0.0.1 Policy for Global Warming with Uncertainty

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Abstract

Global warming is happening, and human activities are its major cause. Projections of warming and its consequences are subject to great uncertainty, especially where long-run outcomes are concerned. Does this uncertainty weaken the case for present expensive interventions? A theoretical examination of the problem is provided with a simple two-period model. A random process determines productivity. The optimal condition for expenditure to ameliorate global warming is derived. The result arises in part because the expected marginal utility of consumption and the productivity of green expenditure co-vary. Another influence is the Mirrlees Insurance Effect (MIE). While risk discourages expenditure due to standard risk aversion, additional expenditure provides insurance against bad outcomes. The MIE only produces increased expenditure with risk when sufficiently powerful to overcome a negative covariance effect.

Key words: Optimal investment Ramsey growth model Risk and covariance Global warming

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0.0.2 Introduction

Open-minded observers agree that global warming is happening, and that human activities are its major cause. Some of its probable effects involve major costs which it will be prudent to ameliorate. That requires current expenditures to moderate greenhouse gas emissions. Expenditures include outlays on projects such as wind-farm construction. But they should also be taken to include consumption activities surrendered for the sake of emission reduction, as when individuals reduce their air travel with their carbon footprint in mind. Then a leading problem is how far to reduce emissions, given that to do so implies a reduction of current consumption. Suppose that an individual declines to purchase an air ticket and buys a bicycle instead, although the air ticket would be preferred if carbon emission was not taken into account. In our analysis that counts as a reduction in current consumption for the sake of the future, although the national income statisticians would not measure a reduction in consumption.

We call the question of how far to reduce current consumption to moderate climate change the *greenhouse policy problem*. Other issues include the implementation of emission reductions, and how to organize the international cooperation that an effective policy demands. These last questions are important but outside the scope of the present paper.

A major headache caused for the design of an optimal greenhouse policy is that great uncertainty attaches to the relationship between current activities and future negative outcomes. This uncertainty is considerable even for a relatively short-term forecast for a few decades. However the worst and most concerning implications of global warming concern a longer-term outlook embracing the world towards the end of the 21st century. In that case there is so much uncertainty that the range of possible outcomes is enormous. That is not to say that anything goes. For example ocean acidification is a serious problem, and while its precise extent is uncertain, no serious model claims that it will become an insignificant issue. Not all the uncertainty takes the form of down-side risks. Suppose, for example, that towards the middle of this century the ever-recalictrant problem of harnessing nuclear fusion (the source of energy that powers the Sun and hydrogen bombs) to provide commercial power generation should be solved. That would transform the climate-change situation, and to a great extent fossil fuels would become redundant.

How should uncertainty concerning the consequences of current emissions affect policy? A simple argument maintains that uncertainty makes emission-reducing policies less attractive. This would be because uncertainty makes the policies in part gambles, and rational agents do not like to gamble where this involves a spread of outcomes around a given mean. They may enjoy a day at the race track, but when it comes to serious matters such as life insurance or saving, risk aversion is the normal feature. However when heavy negative risks are involved increased uncertainty does not always deter investment. Consider the example of fire insurance for homes. Suppose that you learn that all insurance companies have adopted devious mechanisms for loss adjustment, such that with a significant probability they will only pay out 90 percent of the true losses due to a fire. Fire insurance, which is meant to be risk reducing has now become significantly more risky. Do you therefore buy less insurance? Plainly no. A loss of a house through fire is a major personal disaster and you will pay heavily to avoid that risk. If you can offset the new way in which insurance companies are behaving by buying extra insurance that is what you will do. In reality there may be a penalty for over-insuring, but ignoring that point the example shows that increased risk does not always deter involvement. Below we will see that something similar to the fire insurance example applies to greenhouse policy.

The fire insurance example illustrates a point that was made more formally in a crucial paper Mirrlees 1965, unpublished but frequently cited. Its author showed that there is an ambiguity with regard to the response of investment to uncertainty. Risk discourages investment due to standard risk aversion. On the other hand additional investment provides some insurance against bad outcomes. We call the second effect the Mirrlees Insurance Effect (the MIE for short). The relationship between uncertainty and the optimal level of investment is complicated, as there are several effects, sometimes offsetting and sometimes uncertain as to sign. After I had worked out the analysis of a more complicated version of the problem analysed in this paper I discovered that James Mirrlees was aware of my results as far back as the 1960s. He published almost nothing at that time, so my originality was unnecessary.

