A program to implement the Condorcet and Borda rules in a small-*n* election

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Introduction: The Condorcet and Borda criteria

There are two defensible procedures for aggregating votes: the Condorcet rule and the Borda rule. Each may be used either to choose a winner or to rank the alternatives. To choose a winner, the Condorcet rule is:

> *Select the option (if one exists) that beats each other option in exhaustive pairwise comparison*

And the Borda rule is:

> *Select the option that on average stands highest in the voters' rankings.*

To rank the alternatives, the Condorcet (also known as Copeland) rule is:

> *Rank the options in descending order of their number of victories in exhaustive pairwise comparison.*

And the Borda rule is:

> *Rank the options in descending order of their standing in the voters' rankings.*

These choice and ranking rules have properties, and defects, that are now well known. By Arrow's (1951) General Possibility Theorem, neither ranking rule can satisfy the five Arrow conditions, because no ranking rule can. The Condorcet rule fails to satisfy universal domain, because a strong ordering does not always exist. For instance, there may be a top cycle, and no Condorcet winner. The Borda rule fails to satisfy independence of irrelevant alternatives (IIA). For a recent introductory-level discussion, see Dasgupta and Maskin (2004).

However, in the context of selecting a winner, given a set of votes intended to have equal weight, no criterion other than Condorcet and Borda has ever gained acceptance. The Condorcet winner and the Borda winner are the only two methods that must choose an option with a claim to democratic legitimacy. If a method may reject a Condorcet winner, it is inferior to the Condorcet method. If it may reject a Borda winner, it is inferior to the Borda method. If it may select a Condorcet or Borda loser, it is in a worse state. And if it may select an absolute majority loser (an option which a majority of the electorate rank last) it is in the worst state of all.

Thus, regardless of Arrow's impossibility theorem, it is possible to rank choice procedures. Procedures which select the Condorcet winner and/or the Borda winner are superior to those that may fail to choose one of these, which are in turn superior to those that may choose losers.

In real-world elections, however, the Condorcet rule is almost never used. The Borda rule is sometimes used in sporting and entertainment contests (example: the Eurovision Song Contest) bur rarely used in political, economic, or social contexts. Its use in the Eurovision Song Contest is open to the objection that as Borda is the most transparent of all choice rules, and also the one that involves the most extensive possible violation of IIA, it is by the same token the most manipulable. The best-known form of manipulation is to place the most dangerous rival to one's favourite option at the bottom of one's ranking. "My scheme is only intended for honest men", said Borda when this objection to his rule became evidence in the first practical application. (Black [1958] 1998, p. 215).

Usually, elections to a single post (e.g., an executive presidency) and votes on appointments use a majoritarian elimination-based system. This may take place in a single round:

> *Voters each rank the options. If any option has more than half of the first places, it is chosen. If not, the option with the fewest first places is eliminated, and those ballots reallocated in favour of the surviving option that they rank highest. Repeat the process as often as necessary until a candidate has a majority of first places* (known as Alternative Vote (AV) or Single Transferable Vote – the two are equivalent in the single-option case).

Or in multiple rounds:

> *Voters each rank the options. If any option has more than half of the first places, it is chosen. If not, the option with the fewest first places is eliminated, and voters vote again, ranking the surviving options. Repeat the process as*

often as necessary until a candidate has a majority of first places (known as Exhaustive Ballot (EB)).

Real world variants of AV and EB truncate subsequent counts or ballots, for instance by restricting voters to a choice between the top two options in the first count/ballot. The London mayoral election system is such a variant of AV. The French Presidential election system is such a variant of EB. From a design perspective, these systems are dominated by AV and EB respectively. They share the faults of AV and EB, and add the extra fault that they throw away information capriciously. We do not discuss them further.

All elimination systems including AV and EB may reject the Condorcet and the Borda winner. If a Condorcet and/or Borda winner has fewer first places than a Condorcet and/or Borda loser, then the winner is eliminated and fails to survive to the pairwise contest that would prove that he (she, it) was the Condorcet and/or Borda winner. This is, or should be, a fatal objection to using an elimination-based system for either elections to a presidency or appointments to a post.

