1. Chapter 6:

2. High Dimension Models[1]

"With three or more factors of production it is certainly not necessary that the result of trade is to make the ratios of factor prices in the respective countries more closely approach unity. Some may do so, but others may diverge depending upon complicated patterns of complementarity and competitiveness." (Stolper and Samuelson 1941-2, p. 72)

2.1. The Structure of High Dimension Trade Models

In Chapter 5 we saw how the Heckscher-Ohlin-Samuelson trade model (HOS for short) has retained its popularity and usefulness for analysing international trade. It has sometimes seemed necessary to adapt it for such application; the work of Paul Krugman and Adrian Wood provides leading examples of how that might be done. See Krugman (1993) and Wood (1994). The analysis of Chapter 5 indicates that some generalizations of HOS are readily available. We saw for instance that there is a general Rybczynski result, which applies to a small country facing fixed prices for tradeable goods. This theorem associates a vector of output changes, both negative and positive with a small change in factor supplies. The same analysis indicates how factor prices will be unaffected by small changes in factor supplies. provided that the vector of factor supplies lies within a cone of

[1] The central analysis of this chapter is based on Bliss (2003). The argument is reworked and extended. The application is new.

diversification, just as in the standard 2X2 HOS model.

It would be an illusion to suppose that this great leap of generality comes without a cost. We obtain some results which evoke the HOS model, yet on closer inspection they can be seen to be only faint images of the clear and robust results with which 2X2 HOS theory rewards us. Thus factor price equalization can be demonstrated, at least for the square case when the number of factors and goods is equal. However the axiom that supports the result lacks a natural economic interpretation, unlike the factor-intensity condition of HOS theory. Similarly, it is plain that if an increased supply of IT specialists requires increases in specific quantities of the outputs of various goods, then the production of those goods in the said proportions may be said to be intensive in the use of IT specialists. Then, as has been seen, should that particular basket of goods increase in tradeable value, then so will the price of the services of IT specialists. What has been lost here is the simple direct intuition of 2X2 HOS theory. There one good is associated uniquely with one factor, as when agriculture is labour intensive. It is not without reason that economists cling to the root 2X2 model.

Yet that model, for all its convenience, often generates doubts and dissatisfaction. Which precisely are the two goods? Which are the two factors? Is it land and labour, as with Heckscher's original analysis? Or is it capital and labour as most textbooks state? If the distinction between skilled and unskilled labour is of crucial importance in the modern world, does that require a three-factor analysis? Or can we follow Wood and assume perfect capital mobility, dropping influences that may spring from differential endowments of physical capital? If we go beyond 2X2 we face the issue of squareness versus non-squareness. Will the number of factors and goods be expanded in parallel, or will one be larger than the other? As the fine survey by Ethier (1984) makes clear, high dimension results mostly depend upon the imposition of some particular structure. Even 2X2 HOS theory needs structure, in the form of unambiguous factor intensity. When we go into higher dimensions more structure and less generality is required. It seems that there are no models available which are both truly general and also useful. So the choice of a model involves pragmatic compromise between the competing demands of realism and utility.

This chapter makes extensive use of a toy model. What is a toy model? It is a simple low dimension model built for a particular purpose, with no pretensions to mathematical generality. The HOS model started life as a toy model, but so much has it become the reference model of trade theorizing that it is hard to see it today as a toy model. Here the model is formally the same as that exposed in Bliss (2003). In that paper the focus is on the development of real wages in Britain during the second half of the nineteenth century. The same toy model structure lends itself readily to the analysis of contemporary situations, including trade between countries differentially endowed with both human and physical capital. Wood (1994) and Barro, Mankiw and Sala-i-Martin (1994), again BMS, are each models with three factors and two goods (Wood), or one good (BMS). The three factors are physical capital and two other inputs. These are labelled either unskilled labour and skilled labour (Wood), or labour and human capital (BMS). The labels chosen are significant. Unskilled versus skilled labour encourages a relatively short-run view of factor supplies in which the balance of the two types of labour is fixed. This is appropriate for Wood's concern with current trading equilibrium in the world economy. Human capital indicates that labour skill can be and will be accumulated, just like physical capital. And indeed the BMS paper is concerned with the long-run and asymptotic convergence.

