The Reasonableness of Independence:

A Conversation from Condorcet and Borda to Arrow and Saari

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Abstract

A conversation has been going on for over 200 years that is not concluded (nor will this paper conclude it). Is it normatively appropriate to impose a condition of independence of irrelevant alternatives on choice or ordering rules? This paper reviews the conversation since it began in 1784. It sides with Arrow against Saari. The transparency of the Borda rule is simultaneously its theoretical beauty and its practical horror. It is just Borda's violation of IIA that makes it a non-starter for real-world use.

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A conversation has been going on for over 200 years that is not concluded (nor will this paper conclude it). Is it normatively appropriate to impose a condition of independence of irrelevant alternatives on choice or ordering rules?

The conversation began in 1784, with the publication of Borda's *On Elections by Ballot (Mémoire sur les elections au scrutiny*) in the *Mémoires de l'académie Royale des Sciences année 1781.* The *Histoire et Mémoires de l'académie Royale des Sciences* was the leading scientific journal in the world. In 1784 it was running three years in arrears, a feat which puts the current record of, say, the *APSR* into perspective. The layout of the journal was that copied and still followed 200 years later by a number of the top scientific journals today, such as *Nature* and *The Lancet*. The opening section, the *Histoire,* contains the leading articles. These draw the reader's attention to the most important scientific discoveries in the journal, and are written by the editor. The following section, the *Mémoires*, contain the papers themselves.

The editor of the *Histoire et Mémoires* in 1784 was the Perpetual Secretary of the Academy of Sciences, the Marquis de Condorcet. Condorcet was quite self-consciously the gatekeeper of French and therefore of world science (see especially Baker 1975 *passim* esp. pp. 35-55). France was, as the USA is now, the locus of the best academies and the best journals. In the French model, followed in much of Europe and the former Soviet Union, scientific research concentrated not in the universities but in the national academies of science. The royal Academy in Paris was the senior academy, whose secretary exchanged papers and scientists with the daughter academies in St Petersburg, Berlin, Bologna, and Philadelphia (viz., his friend Benjamin Franklin's American Philosophical Society). Thus did Bernoulli and Diderot go to St Petersburg, rather as visiting Americans now go to less enlightened places such as Paris and Oxford.

As gatekeeper, Condorcet could control what counted as science and what did not. Normally, he would not have counted the publications of his enemy Jean-Charles de Borda as science. Borda was a fellow-member of the Academy, but he was there at the wish of the king. Condorcet was there by a combination of eminence and friends. He was no friend of the king, and was to become a bitter enemy during the Revolution which was to cost his life. Borda, on the other hand, was able to move seamlessly from serving the king to serving the one and indivisible French Republic. Very few personal records relating to him have been traced, but one of the few is a payslip in the records of the Ministry of the Marine at Vincennes in the name of the one and indivisible republic. Borda was primarily an engineer, whose service to the monarchy and then the republic was mostly in the form of harbour construction. Condorcet described him as

> what they call 'a good Academician', because he talks in Academy meetings and likes nothing better than to waste his time drawing up prospectuses, examining machines, etc; and especially because, realizing he was eclipsed by other mathematicians, he abandoned mathematics for petty experiments (*la physicaille*). . . . Some of his papers display some talent, although nothing follows from them and nobody has ever spoken of them or ever will" (letter to Turgot 1775, in Henry 1883, 214---5; our translation).

Borda's paper was delivered orally in 1770. It proposed the use of the rank-order method to elect new members of the Academy. It was neither adopted nor printed. Therefore its revival fourteen years later - when Borda may not have been in Paris at all because he was a serving officer of the French navy - was Condorcet's deliberate decision. Condorcet was then finishing his massive *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix* (Condorcet 1785). In the text of the *Essai*, he states

> A famous mathematician (*un Géomètre célebre*) pointed out the drawbacks of the conventional election method before we did, and suggested a system whereby each voter ranks the candidates in order… The elected candidate is the one whose total score, of all the values attributed by all the voters, is the highest… Although the famous mathematician who

> suggested this method has not published anything on the subject, I felt I should mention him here.

