A New Proposal on Special Majority Voting[1]

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Abstract. Special majority voting is usually defined in terms of the *proportion* of the electorate required for a positive decision. This note reassesses that definition, and proposes an alternative. The proposal is to define special majority voting in terms of the *absolute margin* between the majority and the minority required for a positive definition. It is shown that if we use special majority voting for "epistemic" reasons – i.e. because of a concern for reaching "correct" decisions – then the standard definition not only has the wrong focus, but may be even counterproductive, while the proposed alternative definition is "epistemically" sound. Several technical results related to the Condorcet jury theorem are adduced to back up the argument.

Many decision-making bodies employ special majority voting for important decisions. In jury decisions, for example, special majorities of at least 10 out of 12 jurors are often required for a verdict leading to conviction. To change the Basic Law of the Federal Republic of Germany, two thirds majorities in both chambers of parliament, Bundestag and Bundesrat, are required.

Under *simple majority voting*, a positive decision – for example, the accceptance of law – is reached if the decision is supported by more than ½ of the electorate. Under *special majority voting*, given the standard definition, a positive decision is reached if the decision is supported by a proportion of at least[2] q of the electorate, where q is greater than ½, often substantially greater. In the examples above, q equals 5/6 and 2/3, respectively.

In this note, I will reassesses the standard definition of special majority voting, and propose an alternative definition. Under the proposed alternative definition, a positive decision is reached if the *absolute margin* between the majority and the minority is at least[3] m, where m is a positive integer (for example, m = 100 or m = 2). Specifically, I will show that if we use special majority voting because of a concern for reaching "correct" decisions – in short, for "epistemic" reasons –, then the standard definition not only has the wrong focus, but may be even counterproductive. I will further show that the proposed alternative definition remedies these problems, by diverging from the standard characteristic of special majority voting that the support from a certain *proportion* q of the electorate is required, where q may be substantially greater than $^1/_2$.

A concern for reaching "correct" decisions should thus lead us to adopt the alternative definition of special majority voting. If we nonetheless seek to use special majority voting under the standard definition, then we cannot justify this in terms of a concern for reaching "correct" decisions. Rather, a justification would need to appeal to certain "procedural" properties of special majority voting in its standard form which are independent from a concern for "correctness". Amongst those procedural properties might be, for example, the property of giving veto power to minorities.

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[2] Alternatively: greater than.

[3] Alternatively: greater than.

Section 1 situates in the present problem in terms of the debate between "epistemic" and "procedural" conceptions of democracy. Section 2 presents the two definitions of special majority voting more formally, the standard one and the proposed alternative one, and outlines the main argument of this note in greater detail. Sections 3, 4 and 5 then complete the argument by providing the requisite technical results. Proofs are given in an appendix. The technical results are mostly a distillation of existing wisdom on the Condorcet jury theorem. Nonetheless, and surprisingly, it has apparently never been noted in the literature that these results suggests an alternative definition of special majority voting that diverges crucially from what seems universally interpreted as special majority voting.[4]

1. Epistemic and Procedural Justifications for Special Majority Voting

Broadly, there are two types of justification that might be given for the use of special majority voting (under a given definition) rather than simple majority voting in a given context. From an *epistemic perspective*, a decision is right if it is "correct" by some external standard.[5] In a jury decision, for instance, the aim is to convict the defendant if and only if the defendant is guilty, usually with the additional assumption that convicting the innocent is worse than acquitting the guilty. An epistemic justification of special majority voting would thus be an argument that special majority voting is better than simple majority voting at reaching "correct" decisions, or in short, at "tracking the truth".

From a *procedural perspective*, by contrast, the rightness of a decision is not a question of whether that decision is "correct" by some external standard. Rather, its rightness is solely constituted by the fact that the decision has emerged through a procedure that has certain procedural properties.[6] There may be different views on what those procedural properties are. The relevant properties in the case of special majority voting could be giving veto power (and thus special protection) to minorities, or securing the legitimacy of a decision by ensuring the endorsement of that decision by a large proportion of the electorate. A procedural justification, then, would be an argument that special majority voting has certain procedural properties that simple majority voting lacks.

