q \rightarrow R)$  : If compulsory national service reduces the number of crimes committed by young people, then compulsory national service is justifiable.

Although we may immediately notice a logical tension in the initial dispositions to *accept* each of these propositions, we will leave this point aside for the moment and return to it below.

Suppose the individual engages in a modus ponens decision (or argumentation) process over the five propositions (we might also think of this as an example of a process of public deliberation).

**Case 1.** The propositions are considered in the following order:  $P$  at time 1,  $(P \rightarrow \neg R)$  at time 2,  $R$  at time 3,  $Q$  at time 4,  $(Q \rightarrow R)$  at time 5. At times 1 and 2, the individual accepts  $P$  and  $(P \rightarrow \neg R)$ , respectively, simply following his or her initial dispositions on these propositions. At time 3, the individual faces a first dilemma. The individual's initial disposition to accept  $R$  is inconsistent with the implications of his or her previously accepted propositions  $\neg P$  and  $(P \rightarrow \neg R) \neg$ , namely  $\neg R$ . Using the modus ponens rule, the individual overrules the initial positive disposition on  $R$ , and rejects  $R$ . At time 4, the individual accepts  $Q$ , following his or her initial disposition. At time 5, finally, the individual faces another dilemma. Previously accepted propositions are inconsistent with the individual's initial disposition to accept  $(Q \rightarrow R)$ . Using the modus ponens rule again, the individual overrules that disposition, and rejects  $(Q \rightarrow R)$ . The outcome of the argumentation process is  $P$ ,  $(P \rightarrow \neg R)$ ,  $\neg R$ ,  $Q$  and  $\neg(Q \rightarrow R)$ .

**Case 2.** The propositions are considered in the following order:  $Q$  at time 1,  $(Q \rightarrow R)$  at time 2,  $R$  at time 3,  $(P \rightarrow \neg R)$  at time 4,  $P$  at time 5. Here the individual can follow his or her initial dispositions at times 1 to 4, accepting each of  $Q$ ,  $(Q \rightarrow R)$ ,  $R$  and  $(P \rightarrow \neg R)$ . At time 5, however, the individual faces a conflict between the previously accepted propositions and his or her initial disposition to accept  $P$ . Using the modus ponens rule, the individual overrules that initial disposition, and rejects  $P$ . The outcome of the argumentation process is  $Q$ ,  $(Q \rightarrow R)$ ,  $R$ ,  $(P \rightarrow \neg R)$ ,  $\neg P$ .

The two argumentation processes lead to inconsistent conclusions. Only a single individual is involved, and that individual's initial dispositions on the propositions are the same in both cases. In both cases the individual uses only instances of valid logical inferences. The only difference between the two cases is the decision-path. What has happened?

Let us take a closer look at the individual's initial dispositions on the propositions, and at what may have struck us as a logical tension in these dispositions. Are they inconsistent? They are not inconsistent *according to the definition of consistency in section 5 above*: it is not the case that the individual has an initial disposition to accept a proposition and also to accept the negation of the very same proposition simultaneously.

However, they may strike us as *implicitly* inconsistent.<sup>5</sup> In particular, the individual has initial dispositions to accept each of  $Q$ ,  $(Q \rightarrow R)$ ,  $P$ ,  $(P \rightarrow \neg R)$ , where the first pair of propositions implies  $R$  and the second implies  $\neg R$ .

More formally, if we go through the three rationality conditions introduced above, then it turns out that the individual's initial dispositions on the propositions satisfy completeness and consistency (in the narrow sense of that condition, as we have noted), but violate deductive closure with respect to  $\neg R$ : the individual has an initial disposition to accept  $P$ , to accept  $(P \rightarrow \neg R)$ , but no initial disposition to accept  $\neg R$  (and thereby to reject  $R$ ), in spite of the fact that  $P$  and  $(P \rightarrow \neg R)$  together imply  $\neg R$ .

The conclusion the individual draws on  $R$  will now depend on the order in which the individual considers the propositions in a decision or argumentation process. If  $R$  occurs before  $P$  and  $(P \rightarrow \neg R)$  in the decision-path, then the individual will follow his or her initial disposition to accept  $R$ . If, on the other hand,  $R$  occurs only after  $P$  and  $(P \rightarrow \neg R)$  in the decision-path, then the individual will use the modus ponens rule to reject  $R$ .

The problem would not occur if the individual's initial dispositions were to satisfy *all* of completeness, consistency and deductive closure. This suggests that violations of the rationality conditions, particularly of deductive closure, are responsible for the occurrence of path-dependencies. The theoretical result of the next section confirms this suggestion.

## **7. Path-Dependencies at Individual and Collective Levels – A Necessary and Sufficient Condition**

The result to be stated applies to individuals or groups making decisions or arguments over multiple propositions. All we need to assume is that the individual or group has initial dispositions over the propositions as represented by the acceptance/rejection function  $\delta$ . For simplicity, we assume that the individual's or group's initial dispositions over the propositions satisfy completeness. Further, as *overt* inconsistencies (unlike *implicit* ones as discussed in the previous section) may be assumed to be rare, we assume that the initial dispositions satisfy consistency (in the narrow sense of the definition in section 5).

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<sup>5</sup> Compare the two notions of consistency introduced in note 4. The individual's dispositions here do not violate consistency in the narrow (syntactic) sense as defined there (the individual does not explicitly accept a proposition and its negation simultaneously), but they do violate consistency in the broader (semantic) sense (there exists no semantic model that would make all the accepted propositions simultaneously true).

**Theorem 2.** Suppose the individual's or group's initial dispositions over the propositions satisfy completeness and consistency. Suppose the individual or group uses a modus ponens decision process. Then

there exist (at least) two alternative decision-paths such that, under one path,  $\phi$  is accepted and, under the other,  $\neg\phi$  is accepted

if and only if

the initial dispositions are not deductively closed with respect to  $\phi$ .

For a proof, see the appendix. Saying that an individual or group violates deductive closure with respect to  $\phi$  means that the individual or group has initial dispositions to accept a set of propositions  $\Psi$ , where  $\Psi$  implies  $\phi$ , but the individual or group has no initial disposition to accept  $\phi$ . Now the theorem states that, whenever these initial dispositions violate deductive closure with respect to a particular proposition  $\phi$ , then there will exist (at least) two alternative decision-paths such that one path leads to the acceptance of  $\phi$ , whereas the other leads to the acceptance of the negation of  $\phi$ . Informally, this means that, for those propositions with respect to which deductive closure is violated, anything can happen – depending only on the decision-path. But, conversely, if deductive closure is satisfied, there will no longer be any path-dependencies.