With two three-factor models in front of us, it seems that the complexities that come with three factors can be surmounted. On closer inspection it will be seen that each of these models prunes the complications that a three-factor specification implies, so that in effect only one or two of the factors play active roles. Wood assumes that physical capital is perfectly mobile, with the consequence that his two unit cost functions include only two prices which vary by country: wage rates for skilled and unskilled labour. To all intents and purposes we have a two factor model. In the BMS model the input of labour is constant, so that this factor adds no extra complexity. Furthermore, physical capital is again perfectly mobile, as it represents perfect collateral. Then, as was shown in Chapter 4, only one type of capital, human capital, is subject to an optimal accumulation condition.. All the intricate analysis in the BMS model is concerned with one factor: human capital.

With greater generality when the HOS model is expanded by including more factors one has to restrict its generality so as to keep it manageable. This is done in the Ricardo-Viner model. Jones (1971) provides one of the most influential expositions. The model has three factors and two goods. Two of the factors are specific to one sector (a different sector for each such factor). Jones assumes that two of the factors are permanently wedded to their two separate sectors. Many later expositions of the model take it that the long-run equilibrium is that described by the 2X2 HOS model. However following a shock the immobile factor (often taken to be capital) is misallocated between the two sectors. Then the specific quantities of immobile capital can be taken temporarily to be entirely different factors, just as Jones assumes.

The two specifications lead to exactly the same model in the short-run. For a lengthier explanation of the relation between the Ricardo-Viner and HOS models see the survey by Jones and Neary (1984). It is better here to stay with the original Jones specification. So the factors specific to certain sectors are permanently confined to those sectors. Their confinement to one sector is not the result of short-run immobility as they are not productive elsewhere. In this model factor-price equalization does not feature, not even in the long-run. That is the feature which Samuelson (1971) uses in his parallel model to support Bertil Ohlin's contention that factor-price equalization will be incomplete. Also, in this type of specific factors model, Stolper-Samuelson magnification cannot be shown.

Ruffin and Jones (1977) is another small-scale (toy) non-square trade model. A useful general discussion of higher dimension models is included. The only model considered in detail is what the authors call the Ricardo-Viner model. This is the same as the Jones (1971) specification. Closer to the model of this chapter is

Ruffin (1981), as that paper addresses the three factors and two-goods case. Its main concern is the effect of a factor supply change on the price of another factor. However the analysis confirms the point already made by Stolper and Samuelson, that patterns of complementarity and substitutability are crucial for results.

The paper which bears most directly on this chapter is Jones and Easton (1983). In fact our toy model is a special case of Jones-Easton. In a two-goods three-factor set up, these authors impose the restriction:

$$\frac{\theta_{31}}{\theta_{32}} > \frac{\theta_{11}}{\theta_{12}} > \frac{\theta_{21}}{\theta_{22}} \tag{1}$$

where θ_{ij} is the share of factor i in sector j. As in the standard HOS model, (1) should be satisfied at all factor prices. For the sake of a specific case, let sector 1 be a high-tech sector (such as computers) and Sector 2 be a low-tech sector (such as saucepans). The three factors are 1 capital, 2 unskilled labour, and 3 skilled labour. Then (1) implies the inequality $\frac{\theta_{11}}{\theta_{12}} > \frac{\theta_{21}}{\theta_{22}}$. That is a natural factor intensity assumption for capital and unskilled labour. Also implied by (1) is $\frac{\theta_{31}}{\theta_{21}} > \frac{\theta_{32}}{\theta_{22}}$. The hi-tech sector uses skilled labour more intensively relative to unskilled labour than does the low-tech sector. We would hardly consider any other specification. And finally $\frac{\theta_{31}}{\theta_{11}} > \frac{\theta_{32}}{\theta_{12}}$, the high-tech sector uses skilled labour intensively relative to capital than does the low-tech sector. These restrictions might be questioned. What matters is whether these restrictions or any similar can lead to definite analytical conclusions.