It is tempting to seek an epistemic justification for the use of special majority voting. Intuitively, important decisions necessitate stringent "truth-tracking" requirements, which

[4] While there is a vast literature on the Condorcet jury theorem, far less work has been done on applications of the theorem to special majority voting. The technical results most closely related to this note are Nitzan and Paroush (1984) and Ben-Yashar and Nitzan (1997). Fey (2001) provides an extension of these results. All of these papers are concerned with determining the 'optimal' size of a special majority, given various specifications of a Condorcet jury framework. However, unlike the present note, they do not offer an alternative definition of special majority voting in terms of absolute margins. Feddersen and Pesendorfer (1998), Coughlan (2000), Gerardi (2000), and Guarnaschelli, McKelvey and Palfrey (2000) all discuss the Condorcet jury theorem in relation to unanimous jury verdicts. Kanazawa (1998) provides a Condorcet jury theorem for special majority voting with high individual competence. Hawthorne (2001) proves a 'convincing majorities theorem', which does not refer explicitly to special majority voting, but which can provide an alternative derivation of propositions 2 and 3 and the corresponding tables below.

[5] See, amongst others, Cohen (1986); Estlund (1993, 1997); Estlund, Waldron, Grofman and Feld (1989); Grofman and Feld (1988); List and Goodin (2001).

[6] See, amongst others, Dahl (1979); Coleman and Ferejohn (1986).

in turn may seem to necessitate the use of special majority voting. This intuition underlies, for example, the requirement of supermajoritarian or unanimous jury verdicts in capital punishment cases in the US. While the intuition is true with regard to small decision-making bodies,[7] I intend to show that if we want to justify the use of special majority voting under its standard definition in large electorates, as many people believe we should, then that justification must take a procedural rather than epistemic form, contrary to the intuition. I will also show that the proposed alternative definition of special majority voting is epistemically sound.

2. Two Definitions of Special Majority Voting

As noted above, special majority voting is usually defined in terms of the proportion q of the electorate required for a positive decision.

The "proportion" definition.
A special majority rule with parameter q (where $1/2 \leq q \leq 1$, typically $q > ½$).

> A positive decision is reached if and only if the number of individuals supporting a positive decision divided by the total number of individuals n exceeds[8] q.
> (the limiting case $q = 1/2$ is the case of a simple majority.)

This is the definition typically given in constitutions and legal documents that prescribe special majority voting. The proposed alternative definition of special majority voting focuses not on the *proportion* of the majority in the electorate, but rather on the *absolute margin* between the majority and the minority.

The "absolute margin" definition.
A special majority rule with parameter m (where $m \geq 0$, typically $m > 0$).

> A positive decision is reached if and only if the difference between the number of individuals supporting a positive decision and the number of individuals supporting a negative decision exceeds[9] m.
> (the limiting case $m = 0$ is the case of a simple majority.)

The main argument can be summarized as follows. To give an epistemic justification for the use of special majority voting (under a given definition) instead of simple majority voting, we need to show that special majority voting rather than simple majority voting is necessary to implement a given truth-tracking criterion. Estlund and Goodin[10] have pointed out that, according to a formula by Condorcet, the probability that a majority is correct, given the size of that majority, depends only on the absolute margin between the majority and the minority, but neither on the absolute size of the majority, nor on the ratio of the majority to the total electorate.

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[7] But see Feddersen and Pesendorfer (1998) for an argument on why, taking strategic voting into account, unanimous jury verdicts may be less likely to track the truth than special majority verdicts of a size less than unanimity. For a response, see Coughlan (2000).
[8] Alternatively: is at least.
[9] Alternatively: is at least.
[10] Personal correspondence.