The result of theorem 2 is also illustrated by the example (in section 3) of the multi-member government voting on expenditure and taxes. Under the decision-path of case 1,  $\neg S$  (don't increase taxes) is accepted, and under the decision-path of case 2,  $S$  (increase taxes) is accepted. Theorem 2 then implies that the initial dispositions of the group are not deductively closed with respect to  $S$ . And, indeed, we can see in table 2 that there are majorities on each of  $P$ ,  $Q$ ,  $R$  and  $((P \wedge Q \wedge R) \rightarrow S)$ ; further, these propositions imply  $S$ ; and yet there is no majority on  $S$ , a violation of deductive closure with respect to  $S$ .

At an individual level, the response to this path-dependence problem may seem fairly simple. So long as an individual is sufficiently rational – i.e. so long as his or her views satisfy completeness, consistency and deductive closure –, the individual is immune to path-dependencies in a modus ponens decision process. Of course, it is an interesting empirical question whether individuals do in fact satisfy these rationality conditions, or whether violations of deductive closure in individual views are empirically frequent.

At a collective level, on the other hand, the response is much less straightforward. It would be desirable if we could simply construct an aggregation function  $\delta$  which generates acceptance/rejection dispositions that satisfy completeness, consistency and deductive closure. By theorem 2, such an aggregation function would immediately solve the problem of path-dependencies.

We have already seen that majority voting may violate the desired properties. Under majority voting a group's initial dispositions are always complete and consistent (in the narrow sense of that condition), but they may fail to satisfy deductive closure. This is not an accidental property of majority voting. In general, if we demand the minimal conditions of universal domain and anonymity, no aggregation function with the desired properties exists.

**Theorem 3.** (List and Pettit 2002) There exists no aggregation function (satisfying universal domain and anonymity) which generates acceptance/rejection dispositions on the propositions satisfying completeness, consistency and deductive closure.

By theorem 3, any aggregation function (satisfying universal domain and anonymity) which generates complete and consistent acceptance/rejection dispositions – such as majority voting – will *of necessity* generate some violations of deductive closure. And, by theorem 2, this will be sufficient for the possibility of different decision-paths with mutually inconsistent outcomes. This, in a nutshell, is the proof of theorem 1. The technical details are discussed in the appendix.

Does this imply that, at a collective level, path-dependencies are *in principle* unavoidable? And if path-dependence is an unavoidable phenomenon, what is the price we have to pay for this? We now address both of these questions. We first address the second question, and show that path-dependence gives rise to the possibility of two types of strategic manipulation; and we then turn to the first question, and discuss some escape-routes from the problem of path-dependence.

## **8. The Price of Path-Dependence: The Possibility of Strategic Manipulation**

As briefly mentioned in the introduction, path-dependence opens up the possibility of two types of strategic manipulation.

### **Manipulation by agenda setting**

Whenever the initial dispositions are not deductively closed with respect to  $\phi$ , the agenda-setter – whoever can choose the decision-path – may have considerable power in determining whether the outcome of the decision process will be  $\phi$  or  $\neg\phi$ . We know by theorem 2 that, given a violation of deductive closure with respect to  $\phi$ , there exist alternative decision-paths leading to each of these two opposite outcomes on  $\phi$ . Given sufficient information and computational power, the agenda-setter can thus work out the decision-path that is required to bring about the preferred outcome. In our example of the multi-member government in section 3, an agenda-setter who cares a lot about increasing defence spending (proposition  $R$ ) would advocate the decision-path of case 2, which results in the acceptance of proposition  $R$ , whereas an agenda-setter who wants to avoid an increase in defence spending would advocate the decision-path of case 1, which results in the rejection of proposition  $R$ .

### **Manipulation by expression of untruthful views**

Suppose a decision process is path-dependent, and suppose a particular decision-path has been chosen. If some individual (or group of individuals) cares particularly about some propositions that occur at a later point in the decision-path, they might strategically express untruthful views on certain propositions that occur earlier in that path, as the decision on the later propositions will be affected by the decision on the earlier ones. Going back to our example of the multi-member government again, let us suppose that individual 3 cares most about increasing defence spending (proposition  $R$ ), and is quite willing to sacrifice his conviction that spending on health care should also be increased (proposition  $Q$ ), in order to get his or her way on the defence issue. Suppose the decision-path is the one of case 1 in the example. At time 3, when proposition  $Q$  is considered, individual 3 might untruthfully vote against  $Q$ , thus bringing about a majority *rejection* of  $Q$ . At time 4, when proposition  $R$  is finally considered, the government then no longer faces a conflict between its prior commitments and the initial verdict on the new proposition; the government can follow its majority verdict on  $R$ , and accept  $R$ . Without individual 3's strategic intervention, the outcome of the decision process would have been  $P$ ,  $Q$ ,  $\neg R$  and  $\neg S$ . With individual 3's strategic intervention, the outcome is  $P$ ,  $\neg Q$ ,  $R$  and  $\neg S$ . In this case, we say that individual 3 has an incentive to express an untruthful view on proposition  $Q$ . To give a formal definition of strategic manipulability by expression of untruthful views, we assume that each individual,  $i$ , has preferences over all possible outcomes of the decision process. Then individual  $i$  has an *incentive to express untruthful views on some of the propositions* at some profile of individual views

if the following three conditions hold: (i) If individual  $i$  expresses his or her truthful views on the propositions (holding the views of the other individuals fixed), the decision process leads to the overall outcome  $\Phi_1$ . (ii) If individual  $i$  expresses alternative (untruthful) views on some of the propositions (holding the views of the other individuals fixed), the decision process leads to the overall outcome  $\Phi_2$ . (iii) Individual  $i$  strictly prefers outcome  $\Phi_2$  to outcome  $\Phi_1$ . A decision process is said to satisfy *strategy-proofness* at a particular profile of individual views if there exists no individual who has an incentive to express untruthful views on some of the propositions at that profile. Path-dependent decision processes may thus violate strategy-proofness. Note that, in decision processes that violate strategy-proofness, there exists at least one individual for whom expressing truthful views is *not* the dominant strategy (in the game-theoretic sense). In strategy-proof decision processes, by contrast, expressing truthful views is the dominant strategy for every individual.

Neither type of strategic manipulation is possible when a decision process is invariant under changes of the decision-path. In the case of manipulation by agenda setting this is immediately obvious. Agenda setting – in the sense of determining the order in which the propositions are considered – clearly has no effect when the decision process is invariant under changes of the decision-path.

