

Non-Separable Preferences, Fiscal Policy 'Puzzles' and Inferior Goods.

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ABSTRACT. Non-separable preferences over consumption and leisure can generate an increase in private consumption in response to government spending, as found in the data, in a frictionless business cycle model. However, the conditions on preferences required for these result to obtain hold if and only if the consumption good is inferior. Similarly, positive co-movement of consumption and hours worked occurs if and only if either consumption or leisure is inferior.

Keywords: *non-separable preferences; fiscal policy; government spending; private consumption; inferior goods.*

JEL codes: *D11; E21; E62; H31.*

This paper studies purely preference-based explanations of some recent puzzles pertaining to the effects of fiscal policy shocks. The puzzles concern the positive response of private consumption to government spending (and the associated positive co-movement between consumption and hours worked) found by i.a. Blanchard and Perotti (2003), Fatás and Mihov (2001) and Gali, Lopez-Salido and Valles (2005) for US data and by Perotti (2005) for OECD countries. Standard, frictionless business cycle models using separable preferences over consumption and leisure cannot generate either of these observations since they imply that taxation used to finance the spending has a negative wealth effect that translates into an increase in hours worked and a fall in consumption (e.g. Baxter and King (1993)).

In a recent paper, Linnemann (2006) argues that these puzzles can be solved in a frictionless business cycles model if one considers a certain type of non-separability over consumption and leisure in the utility function. A first undesirable feature of the preferences considered by Linnemann is that they rely on a downward-sloping labor supply schedule. I first generalize this result: considering fully general non-separable preferences, I find conditions on preferences under which the puzzles are resolved, without the need of assuming a negative labor supply elasticity, starting from a simple model without capital in which these conditions can be found analytically.

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However, I further show that the conditions under which (fully general) non-separable preferences generate an increase in private consumption in response to government spending are satisfied *if and only if the consumption good is inferior (non-normal)*¹. Moreover, the *positive co-movement* between consumption and hours occurs if and only if either consumption or leisure is inferior. This result is intuitive: an increase in government spending financed by lump-sum taxation generates a fall in income; if consumption is to increase (in the absence of agent heterogeneity and/or demand effects), it must mean that the consumption good is inferior (its demand increases when income falls, i.e. its demand falls when income increases). When leisure is inferior and consumption is not, both consumption and hours fall when government spending increases.

The "puzzles' resolution" based on non-separable preferences may at first sight seem preferable based on simplicity grounds; indeed, *Occam's razor* implies in this context that among theories competing to explain a given set of facts, one should prefer the one that uses the minimal set of unproven assumptions. The results of this paper imply that other theories (reviewed in the final section) purported to explain these puzzles by using more complicated models are in fact preferable since they do not rely on the consumption good being inferior. More generally, this paper implies that the use of non-separable preferences should be taken with care.

1. Non-separable preferences and a possible resolution of the puzzle(s)

Suppose that the representative household maximizes the expected discounted value of future utility, where the momentary utility function at t is of the general non-separable form:

$$U(C_t, L_t),$$

where C_t is consumption, leisure is $L_t = 1 - N_t$, and N_t are hours worked. Assume that $U_C > 0, U_L > 0, U_{CL} = U_{LC} \neq 0$ and U is concave². The household works and receives labor income, saves by buying riskless, one-period, discount government bonds B_{t+1}^d that cost $(1 + R_t)^{-1}$ and deliver one unit of the consumption good next period and pays lump-sum taxes T_t ; the budget constraint is:

$$(1 + R_t)^{-1} B_{t+1}^d + C_t = B_t^d + W_t N_t - T_t.$$

We assume initially for simplicity that firms only employ labor and transform it linearly into the consumption good, so output is given by: $Y_t = N_t$. The government chooses an exogenous stream of spending, and finances it via lump-sum taxes and the issuance of one-period, risk-free discount bonds³:

$$(1 + R_t)^{-1} B_{t+1}^s + T_t = B_t^s + G_t.$$

In this simple model with fully general non-separable preferences, the dynamics of consumption and hours (and hence output) in response to government spending can be summarized by the following Proposition.