In section 3, I will confirm this insight by providing a derivation of a more general version of Condorcet's formula from Bayes's law and the Condorcet jury theorem, showing that Condorcet's own formula is a special case of the more general formula. The insight has crucial implications for the epistemic justification of special majority voting, as I will show in subsequent sections. From an epistemic perspective, the "proportion" definition of special majority voting not only has the wrong focus, but it may also be counterproductive. The "absolute margin" definition avoids both problems.

In section 4, I will explain why the "proportion" definition has the wrong focus, while the "absolute margin" definition has the right one. For any given confidence threshold (for example, 0.9), we can ask how large, for a given size of the electorate, the values of q (in the "proportion" definition) and m (in the "absolute margin" definition) need to be to ensure that the probability that a decision is correct, given that it has received the requisite special majority support, exceeds the given confidence threshold. I will show that, while m is invariant under changes in the size of the electorate, q is not. Further, q converges to $^1/_2$ as the size of the electorate increases, which casts doubt on the epistemic justifiability of a choice of $q > {}^1/_2$ in large electorates.

In section 5, finally, I will show that special majority voting under the "proportion" definition can be even counterproductive from an epistemic perspective, while special majority voting under the "absolute margin" definition is epistemically sound. I will show that, if individuals are better than random at tracking the truth, but not especially good at it, then the probability that the correct decision will emerge under special majority voting in the "proportion" form with $q > {}^1/_2$ may converge to zero as the number of individuals increases, while the probability that a correct decision will emerge under simple majority voting converges to one. If we we use the "absolute margin" definition of special majority voting, by contrast, the probability that the correct decision will emerge converges to one as the number of individuals increases.

3. Condorcet's Insight

We consider a binary choice problem. The two options will simply be called a *positive decision* (in short, option x) and a *negative one* (in short, option *not-x*). For instance, x and *not-x* might represent acceptance and rejection of a proposition, respectively. Suppose further that there is an independent fact of the matter as to which of the two options is the correct one. Suppose there are n individuals, where (i) each individual has probability p of voting for the correct option, and (ii) the votes of different individuals are independent from each other.

According to Condorcet's formula, the probability that the majority of individuals will vote for the correct option equals $p^{h-k} / (p^{h-k}+(1-p)^{h-k})$, where h and k are the numbers of individuals in the majority and in the minority, respectively. As noted by Estlund and Goodin, the structure of the formula immediately implies that

(i) The probability that the majority is correct, given the size of the majority, depends only on the absolute margin between the majority and the minority (i.e. h-k), but neither on the absolute size of the majority (i.e. h), nor on the ratio of the majority to the total size of the electorate (i.e. h/n).

I will confirm this insight by providing a derivation of a more general version of Condorcet's formula from Bayes's law and the Condorcet jury theorem. I will also show that Condorcet's formula is a simplification, leaving out the following complication:

(ii) The probability that the majority is correct, given the size of the majority, depends also on the prior probability r that the specific option supported by that majority is the correct one.

In a jury decision, the prior probability r associated with a "guilty" verdict might be the (very small) probability that a randomly chosen member of the population is guilty of the relevant charge. Condorcet's formula does not explicitly address that dependency, tacitly assuming that the prior probability is always 1/2.

We need to introduce some further formalism. Let *x-correct* refer to the event that option x is correct. Further, let N_x and $N_{not\text{-}x}$ denote the random variables whose values are, respectively, the numbers of individuals supporting options x and *not-x* in an electorate of n individuals. For any event A, let $P(A)$ be the unconditional (or prior) probability of A. For any events A and B, let $P(A|B)$ be the conditional probability of A, given B. Further, let $r := P(x\text{-}correct)$ be the prior probability that option x is correct.

Proposition 1. *Suppose $h > n/2$. Then*

$$P(x\text{-}correct \mid N_x = h) = \frac{r\,p^m}{r\,p^m + (1\text{-}r)\,(1\text{-}p)^m},$$

where $m = 2h\text{-}n$ (in terms of Condorcet's formula, $m = h\text{-}k$ with $k = n\text{-}h$).