The Borda and Condorcet rules are easily programmable, and because of an equivalence theorem due to Borda himself (Borda 1784) a single program can derive Borda and Condorcet (Copeland) rankings from the same input data. The lack of such a publicly available program may have inhibited organisations from using the superior Borda or Condorcet methods rather than AV or EB. The following sections therefore describe such a program and its implementation. The program itself is freely available at www.nuff.ox.ac.uk/users/mclean

Method

A body of electors to a post decided to revive an electoral system first used around 1980, and originally devised by the economist Francis Seton (1920-2002; Scott 2002). The method required the winning candidate to be both the Condorcet and the Borda winner. If no candidate passed this test, the electors were to eliminate any dominated options, discuss the surviving candidates, and vote again. The electors decided to treat "no election" as if it were a candidate, and also to hold a prior, public "signalling" vote declaring their first preference among the candidates. Both of these decisions may be justified for the reasons given by Dodgson (1876) but are extraneous to the programming task. However, they are discussed briefly in *Discussion*, below.

There was no known program to implement the 1980 version of the Seton procedure, so we undertook to write one. The instructions to voters are at Appendix A. The instructions to scrutineers are at Appendix B. A sample worksheet, for 34 voters and four options, is at Appendix C. The underlying code is freely available at www.nuff.ox.ac.uk/users/mclean

The scrutineers first checked each ballot paper to verify that it contained a complete (weak or strong) ordering of the options. A strong ordering ranks the four options {1,2,3,4} in some order of preference. A weak ordering contains ties. Before data entry, all ballots containing weak orderings were scrutinised individually and the correct arithmetic values for weak orderings to ensure that the ordering could be directly summed to a Borda ordering were inserted according to the rules given in Appendix B. Thus, for instance, a ballot containing a first preference for *A* and otherwise blank is coded {1,3,3,3}. This coding gives the ballot paper the same value as one expressing a strong ordering, and expresses the voter's indifference among options *B*, *C*, and *D*.

The next step was data entry direct on to the spreadsheet. The top left block of Appendix C is for data entry. Each ballot was entered directly on the spreadsheet by one scrutineer, with two others watching for any errors. Each ballot must have a total value of 10 i.e., $\frac{1}{2}n(n + 1)$ for $n = 4$. This was verified by the CHECK column to the right of the data entry area. The importance of this stage is that errors can be detected and corrected as they are made. Earlier versions of the software, in which the pairwise comparisons for each ballot are entered directly on the Dodgson matrix described below, were found to give no satisfactory audit trail for data entry errors.

An effect of entering data in this way is that the sum of rankings of each option is itself its Borda score, but notated in the inverse way to the normal one, with the lowest-scoring option being the Borda winner and the highest-scoring being the Borda loser. This is shown in the third-last row on the left, with two check rows added to verify that this conforms to the Borda score as calculated in the conventional direction.

The heart of the spreadsheet is the *Dodgson matrix* at the centre, i.e., the area headed "Paired comparison". We have so named it (cf McLean 1996) because C. L. Dodgson (Lewis Carroll – in Dodgson 1876, Fig. 2) was the first writer to propose aggregation by means of a square matrix in which each option is compared with each other. The principal diagonal of the Dodgson matrix is of course blank. In each other cell are computed the votes *for* the candidate in the column and *against* the candidate in the row. The Borda and Copeland rankings can then be directly read off the matrix, in the rows marked “Borda” and “Copeland”.

By Borda’s (1784) equivalence theorem, if Borda scores are set at 0 for a last place, 1 for a second-last place, …, $n - 1$ for a first place, then the number of pairwise victories for each candidate is also that candidate’s Borda score. The Borda ranking is invariant to any positive linear transformation. Therefore the vertical sum of scores for each option is that option’s Borda ranking. To calculate its Copeland ranking, it is necessary to compare each cell with its diagonal opposite, across the principal diagonal of the Dodgson matrix. As all ties are entered in the matrix of pairwise comparisons as 0.5 for each member of the tying pair, each pair of diagonally opposite cells sums to the total of votes cast (here 34). Any cell, therefore, of magnitude greater than half the total votes cast (here 17) is awarded a Copeland score of 1, signalling a pairwise victory for the option in its column over the option in its row. A sub-matrix on the top right of the spreadsheet computes these Copeland scores. The Copeland score for an option is simply the number of its pairwise victories. Therefore, if there is no cycle in the Condorcet rankings, for four candidates the vector of Copeland scores will be {0,1,2,3}. A tie in the Copeland scores indicates either social indifference or a cycle.

