0.1. Chapter 5 Competitive Trade Theory

0.1.1. Introduction

Competitive trade theory is the application of the competitive general equilibrium model (the GE model), sometimes called the Arrow-Debreu model, to an international economy. That definition fails to note that the GE model is so general as to incorporate only those assumptions essential to prove the existence of an equilibrium, and as such is to a great extent devoid of specific results. For instance, outside of special cases there are no general GE comparative static results. Those that can be derived depend upon particular restrictive assumptions, such as gross substitutability. Trade theory has followed a very different course. Its leading competitive equilibrium models are so special as to make a GE theorist laugh. Yet such mockery might disguise some envy, for the particular lends to trade theory numerous definite results. Then a critical question is whether special models - they will be called "toy models" later - can give insights concerning a world which in its complexity and elaborate structure resembles more the GE model than a simple classroom trade model. We will return to that difficult question more than once below but without reaching any final or definite conclusion.

This chapter may be taken as a review of generally well-known theory. Indeed major sources for its results are the fine surveys of trade theory provided by Chipman (1966) and Dixit and Norman (1980), as well as other standard sources. Arguably the chapter is too dry and technical to provide much pleasure to my readers. A reader with a basic training in trade theory could well skip it. It is always there to be referred to as required.

New value however may be found here because the argument is tailored to the particular focus of the book. Also, surprisingly, to the author at least, there is some meat to be picked from these old bones. While indirect functions, or duality, have been widely employed in expositing trade theory, the nice insights offered by the mixed price-quantity revenue functions have not been fully exploited. I hope to convince the reader of this point below. The concept of a mixed price-quantity revenue function is the same as the normalized restricted profit function of Lau (1976). That writer noted that such a function permits the east derivation of factor shadow prices.

This is chapter with a large, almost an unwieldy, mathematical appendix. That reflects the fact that this part of economic theory is highly technical in certain respects. However all the technical detail has been banished to the appendix because the most important need is to give a picture of the essentials. Given that picture, the interested reader may pursue the details in the appendix, which it is hoped provides an accurate and more complete account.

Diewert (1982) provides a wide-ranging review of duality theory, with many references and some application to trade theory. See also Woodland (1974). The indirect functions that will used here are the *Revenue Function*:

$$R\left[\mathbf{p}\right] \tag{1}$$

which is the maximum net profit when prices are the vector $\mathbf{p}$. $R\left[\mathbf{p}\right]$ is homogeneous of degree one in $\mathbf{p}$, and weakly convex. As a convex function it is continous and is differentiable almost everywhere. The most important property of $R\left[\mathbf{p}\right]$ is this. Its partial derivatives are profit maximizing outputs. This is an envelope property. If outputs are unaffected by a small change in a price, then it is evident that a small increase in the price of slippers increases profit at a rate equal to the output of slippers. Any substitution of production in favour of more highly valued slippers is a second-order effect and may be neglected.

Another indirect function is the *Expenditure Function*:

$$E\left[\mathbf{p},U\right] \tag{2}$$

which gives the minimum cost of buying a basket of goods with utility at least as large as U when prices are $\mathbf{p}$. The function $E[\mathbf{p},U]$ is increasing in U and is concave in $\mathbf{p}$. The partial derivatives of $E[\mathbf{p},U]$ are denoted $E_p[\mathbf{p},U]$ and are demands when prices are $\mathbf{p}$. This again is an envelope result and is valid because any substitution resulting from the price change is a second-order change.

0.1.2. Two Examples with Results

To illustrate how powerful is the indirect function approach this section offers two examples of the method at work. The first is a straightforward generalization of a familiar textbook result for the one-consumer economy. It is followed by a demonstration of the projection of that result to the many-consumer economy, where it is seen that a far weaker conclusion holds.

1. Gains from Trade The proposition that trade is gainful is one that is demonstrated, usually with a simple diagram, in almost any undergraduate textbook. For the sake of diagrammatic simplicity, but also because the result as stated depends upon that assumption, this is a theorem for a one-consumer economy. Figure 5.1 shows one consumer gaining from trade at fixed prices shown by the slope of the red line TT. To allow for a diagram it is a two-good example for which the result is shown. Here we show a result not of gain as such, but of minimum gain against a hostile price-setter, which the same figure can also demonstrate. The production possibility curve is the heavy blue curve PP. The worst possibility for this consumer is relative international prices shown by the slope of the light blue line NT-NT (NT stands for "no trade"). With those bad prices the consumer consumes at A, and the welfare level is that of the lower green indifference curve. With trade the consumer consumes at B on a higher indifference curve.

By mentally adjusting the position of the TT line the reader may confirm that the consumer can never end up at a lower indifference level than that delivered at A. However when the two lines TT and NT-NT co-incide the consumer consumes at A. In other words, rather than showing that trade is a welfare gain, it is shown that autarky is a welfare minimum. So imagine that a demon, who hates our consumer, can choose international prices to make him as badly off as possible. In minimizing the consumer's utility the demon is constrained by the fact that the consumer can optimize at whatever prices obtain. Also the demon cannot loot national resources, for which reason the consumer's expenditure can be as large as the value of national production. The programme is:

Maximize:

$$-U \tag{3}$$

subject to:

$$E(\mathbf{p}, U) - R(\mathbf{p}) \geq 0 \tag{4}$$

Having the agent maximize $-U$ is the same as writing for him a role in which he minimizes U. The use of the indirect functions automatically incorporates maximization on the part of both the consumer and producers in aggregate. The demon is like a chess player planning a move designed to cause maximum harm to his opponent's position, However the choice of that move must take into account that the opponent will select (or must be assumed to select) the best possible reply from his point of view. The situation is that of a game in which the demon must make Nash-optimal choice of the variables $(U, \mathbf{p})$. He selects those variables in the knowledge that given his choice the agent will choose optimal consumption and production levels given $\mathbf{p}$ and U. Those optimal agent replies are already written into the indirect functions. This is because the best reply on the part of the agent to any prices he may face is to maximize the value of production at those prices, and to minimize the cost at those same prices of attaining the given level of utility U.