Here m is precisely the margin between the majority and the minority. Claims (i) and (ii) thus follow immediately from proposition 1. An implication is that a majority of 1010 against 1001 is as likely to be correct as a majority of 10 against 1, although the former is only a 50.22% majority while the latter is a 90.91% majority.

4. Determining the Special Majority Required for a Given "Truth-Tracking" Criterion

Suppose we consider a positive decision (for example, for conviction, or in favour of a new constitutional amendment) to be epistemically justified only if the probability that the decision is correct exceeds a certain fixed confidence threshold c. To capture a "beyond any reasonable doubt" criterion, c could for instance be chosen to be very close to 1.

For any prior probability r that a positive decision is correct and any individual competence level p, we can now use the formula in proposition 1 to determine the margin between the majority and the minority required for the probability that a positive decision supported by that majority is correct to be at least c.

$P(x\text{-}correct \mid N_x - N_{not\text{-}x} = m)$ is the probability that option x is correct, given that it is supported by a majority with a margin of m between the majority and the minority. By proposition 1, we know that $P(x\text{-}correct \mid N_x - N_{not\text{-}x} = m)$ depends only on m, p and r.

Proposition 2.[11] *Let c be a fixed confidence threshold* ($0 \leq c \leq 1$).
$P(x\text{-}correct \mid N_x - N_{not\text{-}x} = m) \geq (>)\ c$ *implies*

$$m \geq (>) \frac{\log\left(\dfrac{r-cr}{c-cr}\right)}{\log\left({}^{1}/_{p} - 1\right)}.$$

The following table reports some sample calculations of the values of m, for different values of p, r and c.

Table 1. Values of *m* corresponding to different values of *p*, *r* and *c*

		$r = 0.001$	$r = 0.01$	$r = 0.25$	$r = 0.4$	$r = 0.5$	$r = 0.6$	$r = 0.75$
$p = 0.51$	$c = 0.5$	173	115	28	11	0	0	0
	$c = 0.75$	201	143	55	38	28	18	0
	$c = 0.99$	288	230	143	125	115	105	88
	$c = 0.999$	346	288	201	183	173	163	146
$p = 0.55$	$c = 0.5$	35	23	6	3	0	0	0
	$c = 0.75$	40	29	11	8	6	4	0
	$c = 0.99$	58	46	29	25	23	21	18
	$c = 0.999$	69	58	40	37	35	33	29
$p = 0.6$	$c = 0.5$	18	12	3	1	0	0	0
	$c = 0.75$	20	15	6	4	3	2	0
	$c = 0.99$	29	23	15	13	12	11	9
	$c = 0.999$	35	29	20	19	18	17	15
$p = 0.75$	$c = 0.5$	7	5	1	1	0	0	0
	$c = 0.75$	8	6	2	2	1	1	0
	$c = 0.99$	11	9	6	5	5	4	4
	$c = 0.999$	13	11	8	7	7	6	6
$p = 0.9$	$c = 0.5$	4	3	1	1	0	0	0
	$c = 0.75$	4	3	1	1	1	1	0
	$c = 0.99$	6	5	3	3	3	2	2
	$c = 0.999$	7	6	4	4	4	3	3

[11] For a related result, see the 'convincing majorities theorem' and the corresponding table in Hawthorne (2001). Hawthorne also shows that the proportion of the electorate required to obtain a 'convincing majority' converges to ½ as the number of individuals increases, similar to proposition 3 below, and finally proves a Condorcet jury theorem on the likelihood of obtaining a 'convincing majority'. At the cost of more complicated mathematics, Hawthorne's framework, unlike the classical Condorcet jury framework used here, allows different competence levels for different individuals and dependencies between the choices of different individuals. Goodin (2002) addresses the question under what conditions a majority can convince someone who initially assigns a high prior probability to the truth of some proposition of the negation of that proposition.