Jones and Easton show that the strong factor intensity conditions implied by a condition such as (1) leave the analysis of particularly output price changes and factor availability changes more complicated and uncertain than in the basic HOS case. Two observations will illustrate the issues. Suppose that at all factor prices $\theta_{21} = \theta_{22}$. That is the share of unskilled labour costs is the same in both sectors. Consider a relative price change between high-tech output and low-tech, say in favour of low-tech. The standard Stolper-Samuelson argument (now applied to capital and skilled labour) goes through virtually unchanged. The only way the relative unit cost of the low-tech output can rise is if the rental of capital rises and the wage of skilled labour falls. As usual these chnages will involve magnification. The factor price of unskilled labour can do what it likes as any change has no effect on the relative cost of the two goods. Now, clearly, if we relax the special assumption $\theta_{21} = \theta_{22}$, matters are far more complicated. If, for instance, the expansion of the low-tech sector and the contraction of high-tech sector following a goods price change greatly lowers the wage rate of unskilled labour this could assist a relative decline in high-tech unit costs independently of a Stolper-Samuelson effect involving capital and skilled labour.

With the consequences of factor supply changes (Rybczynski effects) the two-good three-factor model is again very different from the HOS case. Take the case of fixed coefficients. Outputs are such as to fully employ factors and over a broad range are independent of goods prices. With two goods and three factors, and again assuming fixed coefficients, outputs are uniquely determined by any two fully employed factors. And only a particular supply of the other factor can be fully employed. Then a change in the supply of any factor will destroy full employment of all factors. If factors are sufficiently substitutable, factor price changes may restore full employment of all factors. But even when that is possible the situation is very different from the Rybczynski situation when factor supply changes are fully accomodated by output changes at constant factor prices.

2.2. The Three Dimensioned Factor-Price Frontier

Consider a country producing both goods, high-tech and low-tech, with unit costs equal to given output prices p_1 and p_2. Let the factor prices of specifically capital, unskilled labour and skilled labour be (w_1, w_2, w_3), and the unit cost function for sector i be $c^i [w_1, w_2, w_3]$. We can now show how our simple two-good three-factor model is consistent with the possibility that the North might have higher unskilled wages than the South, and also a higher rate of return on immobile capital than applied in the South. That case might be thought realistic. If so, it comes to be in the present instance when the North has more abundant skilled labour than the South, and for that reason a lower wage rate for skilled labour. We may think of skilled labour as ingenious Yankees, or abundant land. Then the idea employed here is similar to the Temin (1966) explanation of the apparent paradox that the USA in the 19th century had higher real wages and a higher return on capital than did Britain.

The cost-price equations implied by our assumptions are:

$$c^1\left[w_1, w_2, w_3\right] = p_1 \tag{2}$$

$$c^2\left[w_1, w_2, w_3\right] = p_2 \tag{3}$$

Equations (2) and (3) define an implicit relationship between the w values which is the three-dimensional factor price frontier. Now we differentiate (2) and (3) with respect to w_3, with output prices constant. We take into account that that the partial derivative of a unit cost function with respect to an input price is the factor-input unit-output coefficient. Thus we obtain:

$$a_{11}\frac{dw_1}{dw_3} + a_{21}\frac{dw_2}{dw_3} = -a_{31} \tag{4}$$

$$a_{12}\frac{dw_1}{dw_3} + a_{22}\frac{dw_2}{dw_3} = -a_{32} \tag{5}$$

Or, writing (4) and (5) in matrix form:

$$\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} \frac{dw_1}{dw_3} \\ \frac{dw_2}{dw_3} \end{bmatrix} = - \begin{bmatrix} a_{31} \\ a_{32} \end{bmatrix} \tag{6}$$