Let the Lagrange multiplier for the constraint (4) be λ. Then the Lagrangean to be maximized is:

$$-U + \lambda \cdot [E(\mathbf{p}, U) - R(\mathbf{p})] \tag{5}$$

and the first-order conditions are for respectively U and $\mathbf{p}$:

$$-1 + \lambda \cdot \frac{\partial E(\mathbf{p}, U)}{\partial U} = 0 \tag{6}$$

$$\lambda \cdot [E_p(\mathbf{p}, U) - R_p(\mathbf{p})] = 0 \tag{7}$$

The equation (6) just defines the multiplier λ as the inverse of the marginal international money cost of additional utility. Equation (7) is the key. It says that the two vectors $E_p(\mathbf{p}, U)$ and $R_p(\mathbf{p})$ are exactly equal component by component. That is the same as stating that there is national self-sufficiency (i.e. autarky) with regard to each and every good. The demon does his worst by choosing international prices exactly equal to national autarky prices. The best definition of comparative advantage defines it as a difference between world equilibrium prices when the country of interest engages in trade and the autarky prices of the same country. Notice that this is not a small country definition. Thus the demon in doing his worst to a country denies it any comparative advantage whatsoever.

To extend the analysis to many agents, let individual agents $i = 1, ..., N$ be characterized by individual profit and expenditure functions.$R^i(\mathbf{p})$ and $E^i(\mathbf{p}, U^i)$. Before trade prices are $\mathbf{p}$ and give an autarkic general equilibrium.

$$\sum_{i=1}^{N} \left[R_p^i(\mathbf{p}) - E_p^i(\mathbf{p}, U^i) \right] = 0 \qquad (8)$$

where the U^i values are those of the autarkic equilibrium. After trade world prices are $\mathbf{p}^0$ and:

$$\mathbf{p}^0 \cdot \sum_{i=1}^{N} \left[R_p^i(\mathbf{p}^0) - E_p^i(\mathbf{p}^0, U^{Fi}) \right] = 0 \qquad (9)$$

where the U^{Fi} values are those of the free-trade equilibrium.

The following theorem is proved in the Mathematical Appendix. The proof is much the same as a standard proof of the Pareto efficiency of a competitive equilibrium for a closed economy. While not difficult, the proof takes up too much space to allow it to interrupt the flow of the argument here.

Theorem 5.1 Relative to autarky free trade cannot be Pareto inferior.

Theorem 5.1 contrasts sharply with the earlier demonstration that trade is gainful in a one-agent economy. In that case there is only gain, for the unique agent who must gain. Here at least one agent among many, but it could be only one, may gain. That conclusion might come as a shock to one who has been exposed to simple pro-trade advocacy. That kind of argument often makes freerer trade sound like penicillin, a benign innovation that helps many greatly and harms no-one. Some have talked of trade as a win-win policy; a benefit to all and any. Whatever the facts of particular examples, economic theory does not teach that trade by itself is win-win. Probably no real-life change of any complexity benefits all without exception.

In excluding Pareto inferiority Theorem 5.1 is consistent with a situation in which the benefits from trade are concentrated and the costs are widespread. Yet it is also perfectly consistent with the opposite case in which a few lose but many gain. That was the picture of the abolition of the Corn Laws in midnineteenth-century Britain as painted by the abolition lobby, including notably David Ricardo. A similar case is made today by the more sophisticated advocates of increased trade. Economic theory says only that this too is possible. It might seem that the only general conclusion is the weak non-dominance result of Theorem 5.1; not everyone can lose. For a similar reason economic theory also fails to support the antithesis of win-win optimism: "the rich get richer and the poor poorer" pessimism. General theory cannot adjudicate on the costs and benefits of trade. That requires careful empirical analysis of the particular example.

An easy point to make against the model of trade encapsulated in equations (8) and (9) is that it is static, while the world is dynamic. As it stands this argument is incorrect. The many goods and agents of the model could be dated, which would lend to the model a dynamic (strictly an intertemporal) aspect. The dating of goods is a familiar device of general equilibrium theory. Bread becomes bread delivered, or consumed, at various dates. The dating of agents might bring into account the yet unborn agent. For trade, just as much as many other economic policy choices, affects the as yet unborn. The key issue here is not what can be done formally, but rather what is realistic and useful. To take the unborn first, they cannot represent themselves directly in current markets, not even if those markets are present markets for future goods. So their interests inescapably can only be felt through the altruistic actions of the presently-living.

Among the many problems posed by dated goods is included uncertainty and how to deal with it. There is evidence that typical consumers do not behave rationally under uncertainty. So the model of the rational agent comes under serious stress when uncertainty is involved. In addition futures markets can only cover the known and the standardized, and that must exclude the newly-invented. Such considerations make it clear that the competitive model in its simple form has serious limitations. A more realistic treatment requires the inclusion of missing markets which makes for greater realism but also great complications. For a demonstration of how missing markets can change the gains-from-trade analysis, see Newbery and Stiglitz (1984). In conclusion two points may be noted. First, pointing out problems with the competitive model is easier than building a useful alternative. Secondly, the main problems are inherent in the competitive model in any form; they are not difficulties with its extension to international trade.