The model developed below is extremely simple, and some of the subtle and difficult points that are encountered with a multi-period model do not arise. The best models for projecting climate change are complicated, because this is a complicated problem. Economic growth, including population growth; the melting of ice sheets; deforestation; sea-level rise; and technical change, all have to be taken into account. Good complicated models include that used for the Stern report, modelling by the late Alan Manne, and the DICE model developed at Yale by William Nordhaus and his colleagues. In contrast to these weighty efforts, the model employed in this paper is absurdly simple. It is a stripped-down two-period model which includes uncertainty. Simple models sometimes capture the essentials of a problem, and their bareness can allow fundamental principles to come into sharp focus. It is hoped that such is the case with the present paper.

The Model In the first period the world starts with resources R. These resources can be applied to current consumption or to investments that will reduce emissions this period, and consequently allow for a higher level of consumption next period. The resource budget constraint takes the form:

$$R = C_1 + G \tag{1}$$

where C_1 is consumption in the first period, and G is expenditure on climate-change abatement, or green expenditure.

Then first-period world utility is:

$$U(C_1) \tag{2}$$

The resources available for consumption in the second period are:

$$\epsilon F(G) \tag{3}$$

The function $F(\cdot)$ shows the average productivity of green expenditure, and ϵ is a non-negative random variable with mean unity, that shows the presence of uncertainty in the projection and control of climate change. The distribution of ϵ is not necessarily symmetric about its mean, and not necessarily bell-shaped. This is an important point to which I will return.

Discounted world utility in the second period is:

$$\delta U\left[\epsilon F(G)\right] \tag{4}$$

where δ is the utility discount rate. The discounting of future utility is a controversial issue in climate policy design, but I will not go into it. See *inter alia* Ackerman, Nordhaus, and Stern, all (2007). Readers who do not like discounting can set $\delta = 1$. In a two period model that makes little difference.

Now total utility for both periods is:

$$U(R - G) + \delta U\left[\epsilon F(G)\right] \tag{5}$$

Optimal policy with No Uncertainty With no uncertainty total utility for both periods is:

$$U(R - G) + \delta U\left[F(G)\right] \tag{6}$$

In equation (6) the random variable has been replaced by its mean.

The maximization of (6) by choice of G requires:

$$-U_1(R-G)+\delta U_1\left[F(G)\right]F_1(G)=0 \tag{7}$$

where subscripts 1 denote first derivatives.

We can rewrite (7) showing consumption levels explicitly and rearrange it to obtain:

$$\frac{U_1(C_1)}{U_1(C_2)}=\delta F_1(G) \tag{8}$$

where C_2 is consumption in the second period. With consumption in the second period larger than consumption in the first, the left-hand side of (8) shows the rate of decline of the marginal utility of consumption. This is equal to the discounted marginal productivity of green expenditure. This is an entirely familiar and unsurprising result, as it corresponds in the present model to the well-known necessary condition for optimal consumption in the Ramsey growth model.

Optimal policy with Uncertainty With uncertainty we have to maximize expected lifetime utility, and the maximand becomes:

$$U(R-G)+\delta EU\left[\epsilon F(G)\right] \tag{9}$$

where E indicates mathematical expectation. Maximization of (9) by choice of G requires:

$$-U_1(R-G)+\delta E\left\{U_1\left[\epsilon F(G)\right]\epsilon F_1(G)\right\}=0 \tag{10}$$

Now a standard statistical formula[1] allows us to write (10) as:

$$-U_1(R-G)+\delta EU_1\left[\epsilon F(G)\right]F_1(G)+cov\left\{U_1\left[\epsilon F(G)\right],\epsilon F_1(G)\right\}=0 \tag{11}$$

[1] The formula is $E(XY)=E(X)E(Y)+Cov(X,Y)$.

where $cov\left\{x,y\right\}$ is the covariance of the two variables x and y. As above we can rewrite (11) showing consumptions in the two periods explicitly.