In a footnote Condorcet adds 'When this essay was printed, I knew about this method only because various people had mentioned it to me. It has since been published in the *Mémoires de l'Académie 1781*.' (Condorcet 1785 p. clxxix. Translations from McLean and Hewitt 1994 pp. 136-8). This note is thoroughly disingenuous - enough to throw even Black (1958) off the scent. Condorcet published Borda's paper in order to refute it. He had come to realise that the Borda method posed a challenge to his own way of thinking. It would take him four years before he resolved the challenge to his own satisfaction, by making the first clear statement of an axiom of independence of irrelevant alternatives (hereafter IIA).

In his leader introducing Borda's paper, Condorcet observes that the Borda method may be characterised as a method of pairwise comparison:

> Having shown the failings of the conventional method, M. de Borda now goes on to propose a solution. First, he asks each voter to list the candidates in order of merit, or else pronounce on the merit of the candidates, taken two by two. Clearly, once the list of candidates in order of merit has been submitted, we can extract from it each voter's judgment on the relative merits of any two candidates. (Condorcet 1784, translated by McLean and Urken 1995 pp. 81-2).

This is to highlight that Borda's method, although presented as a scoring method, is in fact a method of exhaustive pairwise comparison. Borda, as Black (1958, p. 157), points out, uses the Condorcet criterion to reject plurality rule, by showing that it can fail to select the Condorcet winner.

Condorcet's editorial continues by showing that the Borda scoring method is a strict ordinal method whose results are invariant for any value of the score awarded for last place and any positive value of the score for each interval above last place. He highlights Borda's necessity proof that for a plurality winner to be the certain Condorcet winner, the plurality winner must get more than $(m - 1)/m$ of the vote, where m is the number of voters. The manuscript of Condorcet's editorial concludes

mais elle parassait si éloignée des idées communes que…. [but it <the Borda scheme> seemed so remote from ordinary ideas that…. Institut de France, MS Condorcet 848. Our translation]

but he then deletes this phrase, which does not appear in the published version.

Thus Condorcet's decision to publish the Borda scheme is opaque. By 1785 he had not yet thought his way to a clear refutation of Borda. In his classic discussion of Condorcet's *Essai*, Duncan Black (1958, pp. 163-171) shows that some of the confusion in Condorcet's exposition results from the failure of his probabilistic framework to accommodate the really quite different social choice reasoning that we now associate with the axiomatic theory of voting. Most of the *Essai* is devoted to what we now call the Condorcet jury theorem, viz., the demonstration that the reliability of a majority decision is an increasing function of the individual juror's reliability and the size of the majority. In Condorcet's notation, the reliability of a juror is denoted by v [for *vérité*, truth. $1 - v \equiv e$, for *erreur*]. The number of votes cast in the majority is h and the number in the minority is k. The probability that the majority decision is correct is then, by Bernoulli's theorem,

$$\frac{v^{h-k}}{v^{h-k} + e^{h-k}}$$

The jury result is well-behaved for binary choice, but Condorcet soon discovers that he is in trouble when there are three or more options. As Black (1958, pp. 169-70) beautifully demonstrated, Condorcet came to realise with horror that for low values of v, the jury theorem points not to the Condorcet method but to the Borda method. The practical range of v is from 0.5 to 1. (For $v < 0.5$, jury aggregation simply implies doing the opposite to the aggregate of the jurors' verdict). For $v = 0.5 + \varepsilon$, the jury theorem implies the Borda rule, whose outcome will be more reliable the larger the majority. But this is exactly a description of a mass election - a large number of 'jurors', each with a relatively low probability of being correct (accepting the Condorcet probabilistic framework for sake of argument).

The signs of Condorcet's embarrassment are indirect. He introduces an example where the Borda winner differs from the Condorcet winner (Table 1).

Table 1.