2. Reform sequencing Another example of the application of the indirect function approach is provided by the question of whether capital mobility is a good idea for a distorted economy. The distortion here will be the presence of tariffs, presumably arbitrary tariffs not justified by any second-best consideration. The problem goes back to Johnson (1967) whose diagrammatic treatment is generalized here. The model focusses the issue sharply, because what is analysed is a gift of capital, which must be employed in the home country. Obviously if a free gift of capital can be harmful, as will be shown to be the case, an inflow of return-seeking commercial capital would be even less favourable. This analysis also introduces for the first time a simple case of the mixed price quantity revenue function, which will be used extensively below.

The economy is a single-consumer economy, which makes welfare evaluation easy. World prices are fixed, so this is a small country, and their value is the vector $\mathbf{p}$. Domestic prices are $\mathbf{p}+\mathbf{t}$, where the tariff vector $\mathbf{t}$ is not proportional to $\mathbf{p}$, when it would be without effect. They key innovation here is the use of the revenue function:

$$R\left[\mathbf{p}+\mathbf{t}, K\right] \tag{10}$$

where K in the national capital stock, and the dependence of maxmized value on the quantity of capital is made explicit. Of course other factors may be as important as capital. However as these quantities do not vary they are not shown explicitly as arguments of (10). The trade balance is always zero in this model, so that:

$$\mathbf{p} \cdot \left[R_p\left(\mathbf{p}+\mathbf{t}, K\right) - E_p(\mathbf{p}+\mathbf{t}, U)\right] = 0 \tag{11}$$

where the subscripts p denote partial derivatives with respect to the first arguments of these functions evaluated at $\mathbf{p}+\mathbf{t}$. Differentiating (11) totally with respect to K gives:

$$\mathbf{p} \cdot \left[R_{pK}\left(\mathbf{p}+\mathbf{t}, K\right) - E_{pU}(\mathbf{p}+\mathbf{t}, U)\frac{dU}{dK}\right] = 0 \tag{12}$$

The term $\frac{dU}{dK}$ can be negative if either but not both of the other terms within the square brackets are negative when multiplied by world prices. The first such term $\mathbf{p} \cdot R_{pK}\left(\mathbf{p}+\mathbf{t}, K\right)$ can be negative if additional capital expands capital intensive sectors already over-expanded by tariff protection, so that the value of production at world prices is lowered by a gift of extra capital. The second term $\mathbf{p} \cdot E_{pU}(\mathbf{p}+\mathbf{t}, U)$ can be negative if the value of the increase in demand associated with a higher level of utility, which must be positive at domestic prices, is negative at world prices.

The argument just completed indicates that the problem of how to sequence reform is difficult, and that wrong choices can cause harm. The model shows that liberalizing capital movements first when the economy is distorted by tariffs may not be a good thing. Suppose instead that trade is liberalized first with capital movements restricted. Must that be a good thing? The answer is: not necessarily. Capital inflows are a kind of trade, trade with an inescapable intertemporal aspect. So asking whether tariff liberalization with capital restriction must be good is formally similar to asking whether removing some tariffs while others remain in place is always a good thing. There the well-known conclusion is that it all depends. Imagine that the restriction of capital imports, say by policy hostile to foreign capital, has caused the labour-intensive sector to be overexpanded and the capital-intensive sector to be too small. Then it is possible that tariff reductions would further expand the labour-intensive sector and worsen the existing distortion.

Some commentators on economic reform have proposed a simple way round the difficulties exposed above. They advocate a "big-bang" solution to the problem, Everything should be liberalized immediately. Would that solve the problem? Plainly it would if it implied an instantaneous switch to the first-best equilibrium. Sadly things are seldom that simple. In real life adjustment is sluggish. When prices jump to new values, agents behave as if prices had only adjusted partially. That inescapably entails that the problem of reform sequencing reasserts itself. Suppose, for instance, that capital is "quicker off the mark" once reform is implemented, so that perfect capital mobility becomes a fact soon after reform, while production and consumption decisions take some time to respond. Then we are back with a situation somewhat similar to the original difficulty rehearsed at the beginning of this section. Capital mobility is not necessarily a good thing when the economy is distorted by tariff-affected prices. We just substitute a formally-equivalent conclusion. Capital mobility is not necessarily a good thing when the economy is distorted by partial adjustment.

The exercises above are illustrative of method more than final policy conclusions. Let the discussion conclude therefore on a more positive note. If the analysis undermines some simple rules for reform sequencing, it does not therebye imply the conclusion that anything is as wrong as anything else. In any particular case it may well be possible to work out a good approximation to an optimal transit from distortion to liberalization. Our arguments suggest that a somewhat gradual adjustment on all fronts will often be the best choice. The detailed solution must be left to the policy-maker faced with the facts of a particular case. If that policy-maker understands that simple rules are suspect it is more likely that his decisions will be good.

0.1.3. The simple HOS model

The Hecksher-Ohlin-Samuelson (HOS) model is to the international economist what a sharp knife is to a chef. It is a tool used all the time because nothing else does so much so well. In a skilled hand the chef's knife cuts, chops, trims and shapes. In a clumsy hand it causes injury. Similarly, used well the HOS model is an endlessly flexible device for depicting international trade between countries whose comparative advantage differences are modelled, rather than simply assumed. Used clumsily it is as bad as any other model.

It reflects well on the model that its application has tracked long-term developments in the world economy according to how its basic formal structure is filled out. At its birth it was directed to trade between the New World and the Old World, and the factor endowments that differentiated the trading regions were relative supplies of labour and land. Later when cross trade in manufactures became of central importance, the factors became capital and labour. Leontief's famous test of the model, on which see Chipman (1966), was designed to see whether US exports are capital intensive in comparison with its imports. The point is that the US is taken to be well-endowed with capital. Capital has become something with which a nation is endowed, much like land. In the Leontief exercise land, and also climate, is pushed off stage, although in fact the importance of these two actors for US trade cannot be ignored. The consideration of US exports of timber and imports of coffee makes that clear.

