

### 0.0.1 Prudent Sovereign Debt Borrowing

Christopher Bliss

Nuffield College, Oxford

christopher.bliss@nuffield.ox.ac.uk

#### Abstract

A sovereign borrower is modelled as a rational calculator choosing levels of investment, consumption, and borrowing to maximize the expected level of a valuation function. The borrowing nation knows that it may opt to default on its debt. The lenders understand this and price their loans to fully compensate for the risk of default. A boundary defines a set of states below which the default option will be exercised. Close to that boundary both consumption and investment are suppressed. The influence of the cost of default is most clear via its effects on the marginal valuations of consumption and investment.

Key words: Sovereign debt Default option Optimal growth

JEL Classification F34 F41 G15 G01

## 0.0.2 Introduction

The importance of sovereign debt and sovereign debt defaults does not need to be emphasised. Sovereign debt defaults have generated international crises, as with the Russian default of 1998. It is no surprise then to discover that there is a large literature concerned with sovereign debt. Much of this literature addresses issues outside the scope of the present paper. These include how to predict debt crises, how bankruptcy should be organized, the legal aspects of default, and more besides. These fields apart, it is notable that a majority of the literature, though not all of it, whether empirical or theoretical, looks at sovereign debt from the lender's point of view.

Here the focus is on the borrowing nation and how it decides how much to borrow, how much to invest, and how much to consume. In making those decisions account must be taken of the risk that the nation will be unfortunate enough to default on its loan. Of course while the borrowing nation holds centre stage, the lenders play a role too. But they are modelled in the most simple and formal manner. The world capital market is treated as if it was an individual lender. It is assumed that this actor is risk neutral and has available an unlimited supply of funds at a given rate of interest  $r$ . It is not willing to lend at that rate to any borrower because it is fully aware of the risk of default. Rather the market has full information and can assess precisely the risk of default, when it prices its loans to give an expected return equal to  $r$ . This implies of course that nations in shakey economic circumstances will face a cost of borrowing far higher than the rate  $r$ .

The way in which the lender is modelled here makes it impossible to treat of the important issue of lenders' blame. When default and bankruptcy result it is rather like a road accident. It is rare for one driver to be entirely to blame and the other wholly innocent. Usually blame must be partitioned between the

two parties, and the same is true with sovereign debt default. The example of the economy of Greece illustrates the point. While Greece has not defaulted outright so far, it has had to be rescued from total default by a large infusion of funds, and a severe ‘haircut’ for private lenders, to rescue it from bankruptcy. Here is a clear case where lenders’ blame is involved. The willingness of overseas banks to lend to Greece, taking account only of the denomination of the debt in Euros, and underweighting the financial incontinence of the Greek government, was wildly irresponsible.

On the basis of any reasonable calculation Greece is bankrupt. There is no chance that it can service even its reduced debts. The harsh austerity and huge deflation that it suffers do not help it to do so. Yet there is no formal declaration of bankruptcy, followed by writing down of debts and economic reorganization, as would happen with an individual or a firm. Instead Greece receives a series of bail-out payments to keep it financially afloat. This is not to be kind to Greece, but rather to support the fiction that the large volumes of Greek debt on the balance sheets of European banks are worth something, when without the bail-outs they would not be. Greeks are angry at the moment, and they have every right to be angry, as their interests take second place to the financial politics of Europe.

Another feature that has engaged some of the literature is the term structure of debt, the proportion of long-term debt relative to short-term debt. This is an important area. Generally short-term borrowing is cheaper than long-term borrowing, and this has tempted many borrowing nations to favour the short market. That strategy carries with it a heavy risk, as the nation may be forced to re-finance a large amount of debt at a time of crisis. In the present paper all borrowing is short-term with the loan to be repaid at the start of the following period. This makes for analytical simplicity, as otherwise we would need to model the pricing of debts of different durations.