Difference between the Borda and the Condorcet ordering:

Condorcet's earliest example (1785, p. lxv)

	n of voters in group				
	13	10	13	6	18
Top	*A*	*A*	*B*	*B*	*C*
Middle	*C*	*B*	*C*	*A*	*B*
Bottom	*B*	*C*	*A*	*C*	*A*

It is helpful to construct a Dodgson matrix (cf Dodgson 1876) showing the pairwise outcome among these three candidates, and deriving the Borda and Copeland rankings among them. (The Copeland ranking ranks candidates by the number of the others that they beat in pairwise comparison. A candidate with a unique top Copeland score is therefore the Condorcet winner). Table 2 is the Dodgson matrix for Table 1.

Table 2.

Difference between the Borda and the Condorcet ordering:

Dodgson matrix for Condorcet's earliest example (1785, p. lxv)

		Votes against:				
					Borda	Copeland
		A	B	C	score	score
	A		23	29	52	0
Votes for:	B	37		29	66	1
	C	31	31		62	2

Recall that Condorcet is trying to fit this result into his probabilistic (jury theoretic) framework. By the reasoning above, candidate (or option) B should win. Condorcet therefore drops his prior reasoning, and produces the following justification for a choice of the Condorcet winner C:

> Candidate A clearly does not have the preference, because there is a plurality of vote against him whether he is compared to B or to C (and this is always the case in such situations). The choice is therefore between B and C. As the proposition 'B is better than C' has only minority support, we must conclude that it is C who has plurality support…..
>
> In any decision-making situation where we follow, not the most probable system [viz. Borda - IM), but the result of the two systems which favoured the same candidate (who would therefore be the most probable [i.e., the Condorcet winner - IM]), it seems that the results dictated by the calculus of probabilities contradict simple reason. (Condorcet 1785, pp. lxiv-lxv. Translation from McLean and Hewitt 1994, p. 126).

This is the earliest statement of IIA. Not till 1788, in a (slightly) more popular work called *On the Constitution and the Functions of Provincial Assemblies* (Condorcet 1788) did Condorcet return with a more reasoned justification of IIA. However, the obscurity of the source meant that its significance was

not noticed until 1994. *Provincial Assemblies* is a huge work elaborated with mind-numbing attention to detail. It offers a Condorcet scheme for aggregation from popular judgments, through the provincial assemblies that the king was proposing to set up, to a national judgment. However, before it was even published, the king had dropped the idea of provincial assemblies and decided to call the Estates-General instead, setting the French Revolution in train by doing so.

Condorcet again introduces an example where the Condorcet and Borda winners differ, but with much more confidence than in 1785.

> 3) But this method will not always give us the same result. Suppose that out of eighty-one voters, thirty rank the candidates Peter, Paul, Jack; and one ranks them Peter, Jack, Paul. Twenty-nine rank them Paul, Peter, Jack; and ten rank them Paul, Jack, Peter; ten rank them Jack, Peter, Paul and one ranks them Jack, Paul, Peter.
>
> Peter thus has thirty-one first places, thirty-nine second places, eleven third places and a score of 182. Paul, with thirty-nine first places, thirty-one second places, and eleven third places, scores 190. And Jack, with eleven first places, eleven second places and fifty-nine third places, scores 114. Paul thus beats Peter and Jack, and Peter beats Jack. But if we examine the voting more closely, we find that Peter beats Paul by forty-one votes to forty and Jack by sixty votes to twenty-one, while Paul beats Jack by sixty-nine votes to twelve.
>
> The plurality therefore supports Peter, not Paul, and the above method is erroneous, as is the ordinary one…. (Condorcet 1788, appendix 1. Translation from McLean and Urken 1995, pp. 124-5).

Tables 3 and 4 give the preference profile and Dodgson matrix for this example. The Borda scores in the Dodgson matrix (109, 101, 33) differ from those in Condorcet's text only because in the matrix a last place is scored at 0 and in the text a last place is scored at 1. In both sources the interval between places is scored at 1 and so of course the Borda outcome is invariant.