Given the "truth-tracking" condition that a positive decision be acceptable only if the probability of its correctness is at least *c*, we can thus infer the margin between the majority and the minority required for implementing this truth-tracking condition, for different values of *p* and *r*. Further, we can infer the corresponding necessary minimal size of the electorate. For instance, if $r = 0.25$ and $p = 0.55$, an electorate of at least 29 individuals is required to implement the truth-tracking criterion given by $c = 0.99$. If there are precisely 29 individuals, unanimous support of a positive decision is required to secure the required margin of 29. If there are 600 individuals, 315 out of 600 votes for a positive decision are sufficient to secure a margin of 29 between the majority and the minority. This corresponds to a 52.5% majority.[12]

More generally, given an electorate of size *n*, a margin *m* between the majority and the minority is equivalent to a proportion $q = {}^1/_2({}^m/_n + 1)$ of the electorate. Proposition 2 thus immediately implies the following proposition.

$P(x\text{-}correct \mid N_x/n = q)$ is the probability that option *x* is correct, given that it is supported by a proportion of *q* of an electorate of size *n*.

Proposition 3.[13] *Let c be a fixed confidence threshold* ($0 \leq c \leq 1$).
$P(x\text{-}correct|N_x/n = q) \geq (>)$ *c implies*

$$q \geq (>) \ {}^1/_2 \left(\frac{log\left(\frac{r\text{-}cr}{c\text{-}cr}\right)}{n \ log({}^1/_p - 1)} + 1 \right) .$$

Note that, for any fixed values of *p*, *r* and *c*, the value of *q* tends to $^1/_2$ as the number of individuals *n* increases. To illustrate, if $n > 17300$, the value of *q* will be less than 51% for *all* values of *m* that are shown in table 1. Table 2 reports some sample calculations of *q*, for different values of *p*, *n* and *c*, but with $r = 0.001$.

[12] Whenever $m = 0$ in table 1, this means that, given the individual competence *p* and the prior probability *r*, any majority from 50% onwards (including a tie) will already be sufficient to ensure that the probability of the correctness of a positive decision exceeds *c*.

[13] Proposition 3 can be seen as a slightly more general variant of a theorem by Nitzan and Paroush (1984). Nitzan and Paroush's result concerns the special case $c = {}^1/_2$. Their result, however, focuses entirely on the definition of special majority voting in terms of proportions rather than absolute margins. See also note 11.

Table 2. Values of *q* corresponding to different values of *p*, *n* and *c*, with *r* = 0.001

	$n = 12$	$n = 50$	$n = 100$	$n = 300$	$n = 500$	$n = 1000$	$n = 10000$
$p = 0.51$ $c = 0.5$	n/a	n/a	n/a	78.9%	67.3%	58.7%	50.9%
$c = 0.75$	n/a	n/a	n/a	83.5%	70.1%	60.1%	51.1%
$c = 0.99$	n/a	n/a	n/a	98%	78.8%	64.4%	51.5%
$c = 0.999$	n/a	n/a	n/a	n/a	84.6%	67.3%	51.8%
$p = 0.55$ $c = 0.5$	n/a	85%	67.5%	55.9%	53.5%	51.75%	50.2%
$c = 0.75$	n/a	90%	70%	56.7%	54%	52%	50.2%
$c = 0.99$	n/a	n/a	79%	59.7%	55.8%	52.9%	50.3%
$c = 0.999$	n/a	n/a	84.5%	61.5%	56.9%	53.5%	50.4%
$p = 0.6$ $c = 0.5$	n/a	68%	59%	53%	51.8%	50.9%	50.1%
$c = 0.75$	n/a	70%	60%	53.4%	52%	51%	50.1%
$c = 0.99$	n/a	79%	64.5%	54.9%	52.9%	51.5%	50.2%
$c = 0.999$	n/a	85%	67.5%	55.9%	53.5%	51.8%	50.2%
$p = 0.75$ $c = 0.5$	79.2%	57%	53.5%	51.2%	50.7%	50.4%	50.1%
$c = 0.75$	83.4%	58%	54%	51.4%	50.8%	50.4%	50.1%
$c = 0.99$	95.9%	61%	55.5%	51.9%	51.1%	50.6%	50.1%
$c = 0.999$	n/a	63%	56.5%	52.2%	51.3%	50.7%	50.1%
$p = 0.9$ $c = 0.5$	66.7%	54%	52%	50.7%	50.4%	50.2%	50.1%
$c = 0.75$	66.7%	54%	52%	50.7%	50.4%	50.2%	50.1%
$c = 0.99$	75%	56%	53%	51%	50.6%	50.3%	50.1%
$c = 0.999$	79.2%	57%	53.5%	51.2%	50.7%	50.4%	50.1%