The legal, empirical, and theoretical aspects of sovereign debt borrowing and sovereign debt default are extraordinarily broad and complex. This is well-illustrated by the monumental survey by Panizza et al (2009). These authors make clear the evolution over recent decades of the legal doctrine of sovereign immunity, and the consequences of hold-out strategies adopted by private debt-holders. The paper also provides extensive references. In the present paper there is no distinction between excusable and non-excusable default, as analysed by Grossman and van Huyck (1989). So the costs of default are the same however it comes about, although bad luck always plays a role. Hilscher and Y. Nosbusch (2010) examines how the pricing of foreign debt is affected by macroeconomic fundamentals. Closest to the present paper is Eaton and Fernandez (1995). Their chapter is far more wide-ranging than the discussion provided here. They do however model sovereign borrowing as an intertemporal optimization problem, they include an incentive compatibility condition for repayment, and they allow for the effects of random shocks. On the other hand, their analysis does not go as deeply as does the present paper into the connection between random shocks, default probabilities, and the optimal choices of consumption and investment given default risk. And the explicit connection between the cost of borrowing and the probability of default, which will be seen below, does not figure in the Eaton and Fernandez chapter.

Arellano (2008) includes in a mainly empirical model several of the features of the theoretical analysis that follows. The paper develops a model to study default risk and its interaction with output, consumption, and foreign debt. The model predicts that default incentives and interest rates are higher in recessions. Gordon and Guerron-Quintana (2013) investigate the effect of the level of capital on a developing country's decision to default. From theoretical analysis and simulation they identify two offsetting effects. On the one hand more capital reduces the likelihood of default. But more capital displays diminishing returns.

[On the other hand more capital increases the attractiveness of autarky. In the model developed below the first effect is replicated. However a fixed cost attaches to default. In that respect our analysis is less rich than that provided by Gordon and Guerron-Quintana.

Niemann and Pichler (2015) presents an elaborate model of optimal fiscal policy in a closed economy with the government having a partial default option. This it exercises only given an adverse productivity shock. The model is calibrated to produce an optimal debt level close to 100 per cent of national output. Fiscal policy is distortionary and implemented under lack of commitment. Debt held domestically is valued as an instrument to smooth consumption, and also as a source of collateral and liquidity. There is a debt Laffer curve that induces government to issue bonds to the point where marginal debt has negative welfare effects. Because it presents a closed economy model, the Niemann and Pichler paper has little in common with the present paper. However a fixed cost of default is common to the two models. How and whether the cost of borrowing increases with the level of borrowing is obscure in the Niemann and Pichler model.

The present paper includes a feature that is hinted at in the literature but is never made precise. This is the exact relationship between the cost of borrowing and the probability of default. It is not enough to say that a higher probability of default will increase the cost of borrowing. The theory of sovereign debt default needs to specify an exact formula to determine the cost of borrowing given the probability of default. Only then can the implications of policy choices be made clear. The formula required is simple and intuitive. Another point left unclarified in the literature, but resolved here, is the issue of when more investment at the margin is justified in conditions of economic crisis.

If there were no uncertainty the model of this paper would be empty. Our borrowing nations do not take on debt that they are certain they cannot repay.

So bad outcomes and default arise only because of random misfortunes. National risk is modelled as a random walk process in which the effects of shocks cumulate into the future. Often random shocks are treated more simply as serially uncorrelated levels of national productivity. The reader may satisfy himself that all the results below hold good with the latter specification.

What is the meaning of the word prudent in our title? The dictionary defines the term as "acting with caution and with consideration for the future". That is precisely what the nation modelled here is assumed to do. It chooses its actions with a complete awareness of the risks involved, weighing benefits against costs. Prudence does not imply the complete rejection of any avoidable risk. If that were the case no prudent person would ever cross a road. No, prudence is reasonable and balanced caution.

### 0.0.3 The Model

National output in period  $t$  is:

$$A_t F(K_{t-1}) \tag{1}$$

where  $K_{t-1}$  is the capital stock chosen in period  $t - 1$ , that is the value that determines output in period  $t$ . The function  $F(K)$  gives gross output including the maintenance of the capital stock. So the first derivative of  $F(K)$  (the gross marginal product of capital) is greater than 1. The variable  $A_t$  measures national productivity at  $t$ . With a low value of  $A_t$  the realised gross marginal product of capital may be less than 1. This would correspond for example with the consequences of a huge earthquake that destroys much national capital. The productivity parameter  $A_t$  evolves over time according to the Markov process:

$$A_t = \epsilon_t A_{t-1} \tag{2}$$

where the  $\epsilon_t$  values are positive and are generated by an i.i.d random process known to the national planner.