The Atlantic trade which the original model was designed to explain was accompanied by a high level of labour migration to the labour-scarce New World. A good part of this migration had political causes (Jews fleeing Russian oppression), or economic catastrophe more than economic migration (famine in Ireland and elsewhere). The model depends upon some barriers to factor migration, for without these differences between relative factor endowments would be obliterated. If factor migration is partial the HOS model is still relevant. In that case, as claimed by Heckscher at the beginning, trade substitutes for migration. Thus, rather than migrating to the Americas, European labour produces labour-intensive manufactures and sends those across the sea. In return the Americas produce land-intensive food which is shipped in the opposite direction. Factor prices in the two regions are brought closer together and the economic motivation for migration is weakened.

Thus incomplete economic migration of factors plays an essential part in the HOS model. Where the trans-national migration of labour is concerned this specification is reasonable. What about the trans-national migration of capital? For the greater part of the post-War years capital mobility was highly restricted. The restrictions encompassed formal legal restrictions. These have relaxed over time but have never wholly disappeared. They have certainly declined in significance in relation to the non-formal barriers to capital mobility. The greatest of these is undoubtedly imperfect information. It is always easier to know the local than to know the distant. And typically agency is not the answer because the missing knowledge that the agent might provide is needed to select the agent. Of course economies of scale apply to information as to other areas. So the large international firm can afford to expend the resources needed to cut through the undergrowth of imperfect knowledge and to take advantage of a high return in a strange and distant country. It is large companies, such as motor vehicle builders, or electronics producers, that have opened up the major possibilities for profitable international investment.

For reasons sketched above the HOS model with factors capital and labour has retained great relevance over many years. However capital has increased its international mobility over time, and this has forced the contemporary economic theorist to choose between two options. One choice is to posit that capital is as mobile as desired, that this effectively equalizes the rate of return between countries, and international differences in technology is what explains variations in real wage rates across countries. In solving one problem that creates another. Why is technology not as mobile, or even more mobile, than capital? Lucas (2002) provides an excellent discussion of the issues. An alternative choice is to posit again perfect capital mobility but to let its role be taken on by another immobile factor. This can be done by having two kinds of immobile labour, skilled and unskilled. That route is taken by the Krugman-Wood model, which is examined below.

0.1.4. Equilibrium in the Simple HOS Model

The model has two sectors each of which has its own constant-returns production function with two inputs. The sectors are called food and machines. The factors are capital and labour. Plainly it makes no difference to the formal analysis what are the inputs and outputs, or how they are labelled. Let the fixed factor supplies of capital and labour be respectively K_0 and L_0. Let output prices for respectively food and machines be p_f and p_m. For such a simple model it is easy to write down the revenue function.

$$R\left(p_f, p_m\right) = \mathrm{Max}_{k,l}\left[p_f F^f\left(k, l\right) + p_m F^m\left(K_0 - k, L_0 - l\right)\right] \qquad (13)$$

where the superscripts on the production functions F show the sector, and k and l are the factors employed in the food sector. Equation (13) shows a feature which has already been exploited in the treatment of reform sequencing above, is general, and will be made use of further below. The maximum output value given product prices depends upon total factor supplies. Thus the left-hand side of (13) can be written:|

$$R(p_f, p_m, K_0, L_0) \tag{14}$$

More of the properties of the function (14) will be elucidated below. We may note immediately that it is homogeneous of degree one in the goods prices, holding factor supplies constant, and in the factor supplies, holding goods prices constant. Then when partial derivatives are single-valued we can apply Euler's theorem and the basic properties of indirect functions to deduce:

$$R(p_f, p_m, K_0, L_0) = p_f y_f + p_m y_m = w_K K_0 + w_L L_0 \tag{15}$$

where the y values are optimal outputs of the two sectors, and the w values are shadow prices of the factors indicated by the respective subscripts. Equation (15) shows an ideal accounting balance. Maximized revenue is the sum at given output prices of the value of both optimal output levels. This sum in turn is equal to the value at optimal factor shadow prices of the two given factor supplies.

Figure 5.2 illustrates the form of the revenue function in the ideal case for the HOS model, with factors capital and labour, and when there are no capital-intensity reversals. The space shown in the figure is a Cartesian space of factor quantities. The blue and green curves are isoquants for capital intensive-machines (shown blue) and labour-intensive food (shown green). Of these isoquants those drawn heavily correspond to \$1m worth of total output for the given output prices. All factor pairs on the part-non-linear, and part-linear, curve RSTU, or its extensions beyond the endpoints R and U, can produce \$1m worth of output. With factor supplies anywhere on the line ST an appropriate average of the supply points S and T, chosen to fully employ those factors by mixing the output levels of food and machines as required. When the ratio of factor supplies lies outside the cone shown by the two black rays from O, the factors can still produce $\$1m$ worth of output, but now this is achieved by employing all the factors in one sector only. This is the case of the specialization of production in one product only.

Notice an important property of the locus RSTU: it is everywhere differentiable. This follows from three of its properties. First, the section ST is linear, and therefore evidently differentiable. Secondly, the sections RS and TU are made up of parts of isoquants, assumed to be differentiable. Finally, and critically, at the points of connection S and T the components of RSTU are *smooth pasted*. This means that the slope of RS at S is the same as the slope of the straight line ST. This follows from the fact that ST is the common tangent to the two isoqants. Similarly the slope of TU at T is the same as the slope of the straight line ST.

Maximizing outputs are shown by the intersection of a ray showing the relative factor endowments of the economy with the locus RSTU. Because RSTU is differentiable it follows that: *value-maximizing outputs are differentiable functions of factor supplies.* It is natural to call these changes *Rybczynski effects* as they are just the changes analysed in the famous paper Rybczynski (1955).