Table 3

Difference between the Borda and the Condorcet ordering:

Condorcet's later example (1788)

	n of voters in group					
	30	1	29	10	10	1
Top	*A*	*A*	*B*	*B*	*C*	*C*
Middle	*B*	*C*	*A*	*C*	*A*	*B*
Bottom	*C*	*B*	*C*	*A*	*B*	*A*

Table 4.

Difference between the Borda and the Condorcet ordering:

Dodgson matrix for Condorcet's later example (1788)

	Votes against:				
	A ('Peter')	B ('Paul')	C ('James')	Borda score	Copeland score
A		41	60	101	2
Votes for: B	40		69	109	1
C	21	12		33	0

Condorcet continues:

> However, there are some situations in which, whatever revisions we introduce, this method will always give the wrong result. Example 3 is one of them. In fact, the problem here is nothing to do with assigning values; as both Peter and Paul are ranked last eleven times, altering the values could make no difference. Since Peter has thirty-one first places and Paul thirty-nine, Peter must necessarily score eight times (whatever value we give first place) less

than Paul. And since Peter has thirty-nine second places and Paul has only thirty-one, Peter must score eight times (whatever value we give the second place) more than Paul. To give an accurate expression of the plurality will, Peter's total would have to be greater than Paul's, and we would therefore need to assign values such that eight times (the value of second place) was greater than eight times (the value of first place); we would need, that is, to give second place a higher value than first place, which is not only illogical, but also totally contradicts the basic principles of the method.

But how is it that Paul is not the clear winner when the only difference between himself and Peter is that Peter got thirty-one first places and thirty-nine second, while Paul got thirty-nine first and thirty-one second? Well, out of the thirty-nine voters who put Peter second, ten preferred him to Paul, whereas only one of the thirty-one voters who put Paul second preferred him to Peter. The points method confuses votes comparing Peter and Paul with those comparing either Peter or Paul to Jack and uses them to judge the relative merits of Peter and Paul. ***As long as it relies on irrelevant factors to form its judgments, it is bound to lead to error, and that is the real reason why this method is defective for a great many voting patterns, regardless of the particular values assigned to each place***. The conventional method is flawed because it ignores elements which should be taken into account and the new one because it takes into account elements which should be ignored.

Thus, the only method remaining to be examined is that by which we have been judging the others. Each voter ranks the candidates in order of merit. From his ranking, we can easily extract his opinion of the relative merits of each candidate, and by collating all the individual opinions we can discover the candidate considered best by the plurality. That is, we need only do precisely what we have been doing in our examples in order to find the plurality will. (Condorcet 1788, Appendix 1. Translation from McLean and Urken 1995 p. 126. Our emphasis).

Condorcet continues to admit that the Condorcet procedure fails to satisfy universal domain, because it does not give a clear result in the event of a top cycle. His cycle-breaking proposal is clearer than it was in 1785, but still very obscure. However, exactly two centuries later Young (1988) gave what I believe is the correct characterisation of the general Condorcet ranking method, although this still leaves

unclear whether the Condorcet choice method ought to be read as ‘Select the most probable candidate’ or ‘Select the Condorcet winner if one exists’.

In the next generation only one scholar understood Condorcet’s IIA argument. P.C.F. Daunou (1761-1840) was a historian and literary critic who had both political and intellectual associations with Condorcet in the early 1790s. Like Condorcet, Daunou was expelled from the National Convention after the Jacobin coup in 1793 and imprisoned for some months. However, unlike Condorcet he survived and gained prominent political offices after the fall of the Jacobins on the 10 Thermidor an III (July 1794). He was responsible for having Condorcet's *Esquisse* ('Outline for a history of the progress of the human mind'), which had been written in hiding before Condorcet’s death in the Terror, published at public expense in 1795. He was also a prime mover in the refoundation of the Academy of Science and its sister academies as the Institut de France, and he was on the committee which drafted its constitution. This constitution instituted the Borda rule for the election of academicians in 1796. However, the Borda rule was abandoned following what is said to have been Napoleon's only intervention in the affairs of the Institut. One reason for abandoning it was its susceptibility to manipulation ('My election method is only for honest men', said Borda when this was pointed out). Daunou's paper (Daunou 1803; McLean and Urken 1995 pp. 237-76)) was written when a consensus was growing for replacing the Borda method by one in which candidates for a vacancy in one of the academies must obtain an absolute majority of the votes, and the place remain unfilled if no candidate did so. In spite of Daunou's opposition, this system did indeed supplant the Borda count in 1804.