¹Recall that a *normal good* is defined as having a *positive* income elasticity of demand, whereas an *inferior good* has a *negative* elasticity of demand.

²Specifically, the Hessian of $U()$ has to be negative semidefinite, i.e. $U_{CC} \leq 0; U_{LL} \leq 0$ and $U_{CC}U_{LL} - (U_{CL})^2 \geq 0$. Strict concavity requires that Hessian be negative definite, i.e. only the first and third inequality be satisfied with strong inequality.

³Since this is a Ricardian model the intertemporal path of taxation T_t (and hence the amount of debt issued B_t^s) will be irrelevant.

PROPOSITION 1. *In response to a government spending increase:*

(i) *there is **positive co-movement between consumption and hours** if and only if*

$$(1.1) \quad \delta > 0, \text{ where } \delta \equiv \frac{U_{CL} - U_{CC}}{U_{LL} - U_{CL}};$$

(ii) *private consumption **increases** if and only if:*

$$(1.2) \quad \delta > 1,$$

which is a necessary and sufficient condition for both puzzles to be solved.

Maximizing lifetime utility subject to the budget constraint, taking prices as given, we obtain:

$$(1.3) \quad \begin{aligned} L_t & : U_L(C_t, L_t) = U_C(C_t, L_t) W_t \\ B_{t+1} & : U_C(C_t, L_t) = \beta(1 + R_t) E_t U_C(C_{t+1}, L_{t+1}) \end{aligned}$$

The firm's problem is trivial, and simply implies a flat labor demand curve: $W_t = 1$. Equilibrium is completed by requiring market clearing for labor and bonds $B_{t+1}^s = B_{t+1}^d$, and implies -by Walras' Law- that the goods market also clears: $Y_t = C_t + G_t$. Following the by now standard approach in business cycle models, I loglinearize the intratemporal optimality condition⁴ (1.3) letting small case letters denote log-deviations from steady state (unless specified otherwise), e.g. for any variable $x_t \equiv \ln(X_t/X)$:

$$(1.4) \quad U_L() \left[1 + \frac{U_{CL}C}{U_L} c_t + \frac{U_{LL}L}{U_L} l_t \right] = W U_C() \left[1 + w_t + \frac{U_{CC}C}{U_C} c_t + \frac{U_{CL}L}{U_C} l_t \right].$$

Since the real wage is constant, we have $w_t = 0$ and $U_C = U_L$, so this expression reduces to:

$$[U_{CL} - U_{CC}] C c_t = [U_{CL} - U_{LL}] L l_t = [U_{LL} - U_{CL}] N n_t$$

where the second equality has used that log-deviations of leisure are related to log-deviations of hours by $L l_t = -N n_t$. Let us further make the innocuous but simplifying assumption that in steady state $G = 0$ and $B = 0$, implying $T = 0$ and $C = N = 1 - L$, so:

$$n_t = \delta c_t$$

where δ is given in the Proposition. It is clear that positive co-movement between hours and consumption requires $\delta > 0$ ⁵. Using the loglinearized versions of the

⁴The Euler equation merely serves here to pin down the interest rate, so I will ignore it in the remainder.

⁵For the specific functional form considered by Linnemann, $U(.) = [C_t^{1-\sigma} v(L_t)] / (1-\sigma)$ it can be easily shown that the condition for positive comovement boils down to:

$$\frac{v''}{v'} - \frac{v'}{v} > 0,$$

which is the same as the condition obtained by Linnemann (eq. 29) when one sets the labor share to 1. In Linnemann's setup, the same condition implies that labor supply elasticity is negative.

production function $y_t = n_t$ and the resource constraint⁶ $y_t = c_t + g_t$ we obtain a reduced-form expression for consumption:

$$c_t = \frac{1}{\delta - 1} g_t,$$

so a necessary and sufficient condition for crowding in of consumption is: $\delta > 1$.