A basic symmetry condition is explored in the mathematical appendix. This is similar to the familar symmetry property of consumer demand functions, according to which $\frac{\partial x_i}{\partial p_j} = \frac{\partial x_j}{\partial p_i}$ where the xs are demands and the ps are prices. The response of outputs to factor supplies is obtained by differentiating $R(p_f, p_m, K_0, L_0)$ first with respect to prices and then with respect to factor quantities. Differentiating the same function first with respect to factor quantities and then with respect to prices gives the same quantitative result. Now however the resulting term measures the response of a factor shadow price to an alteration in an output price. This is a *Stolper-Samuleson effect*, so called after Stolper-Samuleson (1941-42).

0.1.5. Cross Partial Derivatives

The differentiability of outputs with respect to factor supplies implies that a second order partial derivative such as:

$$\frac{\partial^2 R(p_f, p_m, K_0, L_0)}{\partial p_f \partial K_0} \tag{16}$$

is uniquely defined. And in the case of the basic HOS model it will equal:

$$\frac{\partial^2 R(p_f, p_m, K_0, L_0)}{\partial K_0 \partial p_f} \tag{17}$$

This symmetry property, which appears puzzling at first sight, has a clear intuition. If the price of food increases slightly, with all other prices constant, the only effect on the shadow price of capital is the rate at which extra capital leads to more food (or less food as the case may be) being produced. Below in the section of this chapter entitled Generalizations it will be seen that this last result is perfectly general, and always equally intuitive. Here is an instance of the point.

Proposition *The additional quantity of pianos which the economy produces when there is a small increase in the supply of french-polishers is equal to the increase in the shadow-price of french-polishers when there is a small increase in the price of pianos.*

The intuition is the same as before. The shadow-price of french-polishers is the market value of the marginal change in all outputs when there is a marginal increase in their supply. The price of pianos affects only the piano component of this vector. Hence the result.

We can use the equality of (16) and (17) to show conveniently a crucial point in the original Stolper-Samuelson analysis. This is the famous *magnification* property. When protection is removed from the labour-intensive good the price of that same good falls and the wage rate falls even more proportionately than the fall in the output price. Let the price of the labour-intensive good in terms of the other good be p. The price of the other (capital-intensive) good is 1. The effect on the wage of a small change in p is then:

$$\frac{\partial^2 R(p,1,K_0,L_0)}{\partial L_0 \partial p} \tag{18}$$

and this is equal to:

$$\frac{\partial^2 R(p,1,K_0,L_0)}{\partial p \partial L_0} \tag{19}$$

which measures the effect on the output of the labour-intensive good of an increase in the supply of labour. Provided only that both outputs are produced, (19) is positive and the increase in output is more than proportional to the increase in labour supply. The last property follows from the fact that when more labour expands one sector the other sector must contract to provide the expanding sector with the additional capital that it requires. Then (18), which takes the same value, implies that a rise in p raises the wage rate more than proportionately, and that is magnification.

What happens to factor prices when factor supplies change? That involves evaluating terms such as:

$$\frac{\partial^2 R(p_f,p_m,K_0,L_0)}{\partial L_0^2} \tag{20}$$

In that particular case the term (20) is less than or equal to zero. This follows immediately from $R(\cdot)$ being concave in the factor quantities. Note however that this same term may be zero. That will be the case when factor supplies are on the line ST.

0.1.6. Key Conclusions

The time has come to gather together some key conclusions concerning the simple HOS model. Its implications for inequality are examined below in a section devoted to that issue. Here we focus on what the model implies for convergence of prices through trade, especially the prices of factors. There are no large new insights here. The model is old and well-used and anyone who thinks that he has a big new result from the model has surely made a mistake. That said. the emphasis here is different from that which is sometimes encountered in the literature. There has been a tendency to concentrate on the canonical version of the 2X2 model, and then to suggest that matters are much more complex and difficult in broader models. In the canonical model there are no factor intensity reversals and all countries are diversified in production. Then there is factor-price equality between all such countries. In Figure 5.2 all countries are found somewhere on the linear segment ST.

One can characterize these same conclusions in such a way that broader generalization is immediately possible.

- If two countries are sufficiently similar with regard to their factor supplies, but share a common technology, they will almost certainly have the same factor prices. In particular every country diversified in production is in a cone which contains other possible countries with different factor supplies but the same factor prices. The almost certainly qualification above allows for two countries on different sides of a point like S or T in Figure 5.2.

- If the gain-loss evaluation of a change is output prices is calculated from the point of view of pure factor interests, then such a computation always shows that price changes, and therefore trade liberalization, are conflictual. That means that there are both gainers and losers. This is stronger than the demonstration above that trade is a Pareto improvement. That is consistent with all gaining. Now we see that if individuals are associated with the ownership of particular factors in the simple HOS model, one factor or other will lose.

- Changes in factor supplies cause changes in outputs. In the simple HOS model with diversified production, the change of outputs takes place at constant factor prices. In other cases factor prices change.

0.1.7. The Krugman-Wood model

It has been noted already that the designation of the two factors in the simple HOS model has varied over time. Where once the two factors were land and labour, they later became capital and labour. More recently, in an attempt to pin-point a crucial force operating on contemporary international trade, the two factors have become skilled and unskilled labour. The idea is that the most important distinction between the "North" (the rich industrial countries) and the "South" (the less developed countries) is their relative endowments of highly skilled educated labour, on the one hand, and basic labour with few formal skills, on the other. Adrian Wood promoted this approach from the late 1980s. See Wood (1994). Paul Krugman has also used the approach. See for instance Krugman (1993). Certainly other writers have made use of the model, but the short title Krugman-Wood model is convenient.