National consumption in period  $t$  is  $C_t$ , and  $K_t$  is investment in the same period. Then the budget constraint for period  $t$  is:

$$A_t F(K_{t-1}) + B_t = C_t + K_t + P_t \quad (3)$$

where  $B_t$  is new borrowing in period  $t$ , and  $P_t$  is the repayment of last period's loan in the same period.

The nation adopts a Markov strategy to maximize:

$$E \sum_{t=1}^{\infty} \delta^{t-1} (1 - \pi_t) U(C_t) + \pi_t V^* \quad (4)$$

where  $0 < \delta < 1$ ,  $\pi_t$  is the probability of default in period  $t$ , and  $E$  stands for mathematical expectation. Here  $U(C_t)$  is an increasing strictly concave function. A Markov strategy is a mapping from the state of the economy, which is fully-specified by three values ( $A_t$ ,  $A_t F(K_{t-1})$ , and  $P_t$ ) to  $C_t$ ,  $K_t$ , and  $B_t$ . The infinite summation (4) is terminated in the event of a default. The number  $V^*$  measures the value in terms of units of the objective function of the exercise of the default option. Maximization of (4) is subject to the budget constraint (3).

A sufficient condition for convergence is that the range of the random variable  $\epsilon$  is bounded above at a level smaller than the inverse of  $\delta$ . Clearly weaker conditions would suffice, but their development and justification would take up more space than is justified.

The budget constraint shows the value of  $B_t$  given  $P_t$ . Then  $P_{t+1}$  can be computed from the lender's requirement that the expected return from the loan shall be  $r$ .

$$(1 - \pi_{t+1}) P_{t+1} = (1 + r) B_t \quad (5)$$

If  $\pi_{t+1} = 0$  there is no probability of default, and the nation is borrowing at rate  $r$ . For nations with shaky finances the value of  $\pi_{t+1}$  will be high, and in consequence the cost of borrowing will be high.

#### 0.0.4 When does the Nation Default?

No nation decides to default just because it frees it from the obligation to repay its debt. Default is a costly unattractive option. It is often followed by severe penalties, such as the confiscation of external assets. Also capital markets will be closed to the defaulting nation. Yet history teaches us that default is not always a total disaster the risk of which should be avoided any cost. In some cases a defaulting nation has been able to re-negotiate its obligations with its creditors, and has been re-admitted to the capital markets after a surprisingly short time. That description applies to Russia following its 1998 default. The aftermath of the Argentinian default of 2002 was far more messy and prolonged. The current problems for Argentine are exacerbated by some hedge funds, a minority of bond-holders, who refuse to accept partial repayment<sup>1</sup> and litigate aggressively. This has produced the result that Argentina has again defaulted in August 2014. At the time of writing a US court has found in favour of the hedge funds. This has gained for Argentina readmission to the international capital market. It will need to borrow now to repay its debts to the hedge funds. However many problems created by this situation persist into the future. The world badly needs a system of national bankruptcy rules to enable unmanageable debt to be unwound in a fair and efficient manner. See Bleger (2016). Argentina is an interesting case because it is a serial defaulter, which underlines the point that

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<sup>1</sup>The IMF always does the same by making itself a priority lender. For some reason the IMF is seldom described as a "vulture", a description that has been applied to grasping private creditors.

the world capital market is repeatedly forgiving.

The realities of default are far more complicated than its simple treatment in this paper. Again Panizza et als (2009) provide helpful details of a complex reality. For example, default is typically partial, and often negotiated with lenders before the plug is pulled. The conclusion seems to be that predicting the cost of default is difficult in reality. Below a definite valuation on default is assumed. This could be the borrower's valuation of the lottery that default represents. It is not necessary that the lender should evaluate the costs of default equally, but he should know the borrower's valuation. It is not assumed that default extinguishes all debt, but only that it represents a break in the intertemporal planning that is the maximization of (4).