2. Why non-separable preferences do not really solve the puzzle(s): good inferiority

PROPOSITION 2. *In response to a government spending increase: (i) Positive co-movement of consumption and hours requires that either consumption or leisure be inferior.*

(ii) Private consumption increases if and only if consumption is an inferior good.

To study good inferiority, rewrite the budget constraint as:

$$C_t + W_t L_t \leq E_t,$$

where E_t is total expenditure/income, given in this case by $E_t = W_t + B_t - T_t - (1 + R_t)^{-1} B_{t+1}$. A good is inferior if its income elasticity of demand is negative. We find demands for leisure and consumption for a given level of income E by solving the following static optimization problem:

$$\max U(C, L) \text{ s.t. } C + WL \leq E$$

which leads to:

$$\begin{aligned} WU_C(C, L) - U_L(C, L) &= 0 \\ C + WL - E &= 0 \end{aligned}$$

We apply the implicit function theorem to this system to study variations in consumption demand to changes in income E . Totally differentiating the above (keeping the price of leisure W fixed) we get the system:

$$\begin{aligned} [WU_{CC} - U_{CL}] \frac{\partial C}{\partial E} + [WU_{CL} - U_{LL}] \frac{\partial L}{\partial E} &= 0, \\ \frac{\partial C}{\partial E} + W \frac{\partial L}{\partial E} &= 1, \end{aligned}$$

whose solution is:

$$\begin{aligned} \frac{\partial C}{\partial E} &= \left[1 - W \frac{U_{CL} - WU_{CC}}{U_{LL} - WU_{CL}} \right]^{-1}; \\ \frac{\partial L}{\partial E} &= \left[W - \frac{U_{LL} - WU_{CL}}{U_{CL} - WU_{CC}} \right]^{-1}. \end{aligned}$$

Inferiority of consumption implies by definition $\frac{\partial C}{\partial E} < 0$, i.e.

$$(2.1) \quad W \frac{U_{CL} - WU_{CC}}{U_{LL} - WU_{CL}} > 1.$$

Since $W = U_L/U_C = 1$, consumption is inferior if and only if

$$\delta > 1,$$

⁶Since $G = 0$, we have $C = Y$ and g_t is defined as a share of steady-state output $g_t \equiv (G_t - G)/Y = G_t/Y$.

which is precisely the same condition needed to ensure a positive response of consumption to government spending in Proposition Iii, (1.2). Furthermore, leisure is inferior if and only if $\frac{\partial L}{\partial E} < 0$, so

$$(2.2) \quad \frac{U_{LL} - WU_{CL}}{U_{CL} - WU_{CC}} > W,$$

so using again the expression for W leisure is inferior if and only if:

$$\frac{1}{\delta} > 1$$

Therefore, *neither* good is inferior if and only if $\delta \leq 1$ and $\frac{1}{\delta} \leq 1$ *simultaneously*, which instead requires $\delta < 0$. But by Proposition Ii, whenever $\delta < 0$, hours and consumption are *negatively* correlated. So for consumption and hours to be positively correlated in response to government spending ($\delta > 0$), either consumption or leisure must be an inferior good (either $\delta > 1$ or $\delta^{-1} > 1$). For consumption to increase in response to government spending, the consumption good has to be inferior.

2.1. Adding capital. Consider now a version of the model with physical capital, as in Linnemann, whereby $\alpha \in (0, 1)$ is the labor share; the production function in log-linear form becomes $y_t = \alpha n_t + (1 - \alpha) k_t$ and the real wage equation/labor demand curve is: $w_t = y_t - n_t = (\alpha - 1) n_t + (1 - \alpha) k_t$, and is downward sloping in the (w, n) space.