$$-U_1(C_1)+\delta EU_1\left[C_2\right]F_1(G)+cov\left\{U_1\left[C_2\right],\epsilon F_1(G)\right\}=0 \tag{12}$$

Care must be taken when interpreting equation (12). First the magnitude of $EU_1(C_2)$ is unclear. With $U(C_2)$ a concave function the expectation of its slope is uncertain. A high value of ϵ decreases the slope of $U(C_2)$; a low value of ϵ increases the slope of $U(C_2)$.What results on average is unclear. This is the source of the ambiguity that underlies the MIE. When the MIE predominates we will have:

$$\frac{U_1\left[EC_2\right]}{EU_1\left[C_2\right]}<1 \tag{13}$$

There is no ambiguity concerning the sign of the final covariance term. It is negative. A high value of ϵ will increase the level of C_2, and therefore lower the value of $U_1(C_2)$. The same high value of ϵ will increase the level of $\epsilon F_1(G)$. With a low value of ϵ we have a lower level of C_2, and therefore lower a higher value of $U_1(C_2)$. The same low value of ϵ will decrease the level of $\epsilon F_1(G)$.

Rearranging equation (12) gives:

$$\frac{U_1(C_1)}{EU_1(C_2)}=\delta F_1(G)+\frac{Cov}{U_1(C_2)} \tag{14}$$

We call equation (14) *the necessary condition for optimal green policy*, or *the necessary condition* for short.

The Interpretation of the Necessary Condition In the model with no uncertainty but otherwise as the above, the necessary condition equation (8), repeated here for convenience, is:

$$\frac{U_1(C_1)}{U_1(C_2)}=\delta F_1(G) \tag{15}$$

Then a comparison with equation (14) reveals that uncertainty makes two differences. First, on the left-hand side of the equation the denominator $U_1(C_2)$

is replaced by $EU_1(C_2)$. In general what difference this makes is ambiguous. However if low values of ϵ, that is bad climate change developments, entail disaster, then $EU_1(C_2)$ will be far greater than $U_1(C_2)$ without uncertainty. This is the MIE with climate change. In that case uncertainty makes the left-hand side of (14) smaller.

The other difference between equations (14) and (15) is that the former includes the covariance term on its right-hand side. We have seen that this term is negative, so that makes the right-hand side of (14) smaller relative to its value with no uncertainty. These two effects of uncertainty, one influencing the left-hand side of equation (14), the other the right-hand side, are not independent of each other. The definition of the covariance term is:

$$cov\left\{U_1\left[C_2\right], \epsilon F_1(G)\right\} = E\left\{(U_1\left[C_2\right] - EU_1\left[C_2\right])(\epsilon - 1)\right\} F_1(G) \qquad (16)$$

In the case in which the MIE is strong the term $U_1\left[C_2\right] - EU_1\left[C_2\right]$ is on average strongly negative, and this correlates with the term $\epsilon - 1$ which is zero on average. So when the MIE applies the covariance effect is strongly negative. The left-hand side of equation (14) is not identical to the left-hand side of equation (15). The former may be rearranged as follows:

$$\frac{U_1(C_t)}{EU_1(C_{t+1})} = \frac{U_1(C_t)}{U_1(EC_{t+1})} \frac{U_1(EC_{t+1})}{EU_1(C_{t+1})} \qquad (17)$$

We noted above that the term $\frac{U_1(C_1)}{U_1(C_2)}$ shows the rate at which the marginal utility of consumption declines when there is no uncertainty. This rate of decline is equal to the discounted marginal productivity of green investment G.