We have seen that, under the "proportion" definition of special majority voting, for any fixed truth-tracking criterion, the value of q required for implementing that criterion depends on the size of the electorate and converges to $^1/_2$ as the size of the electorate increases. In a sufficiently large electorate, q thus approximates $^1/_2$. The epistemic justifiability of special majority voting under the "proportion" definition, particularly with q significantly larger than $^1/_2$, is therefore questionable.

What matters from an epistemic perspective is not the proportion of the electorate supporting a positive decision, but rather the absolute margin between the majority and the minority.[14] The results of this section thus show that, from an epistemic perspective, the "absolute margin" definition of special majority voting is more robust than the "proportion" definition. Regardless of how large the electorate is and regardless of how large the majority is in proportional terms, the margin between the majority and the minority required for implementing a given truth-tracking criterion remains the same.

5. A Condorcet Jury Theorem for Special Majority Voting

We will now see that special majority under the "proportion" definition voting may be even counterproductive from the perspective of "truth-tracking", while special majority voting under the "absolute margin" definition is conducive to "truth-tracking". So far we have been concerned with the probability that option x is correct, given that it is supported by an appropriate special majority. In this section we reverse the order of

[14] If the Florida presidential election debacle in 2000 were interpreted in epistemic terms (which is of course a problematic interpretation in that context), then the appropriate critique would be not that the election was too close in proportional terms, but rather that it was too close in terms of the absolute margin between the majority and the minority.

conditionality, and consider the probability that option x will be supported by an appropriate special majority, given that it is correct. Even if the probability that option x is correct, given that it has been selected under special majority voting, is high, the probability that it will be selected under special majority voting in the first place may be low, even when x is correct. The following proposition highlights precisely this possibility.

$P(N_x/n \geq q \mid x\text{-}correct)$ is the probability that option x will be supported by a proportion of at least q of an electorate of size n, given that x is correct.

Proposition 4.[15]

- *If* $^1/_2 < p < q$, *then* $P(N_x/n \geq q \mid x\text{-}correct)$ *converges to* 0 *as n increases.*
- *If* $p > q$, *then* $P(N_x/n \geq q \mid x\text{-}correct)$ *converges to* 1 *as n increases.*

Under the "proportion" definition of special majority voting, in a large electorate, the probability that the correct option will be selected may thus be very low, unless the competence of individuals is very high (i.e. unless $p > q$). In particular, the probability that the correct option will be selected may even converge to 0 as the number of individuals increases, even when option x is correct and each individual is better than random at tracking the truth (i.e. when $^1/_2 < p < q$).

In defence of special majority voting under the "proportion" definition, it might be argued that special majority voting is an effective method of avoiding 'false positives', i.e. decisions in favour of option x when option x is not correct. Given that convicting the innocent is worse than acquitting the guilty, as noted above, this might seem to be an attractive property of special majority voting under the "proportion" definition. However, proposition 4 shows that the avoidance of 'false positives' may come at the expense of almost never obtaining 'true positives' either.