Daunou's review is very sophisticated. Broadly speaking, he sides with Condorcet and against Borda, although he was probably the author of the Borda scheme in the constitution of the Institut. His attack on the Borda count opens by showing that a voting system cannot measure the degrees of intensity of preference sincerely held by the voters. Therefore, rank orderings, as in the Borda count, cannot be held to be measures of intensity. The thrust of this is towards reasserting Independence of Irrelevant Alternatives. Borda (and Laplace - for whom see Black 1958 pp. 181-2) wish to claim that if, say, A is 6 places ahead of B on one ballot paper and B is one place ahead of A on another, that is some evidence that A is socially preferred to B. Condorcet, Daunou, and all those who believe that a choice system should respect Independence, deny the claim.

Daunou goes on to analyse existing and proposed procedures. He gives plurality and runoff methods very short shrift: 'I do not consider it necessary to prove' their defects, but discuss them 'only because we constantly resort' to them. He rejects Condorcet's supplementary list scheme on the same grounds as had S. F. Lhuilier (for whom see McLean and Urken 1995, pp. 151-95), though without reference to him. He points out that qualified-majority schemes can be minority-veto schemes. He confirms that the Borda count had been 'abused' in the Institute by voters' 'deliberately ranking [their favourite's] most dangerous opponents last'. Like Condorcet, he points out that the Borda count violates (Nash) Independence and expansion-consistency (Sen's (1982 pp. 171-2) condition γ):

> But how can the intervention of another candidate alter or reverse the relationship established by the voters between ... two candidates? .... [as if to] say "if the choice is between just A and B, then we categorically prefer A, but if it is between A, B, and also C, then we consider that B beats not only C, but A as well".

Discussing Borda's proof that only a unanimity rule would guarantee that a majority winner was also the Borda winner, Daunou argues that this should have caused Borda to re-evaluate a method which had such a perverse implication: 'we must judge the method by the maxim, and not the maxim by the method'. He continues boldly by attacking 'Citizen Laplace ... this wise teacher', who had justified the Borda count from a more explicit axiom of equiprobability than Borda's (cf Black 1958, pp 181-2). Laplace's justification produces a geometrical, not arithmetical, progression of numbers, and there can be no reason to regard this progression as a measure of the unmeasurable 'true' intervals between candidates in voters' minds. (For more on Daunou and his contemporaries see McLean 1995a, b).

After Daunou, the IIA criterion disappears for over a century. Neither of the only two people who understood Condorcet in the later 19th century, viz., Charles Lutwidge Dodgson and E. J. Nanson) addresses IIA. Although Dodgson switched from a Borda scheme to a Condorcet scheme between his first pamphlet of 1873 and his final pamphlet of 1876, he does not give the reasons for doing so. He seems to have discovered the conflict between the criteria some time after writing his first pamphlet - for the evidence see McLean and Urken 1995, p. 283 fn 3).