The final fraction in (17) may be greater or less than 1 according to the form of the function $U(C)$ and the distribution of ϵ. Therefore we denote this term Q, bearing in mind that the sign of $Q - 1$ is unknown. Now equation (14) can be rewritten as:

$$\frac{U_1(C_1)}{U_1(EC_2)} = \frac{\delta F_1(G) + \frac{Cov}{U_1(C_2)}}{Q} = \frac{\delta F_1(G)}{Q} + \frac{\frac{Cov}{U_1(C_2)}}{Q} \tag{18}$$

This version allows a direct comparison of the rate of decline of marginal utility with or without uncertainty. Comparing equations (15) and (19) we can see clearly what effect uncertainty has on the necessary condition for optimal green expenditure. First $U_1(C_2)$ is replaced by $U_1(EC_2)$. This is a relatively small amendment, as it amounts to using an expected value in place of a non-random value. A more radical effect is seen on the right-hand side of equation (19). The value $\delta F_1(G)$ has been replaced by $\frac{\delta F_1(G)}{Q}$. The size of Q is ambiguous as to its effect because it may be less than or greater than unity. However in a case of great interest, when the downside risk associated with uncertainty is great, we will have $Q < 1$ and this by itself will increase the value of $\frac{U_1(C_1)}{U_1(EC_2)}$ relative to its value without uncertainty. Also we have seen that $Cov < 0$. So this reduces the value of $\frac{U_1(C_1)}{U_1(EC_2)}$ relative to its value without uncertainty.

With this first effect, where the MIE increases the value of $\frac{U_1(C_1)}{U_1(EC_2)}$, we have first period marginal utility larger relative to second period marginal utility. That implies first period consumption smaller relative to average second period consumption. The only way that this higher average rate of consumption can be achieved is by devoting more resources to green investment in the first period. Now even with a strong MIE the right-hand side of (18) remains negative, and this pushes in the opposite direction to the MIE effect, and points to lower resources devoted to green investment. In general the complete consequences of uncertainty are ambiguous.

It is important to be clear concerning the nature of the comparison detailed in the following theorem. We look at two economies which at the same time have reached identical states. They have the same utility functions, the same resources, and the same function $F(G)$. The only difference between the two cases is that in the first case ϵ is a random variable with mean 1. In the second case ϵ always takes the value 1. The question that now arises is how the level of consumption will compare in the two cases, and the theorem provides the key to answering that question.

Theorem 1 *Provided that Q is well below unity, so that the MIE is powerful, then relative to a position with no uncertainty the addition of uncertainty results in the marginal utility of consumption being expected to decline more rapidly than would otherwise be the case. Uncertainty increases the optimal level of green investment.*

Proof: The proof is provided by equation (18).□

Concluding Remarks A few outright climate change deniers reject the claim that there is human-induced climate change. This view can reasonably be ignored. However a more subtle argument accepts that climate change is happening, but argues that its long-term consequences are extremely uncertain. This can lead on to the claim that investing to mitigate an uncertain problem is less worthwhile than investing to mitigate a fairly certain difficulty. We can think of interventions to control greenhouse emissions as current investments. That is a valid way of looking at the issues even when the investment takes the form of moderating or adapting current consumption. We can imagine changes of this type as lowering both the utility generating value of current consumption, and also moderating greenhouse emissions.

The analysis above now allows us to draw a clear conclusion concerning how the uncertainty that attaches to climate change in the future, and hence to the pay-off from policies to control emissions, should affect the level of investment in emission control interventions. The covariance effect will be present, and may push in the direction of reduced investment. Yet huge weight will attach to the insurance effect. If current investments can moderate the horrors that will be experienced should the worst possiblilities implied by uncertainty be realised, then there is an overwhelming case for those investments. The uncertainties that are to be seen where the consequences of carbon emissions are concerned do not support a case for either doing less or for postponing action.

Nordhaus (2013) in his outstanding review of climate change economics refers to a Climate Casino. He means that we face great uncertainty. One can add that uncertainty at issue is not a standard normally-distributed risk, with extreme outcomes, positive or negative having low probabilities. Rather we may face fat tails, meaning that extreme outcomes, especially extreme negative results, may be quite probable. On this see Weitzman (2008) and Pindyck (2011). For instance, global warming might produce the melting of the Siberian permafrost, leading to a huge release of methane, a toxic greenhouse gas. This result is extreme, on the tail of possibilities arranged by probability. Yet it may not be a low-probability outcome. This is a fat tail. The fat tail possibility reinforces the potential power of the MIE. As noted above it would also increase the scale of the covariance effect. Yet it seems reasonable to suppose that the MIE will prove more powerful. Then uncertainty is a reason for more green expenditure, not for less.

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