In the case of manipulation by expression of untruthful views, the following result is proved in the appendix:

**Theorem 6.** Suppose the preferences of all individuals are *weakly monotonic*, and the aggregation function used to determine the group’s initial dispositions also satisfies a *weak monotonicity* condition.<sup>6</sup> Then any modus ponens decision process that is invariant under changes of the decision-path satisfies strategy-proofness.<sup>7</sup>

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<sup>6</sup> Individual  $i$ ’s preferences over the possible outcomes of the decision process are *weakly monotonic* if the following holds: if one possible outcome  $\Phi_1$  of the decision-process is *at least as close* (in a sense defined in the appendix) to individual  $i$ ’s views as another possible outcome  $\Phi_2$ , then individual  $i$  weakly prefers  $\Phi_1$  to  $\Phi_2$ . Weak monotonicity is intended to be an undemanding minimal condition on the preferences of an individual; typically, we expect the preferences of an individual to satisfy stronger conditions. An aggregation function is *weakly monotonic* if the following holds. Consider the result of aggregating all individuals’ initial dispositions on some proposition  $\phi$  into an overall collective disposition on  $\phi$ . Supposing the dispositions of all other individuals remain the same, if some individual(s) change(s) their disposition(s) on  $\phi$  in favour of the (previous) ‘winning’ collective disposition on  $\phi$ , then that collective disposition is *not* reversed. Weak monotonicity is also intended to be an undemanding *minimal* condition. Majority voting, for instance, satisfies weak monotonicity.

<sup>7</sup> Note that, in spite of the impossibility result of theorem 1 above, theorem 6 is *not* vacuous. In particular, under the conditions of theorem 6, the set of modus ponens decision processes that are invariant under

## 9. Escape-Routes from Path-Dependencies at a Collective Level

We suppose again that the views of individuals are complete, consistent and deductively closed. Let us first remind ourselves of theorem 1. According to that theorem, there exists no aggregation function (satisfying universal domain and anonymity) for which a modus ponens decision process (satisfying decisiveness) is invariant under changes of the decision-path.

To avoid path-dependencies it is therefore necessary to relax at least one of the conditions of theorem 1, namely universal domain, anonymity or decisiveness. More specifically, by theorem 2, we know that we need to relax these conditions in such a way that we can find an aggregation function which generates deductively closed acceptance/rejection dispositions for the group.

### Relaxing decisiveness

Suppose we do not insist that the modus ponens decision process should produce a determinate verdict (acceptance or rejection) on every proposition, but we allow that it may fail to produce a verdict on some propositions. We can then define the group's initial disposition on each proposition by the unanimity rule. For each proposition  $\phi$ , the group will have a disposition to accept  $\phi$  if and only if *every* individual accepts  $\phi$ . The group's initial dispositions are then consistent and deductively closed (so long as the views of the individuals are consistent and deductively closed), but not necessarily complete. (They are incomplete to the extent that there is disagreement between individuals.) The resulting modus ponens decision process will be invariant under changes of the decision-path, but it may fail to produce a verdict on some propositions – namely on those propositions which are neither unanimously accepted nor unanimously rejected by the individuals.

### Relaxing anonymity

Suppose we do not insist that all individuals should have equal weight in determining the group's initial disposition on any proposition, but we allow that a fixed single individual may be dictatorial. Specifically, we choose one fixed single individual, the *dictator*, and

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changes of the decision-path is *not* empty. If at least one of universal domain, anonymity or decisiveness is relaxed, there do exist such decision processes. *All* of the examples of (path-independent) modus ponens decision processes discussed in section 9 below satisfy the conditions of theorem 6.

define the group's initial disposition on each proposition simply to be the view of the dictator. Under this definition, so long as the views of the dictator satisfy completeness, consistency and deductive closure, so will the dispositions of the group. The resulting modus ponens decision process will be invariant under changes of the decision-path, but of course it may fail to reflect the views of any individuals other than the dictator.

### Relaxing universal domain

Suppose we have reasons to believe that not all logically possible profiles of individual views over the propositions will occur. There are two conceivable such reasons. Either certain profiles of individual views are *explicitly ruled out* by restrictions on the views that individuals can express (the coercive solution); or certain profiles of individual views simply *happen*, as a matter of fact, *not to occur* in practice (the empirical solution).

We can identify a certain structure condition on the profile of individual views such that, if the domain of admissible individual views includes only profiles satisfying that structure condition, then the impossibility result of theorem 3 and the path-dependency result of theorem 1 can be avoided. Specifically, suppose that the profile of individual views satisfies the following property: there exists a single linear ordering of the individuals from 'left'-most to 'right'-most (called a *structuring ordering*) such that, for *every* proposition in  $X$ , the individuals accepting that proposition are either all to the left, or all the right, of those rejecting it. Then we say that the profile of individual views satisfies *unidimensional alignment*. To illustrate, the profile of individual views shown in table 3 satisfies unidimensional alignment.

	Individual 4	Individual 1	Individual 5	Individual 2	Individual 3
$P$	1	1	1	1	0
$Q$	1	1	1	0	0
$R$	1	1	0	0	0
$S$	1	1	0	0	0
$((P \wedge Q \wedge R) \rightarrow S)$	1	1	1	1	1

**Table 3**

For simplicity, we assume that the number of individuals is odd. The individual who has an equal number of individuals to the left and to right on the structuring ordering is called the *median individual* with respect to the structuring ordering.

The following results hold:

**Theorem 4.** (List 2001) (a) Majority voting is an aggregation function (defined on the domain of unidimensionally aligned profiles of individual views and satisfying anonymity) which generates acceptance/rejection dispositions on the propositions satisfying completeness, consistency and deductive closure. (b) For each profile of individual views satisfying unidimensional alignment, the group's acceptance/rejection dispositions will coincide with the views of the median individual with respect to the relevant structuring ordering.

**Theorem 5.** (a) Majority voting is an aggregation function (defined on the domain of unidimensionally aligned profiles of individual views and satisfying anonymity) for which a modus ponens decision process (satisfying decisiveness) is invariant under changes of the decision-path. (b) For each profile of individual views satisfying unidimensional alignment, the outcome of the modus ponens decision process will coincide with the views of the median individual with respect to the relevant structuring ordering.

Is unidimensional alignment just an artificial condition, or can we imagine plausible circumstances in which unidimensional alignment might arise? Suppose we have a situation in which the individuals do not agree “substantively” on what set of views on the propositions to endorse, but they reach agreement “at a meta-level”, as follows. Suppose, firstly, that they agree on a single linear dimension (such as from “most liberal” to “most conservative”) that characterizes the range of their disagreement; in particular, suppose that each individual takes a certain position on that dimension. For simplicity, we will call it a 'left'/right' dimension, but a range of interpretations is possible. And suppose, secondly, that, for each proposition, the extreme positions on the 'left'/right' dimension correspond to either clear acceptance or clear rejection of the proposition and there exists an 'acceptance threshold' on the dimension (possibly different for different propositions) such that all the individuals to the left of the threshold accept the proposition and all the individuals to its right reject it (or vice-versa). These two conditions then entail unidimensional alignment. In other words, agreement at a meta-level (as described) can induce unidimensional alignment. This suggests that, to the extent that the individuals reach agreement at a meta-level, the escape-route from the path-dependency result opened up by theorem 5 becomes available. For the present purposes, however, the specific interpretation of unidimensional alignment is less

relevant than the more general insight that path-dependencies can be avoided if there is a sufficient level of structure or cohesion among the views of different individuals.