The following simple diagrammatic representation of the labor market gives an intuitive understanding of what is required for positive co-movement of consumption and hours. We plot the labor demand curve along with the labor supply curve, which is obtained by rewriting (1.4):

$$(2.3) \quad \left(\frac{U_{CL}}{U_C} - \frac{U_{LL}}{U_L} \right) N n_t = w_t + \left[\frac{U_{CC}}{U_C} - \frac{U_{CL}}{U_L} \right] C c_t$$

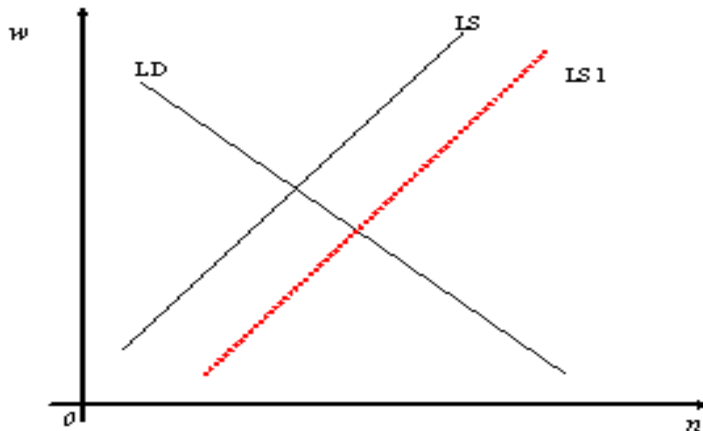


Figure 1: The positive co-movement of consumption and hours and the labor supply curve.

An increase in government spending leaves the labor demand curve unaltered and implies that labor supply shifts. If labor supply is upward sloping, implying $\frac{U_{CL}}{U_C} > \frac{U_{LL}}{U_L}$, a positive response of both consumption and hours requires that

$\frac{U_{CC}}{U_C} > \frac{U_{CL}}{U_L}$ ⁷. Consider also the cases where labor supply is downward-sloping (not pictured above). In Linnemann (2006), labor supply is in fact downward sloping, and its slope is more negative than that of labor demand (in particular, labor supply elasticity is -1). When this is the case, $\frac{U_{CL}}{U_C} - \frac{U_{LL}}{U_L} < \alpha - 1 < 0$, and the condition for positive responses of both n and c is: $\frac{U_{CC}}{U_C} < \frac{U_{CL}}{U_L}$. Finally, even if labor supply were downward sloping but less so than labor demand, $\alpha - 1 < \frac{U_{CL}}{U_C} - \frac{U_{LL}}{U_L} < 0$, the positive response requires a leftward shift in labor supply, implying $\frac{U_{CC}}{U_C} > \frac{U_{CL}}{U_L}$. A compact way, covering all these three cases, to write the positive co-movement condition can be found by substituting labor demand into labor supply to obtain:

$$\left[\frac{U_{CL}}{U_L} - \frac{U_{CC}}{U_C} \right] C c_t = \left[(\alpha - 1) + \left(\frac{U_{LL}}{U_L} - \frac{U_{CL}}{U_C} \right) N \right] n_t + (1 - \alpha) k_t$$

Since capital is fixed in the short run, positive co-movement therefore requires⁸:

$$(2.4) \quad \frac{\left[\frac{U_{CL}}{U_L} - \frac{U_{CC}}{U_C} \right] C}{\left[(\alpha - 1) + \left(\frac{U_{LL}}{U_L} - \frac{U_{CL}}{U_C} \right) N \right]} > 0$$

However, as in the case without capital, this fulfillment of (2.4) requires a violation of the non-inferiority requirement. The conditions for non-inferiority of consumption and leisure respectively obtained above (without the assumption $W = 1$, but using $W = U_L/U_C$) are:

$$\frac{U_L}{U_C} \frac{U_{CL} - \frac{U_L}{U_C} U_{CC}}{U_{LL} - \frac{U_L}{U_C} U_{CL}} > 1 \quad \text{and} \quad \frac{U_C}{U_L} \frac{U_{LL} - \frac{U_L}{U_C} U_{CL}}{U_{CL} - \frac{U_L}{U_C} U_{CC}} > 1,$$

implying:

$$(2.5) \quad \frac{\frac{U_{CL}}{U_L} - \frac{U_{CC}}{U_C}}{\frac{U_{LL}}{U_L} - \frac{U_{CL}}{U_C}} < 0.$$