#### 0.0.5 The Default Frontier

At the start of period  $t$  the level of  $\epsilon_t$  is revealed and the nation observes the state of the economy as:

$$\epsilon_t A_{t-1}, \epsilon_t A_{t-1} F(K_{t-1}) - P_t \tag{6}$$

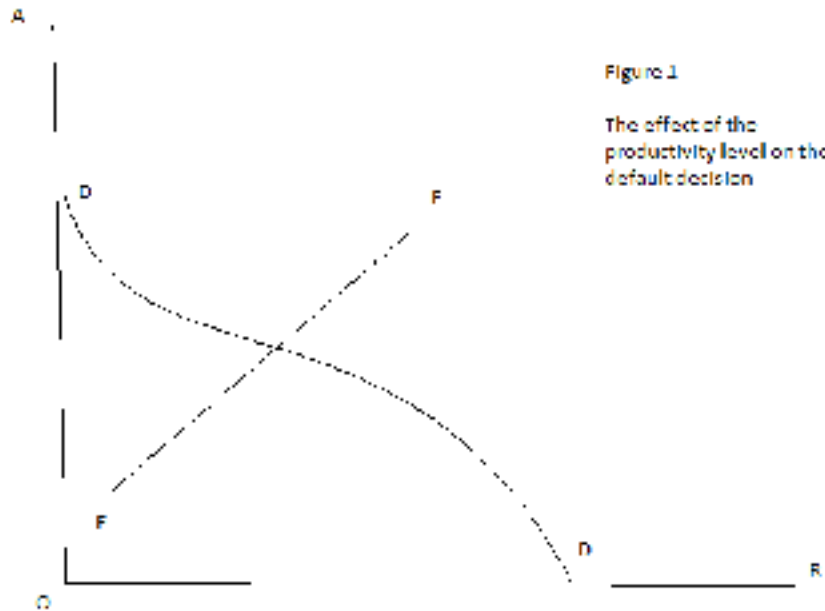
Then there exists a set of these last two values such that default will follow. This set will be closed and bounded above. Default will occur if:

$$D[\epsilon_t A_{t-1}, \epsilon_t A_{t-1} F(K_{t-1}) - P_t] \leq 0 \tag{7}$$

Notice that the arguments of  $D[\cdot, \cdot]$  decline linearly with the value of  $\epsilon_t$ . The value of  $D$  is a level of expected utility looking forward that is so low that the nation prefers default. What do we know about the form of the function  $D[\cdot, \cdot]$ ? In particular what does the upper boundary of the default set defined by equation (7) look like? From the definition of  $D$  as expected utility looking forward it is clear that  $D$  must be increasing in each of its arguments. A higher level of productivity, or a larger level of initial resources, can only increase

expected utility. Hence if we graph the frontier of the set defined by (7) on a space with axes  $A$  for productivity and  $R$  for initial resources, the frontier will be a downward sloping curve.

But what about the curvature of the frontier? Can we show that the curve will be bowed towards the origin, meaning that unbalanced initial conditions cost utility? That is the case with simple consumer preferences, but the issue here is far more complicated. If we take an average of two points in  $(A, R)$  space we can average policy choices in any subsequent state of the world if there is no default. But default may occur, and averaging default probabilities is horribly complicated, if it is feasible at all. Fortunately the exact shape of the default frontier is immaterial given its certain downward slope. As the value of  $\epsilon_t$  declines the state of the economy declines linearly as noted above. Then the line of declining outcomes will cross the default frontier at one unique value of  $\epsilon_t$ . This will be the case whatever the curvature of the default frontier.



The figure illustrates the point. The curve DD shows the default frontier. The line EE shows the path taken by the economy as  $\epsilon_t$  takes various values. Smaller values of  $\epsilon_t$  imply lower points on the EE line closer to the origin. Clearly there can be only one point at which DD and EE meet, and adjusting the curve DD cannot change this conclusion as long as its downward slope at any point is preserved.