Finally, we should note that in some decision problems the subject-matter of the decision might by itself single out a specific decision-path as the appropriate one. Some propositions might, for instance, be regarded as “weightier than”, or “prior to”, others, in which case the order in which the propositions are to be considered might be uncontroversial. But even in those cases it is of interest to ask whether or not such a privileged decision-path makes a difference to the outcome. If there are no path-dependencies, then, regardless of whether there is a dispute about the decision-path, the perceived legitimacy of a particular outcome would be under no threat – the choice of a decision-path would be irrelevant to the outcome. If there are path-dependencies, on the other hand, then a justification of the chosen decision-path becomes crucial. Moreover, in such cases, even if agreement can be reached on a privileged decision-path, this would solve only one of the two identified problems of strategic manipulation. It would solve the problem of agenda manipulation, as an agenda-setter would be constrained by the agreed choice of a decision-path. But agreement on a privileged decision-path would still not solve the problem of manipulation by the expression of untruthful views. As we have seen in section 8, when a decision process merely satisfies the property of being path-dependent (even if this path-dependence property is not being “acted upon”), individuals may already have incentives to express untruthful views. In our example in section 8, individual 3 has an incentive to express untruthful views, even though the decision-path remains completely fixed throughout the example. So, curiously, path-dependence does matter even if the actual decision-path is not up for grabs.

## 10. Concluding Remarks

We have discussed *modus ponens* decision processes as a model of sequential decision or argumentation processes which are both aggregation-driven and consistency-driven. We have seen that, in a large class of such decision processes, path-dependencies are unavoidable (theorem 1). The occurrence of path-dependencies is linked with the violation of deductive closure by the individual’s or group’s initial dispositions on the relevant propositions (theorem 2). Whenever the individual or group fails to endorse the

logical implications of other propositions they endorse,<sup>8</sup> there will exist (at least) two alternative decision-paths which lead to mutually inconsistent outcomes.

The problem is particularly likely to occur at a collective level. While individuals might simply try to avoid path-dependencies through a self-imposed ‘discipline’ of rationality, no such option is generally available to groups. Any group that determines its initial disposition on the propositions by *aggregation* over individual views will *of necessity* run the risk of violations of completeness, consistency and deductive closure (theorem 3). And such violations are in turn directly responsible for the occurrence of path-dependencies (theorem 2 again). The escape-routes from path-dependencies available to groups are limited: we have discussed relaxations of anonymity, decisiveness and universal domain. Relaxing anonymity would involve the rather undemocratic solution of a dictatorship. Relaxing decisiveness would involve the risk of ‘stalemate’ in that the decision process would not necessarily produce a determinate verdict (acceptance or rejection) on every proposition. Relaxing universal domain, by contrast, may be a promising escape-route from path-dependency problems (theorems 4 and 5). Future research will have to focus on the question of how likely it is (theoretically and empirically) that the relevant structure conditions on profiles of individual views are satisfied and how they can be brought about, and thus whether relaxing universal domain is defensible.

Improving our understanding of path-dependence is an important challenge in the theory of democracy. Many democratic decision processes are sequential, and hence it is important to learn whether, and how, the decision-path matters, and what the consequences, and potential risks, of path-dependencies are.

## Appendix

### A1: Decisions over Multiple Interconnected Propositions

Let  $N = \{1, 2, \dots, n\}$  be a set of individuals ( $n \geq 2$ ). Let  $X$  be a set of propositions from the propositional calculus, where  $X$  is interpreted as the set of those propositions on which decisions are to be made.  $X$  includes atomic propositions, such as  $P$ ,  $Q$ ,  $R$  or  $S$ , as well as compound propositions, such as  $(R \leftrightarrow (P \wedge Q))$  or  $(S \leftrightarrow (P \wedge Q \wedge R))$ . We assume that  $X$

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<sup>8</sup> Supposing the individual or group satisfies the conditions of consistency and completeness.

contains at least two distinct atomic propositions,  $P$  and  $Q$ , and their conjunction,  $(P \wedge Q)$ .<sup>9</sup> Moreover, we assume that  $X$  contains proposition-negation pairs: specifically, whenever  $\phi \in X$ , we also have  $\neg\phi \in X$ . For simplicity, for every  $\phi \in X$ , we identify  $\neg\neg\phi$  with  $\phi$ .<sup>10</sup>

The views of each individual,  $i$ , over the propositions in  $X$  are represented by an *acceptance/rejection function*  $\delta_i : X \rightarrow \{1, 0\}$ . For each proposition  $\phi \in X$ ,  $\delta_i(\phi)=1$  means that individual  $i$  accepts  $\phi$ , and  $\delta_i(\phi)=0$  means that individual  $i$  rejects  $\phi$ . Note that  $\delta_i(\phi)=0$  does not by itself mean that individual  $i$  accepts the negation of  $\phi$ . Individual  $i$  accepts that negation only if  $\delta_i(\neg\phi)=1$ .

The acceptance/rejection function of individual  $i$ ,  $\delta_i$ , may or may not satisfy three rationality properties: *completeness*, *consistency* and *deductive closure*.  $\delta_i$  is *complete* if, for all  $\phi \in X$ , either  $\delta_i(\phi)=1$  or  $\delta_i(\neg\phi)=1$ .  $\delta_i$  is *consistent* if there exists no  $\phi \in X$  such that  $\delta_i(\phi)=1$  and  $\delta_i(\neg\phi)=1$ .  $\delta_i$  is *deductively closed* if, whenever  $S = \{\phi \in X : \delta_i(\phi)=1\}$  logically entails some other  $\psi \in X$ , then  $\delta_i(\psi)=1$ .

$\delta_i$  violates deductive closure with respect to a proposition  $\psi \in X$  if  $\{\phi \in X : \delta_i(\phi)=1\}$  logically entails  $\psi$ , and  $\delta_i(\psi) = 0$ .

A *profile of individual acceptance/rejection functions* is an assignment of one acceptance/rejection function to each individual,  $\{\delta_i\}_{i \in N} = \{\delta_1, \delta_2, \dots, \delta_n\}$ .

An *aggregation function* is a function  $\delta : \{0,1\}^n \rightarrow \{0,1\}$ . For each profile of individual acceptance/rejection functions  $\{\delta_i\}_{i \in N}$ ,  $\delta$  induces a *collective acceptance/rejection function*  $\delta : X \rightarrow \{0,1\}$  defined as follows:

for each  $\phi \in X$ ,  $\delta(\phi) := \delta(\delta_1(\phi), \delta_2(\phi), \dots, \delta_n(\phi))$ .