Results

With four options, 34 ballots, and three scrutineers, the processes of checking weak ballots, data entry, and computation of the result took about 10 minutes. If a second ballot is required because of a conflict between the Borda rankings and the Condorcet (Copeland) rankings (cf Appendix B, rules 9 and 10), it is simple to rerun the ballot on the same template, setting all votes for any excluded option at a value of 4 in the data entry columns (or for any two excluded options at 3.5).

In the case discussed, the number of options and the number of eligible voters were both known in advance. The spreadsheet is flexible for three of the four possible changes in numbers of either voters or options. If fewer votes than the pre-set number are cast, it is easy to record the missing votes as completely indifferent among the options by entering the required value of $(n + 1)/2$ (here 2.5) in each of the appropriate cells. If more votes than the preset number are cast, then altering the number in the line "*m* valid votes cast" to the required *m*, plus adding rows in the data entry and computation columns for the required extra ballots, is straightforward.

If fewer options than the pre-set number are to be voted on, then, as just discussed, it is simple to set the value for excluded options on ballot papers as a forced last (etc.) place. The only change from the pre-sets that would be difficult to incorporate in real time while an election is in progress is an increase in the number of options. Users of the spreadsheet would therefore be advised to adapt it in advance for the largest number of options that can reasonably be predicted to be under discussion. Careful copying and pasting of the Excel "=IF" functions in the spreadsheet is required, but the logic is straightforward.

Discussion

Previous attempts to combine the Borda and Condorcet rules have been made by Nanson (1882) and Black (1958/1998). Nanson gives priority to Borda; Black to Condorcet. Nanson's rule may be informally summarised as "Select the Borda winner, subject to the constraint of eliminating any Borda winner that is not a Condorcet winner". Black's rule may be informally summarised as "Select the Condorcet winner if one exists. If there is a top cycle and hence no Condorcet winner, select the Borda winner among the cycling options". The rule we present has no formal procedure for resolving either the case where the Borda and Condorcet winners differ, or the case of a top cycle, in which there is no Condorcet winner. The rules (Appendix B) treat these as problems in deliberative democracy, to be solved by further deliberation. A fuller implementation would have to contain a stopping rule to the effect "if no decision is reached by [insert number of iterations or amount of time], then the outcome is no election". In the case described, there was an understanding among the electors to this effect, but no formal stopping rule was agreed in advance.

It is well known in social choice, and among sophisticated electorates, such as Eurovision Song Contest juries, that the Borda rule is highly manipulable. This implies that any procedure which uses Borda should be used with caution. In particular, it should not be used directly in the aggregation of interests. It is more suitable for the aggregation of judgements. It is most suitable for a case in which the voters are a permanent body who have to live with one another, and with the result of their choice, and less suitable for a case in which the voters assemble for the election only, and then disperse. The first two social choice theorists since classical times, Ramon Lull and Nicolas of Cusa (Cusanus), discussed respectively the choice of an abbess by her convent and the election of a Holy Roman Emperor by the Electors (who were mostly German princes, such as the elector of Hanover). Lull recommends a pairwise comparison method with voting in public; Cusanus, the Borda method with voting in secret (McLean and Urken 1995, Introduction and chs 3-4). The context of the election discussed in this paper was closer to Lull's than to Cusanus'; appropriately, so were the choice mechanisms. The knowledge that the electors must live with one another and with their choice is, as Lull implied, a constraint on strategic voting. Some such reasoning probably influenced the choice of an open signalling round, as it probably influenced Dodgson's (1876) recommendation of the same step for elections in his Oxford college. However, the reasoning of the proponents of signalling was not made explicit in either case.