While most would agree that skill differences are of great importance in today's world, what about other essential differences between trading nations? In particular differences in national endowments of capital and variations of technology come to mind, Wood has argued vigorously that capital is a fully mobile in the modern world, and that technology is equally footloose. A major problem with supposing of perfect capital mobility is that it seems to depend on perfect information, which is a most questionable assumption. It is possible also that some countries have better technological knowledge, or better delivery of technology, than others. Again variations in economic environments, a concept that will be developed below, inhibit the free mobility of the application of technology, if not of pure technological knowledge. These questions will receive detailed examination in later chapters. Chapter 6 will establish a basic framework which will permit trade with skilled and unskilled labour to be analysed together with imperfect capital mobility.

Notwithstanding the doubts just expressed, a version of the simple HOS model with just two immobile factors, skilled and unskilled labour, is attractive. Not only does it permit new insights to be obtained from a model which in its formal aspect is old and familiar. It also produces conclusions which make sense and which indicates directions for policy. In a standard application of the model, two countries (actually groups of countries), North and South, are differently endowed with skilled and unskilled labour, the North being relatively skilled labour abundant. There are two products, called for convenience bicycles and computers. The production of bicycles is unskilled labour intensive. The production of computers is skilled labour intensive.

Now trade between the two countries becomes easier, so that their relative domestic goods prices move closer together. If the two countries are diversified and producing both the products, their respective ratios of skilled to unskilled wage rates will move closer together. That ratio will rise in the North, as bicycles become cheap relative to computers. The same ratio will fall in the South, as the improved trading opportunities raises the relative price of bicycles to computers. Note that under the maintained assumption of perfect capital mobility, the international flow of capital could even be from South to North. That would happen if the computer sector were to be capital-intensive as well as skilled-labour intensive.

This analysis holds out the hope of explaining the growing inequality of incomes that has characterized the North over recent decades, as well as, in an ambitious extension, the so-called "reverse capital flow paradox".

0.1.8. Trade and Inequality in the Competitive Model

A brief examination of the Krugman-Wood model has brought the dry technical analysis of this chapter face to face with the question of income inequality. Inequality is a central issue for this volume. So now is a good moment to look back and ask what competitive trade theory has to say about inequality. To return to a point with which this chapter opened, there are two most different approaches to trade theory. General equilibrium theory is, as its name implies, exceedingly general. Even so it delivers a clear and strong message where inequality is concerned. It says that what individuals get from a market economy is a function simply of the market value of the resources that they own. Those who come armed with valuable resources, be those physical assets or marketable skills, do well. Those who come with little get little. In line with the rich generality of general equilibrium theory the equilibrium may not be unique. So what individuals get from a market economy may depend upon happenstance,

In Chapter 4 we have already seen how the rule that what the agent gets springs from what the agent has applies even to a dynamic growing economy. The market will not iron out initial inequalities in starting positions, not even in the long run. From these ideas there follows a clear corollary: the effect of any change on inequality is determined solely by how that change affects the market value of the resources commanded by specific individuals. As the opening up of trade for whatever reason, reduced protection or lower transport costs, is an instance of economic change, it follows the same rule. Trade can increase inequality if it raises the market value of resources owned by the already-well-off (the skilled in the North in the Krugman-Wood model). Trade can reduce inequality if it raises the market value of resources owned by the badly-off (the unskilled in the South in the Krugman-Wood model).

It is plain that there can be no general mathematical demonstration that freerer trade is good for either the rich or good for the poor. Yet many students of particularly less developed countries will feel that this argument is too abstract and general to capture an important feature of many actual situations. Very often, it will be claimed, less developed countries (LDCs) have adopted policies that restrict trade in a manner that is especially costly to the poor. For this reason the position in the South as depicted by the Wood model is singularly appropriate. Why might that be the case?

The answer would come from the political economy of protection. In most LDCs mechanisms of political accountability, leave alone formal democracy, are weak if not absent. That leaves policy formation in the control of powerful and to a great extent non-accountable groups, and these are typically the rich. It is no surprise then if the type of protection and trade restriction chosen is not too onerous for the rich and falls with a heavy weight on the poor. If that is the case then it follows that a reversal of restrictive policy will impose a cost on the rich and come as a relief to the poor.

That is a neat argument, but is it not met by a telling objection? Suppose that political economy drives trade policy. If restraints on trade in the North harm the rich particularly, as the other side of the Krugman-Wood model indicates, then why do we see some powerful protection in the North, notably for agriculture and fibres? Possibly this asymmetry, that the rich choose policy to suit themselves in the South, but do not do so in the North, may be explained by political accountability again. In the North democracy, in the messy imperfect form that it takes everywhere, is the norm. For that reason protection is often a response to populist pressure or vote seeking. It may then be aimed, or be supposed to be aimed, to benefit certain poor interests. Agricultural protection in the North is the perfect example of this story. It is meant to benefit "poor" farmers, and by protecting these it necessarily anti-protects other activities, and in doing that it impacts negatively on the interest of rich groups. If "big business", meaning here the interests of rich capitalists, really did run things in industrial countries, we would not see heavy agricultural protection.

That said, note that agricultural protection in the North, while it is not good for the rich, is strongly regressive in its effects, so that the cost that it entails is paid particularly by the poor. The single mother in the North, struggling to feed her family, feels the weight of agricultural protection far more sharply than does the rich company executive deciding what to order for his dinner in a fashionable restaurant.

0.1.9. Generalizations

Many of the results shown above do generalize. Some of these generalizations are explored in the appendix. Two prominent instances are listed here.

1. It is generally true that for a full-rank case any national equilibrium will have close neighbours (whether occupied by any concrete country or not) which share techniques and factor prices, though not of course production levels.

2. It is generally true that any small change in factor supplies is almost always associated with a unique vector of output changes (with positive and negative elements).