The first 20th-century discussion of IIA was by Huntington (1938). I introduce it by means of the following crux, and much disputed, extract from Arrow (1951, pp. 26-7):

> [T]he choice made from any fixed environment *S* should be independent of the very existence of alternatives outside of *S*. For example, suppose ... an election system ... whereby each individual lists all the candidates in order of his preference and then, by a preassigned procedure, the winning candidate is derived from these lists. (All actual election procedures are of this type, although in most the entire list is not required for the choice). Suppose that an election is held, with a certain number of candidates in the field, ... and then one of the candidates dies. Surely the social choice should be made by taking each of the individual's [sic] preference lists, blotting out completely the dead candidate's name, and considering only the orderings of the remaining names in going through the procedure of determining the winner. That is, the choice to be made among the set *S* of surviving candidates should be independent of the preferences of individuals for candidates not in *S*. To assume otherwise would be to make the result of the election dependent on the obviously accidental circumstance of whether a candidate died before or after the date of polling. Therefore, we may require of our social welfare function that the choice made by society from a given environment depend only on the orderings of individuals among alternatives in that environment....
>
> CONDITION 3: *Let R_1, ..., R_n and R_1', ..., R_n' be two sets of individual orderings and let C(S) and C'(S) be the corresponding social choice functions. If, for all individuals i and all x and y in a given environment S, x R_iy if and only if x R_i'y, then C(S) and C'(S) are the same*
> (independence of irrelevant alternatives).
>
> The reasonableness of this condition can be seen by consideration of the possible results in a method of choice which does not satisfy Condition 3, the rank-order method of voting used in clubs.... [An example follows in which the withdrawal of a candidate alters the Borda scores and ranks of those left in contention.]

> A similar problem arises in ranking teams in a contest which is essentially individual, e.g., a foot race in which there are several runners from each college, and where it is desired to rank the institutions on the basis of the rankings of the individual runners. This problem has been studied by Professor E.V. Huntington, who showed ... that the usual method of team scoring in those circumstances, a method analogous to the rank-order method of voting, was inconsistent with a condition analogous to Condition 3, which Huntington termed the postulate of relevancy.

Huntington's (1938) rediscovery of paradoxes that arise when procedures fail to respect IIA was unprecedented. There are no citations in Huntington's paper and thus no direct clue to the sources of his thinking. It is probably significant, however, that by 1938 Huntington had been involved for many years in the mathematics of apportionment (for which see Balinski and Young 1982). In his efforts to define criteria of fairness among units in the apportionment problem, Huntington had been drawn to a principle of exhaustive pairwise comparison. A fair system of apportionment was one that minimized the inequality among all pairs of states (Huntington 1928). Thus it is likely that Huntington's thought about choice procedures was cast in a binary mould, and that therefore he, alone of the post-Daunou and pre-Arrow commentators on choice rules, saw the problem of IIA. The eighteenth-century Frenchmen wrote about the election of members of scientific academies; the twentieth-century American wrote about inequities in judging inter-collegiate sporting competitions. This may say something about the differences in their surrounding cultures, but should not detract from Huntington's originality.

As is well known, Arrow' exposition confounds two conditions, both of which have been labelled independence of irrelevant alternatives. Radner and Marschak (1954, p.63), discussing Savage's (1947) proposal of minimax regret as a criterion in decision theory, wrote:

> ... there are cases in which, if the domain S of the player's available strategies is enlarged, a new minimax regret solution is obtained which differs from the old one, yet is contained in the original S.... It is interesting to note that the idea behind ... [this] objection has analogues in

> Nash's treatment of the bargaining problem ... and Arrow's discussion of social welfare functions.... Borrowing Arrow's terminology we shall say that in the kind of cases described above the minimax regret solution is "dependent upon irrelevant alternatives".

However, Nash's condition, as applied to decision theory by Radner and Marschak, and Arrow's condition are not the same. Following Ray (1973), I label the two conditions IIA(A) and IIA(RM). IIA(A) is as stated in Arrow's statement of his Condition 3. But Arrow surrounds this definition of IIA(A) with two examples of IIA(RM) and one of IIA(A). The case of athletic scoring systems is interesting. They may violate IIA(RM), as shown by MacKay (1980, pp. 33--4) and by the inconsistency in the European ice-skating championships in 1994 which enabled Jane Torvill and Christopher Dean to win. They may alternatively be shown to violate IIA(A) when the question is the grading of a subset of the teams whose relative performance does not change, given changes in the performance of members of teams not in the subset. This is the case considered by Huntington (1938), whose 'postulate of relevancy' is the same as Arrow's Condition 3. Arrow records that it was Marschak who drew Huntington's paper to his attention.