The "absolute margin" definition, by contrast, creates none of these problems. The results of section 4 already imply that the probability of 'false positives' under the "absolute margin" definition of special majority majority can be made as small as we like, simply by choosing a sufficiently large parameter m. Under the "absolute margin" definition of special majority voting, however, the avoidance of 'false positives' does *not* come at the expense of almost never obtaining 'true positives'. Under the "absolute margin" definition of special majority voting, option x, if correct, is very likely to obtain the required special majority support in a large electorate, so long as individual competence exceeds ½ -- *regardless of how large the parameter m is*. This is established by proposition 5, a Condorcet jury theorem for special majority voting under the "absolute margin" definition.[16]

$P(N_x - N_{not\text{-}x} \geq m \mid x\text{-}correct)$ is the probability that x will be supported by a majority with a margin of at least m between the majority and the minority, given that x is correct.

[15] For result related to the second part of this proposition, see Kanazawa (1998).
[16] Proposition 5 also implies the Condorcet jury theorem for simple majority voting.

Proposition 5. *For any* $m > 0$, *if* $p > {}^1/_2$, *then* $P(N_x - N_{not\text{-}x} \geq m \mid x\text{-}correct)$ *converges to* 1 *as n increases*.

The results of this section support the claim that the "absolute margin" definition of special majority voting is conducive to tracking the truth, as soon as each individual is better than random at tracking the truth, while the "proportion" definition may be counterproductive, unless individual competence is very high.

6. Conclusion

I have argued that, if we care about special majorities *because and only because* we care about truth-tracking, then the "absolute margin" definition is the appropriate definition of special majority voting.

Adopting the "absolute margin" definition would require substantial modifications of those constitutions and legal documents that prescribe the use of special majority voting. It would no longer be appropriate to say, for instance, that support from two thirds of the members of the German Bundestag is required to pass a certain law, independently of the actual number of members (which may vary from election to election). Instead, the relevant special majority criterion would have to be spelled out in terms of a required margin of, say, 222 votes between the majority and the minority. Were the size of the Bundestag to be increased or reduced, the "absolute margin" criterion would nonetheless have to remain the same. Similarly, for referenda in large electorates, a special majority criterion would have to be stated in terms of the required margin between the majority and the minority, independently of the total population size and the size of the majority in proportional terms.

If we want to resist this conclusion, and to defend the use of special majority voting in a large electorate under the more familiar "proportion" definition, possibly with a parameter q significantly greater than ${}^1/_2$, then we can immediately infer that our justification cannot be an epistemic one. If we justify the use of special majority voting by an appeal to minority protection or by an appeal to legitimacy considerations, then this is a clear instance of a procedural justification.

If, on the other hand, our use of special majority voting under the "proportion" definition is driven by an intuition about "truth-tracking", then the present argument challenges us to question that intuition, and either to search for a procedural alternative to our justification for special majority voting under that definition, or to switch to the "absolute margin" definition.

Appendix

Proof of Proposition 1. Suppose $h > n/2$. By Bayes's law,

$P(x\text{-}correct \mid N_x = h) = P(x\text{-}correct)\, P(N_x = h \mid x\text{-}correct) / P(n_x = h)$.

Now, as N_x is binomially distributed,

$P(N_x = h \mid x\text{-}correct) = \binom{n}{h} p^h (1\text{-}p)^{n\text{-}h}$.

Further, by elementary probability theory,

$P(N_x = h) = P(N_x = h \mid x\text{-}correct)P(x\text{-}correct) + P(N_x = h \mid not\text{-}x\text{-}correct)(1\text{-}P(x\text{-}correct))$.

Also, as $N_{not\text{-}x}$ is binomially distributed,

$P(N_x = h \mid not\text{-}x\text{-}correct) = \binom{n}{h} (1\text{-}p)^h p^{n\text{-}h}$.

Hence

$P(N_x = h) = \binom{n}{h} p^h (1\text{-}p)^{n\text{-}h} P(x\text{-}correct) + \binom{n}{h} (1\text{-}p)^h p^{n\text{-}h} (1\text{-}P(x\text{-}correct))$.