The electors' decision to treat "no election" as if it were the name of a candidate merits brief discussion. The reasons for doing so are fully set out by Dodgson (1876), to which nothing need be added. However, there is an issue of neutrality to discuss. The rules for the election are, on the face of it, strictly neutral among options. Thus, if "no election" wins, there is no election – end of discussion. If a person wins by both criteria, that person is chosen – again, end of discussion. However, the rules are implicitly non-neutral (cf May 1952) because they are formally incomplete. If the meeting ran out of time or reached the agreed maximum number of iterations without being able to select a candidate who met both the Borda and the Condorcet criteria, the outcome would be "no election" – a violation of neutrality in favour of "no election", but one which might be felt normatively acceptable. Furthermore, a body of electors might feel that to be deemed elected a candidate must score more than a bare majority – must exceed the next-place by a certain minimum Borda score, and-or must win each (or some) pairwise contest by a specified majority. Such qualified-majority rules are common in the real world (e.g., jury rules; the weighted voting rules for the Council of Ministers of the European Union) and are designedly non-neutral. Again, in this context, they privilege “no election” over each candidate. They would not be difficult to program into the Dodgson matrix.

In conclusion, we have shown that a hybrid Borda-Condorcet choice rule is normatively more acceptable than either the Alternative Vote or the exhaustive ballot methods more common in small-*n* elections. We have devised and tested a method of recording data and making an election under this hybrid rule. We would be pleased to hear of other implementations, and of any problems that are encountered.

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Appendix A

Ballot for xxx election, 2004

Please rank the following options in order.

1 means your most-liked option

4 means your least-liked option

Ties are permitted.

If you rank two or more options equally, you may enter the average of the positions they occupy..

E.g., {1.5, 1.5, 3,4}; {1, 2.5, 2.5, 4} are permitted rankings.

A blank will be treated as a (joint-) last place

	Rank
A	
B	
C	
D	

Appendix B

The Seton procedure: Instructions for scrutineers.

1. Check that each ballot shows a legitimate ranking. Refer any ballot that does not show a legitimate ranking to Presiding Officer for a ruling on its admissibility.

2. Count the number of valid votes. By default, the "Seton" spreadsheet records 34 votes. If 34 valid votes have been cast, do not alter the spreadsheet. If fewer than 34 valid votes have been cast, then treat all the missing votes as recording four-way ties by entering 2.5 in each cell for each of them.

3. For any ballot that records a tie, and any short ballot, write the exact arithmetic value of the tied or missing places on the ballot. ****The sum of ranks on every ballot must equal 10.****

    3.1. Example 1. A ballot ranks options A and B as "1=" and options C and D as "3=". Enter 1.5 against A and B, and 3.5 against C and D.

    3.2. Example 2. A ballot has ranked option A as "1" and is otherwise blank. Enter 3 against each of B, C, and D.

4. Record each ballot in turn on the "Seton" spreadsheet. For ballot 1, use the row labelled 1 in column A. Enter 1 in the column for the option that the ballot ranks first, 2 in the column for the option that the ballot ranks 2^{nd} etc. Check (column G) that the total value of the ballot = 10. **NB VERY IMPORTANT**. **This procedure enters the ranks directly, not inverted. 1 = TOP PLACE. 4 = LAST PLACE!!!.**