### 0.0.6 Policy Decisions and How These Affect the Probability of Default

Consider the nation at the start of period  $t$ . It observes the state of the economy as national productivity, and the total level of resources available, without reference to how these numbers were generated. In a forward-looking analysis the last point is irrelevant. Having observed the economy the nation decides upon the levels of  $C_t$ ,  $K_t$ , and  $B_t$ . These values must comply with the budget constraint (3), written here in a compact form as:

$$R_t + B_t = C_t + K_t \quad (8)$$

From the decisions concerning  $C_t$ ,  $K_t$ , and  $B_t$ , values of  $P_{t+1}$  and  $\pi_{t+1}$  follow. These values will be consistent with equation (5).

In computing the marginal effect of changes in  $C_t$  and  $K_t$  it is assumed with no loss of generality that the marginal change is financed by additional borrowing. Consider a marginal increase in  $C_t$  or  $K_t$ . Then  $B_t$  increases at the same rate as both  $C_t$  and  $K_t$ , and from (5):

$$(1 - \pi_{t+1}) \frac{dP_{t+1}}{dK_t} - \frac{d\pi_{t+1}}{dK_t} P_{t+1} = (1 + r) \quad (9)$$

Or,

$$\frac{dP_{t+1}}{dK_t} = \frac{d\pi_{t+1}}{dK_t} \frac{P_{t+1}}{1 - \pi_{t+1}} - \frac{1 + r}{1 - \pi_{t+1}} \quad (10)$$

Similarly from (5):

$$\frac{dP_{t+1}}{dC_t} = \frac{d\pi_{t+1}}{dC_t} \frac{P_{t+1}}{1 - \pi_{t+1}} - \frac{1 + r}{1 - \pi_{t+1}} \quad (11)$$

Since  $\frac{d\pi_{t+1}}{dC_t} > \frac{d\pi_{t+1}}{dK_t}$ , as will be shown shortly, it follows that  $\frac{dP_{t+1}}{dC_t} > \frac{dP_{t+1}}{dK_t}$ .

Given the nation's choices at  $t$ , what is the probability that it will enter the default set next period? This depends upon the level of  $P_{t+1}$ . Given that value the state of the economy next period will be:

$$[\epsilon_{t+1}A_t, \epsilon_{t+1}A_tF(K_t) - P_{t+1}] \quad (12)$$

and the value of  $D[\cdot]$  will be:

$$D[\epsilon_{t+1}A_t, \epsilon_{t+1}A_tF(K_t) - P_{t+1}] \quad (13)$$

There is a particular value of  $\epsilon$ , denoted  $\epsilon_0$ , which causes (13) to take the value zero. Any value of  $\epsilon$  equal or below  $\epsilon_0$  will induce default<sup>2</sup>. So:

$$D[\epsilon_0A_t, \epsilon_0A_tF(K_t) - P_{t+1}] = 0 \quad (14)$$

Then the probability of default is  $E(\epsilon_0)$ , where  $E(x)$  is the cumulative distribution of  $\epsilon$ , the probability that  $\epsilon$  will take a value less than or equal to  $x$ . So we have:

$$\pi_{t+1} = E(\epsilon_0) \quad (15)$$

And differentiating (15) with respect to  $K_t$  gives:

$$\frac{d\pi_{t+1}}{dK_t} = \frac{dE(\epsilon_0)}{d\epsilon_0} \frac{d\epsilon_0}{dK_t} \quad (16)$$

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<sup>2</sup>It makes no difference whether default follows on the boundary of the default set, or only in its interior. The value of  $\epsilon$  that produces the boundary outcome has zero measure.

The first term of the product on the right-hand side of (16) is the probability density of the distribution of  $\epsilon$  values at  $\epsilon = \epsilon_0$ . Differentiating (14) totally with respect to  $C_t$  and then to  $K_t$  gives:

$$D_1 A_t \frac{d\epsilon_0}{dC_t} + D_2 \left[ \frac{d\epsilon_0}{dC_t} A_t F(K_t) - \frac{dP_{t+1}}{dC_t} \right] = 0 \quad (17)$$

$$D_1 A_t \frac{d\epsilon_0}{dK_t} + D_2 \left[ \frac{d\epsilon_0}{dK_t} A_t F(K_t) + \epsilon_0 A_t F_1(K_t) - \frac{dP_{t+1}}{dK_t} \right] = 0 \quad (18)$$

where subscripts denote partial derivatives.