Therefore

$$P(x\text{-}correct \mid N_x = h) = \frac{P(x\text{-}correct) \binom{n}{h} p^h (1\text{-}p)^{n\text{-}h}}{\binom{n}{h} p^h (1\text{-}p)^{n\text{-}h} P(x\text{-}correct) + \binom{n}{h} (1\text{-}p)^h p^{n\text{-}h} (1\text{-}P(x\text{-}correct))}$$

$$= \frac{P(x\text{-}correct)\, p^{2h\text{-}n}}{P(x\text{-}correct)\, p^{2h\text{-}n} + (1\text{-}P(x\text{-}correct))\,(1\text{-}p)^{2h\text{-}n}}$$

$$= \frac{r\, p^{h\text{-}k}}{r\, p^{h\text{-}k} + (1\text{-}r)\,(1\text{-}p)^{h\text{-}k}},$$

where $r = P(x\text{-}correct)$ (the prior probability that option x is correct)
$k = n\text{-}h$ (number of individuals in the minority).

Q.E.D.

Proof of Proposition 2. By proposition 1, we know that

$$P(x\text{-}correct \mid N_x - N_{not\text{-}x} = m) = \frac{r\,p^m}{r\,p^m + (1\text{-}r)\,(1\text{-}p)^m}.$$

Hence $P(x\text{-}correct \mid N_x - N_{not\text{-}x} = m) \geq (>)\ c$ implies

$$\frac{r\,p^m}{r\,p^m + (1\text{-}r)\,(1\text{-}p)^m} \geq (>)\ c,$$

which implies

$$\frac{r\text{-}cr}{c\text{-}cr} \geq (>)\ ({}^1/_p - 1)^m,$$

which in turn implies

$$m \geq (>)\ \frac{log\left(\frac{r\text{-}cr}{c\text{-}cr}\right)}{log({}^1/_p - 1)},$$

as required. **Q.E.D.**

A condition ϕ on the probability p is *consistent* if there exists a value of p satisfying ϕ. A condition ϕ on p is *strict* if, for every value of p satisfying ϕ, there exists an $\varepsilon > 0$ such that, whenever $|p^*\text{-}p|<\varepsilon$, then p^* also satisfies ϕ. An example of a consistent strict condition on p is $p > {}^1/_2$. The condition $p = {}^1/_2$ is consistent, but not strict. The condition $p < 0$ is not consistent.

Lemma 1 (Convergence Lemma).[17] *Suppose p satisfies the consistent strict condition ϕ. Then $P(N_x/n$ satisfies $\phi \mid x\text{-}correct)$ converges to 1 as n tends to infinity.*

Prooof of Proposition 4. Let $q > {}^1/_2$.

- Suppose p satisfies ${}^1/_2 < p < q$. As this condition is consistent and strict, lemma 1 implies that $P({}^1/_2 < N_x/n < q \mid x\text{-}correct)$ converges to 1 as n increases. The result follows.
- Suppose p satisfies $p > q$. As this condition is consistent and strict, lemma 1 implies that $P(N_x/n > q \mid x\text{-}correct)$ converges to 1 as n increases. The result follows.

Q.E.D.

Proof of Proposition 5. Suppose that $p > {}^1/_2$. Let $m > 0$. Then $2p > 1$, and $2p - 1 > 0$. In particular, there exists $\varepsilon > 0$ such that $2p - 1 > \varepsilon$. As this condition is consistent and strict,

[17] This lemma can be derived using the central limit theorem. For a proof, see List (2001).

lemma 1 implies that $P(2N_x - n > n\varepsilon \mid x\text{-}correct)$ ($= P(2N_x/n - 1 > \varepsilon \mid x\text{-}correct)$) converges to 1 as n increases. But, when $n > m/\varepsilon$, we have $n\varepsilon > m$. Then $P(2N_x - n > m \mid x\text{-}correct)$ converges to 1 as n increases. But $2N_x - n = N_x-(n-N_x)$ is precisely the margin between the majority and the minority. The result follows. **Q.E.D.**

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