5. Repeat step 4 for ballots 2, 3, … *m*.

6. When all ballots have been recorded, the row marked “Borda” (row 16) will show the Borda score for each option. This records the options’ ranking on the first Seton criterion (“Borda count”: the average position of each option. **Highest = best.** The coding has inverted the ranks entered at step 4.).
7. When all ballots have been recorded, the row marked “Copeland” (row 17) records the options’ ranking on the second Seton criterion (“Condorcet criterion”: rank first the option that beats all others in pairwise comparison, rank second the option that beats all others bar one, and so on). The Copeland score implements the Condorcet criterion by counting the number of victories scored in pairwise contests. If there are no cycles and no ties in the data, the vector of Copeland scores will be {3,2,1,0}. The Copeland scores must sum to 6. A cycle will be marked by ties in the Copeland score e.g., {2,2,2,0}. Highest = best.
8. If the rankings by the two Seton criteria are the same, the Presiding Officer will announce that the top option by both methods has been chosen.
9. If the top option by the two Seton criteria is the same but lower options differ, the Presiding Officer will announce this. The Meeting will have to decide, then or later, how to proceed if the top-ranked candidate declines appointment.
10. If the top option by the two Seton criteria differs, the Presiding Officer will announce the fact. The Meeting must then decide how to proceed among the candidates in the top set. Dominated options are excluded from further discussion. An option is dominated if by both criteria it ranks lower than all options which are ranked (joint-) highest on at least one criterion.

Appendix C. A sample worksheet for 4 options and 34 voters

In this example, the outcome, by both Condorcet and Borda criteria, is B > A > D > C.

#	**A**	**B**	**C**	**D**	CHECK	A>B	B>C	C>D	A>C	B>D	A>D
1	1	2	3	4	10	1	1	1	1	1	1
2	4	1	3	2	10	0	1	0	0	1	0
3	1	3	3	3	10	1	0.5	0.5	1	0.5	1
4	3.5	1.5	3.5	1.5	10	0	1	0	0.5	0.5	0
5	3	2	4	1	10	0	1	0	1	0	0
6	4	3	1	2	10	0	0	1	0	0	0
7	3	2	1	4	10	0	0	1	0	1	1
8	2	1	4	3	10	0	1	0	1	1	1
9	2	3	1	4	10	1	0	1	0	1	1
10	3	1	4	2	10	0	1	0	1	1	0
11	3	1	3	3	10	0	1	0.5	0.5	1	0.5
12	1	4	3	2	10	1	0	0	1	0	1
13	4	2	3	1	10	0	1	0	0	0	0
14	3	2	4	1	10	0	1	0	1	0	0
15	2	2	2	4	10	0.5	0.5	1	0.5	1	1
16	1.5	3.5	1.5	3.5	10	1	0	1	0.5	0.5	1
17	1	3	2	4	10	1	0	1	1	1	1
18	1	2	3	4	10	1	1	1	1	1	1
19	4	3	2	1	10	0	0	0	0	0	0
20	4	1	2	3	10	0	1	1	0	1	0
21	2	1	3	4	10	0	1	1	1	1	1
22	2	3	4	1	10	1	1	0	1	0	0
23	3	4	2	1	10	1	0	0	0	0	0
24	1	3	4	2	10	1	1	0	1	0	1
25	3	3	1	3	10	0.5	0	1	0	0.5	0.5
26	3.5	1.5	1.5	3.5	10	0	0.5	1	0	1	0.5
27	4	2	3	1	10	0	1	0	0	0	0
28	3	1	2	4	10	0	1	1	0	1	1
29	3	1	2	4	10	0	1	1	0	1	1
30	1	2	4	3	10	1	1	0	1	1	1
31	2	1	4	3	10	0	1	0	1	1	1
32	1	4	3	2	10	1	0	0	1	0	1
33	2	3	4	1	10	1	1	0	1	0	0
34	2.5	2.5	2.5	2.5	10	0.5	0.5	0.5	0.5	0.5	0.5
Sum	**84**	**75**	**93**	**88**	**340**	**15**	**22**	**16**	**19**	**20**	**19**
comp	52	61	43	48							
chec	136	136	136	136							

Need to have smallest sum to get elected

Paired comparison

Wins

	A	**B**	**C**	**D**
A	**<**	**19.5**	**15.5**	**15**
B	**14.5**	**<**	**12**	**14.5**
C	**18.5**	**22**	**<**	**18.5**
D	**19**	**19.5**	**15.5**	**<**

					Criteria
Borda	**52**	**61**	**43**	**48**	*highest*
Copeland	**2**	**3**	**0**	**1**	*3*

34 valid votes cast

Copeland

	A	**B**	**C**	**D**
A	<	1	0	0
B	0	<	0	0
C	1	1	<	1
D	1	1	0	<