The confusion between two concepts of IIA has generated a huge literature. In my opinion, it is irrelevant to the issue between Borda and Condorcet methods. However conceptualised, the Borda rule fails to observe IIA. If conceptualised in the normal version, in which the death of a candidate would lead the rankings to be re-entered over the reduced set of candidates, it violates IIA(RM) because it is not contraction-consistent. If conceptualised in a version in which the rankings were not re-entered, it violates IIA(A). In either case, the arguments of Condorcet and Daunou against the Borda rule apply, and it is for the proponents of Borda systems to respond.

Lately, this has been a dialogue of the deaf. Kenneth Arrow and Donald Saari both made presentations to plenary sessions of the 2001 meetings of the Public Choice Society in San Antonio, TX. They conspicuously failed to engage with one another. Arrow re-stated the reasonableness of his IIA condition. Saari brushed it aside, and maintained that Condorcet efficiency is not an appropriate criterion for judging choice or ordering systems. In the abstract of the paper which formed the basis of his San Antonio presentation, Saari maintains:

> A theory is developed to explain all possible three-alternative (single-profile) pairwise and positional voting outcomes. This includes all preference aggregation paradoxes, cycles, conflict between the Borda and Condorcet winners, differences among positional outcomes (e.g., the plurality and antiplurality methods), and differences among procedures using these outcomes (e.g., runoffs, Kemeny's rule, and Copeland's method). It is shown how to identify, interpret, and construct all profiles supporting each paradox. Among new conclusions, it is shown why a standard for the field, the Condorcet winner, is seriously flawed. (Saari 1999, p. 313).

In my view, however, Saari's argument begs the question. The Condorcet criterion is seriously flawed if you do not accept the reasonableness of IIA. But the reasonableness of IIA is the very question that must be addressed first. Here the contribution of Satterthwaite (1975) seems key. His version of the Gibbard-Satterthwaite result that all determinate choice systems are either dictatorial or manipulable proceeds by aligning the conditions for manipulability with the Arrow axioms. He shows that the proposition 'a determinate choice procedure is non-manipulable' corresponds exactly with the proposition 'a choice procedure conforms to Arrow's conditions of transitivity, universal domain, weak Pareto condition, and independence of irrelevant alternatives'. Therefore if a procedure is non-manipulable, it is dictatorial. This goes to show, as Amartya Sen has often said, how *economical* are Arrow's conditions. IIA is in there for a good reason, as Satterthwaite indirectly shows. Take out IIA and you have gross manipulability.

Saari is clearly right that Borda does the most with the least. The Borda rule beats all other ranking systems by its simplicity and elegance, and its superiority can be shown axiomatically in many ingenious ways, as Saari and his associates have done. But that cuts both ways. The very fact that it conveys information more efficiently and more transparently than any other ranking rule means that it is the easiest to manipulate of all ranking rules. The academicians of 18th-century France saw this with perfect clarity. Why did Borda say 'My scheme is only intended for honest men'? Why did Napoleon propose the abolition of the Borda rule for the election of academicians? Why did Daunou abandon that same rule, of which he had been the proponent, in favour of the Condorcet criterion? Why did Bill

Riker manipulate the *US News and World Report* ranking of political science graduate schools by ranking those that he thought other voters would think were the most dangerous rivals to the University of Rochester at the bottom of the ranking which he reported as head of the Rochester department?

To all four questions I believe the answer is the same. Intelligent people can see that the most transparent of all ranking systems is also the most manipulable. the fact that Borda can be so easily manipulated by the 'most dangerous rival' strategy of course flows directly from its violation of IIA. So do other clever ideas that people have had over the past two centuries, such as adding or withdrawing dummy candidates. The very suitability of Borda as a theoretical choice procedure rules it out absolutely as a practical choice procedure - at least, for humans.

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