Given a profile  $\{\delta_i\}_{i \in N}$ , we use the label  $\delta$  to denote not only the aggregation function  $\delta : \{0,1\}^n \rightarrow \{0,1\}$ , but also the collective acceptance/rejection function over  $X$  induced by  $\delta$ .<sup>11</sup>

The collective acceptance/rejection function  $\delta$  may or may not satisfy completeness, consistency and deductive closure, as defined above. The function  $\delta$  satisfies *anonymity* if, for any  $(d_1, d_2, \dots, d_n) \in \{0,1\}^n$  and any permutation  $\sigma : N \rightarrow N$ ,  $\delta(d_1, d_2, \dots, d_n) = \delta(d_{\sigma(1)}, d_{\sigma(2)}, \dots, d_{\sigma(n)})$ .

<sup>9</sup> The use of conjunction ( $\wedge$ ) is not essential, and the use of other logical connectives would yield a similar result. As the set of connectives  $\{\neg, \wedge\}$  is expressively adequate, any logically possible proposition from the propositional calculus can be expressed as a proposition using  $\neg$  and  $\wedge$  as the only connectives.

<sup>10</sup> For technical reasons, we assume that the propositions in  $X$  are neither tautologies nor inconsistencies.

<sup>11</sup> A more pedantic notation would draw a difference between the *aggregation function*  $\delta : \{0,1\}^n \rightarrow \{0,1\}$  and the *collective acceptance/rejection function*  $\delta_{\{\delta_i\}_{i \in N}} : X \rightarrow \{0,1\}$  induced by  $\delta$  for a given profile  $\{\delta_i\}_{i \in N}$ . To simplify the notation, we here drop the subscript  $\{\delta_i\}_{i \in N}$  and simply write  $\delta$  for  $\delta_{\{\delta_i\}_{i \in N}}$ .

Let  $U$  be the set of all logically possible profiles of individual acceptance/rejection functions satisfying completeness, consistency and deductive closure.

**Theorem 3.** (List and Pettit 2002) There exists no aggregation function  $\delta : \{0,1\}^n \rightarrow \{0,1\}$  satisfying anonymity which induces, for every  $\{\delta_i\}_{i \in N} \in U$ , a complete, consistent and deductively closed collective acceptance/rejection function  $\delta : X \rightarrow \{0,1\}$ .

Propositionwise majority voting is the aggregation function  $\delta : \{0,1\}^n \rightarrow \{0,1\}$  defined as follows:

$$\text{for any } (d_1, d_2, \dots, d_n) \in \{0,1\}^n,$$

$$\delta(d_1, d_2, \dots, d_n) = \begin{cases} 1 & \text{if } \sum_{i \in N} d_i > n/2 \\ 0 & \text{otherwise} \end{cases}$$

(to avoid ties, we suppose that  $n$  is odd). Note that while  $\delta$  satisfies anonymity and induces collective acceptance/rejection functions which are always complete and consistent (so long as  $n$  is odd), these acceptance/rejection functions may not be deductively closed (compare in particular table 2 in section 3 above).

## A2: Sequential Decision Processes

A *decision-path* on  $X$  is a bijective mapping  $\Omega : \{1, 2, \dots, k\} \rightarrow X$ , where  $k = |X|$ .  $\Omega$  represents the order in which the propositions in  $X$  are considered; i.e.  $\Omega(1)$ ,  $\Omega(2)$ , ...,  $\Omega(k)$  are, respectively, the first, second, ...,  $k$ -th propositions to be considered.

We can now define a *modus ponens decision process*.

For each step  $t \in \{1, 2, \dots, k\}$ ,  $\Phi(t)$  denotes the set of propositions accepted in steps 1, 2, ...,  $t$ . We define  $\Phi(t)$  inductively as follows (adding step 0):

$t=0$ :  $\Phi(0)$  is the empty set.

$t > 0$ : Let  $\phi = \Omega(t)$ .

Either  $\Phi(t-1)$  has a logical implication for  $\phi$  (yes)

or  $\Phi(t-1)$  has no logical implication for  $\phi$  (no).

If yes:

$$\Phi(t) := \begin{cases} \Phi(t-1) \cup \{\phi\} & \text{if } \Phi(t-1) \text{ logically entails } \phi \\ \Phi(t-1) \cup \{\neg\phi\} & \text{if } \Phi(t-1) \text{ logically entails } \neg\phi. \end{cases}^{12}$$

If no:

$$\Phi(t) := \begin{cases} \Phi(t-1) \cup \{\phi\} & \text{if } \delta(\phi) = 1 \\ \Phi(t-1) & \text{if } \delta(\phi) = 0. \end{cases}^{13}$$

The *outcome set* of the decision process, using the collective acceptance/rejection function  $\delta$  and the decision-path  $\Omega$ , is defined to be  $M(\delta, \Omega) := \Phi(k)$ .<sup>14</sup>  $M(\delta, \Omega)$  is *decisive* if, for every  $\phi \in X$ , either  $\phi \in M(\delta, \Omega)$  or  $\neg\phi \in M(\delta, \Omega)$ .  $M(\delta, \Omega)$  is *consistent* if, for every  $\phi \in X$ , it is not the case that both  $\phi \in M(\delta, \Omega)$  and  $\neg\phi \in M(\delta, \Omega)$ . Note that, by its definition,  $M(\delta, \Omega)$  is always consistent.

### A3: A Necessary and Sufficient Condition for Path-Dependence

**Theorem 2.** Suppose the acceptance/rejection function  $\delta: X \rightarrow \{0,1\}$  is complete and consistent. For any  $\phi \in X$ , the following two statements are equivalent:

- (i)  $\delta$  is not deductively closed with respect to one of  $\phi$  or  $\neg\phi$ .
- (ii) There exist (at least) two alternative decision-paths  $\Omega_1$  and  $\Omega_2$  such that  $\phi \in M(\delta, \Omega_1)$  and  $\neg\phi \in M(\delta, \Omega_2)$ .

**Proof.** Suppose  $\delta$  is complete and consistent.

(i) implies (ii):

Suppose that  $\delta$  is not deductively closed with respect to  $\phi$  (an analogous argument can be given for  $\neg\phi$ , simply identifying  $\phi' := \neg\phi$ ). This means that  $\{\psi \in X: \delta(\psi)=1\}$  logically

<sup>12</sup> If we already have  $\phi \in \Phi(t-1)$  or  $\neg\phi \in \Phi(t-1)$ , then  $\Phi(t) = \Phi(t-1)$  under this definition.