The evaluation of equation (18) depends upon the sign of the term:

$$\epsilon_0 A_t F_1(K_t) - \frac{dP_{t+1}}{dK_t} \quad (19)$$

The value (19) has a simple interpretation. It is the marginal cost-benefit value of investment at the default point. We call it the MCBV for convenience. As the value  $\epsilon_0$  corresponds to a bad outcome triggering default, it is natural to suppose that the MCBV will be small. That does not imply that this value will be negative.

Inspection of (18) shows that if the MCBV is negative, then  $\frac{d\epsilon_0}{dK_t}$  must be positive. Then from (16) it will be seen that  $\frac{d\pi_{t+1}}{dK_t}$  must be positive. If the MCBV is positive then  $\frac{d\epsilon_0}{dK_t}$  must be negative, and it follows that  $\frac{d\pi_{t+1}}{dK_t}$  will be negative.

Proposition 1 *Provided that the nation is not too far from the default frontier, at the margin additional investment may increase or decrease the probability of default. Which will be the case is determined by the sign of the MCBV term. The change in the probability of default has the opposite sign to the sign of the MCBV term.*□

The demonstration of the proposition is provided by the analysis immediately proceeding its statement. Notice that the qualification in the statement

is required, because if the nation is far from the default frontier the value  $\epsilon_0$  defined by equation (14) might be outside the range of values that  $\epsilon$  can take.

How does the effect on  $\pi_{t+1}$  of a little extra investment compare with the same effect when a little extra consumption is examined? With a marginal increase in  $C_t$  it is again the case that  $B_t$  increases at the same rate as  $C_t$ , but the implication for the following period is different. What can be established is contained in the following proposition.

Proposition 2 *Provided that the nation is not too far from the default frontier, at the margin additional consumption increases the probability of default more than does the same volume of investment. This is the case regardless of the sign of the MCBV term.*

Proof: We have default if  $\epsilon_{t+1}$  takes a value less than or equal to the value  $\epsilon_0$  that satisfies:

$$D[\epsilon_0 A_t, \epsilon_0 A_t F(K_t) - P_{t+1}] = 0 \quad (20)$$

Compare the implications for equation (20) of a given amount of extra consumption expenditure and the same amount of extra investment expenditure. There are two possibilities. The consumption expenditure might cause the second argument of  $D[\cdot, \cdot]$  in (20) for a fixed value of  $\epsilon_0$  to be smaller relative to the value it takes for the same level of investment expenditure. This is natural because the term  $F(K_t)$  appears in that second argument. However that does not guarantee that the said second argument will be smaller with consumption expenditure because consumption might increase  $P_{t+1}$  less than the investment expenditure. If the second argument is smaller in the case of consumption expenditure  $\epsilon_0$  must take a larger value to restore equality in equation (20). This is the same as saying that the probability of default will be larger. In the second case, where  $P_{t+1}$  is increased less by consumption expenditure, it follows

from equation (5) that  $\pi_{t+1}$  will be increased by more by consumption than by investment. Taking the two cases together we reach the conclusion that a fixed extra expenditure increases the probability of default by a greater amount if it is applied to consumption than if it is applied to investment, as required.  $\square$

From Proposition 2 we reach the conclusion that a nation in danger of forced default and anxious to avoid it should moderate consumption and possibly also investment. Investment can be justified at the margin even in desperate circumstances if the MCBV value is positive. It remains to examine the condition of a nation in a state of financial crisis. This means that it is dangerously close to the default frontier. As a consequence the value of  $\epsilon_0$  in equation (14), repeated in equation (20), being the highest value of  $\epsilon$  at which default will occur, is high in the range of  $\epsilon$  values. Then the value of  $E(\epsilon_0)$  in equation (15) is large, and the probability of default is high. The nation faces a stark choice. It must either go for harsh austerity, cutting consumption in particular, and possibly also investment, ‘to the bone’. Or it must borrow to sustain consumption and run a high risk of defaulting next period. In this last case the cost of borrowing will escalate. It will be seen that the nation does not choose to default directly. When default happens it is partly the consequence of heavy borrowing costs.