0.1.10. References

References

[1] Chipman, John (1966), "A survey of international trade theory: Part 3, The modern theory", *Econometrica*, 34 (1), 18-76

[2] Diewert, W.E. (1982), "Duality approaches to Microeconomic Theory", Chapter 12 in Arrow, Kenneth J. and Intriligator, Michael D., *Handbook of Mathematical Economics*, Volume II, North-Holland Publishing Company

[3] Dixit, A.K. and Norman, V. (1980), *The Theory of International Trade: A Dual Equilibrium Approach*, Cambridge University Press

[4] Eggleston, H.G. (1963), *Convexity*, Cambridge University Press

[5] Johnson, Harry (1967), "The possibility of income losses from increased efficiency with capital accumulation in the presence of tariffs", *Economic Journal*, 77, 151-154

[6] Lau, L. J. (1976), "A Characterization of the Normalized Restricted Profit Function", *Journal of Economic Theory,* 12, 131-163

[7] Krugman, Paul (1993), "Trade, Jobs and Wages", *NBER Discussion Paper* 4478

[8] Lucas, Robert (2002), *Lectures on Economic Growth*, Harvard University Press

[9] Newbery, David and Stiglitz, Joseph (1984), "A Pareto Inferior Trade", *Review of Economic Studies*, 1-12

[10] Rybczynski, T.M. (1955), “Factor endowments and relative commodity prices” *Economica,* 22:336-341

[11] Stolper, W.F. and P.A. Samuelson 1941-2, “Protection and Real Wages”, *Review of Economic Studies,* Volume IX, 58-73

[12] Wood, Adrian (1994), *North-South Trade, Employment and Inequality*, Clarendon Press, Oxford

[13] Woodland A.D. (1974), "Demand Conditions in International Trade Theory", *Australian Economic Papers*, 13, 209-224

0.1.11. Mathematical Appendix to Chapter 5

Indirect Functions We have already met the *Revenue Function*:

$$R[\mathbf{p}] \tag{A.1}$$

which is the maximum net profit when prices are the vector $\mathbf{p}$. $R[\mathbf{p}]$ is homogeneous of degree one in $\mathbf{p}$, and weakly convex. As a convex function it is continous and everywhere has directional derivatives. Directional derivatives means that a convex function has left-hand and right-hand derivatives everywhere. See Eggleston (1963). Then a concave function, which is the negative of a convex function, also has directional derivatives. For a convex function, naturally, the left-hand derivative is less than or equal to the right-hand derivative.

This result allows us to talk of the derivatives of a convex function, recognizing that such derivatives may be ranges of values rather than single numbers. That permits a simple statement of the most important property of $R[\mathbf{p}]$. Its partial derivatives are profit maximizing outputs. Then of course when left-hand and right-hand derivatives are unequal, profit maximizing ouputs are non-unique. The partial derivatives of $R[\mathbf{p}]$ are denoted $R_p[\mathbf{p}]$ which is a vector of the same dimension as the number of commodities. with a positive element for a net output and a negative element for a net input.

Another indirect function is the *Expenditure Function*:

$$E[\mathbf{p},U] \tag{A.2}$$

which gives the minimum cost of buying a basket of goods with utility at least as large as U when prices are $\mathbf{p}$. The function $E[\mathbf{p},U]$ is increasing in U and is concave in $\mathbf{p}$. The partial derivatives of $E[\mathbf{p},U]$ are denoted $E_p[\mathbf{p},U]$ and are demands when prices are $\mathbf{p}$.

Where the question of the second order partial derivatives of a convex function is concerned there is no general result for an arbitrary convex function. While a convex function always has first-order directional derivatives, it may not have second-order partial derivatives, not even with the weak directional definition. Nevertheless, such second-order directional derivatives are well-defined for trade models of interest, and in those cases they satisfy the Young's theorem property so that the order of partial differentiation makes no difference to the result. All this is most readily appreciated through the basic 2X2 HOS model as the treatment above has shown.

Gains from Trade Theorem 5.1 Relative to autarky free trade cannot be Pareto inferior.

Proof: Let U_i^A be the utility level of agent i under autarky, and U_i^F the utility of the same agent under free trade. If the theorem is false there must be a case in which:

$$U_i^F \leq U_i^A \tag{A.3}$$

with strict inequality for at least one agent. Then (A.3) implies:

$$E^i(\mathbf{p}^0, U_i^A) \geq E^i(\mathbf{p}^0, U_i^F) \tag{A.4}$$

and because of (A.4) with at least one strict inequality:

$$\sum_i E^i(\mathbf{p}^0, U_i^A) > \sum_i E^i(\mathbf{p}^0, U_i^F) \tag{A.5}$$

Now (A.5) and homogeneity of an expenditure function imply:

$$\sum_i^0 \mathbf{p} \cdot E_p^i(\mathbf{p}^0, U_i^A) > \sum_i^0 \mathbf{p} \cdot E_p^i(\mathbf{p}^0, U_i^F) \tag{A.6}$$

With free trade the balance of payments condition is:

$$R(\mathbf{p}^0) - \mathbf{p}^0 \cdot \sum_i E_p^i(\mathbf{p}^0, U_i^F) = 0 \tag{A.7}$$

Therefore from (A.6) and (A.7):

$$\sum_i^0 \mathbf{p} \cdot E_p^i(\mathbf{p}^0, U_i^A) > R(\mathbf{p}^0) \tag{A.8}$$

As $\sum_i E_p^i(\mathbf{p}^0, U_i^A)$ is a feasible production, (A.8) contradicts the definition of a revenue function.□