Finally, note that since $\alpha < 1$ the following holds:

$$\frac{\left[\frac{U_{CL}}{U_L} - \frac{U_{CC}}{U_C} \right] C}{\left[\alpha - 1 + \left(\frac{U_{LL}}{U_L} - \frac{U_{CL}}{U_C} \right) N \right]} < \frac{\left[\frac{U_{CL}}{U_L} - \frac{U_{CC}}{U_C} \right] C}{\left(\frac{U_{LL}}{U_L} - \frac{U_{CL}}{U_C} \right) N} < 0.$$

Therefore, the positive co-movement condition (2.4) in the case with capital still violates the non-inferiority restrictions(2.5), regardless of whether labor supply elasticity is positive or negative (as in Linnemann (2006)).

⁷In fact, the same condition holds in general for positive co-movement of consumption and hours, whether both respond positively or negatively.

⁸For the preferences considered by Linnemann, implying sharply negatively sloped labor supply, the relevant condition for positive co-movement is:

$$\frac{v''}{v'} - \frac{v'}{v} > \frac{1 - \alpha}{N} (> 0).$$

3. Final comments

Non-separable preferences can generate an increase in private consumption in response to government spending, as found in the data, in a frictionless business cycle model. However, the conditions on preferences required for these result to obtain hold if and only if the consumption good is inferior. Similarly, positive co-movement of consumption and hours worked occurs if and only if either consumption or leisure is inferior. Our results can be useful in distinguishing between competing theories purported to solve the aforementioned puzzles. For example, two resolutions of the puzzle exist in the literature that do not require that either consumption or leisure be inferior - and indeed 'work' for separable preferences, $U_{CL}(C, L) = 0$.

The first, proposed by Galí, J., López-Salido, D. and Vallés, J. (2004) and developed in some directions⁹ by Bilbiie and Straub (2004), is based on consumer heterogeneity (or limited asset market participation) coupled with price stickiness and deficit financing. That framework requires a strong increase in the real wage, which increases consumption of households without assets enough to compensate the fall in consumption of ('Ricardian') households who accumulate assets, and induce an increase in aggregate consumption. The second framework was developed by Devereux, Head and Lapham (1996) and Bilbiie, Ghironi and Melitz (2005) based on (different mechanisms of) firm entry and exit driven by consumers' love for variety; these models also rely on a strong enough increase in the real wage (due to an effect on labor demand coming from an increase in the number of firms) that induces households to substitute from leisure into consumption. Leisure falls due to a negative wealth effect (so its marginal utility increases) but consumption can increase (and the marginal utility of consumption decrease) if the increase in wage is large enough, i.e. if the expansion of labor demand coming from the increase in the number of firms is large enough. While both these frameworks rely on a strong response of the real wage, this is consistent with empirical findings, who also find a positive response of real wage to government spending (see e.g. Fatás and Mihov (2001) and Bilbiie, Meier and Mueller (2006)). In contrast, preference-based explanations imply that the real wage will fall¹⁰ and are hence also inconsistent with this fact.

In general, this paper can be viewed as drawing attention to some caveats of using non-separable preferences in business cycle models. Hintermaier (2003) has shown that models of equilibrium indeterminacy using non-separable utility such as Bennett and Farmer (2000) imply that the utility function is *not concave*. This paper warns that, when building models with non-separable preferences to analyze business cycle fluctuations, one needs to impose restrictions that ensure that *consumption and leisure are normal goods*.

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⁹Namely, to study distortionary taxation and non-Keynesian effects of fiscal policy.

¹⁰This is clear from Linnemann's (2006) numerical simulations and also from the discussion of this paper for the case with capital: labor supply shifts right along a downward-sloping labor demand.

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