<sup>13</sup> The reason that  $\Phi(t)$  is *not* defined to be  $\Phi(t-1) \cup \{\neg\phi\}$  if  $\delta(\phi) = 0$  is that  $\neg\phi$  will occur at a different step from  $\phi$  in the decision-path. The definition of a modus ponens process given in section 4 in the main text is, strictly speaking, a simplification, assuming that  $\phi$  and  $\neg\phi$  are considered at the same step in the decision-path. Whether or not we use the simplified definition makes no difference to the actual results.

<sup>14</sup> If the collective acceptance/rejection function  $\delta$  is the result of applying an aggregation function  $\delta$  to a given profile  $\{\delta_i\}_{i \in N}$ , then  $M(\delta, \Omega)$  is of course dependent on that profile. A more pedantic notation would express this profile-dependency by using the label  $M_{\{\delta_i\}_{i \in N}}(\delta, \Omega)$  to denote the outcome set of the decision process, using the aggregation function  $\delta$  and the decision-path  $\Omega$ , for a given profile  $\{\delta_i\}_{i \in N}$ . To simplify the notation, we again drop the subscript  $\{\delta_i\}_{i \in N}$  and simply write  $M(\delta, \Omega)$  for  $M_{\{\delta_i\}_{i \in N}}(\delta, \Omega)$ .

entails  $\phi$ , and  $\delta(\phi)=0$ . By completeness of  $\delta$ ,  $\delta(\neg\phi)=1$ . Define  $\Omega_1$  as follows: on  $\{1, 2, \dots, k-2\}$ , let  $\Omega_1$  be any bijective mapping from  $\{1, 2, \dots, k-2\}$  into  $X \setminus \{\phi, \neg\phi\}$ ; let  $\Omega_1(k-1) := \phi$ , and  $\Omega_1(k) := \neg\phi$ . Define  $\Omega_2$  as follows: let  $\Omega_2(1) = \neg\phi$ , and  $\Omega_2(2) = \phi$ ; on  $\{3, \dots, k\}$ , let  $\Omega_2$  be any bijective mapping from  $\{3, \dots, k\}$  into  $X \setminus \{\phi, \neg\phi\}$ . Then  $\Omega_1$  and  $\Omega_2$  have the properties required by (ii).

(ii) implies (i):

Suppose that there exist two alternative decision-paths  $\Omega_1$  and  $\Omega_2$  such that  $\phi \in M(\delta, \Omega_1)$  and  $\neg\phi \in M(\delta, \Omega_2)$ . We need to distinguish two cases: either  $\delta(\phi) = 0$  or  $\delta(\phi) = 1$ .

Case  $\delta(\phi) = 0$ . We show that  $\{\psi \in X : \delta(\psi)=1\}$  logically entails  $\phi$ . Consider path  $\Omega_1$ . Choose  $t_1, t_2$  such that  $\Omega_1(t_1) = \phi$  and  $\Omega_1(t_2) = \neg\phi$ . If  $\Phi(\min(t_1, t_2)-1)$  logically entails  $\phi$ , then  $\{\psi \in X : \delta(\psi)=1\}$  also logically entails  $\phi$ , since  $\Phi(\min(t_1, t_2)-1)$  contains only propositions  $\psi \in X$  for which  $\delta(\psi)=1$  and implications of such propositions. If  $\Phi(\min(t_1, t_2)-1)$  does not logically entail  $\phi$ , then, by the definition of the modus ponens decision process,  $\phi \notin \Phi(\min(t_1, t_2))$ , as  $\delta(\phi) = 0$ . However, we know that  $\phi \in M(\delta, \Omega_1)$ , and hence we must then have  $\phi \in \Phi(\max(t_1, t_2))$ , since  $\phi$  and  $\neg\phi$  occur only at  $t_1$  or  $t_2$  in the decision-path. But, as  $\delta(\phi) = 0$ , a necessary condition for  $\phi \in \Phi(\max(t_1, t_2))$  is that  $\Phi(\max(t_1, t_2)-1)$  logically entails  $\phi$ . But this means that  $\{\psi \in X : \delta(\psi)=1\}$  also logically entails  $\phi$ .

Case  $\delta(\phi) = 1$ . Then  $\delta(\neg\phi) = 0$ , by consistency of  $\delta$ . We show that  $\{\psi \in X : \delta(\psi)=1\}$  logically entails  $\neg\phi$ . Consider path  $\Omega_2$ . Choose  $t_1, t_2$  such that  $\Omega_2(t_1) = \phi$  and  $\Omega_2(t_2) = \neg\phi$ . If  $\Phi(\min(t_1, t_2)-1)$  logically entails  $\neg\phi$ , then  $\{\psi \in X : \delta(\psi)=1\}$  also logically entails  $\neg\phi$ , as before. If  $\Phi(\min(t_1, t_2)-1)$  does not logically entail  $\neg\phi$ , then  $\neg\phi \notin \Phi(\min(t_1, t_2))$ , as  $\delta(\neg\phi) = 0$ . However, we know that  $\neg\phi \in M(\delta, \Omega_2)$ , and hence we must then have  $\neg\phi \in \Phi(\max(t_1, t_2))$ , since  $\phi$  and  $\neg\phi$  occur only at  $t_1$  or  $t_2$  in the decision-path. But, as  $\delta(\neg\phi) = 0$ , a necessary condition for  $\neg\phi \in \Phi(\max(t_1, t_2))$  is that  $\Phi(\max(t_1, t_2)-1)$  logically entails  $\neg\phi$ . Again, this means that  $\{\psi \in X : \delta(\psi)=1\}$  also logically entails  $\neg\phi$ . ■

#### **A4: An Impossibility Result**

We say that  $M(\delta, \Omega)$  is *invariant under changes of  $\Omega$*  if, for any two decision-paths  $\Omega_1$  and  $\Omega_2$ ,  $M(\delta, \Omega_1) = M(\delta, \Omega_2)$ .

**Theorem 1.** There exists no aggregation function  $\delta: \{0,1\}^n \rightarrow \{0,1\}$  satisfying anonymity such that, for every  $\{\delta_i\}_{i \in N} \in U$ ,  $M(\delta, \Omega)$  is decisive and invariant under changes of  $\Omega$ .

We will use a lemma to prove theorem 1.

**Lemma 1.** If  $M(\delta, \Omega)$  is invariant under changes of  $\Omega$ , then, for every  $\phi \in X$  (where  $\phi$  is neither a tautology nor an inconsistency) (and any decision-path  $\Omega$ ),  $\phi \in M(\delta, \Omega)$  if and only if  $\delta(\phi)=1$ .