### 0.0.7 The Effect of the Default Cost on Consumption and Investment

Define  $X(C, K, P_{t+1})$  to be:

$$X(C, K, P_{t+1}) = U[C] + \delta EV [A_t \epsilon_{t+1}, A_t \epsilon_{t+1} F(K) - P_{t+1}] \quad (21)$$

That means that (21) is the average pay-off when consumption and investment values  $C$  and  $K$  are chosen, there is no default, and the loan repayment next period is  $P_{t+1}$ . Now we can express the total expected pay-off as:

$$(1 - \pi) X(C, K, P) + \pi V^* \quad (22)$$

where the subscript  $t + 1$  has been dropped for convenience.

Maximization of (22) by choice of  $C$  and  $K$  requires:

$$(1 - \pi) \left\{ X_C - X_P \frac{dP}{dC} \right\} - (X - V^*) \frac{d\pi}{dC} = 0 \quad (23)$$

and:

$$(1 - \pi) \left\{ X_K - X_P \frac{dP}{dK} \right\} - (X - V^*) \frac{d\pi}{dK} = 0 \quad (24)$$

where subscripts denote partial differentiation, and the arguments of  $X(\cdot)$  have been omitted for convenience.

But from (10) and (11) we have:

$$\frac{dP}{dZ} = \frac{d\pi_{t+1}}{dZ} \frac{P}{1 - \pi} - \frac{1 + r}{1 - \pi} \quad (25)$$

where  $Z$  equals either  $C$  or  $K$ . From this it follows that:

$$(1 - \pi) \left[ X_Z + X_P \left( \frac{d\pi}{dZ} \frac{P}{1 - \pi} - \frac{1 + r}{1 - \pi} \right) \right] = (X - V^*) \frac{d\pi}{dZ} \quad (26)$$

Or,

$$\frac{(1 - \pi) X_Z - (1 + r) X_P}{\frac{d\pi}{dZ}} = X - P X_P - V^* \quad (27)$$

Since  $\frac{d\pi}{dC} > \frac{d\pi}{dK}$ , we must have  $X_C > X_K$ .

Proposition 3 *The marginal valuation of consumption always exceeds the marginal valuation of investment.*  $\square$

At the margin consumption carries with it a higher risk of default, and for that reason is not taken as far as investment. Now inspection of equation (27) shows that an increase in  $V^*$  makes the difference between the two valuations smaller. This is natural. When the penalty for default is smaller ( $V^*$  larger) the relative appeal of investment over consumption, on account of default risk, declines.

Proposition 4 *A larger value of  $V^*$  (a lower cost of default) narrows the difference between the marginal valuation of consumption and the marginal valuation of investment.*□

It is tempting to infer from this last proposition that an increase in  $V^*$ , by raising the marginal valuation of consumption relatively to the marginal valuation of investment, will cause the ratio of consumption to investment to rise. No such inference is valid. The implications of an alteration to  $V^*$  depend upon the second derivatives of the utility and production functions, and for that reason they are inherently ambiguous.

#### **0.0.8 Concluding Remarks**

Our analysis has shown that the computation of a prudent sovereign debt policy with the possibility of default is a complicated exercise. Is it reasonable to suppose that even national treasuries, leave alone unprincipled politicians, can do this job? As with many examples of models of economic optimization, the hope must be that while they do not provide exact descriptions of reality, they may capture some essential features that will be reflected in what actually happens. Indeed some of our conclusions may strike the reader as rather obvious. A case in point would be the demonstration of the riskiness of extra consumption relatively to the riskiness of extra investment. The conclusion may be trivial, the demonstration was not.

The paper places all its emphasis on the decisions of a borrower faced with a perfectly informed, but essentially mechanical lender. In the real world both lenders and borrowers are active players, and the interactions between them form the drama which is the international capital market.

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