Stolper-Samuelson and Rybczynski Consider the technology defined by the revenue function::

$$R(\mathbf{p}, \mathbf{z}) \tag{A.9}$$

where $\mathbf{p}$ is the vector of goods prices, and $\mathbf{z}$ is the vector of factor quantities. The optimal production of the economy, $\mathbf{y}$, is given by:

$$\mathbf{R}_p(\mathbf{p}, \mathbf{z}) \tag{A.10}$$

The shadow prices of the non-tradeables (factors) are given by:

$$\mathbf{R}_z(\mathbf{p}, \mathbf{z}) \tag{A.11}$$

In (A.10) and (A.11) the printing of R bold reminds us that this is now a vector of partial derivatives with respect to $\mathbf{p}$ or $\mathbf{z}$. The changes in production levels caused by marginal changes in factor supplies (Rybczynski effects) are given by elements of the matrix:

$$\begin{bmatrix} R_{p_1 z_1} & R_{p_1 z_2} & \cdot & R_{p_1 z_n} \\ R_{p_2 z_1} & R_{p_2 z_2} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ R_{p_n z_1} & \cdot & \cdot & R_{p_n z_n} \end{bmatrix} \tag{A.12}$$

The changes in factor shadow prices caused by marginal changes in goods prices (Stolper-Samuelson effects) are given by elements of the matrix:

$$\begin{bmatrix} R_{z_1 p_1} & R_{z_2 p_1} & \cdot & R_{z_n p_1} \\ R_{z_1 p_2} & R_{z_2 p_2} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ R_{z_1 p_n} & \cdot & \cdot & R_{z_n p_n} \end{bmatrix} \tag{A.13}$$

With strong differentiability, by Young's Theorem, these two matrices transposed are identical.

Factor-Price Equalization The standard Heckscher-Ohlin result generalizes for the square case as follows. Let:

$$c^i \left(w^1, w^2, ..., w^n \right) \tag{A.14}$$

for $i = 1, ..., n$, be unit cost functions. If factor price equalization does not apply there must exist values $w^1, w^2, ..., w^n$ and $w'^1, w'^2, ..., w'^n$ different from each other such that:

$$c^i \left(w^1, w^2, ..., w^n \right) = c^i \left(w'^1, w'^2, ..., w'^n \right) \qquad \text{A.15}$$

all i.

By the generalized mean-value theorem there exist values $w^{01}, w^{02}, ..., w^{0n}$ denoted by the vector $\mathbf{w}^0$ such that:

$$\mathbf{A} \cdot \mathbf{w}^0 = \begin{bmatrix} a_{11} & a_{12} & \cdot & \cdot & a_{1n} \\ a_{21} & a_{22} & & & \\ \cdot & & & & \\ \cdot & & & & \\ a_{n1} & & & & a_{nn} \end{bmatrix} \cdot \mathbf{w}^0 = 0 \qquad \text{(A,16)}$$

where a_{ij} is the input of factor j into unit production of good i, and the values in the matrix are those for factor prices $\mathbf{w}^0$.

Theorem A.1: If the Matrix A is everywhere non-singular, (A.15) has no solution in distinct w values. That entails global factor-price equalization over all full-rank equilibria.

Proof: Is immediate from (A.16).□

The theorem is not particularly interesting and this for two reasons. First, the high-dimension square case is, as it appears to be, quite special. It may well be that given N factors many countries will produce exactly N goods. Yet these need not be the same N goods in each case, and the factor-price equalization result will apply only between countries which produce the same menu of goods. Secondly, a nowhere-singular property for the unit factor input matrix lacks the simple intuitive interpretation of the same property in the $2X2$ case. There it indicates no factor intensity reversals. But the everywhere non-singular feature in the square NXN case is a spare mathematical property. Just imagine that you are involved in an argument over whether this property will obtain. What kind of points would you make? What evidence would you bring to bear? You might even have the print-out of an observed A matrix, but that is only a point observation, and if that particular matrix should be non-singular, as are nearly all square matrices, what does it prove?

The following theorem, which is a direct generalization of the discussion above of the simple HOS model, is perhaps of more interest. A full-rank equilibrium is one in which all goods are produced and all factors are fully employed. The definition does not require that the number of goods and factors be equal, although that may indeed be the case.

Theorem 5.2: Let E be a full-rank equilibrium with given product prices $\mathbf{p}$. Let the activity levels of the processes producing the outputs be $\mathbf{x}^1$, and let the total factor inputs be $\mathbf{z}^1$. Let $\mathbf{A}$ be the possibly non-square matrix of unit factor inputs per unit output. If $\mathbf{z}^2$ is a non-negative vector which solves the equation:

$$\mathbf{z}^2 = \mathbf{A} \cdot \mathbf{x}^2 \tag{A.17}$$

where $\mathbf{x}^2$ is non-negative, then $\mathbf{x}^2$ maximizes $\mathbf{p} \cdot \mathbf{x}$ subject to factor supplies $\mathbf{z}^2$.

Proof: It must be the case that E defines a linear programme:

$$\text{Max } \mathbf{p} \cdot \mathbf{x} \tag{A.18}$$

subject to:

$$\mathbf{A} \cdot \mathbf{x} \leqq \mathbf{z}^1 \tag{A.19}$$

to which a solution is $\mathbf{x}^1$. If $\mathbf{x}^1$ is not a solution to (A.18)-(A.19) then higher value can be obtained from the same factors either by altering $\mathbf{x}$, or by using a feasible linear activity not included in the rows of $\mathbf{A}$. In either instance E is not an equilibrium, contrary to assumption.

The solution E satisfies:

$$A \cdot x = z^1 \tag{A.20}$$

and:

$$w \cdot A = p \tag{A.21}$$

where w is a vector of shadow prices.

Now by definition, equation (A.17):

$$\mathbf{z}^2 = \mathbf{A} \cdot \mathbf{x}^2 \tag{A.22}$$

and this together with (A.20) and the linear programming duality theorem implies that $\mathbf{x}^2$ maximizes $\mathbf{p} \cdot \mathbf{x}$ subject to factor supplies $\mathbf{z}^2$, as required.□