**Proof of lemma 1.** Let  $\phi \in X$  (where  $\phi$  is neither a tautology nor an inconsistency). As  $M(\delta, \Omega)$  is invariant under changes of  $\Omega$ , we may consider any path  $\Omega$ . For each  $\phi \in X$ , define a specific path  $\Omega_\phi$  as follows: let  $\Omega_\phi(1) := \phi$ , and  $\Omega_\phi(2) := \neg\phi$ , on  $\{3, \dots, k\}$ , let  $\Omega_\phi$  be any bijective mapping from  $\{3, \dots, k\}$  into  $X \setminus \{\phi, \neg\phi\}$ . Now suppose  $\delta(\phi)=1$ . Then, by the definition of a modus ponens decision process, we have  $\phi \in M(\delta, \Omega_\phi)$ . Suppose, conversely,  $\phi \in M(\delta, \Omega_\phi)$ . If  $\delta(\phi)=0$ , then, as  $\Phi(0)$  does not imply  $\phi$ ,  $\phi \notin \Phi(1)$  and also  $\phi \notin \Phi(2)$ . But since  $\phi$  or  $\neg\phi$  do not occur elsewhere in the decision-path  $\Omega_\phi$ , we have  $\phi \notin M(\delta, \Omega_\phi)$ , contrary to the assumption that  $\phi \in M(\delta, \Omega_\phi)$ . Hence  $\delta(\phi)=1$ , as required. ■

**Proof of theorem 1.** Suppose, for a contradiction, that  $\delta$  is an aggregation function which satisfies the conditions of theorem 1. Consider a set of propositions  $X$  which does not include any tautologies or contradictions. Lemma 1 implies that, for every  $\phi \in X$ ,  $\phi \in M(\delta, \Omega)$  if and only if  $\delta(\phi)=1$ . As  $M(\delta, \Omega)$  is decisive and consistent,  $\delta$  must then be complete and consistent. By theorem 3, as  $\delta$  also satisfies anonymity, there must exist at least one profile  $\{\delta_i\}_{i \in N} \in U$  such that  $\delta$  is not deductively closed with respect to some proposition  $\phi$ . But then, by proposition 2, there exist (at least) two alternative decision-paths  $\Omega_1$  and  $\Omega_2$  such that  $\phi \in M(\delta, \Omega_1)$  and  $\neg\phi \in M(\delta, \Omega_2)$ . This contradicts the assumption that  $M(\delta, \Omega)$  is invariant under changes of  $\Omega$ . ■

## A5: Unidimensional Alignment

For each  $\phi \in X$ , define  $N_{\text{accept-}\phi} := \{i \in N : \delta_i(\phi)=1\}$  and  $N_{\text{reject-}\phi} := \{i \in N : \delta_i(\phi)=0\}$ . Further, given any linear ordering  $\Omega$  on  $N$  and any  $N_1, N_2 \subseteq N$ , we write  $N_1 \Omega N_2$  as an abbreviation for [for all  $i \in N_1$  and all  $j \in N_2$ ,  $i \Omega j$ ].

A profile of individual acceptance/rejection functions  $\{\delta_i\}_{i \in N}$  satisfies *unidimensional alignment* if there exists a linear ordering  $\omega$  on  $N$  such that, for every  $\phi \in X$ , either  $N_{\text{accept-}\phi} \omega N_{\text{reject-}\phi}$  or  $N_{\text{reject-}\phi} \omega N_{\text{accept-}\phi}$ .<sup>15</sup> An ordering  $\omega$  with this property will

<sup>15</sup> Note that this definition permits  $N_{\text{accept-}\phi} = \emptyset$  or  $N_{\text{reject-}\phi} = \emptyset$ .

be called a *structuring ordering* of  $N$  for  $\{\delta_i\}_{i \in N}$ . Individual  $m \in N$  is the *median individual* with respect to  $\omega$  if  $|\{i \in N : i \omega m\}| = |\{i \in N : m \omega i\}|$ .

Compare the profile of individual acceptance/rejection functions as shown in table 3 in section 9 above. The profile satisfies unidimensional alignment. The corresponding structuring ordering ( $\omega$ ) is 4, 1, 5, 2, 3; and the median individual ( $m$ ) is individual 5.

Let  $R$  be the set of all logically possible profiles of individual acceptance/rejection functions satisfying completeness, consistency and deductive closure and unidimensional alignment.

**Theorem 4.** (List 2001) Suppose  $n$  is odd. Propositionwise majority voting is an aggregation function  $\delta : \{0,1\}^n \rightarrow \{0,1\}$  satisfying anonymity which induces, for every  $\{\delta_i\}_{i \in N} \in R$ , a complete, consistent and deductively closed collective acceptance/rejection function  $\delta : X \rightarrow \{0,1\}$ . Specifically, for each  $\{\delta_i\}_{i \in N} \in R$  with corresponding structuring ordering  $\omega$ , the function  $\delta : X \rightarrow \{0,1\}$  induced by  $\delta$  is  $\delta_m$ , where  $m$  is the median individual with respect to  $\omega$ .

**Theorem 5.** Suppose  $n$  is odd. Propositionwise majority voting is an aggregation function  $\delta : \{0,1\}^n \rightarrow \{0,1\}$  satisfying anonymity such that, for every  $\{\delta_i\}_{i \in N} \in R$ ,  $M(\delta, \Omega)$  is decisive and invariant under changes of  $\Omega$ . Specifically, for each  $\{\delta_i\}_{i \in N} \in R$  with corresponding structuring ordering  $\omega$ ,  $M(\delta, \Omega) = \{\phi \in X : \delta_m(\phi)=1\}$ , where  $m$  is the median individual with respect to  $\omega$ .

**Proof of theorem 5.** Suppose  $n$  is odd. Let  $\delta$  be propositionwise majority voting. By theorem 4,  $\delta$  induces, for every  $\{\delta_i\}_{i \in N} \in R$ , a complete, consistent and deductively closed collective acceptance/rejection function  $\delta : X \rightarrow \{0,1\}$ . Moreover,  $\delta$  satisfies anonymity, and it is easy to see that  $M(\delta, \Omega)$  is decisive. Assume, for a contradiction, that, for some  $\{\delta_i\}_{i \in N} \in R$ ,  $M(\delta, \Omega)$  is not invariant under changes of  $\Omega$ . By theorem 2, this implies that  $\delta$  is not deductively closed with respect to some  $\phi \in X$ , a contradiction. Thus  $M(\delta, \Omega)$  is invariant under changes of  $\Omega$ . By lemma 1, for every  $\phi \in X$  (where  $\phi$  is neither a tautology nor an inconsistency),  $\phi \in M(\delta, \Omega)$  if and only if  $\delta(\phi)=1$ . But, by theorem 4, for all  $\phi \in X$ ,  $\delta(\phi) = \delta_m(\phi)$ , where  $m$  is the median individual with respect to  $\omega$ . ■

### A6: A Simple Result about Strategy-Proofness

The aggregation function  $\delta$  is *weakly monotonic* if, for any  $(d_1, d_2, \dots, d_n)$ ,  $(e_1, e_2, \dots, e_n) \in \{0,1\}^n$ , [for every  $i$ ,  $d_i \geq e_i$ ] implies  $\delta(d_1, d_2, \dots, d_n) \geq \delta(e_1, e_2, \dots, e_n)$ . Propositionwise majority voting is an example of a weakly monotonic aggregation function  $\delta$ .

The set of all possible outcome sets of a modus ponens decision process,  $\mathcal{E}$ , is the set of all possible sets  $M(\delta, \Omega)$  (each satisfying  $M(\delta, \Omega) \subseteq X$ ). To each individual  $i$ , we assign a utility function  $u_i : \mathcal{E} \rightarrow \mathbf{R}$ . For each possible outcome set  $\Phi \in \mathcal{E}$ ,  $u_i(\Phi)$  represents the utility that individual  $i$  would get from the outcome set  $\Phi$ . We assume that  $u_i(\Phi)$  satisfies the following weak monotonicity assumption. We say that an outcome set  $\Phi_1 \in \mathcal{E}$  is *at least as close* to  $\delta_i$  (individual  $i$ 's acceptance/rejection function) as another outcome set  $\Phi_2 \in \mathcal{E}$  if, for every  $\phi \in X$ ,  $|\Delta_1(\phi) - \delta_i(\phi)| \leq |\Delta_2(\phi) - \delta_i(\phi)|$ , where, for each  $j \in \{1,2\}$ ,  $\Delta_j(\phi) = 1$  if  $\phi \in \Phi_j$  and  $\Delta_j(\phi) = 0$  if  $\phi \notin \Phi_j$ . Now we say that  $u_i$  is *weakly monotonic* if, whenever  $\Phi_1$  is as least as close to  $\delta_i$  as  $\Phi_2$ , then  $u_i(\Phi_1) \geq u_i(\Phi_2)$ . Informally, the weak monotonicity of  $u_i$  means that, if an outcome set  $\Phi_1$  of the decision-process is at least as close to individual  $i$ 's views as another outcome set  $\Phi_2$ , then individual  $i$  should get at least as much utility from  $\Phi_1$  as he or she gets from  $\Phi_2$ .

In the decision process  $M(\delta, \Omega)$ , individual  $i$  with utility function  $u_i$  is said to have an *incentive to express an untruthful acceptance/rejection function* at the profile  $\{\delta_i\}_{i \in N}$  if there exists an acceptance/rejection function  $\delta^*_i (\neq \delta_i)$  such that  $u_i(\Phi^*) > u_i(\Phi)$  where  $\Phi^* = M(\delta, \Omega)$  for the profile  $\{\delta_1, \dots, \delta^*_i, \dots, \delta_n\}$  and  $\Phi = M(\delta, \Omega)$  for the profile  $\{\delta_i\}_{i \in N}$ .

Given a profile of utility functions across individuals  $\{u_1, u_2, \dots, u_n\}$ ,  $M(\delta, \Omega)$  is said to be *strategy-proof* in a domain of profiles of individual acceptance/rejection functions  $D$  if there exists no individual  $i \in N$  and no profile  $\{\delta_i\}_{i \in N} \in D$  such that  $i$  has an incentive to express an untruthful acceptance/rejection function at  $\{\delta_i\}_{i \in N}$ .

**Theorem 6.** Suppose  $\delta$  is weakly monotonic, and each individual has a weakly monotonic utility function. Suppose further that, for every  $\{\delta_i\}_{i \in N}$  in some domain  $D$ ,  $M(\delta, \Omega)$  is invariant under changes of  $\Omega$ . Then  $M(\delta, \Omega)$  is strategy-proof in the domain  $D$ .

**Proof.** Suppose the assumptions of theorem 6 hold. Assume, for a contradiction, that some individual  $i$  has an incentive to express an untruthful acceptance/rejection function  $\delta^*_i (\neq \delta_i)$  at some profile  $\{\delta_i\}_{i \in N} \in D$ . Then  $u_i(\Phi^*) > u_i(\Phi)$ , where  $\Phi^* = M(\delta, \Omega)$  for the

profile  $\{\delta_1, \dots, \delta_i^*, \dots, \delta_n\}$  and  $\Phi = M(\delta, \Omega)$  for the profile  $\{\delta_i\}_{i \in N}$ . To deduce a contradiction, we show that the outcome set  $\Phi$  is at least as close to  $\delta_i$  as the outcome set  $\Phi^*$ . As  $u_i$  is weakly monotonic, this implies that  $u_i(\Phi) \geq u_i(\Phi^*)$ , contradicting  $u_i(\Phi^*) > u_i(\Phi)$ . To show that the outcome set  $\Phi$  is at least as close to  $\delta_i$  as the outcome set  $\Phi^*$ , note first that, by lemma 1, for every  $\phi \in X$  (where  $\phi$  is neither a tautology nor an inconsistency),  $\phi \in M(\delta, \Omega)$  if and only if  $\delta(\phi) = 1$ . Let  $\delta$  and  $\delta^*$  be the collective acceptance/rejection functions induced by  $\delta$  for profiles  $\{\delta_i\}_{i \in N}$  and  $\{\delta_1, \dots, \delta_i^*, \dots, \delta_n\}$ , respectively. Then the outcome set  $\Phi$  is at least as close to  $\delta_i$  as the outcome set  $\Phi^*$  if, for every  $\phi \in X$ ,  $|\delta(\phi) - \delta_i(\phi)| \leq |\delta^*(\phi) - \delta_i(\phi)|$ . Now take any  $\phi \in X$ . Let  $(d_1, d_2, \dots, d_n) = (\delta_1(\phi), \delta_2(\phi), \dots, \delta_n(\phi))$ , and let  $d_i^* = \delta_i^*(\phi)$ . We have  $\delta(\phi) = \delta(\delta_1(\phi), \dots, \delta_i(\phi), \dots, \delta_n(\phi)) = \delta(d_1, \dots, d_i, \dots, d_n)$  and  $\delta^*(\phi) = \delta(\delta_1(\phi), \dots, \delta_i^*(\phi), \dots, \delta_n(\phi)) = \delta(d_1, \dots, d_i^*, \dots, d_n)$ . As  $\delta$  is weakly monotonic, we have  $|\delta(d_1, \dots, d_i, \dots, d_n) - d_i| \leq |\delta(d_1, \dots, d_i^*, \dots, d_n) - d_i|$ , and thus  $|\delta(\phi) - \delta_i(\phi)| \leq |\delta^*(\phi) - \delta_i(\phi)|$ . Hence the outcome set  $\Phi$  is at least as close to  $\delta_i$  as the outcome set  $\Phi^*$ , as required. ■

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