Let D be the determinant of the matrix $\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$. Then:

$$D = a_{11}a_{22} - a_{12}a_{21} \tag{7}$$

Notice that the factor shares θ are just the input-output coefficients a multiplied by the factor price w and divided by the output price p. For that reason ratios of input-output coefficients satisfy the same inequality conditions as do the θ values. For this reason, because $\frac{\theta_{11}}{\theta_{12}} > \frac{\theta_{21}}{\theta_{22}}$, as stated above, D must be positive. Now solving the linear equations (6) we obtain:

$$\frac{dw_1}{dw_3} = \frac{-\begin{vmatrix} a_{31} & a_{21} \\ a_{32} & a_{22} \end{vmatrix}}{D} \tag{8}$$

Because $\frac{\theta_{31}}{\theta_{32}} > \frac{\theta_{21}}{\theta_{22}}$, as detailed above, $a_{31}a_{22} - a_{32}a_{21} > 0$. Therefore $\frac{dw_1}{dw_3} < 0$, and we conclude.

<u>Result 6.1</u> *Under the maintained assumptions concerning relative factor intensities, a country which has a high wage for skilled labour will have a low wage for unskilled labour.*

Now solving (6) for $\frac{dw_1}{dw_3}$ we obtain:

$$\frac{dw_1}{dw_3} = \frac{-\begin{vmatrix} a_{31} & a_{21} \\ a_{32} & a_{22} \end{vmatrix}}{D} \tag{9}$$

Because $\frac{\theta_{31}}{\theta_{32}} > \frac{\theta_{21}}{\theta_{22}}$, $a_{31}a_{22} - a_{32}a_{21} > 0$. Therefore with $D > 0$, $\frac{dw_1}{dw_3} < 0$, and we conclude.

<u>Result 6.2</u> *Under the maintained assumptions concerning relative factor intensities, a country which has a high wage for skilled labour will have a low return to capital.*

2.3. Assessing the Results

Results 6.1 and 6.2 are appealing in that they seem to depict a situation similar to reality. Think of the rich industrial world (the North) as well-endowed with skilled labour, and the developing countries (the South) as poorly-endowed with skilled labour. It is plain that the South has a low wage rate for unskilled labour, so in that regard the model does well. It is less plausible to suppose that the South has a low return to capital, as that would lead to a capital outflow in the direction South to North. Something like that does occur. However a long-run equilibrium would not allow of different returns to a mobile factor in different regions.

If capital is perfectly mobile we are back to essentially a two-factor (skilled and unskilled labour) model, as analysed by Wood. Suppose then that technology is everywhere the same, that goods are freely mobile, and if there is no specialization. Then the abundance of skilled labour in the North will have no effect on the unskilled wage rate. Only national production levels will be affected. The North will produce more of the high-tech good relative to the South. It is the Rybczynski story again.

How reasonable are the conditions (1), repeated here for convenience in terms of input-output coefficients?

$$\frac{a_{31}}{a_{32}} > \frac{a_{11}}{a_{12}} > \frac{a_{21}}{a_{22}} \tag{10}$$

Now (10) implies three separate inequalities:

$$\frac{a_{31}}{a_{21}} > \frac{a_{32}}{a_{22}} \tag{11}$$

This says that the high-tech sector uses skilled-labour intensively relative to unskilled labour when compared with the low-tech sector. Below we look at a case in which $a_{32} = 0$ which guarantees the inequality. In general we would hardly consider any other specification, as intensive use of skilled labour is almost a definition of the high-tech sector..

$$\frac{a_{11}}{a_{21}} > \frac{a_{12}}{a_{22}} \tag{12}$$

This says that the high-tech sector uses capital intensively relative to unskilled labour when compared with the low-tech sector. That is a most reasonable condition if only because the high-tech sector makes moderate use of unskilled labour.

$$\frac{a_{31}}{a_{11}} > \frac{a_{32}}{a_{12}} \tag{13}$$

This says that the high-tech sector uses skilled labour intensively relative to capital. Again the condition is guaranteed if $a_{32} = 0$. And it is reasonable even if the low-tech sector makes some use of skilled labour.

In summary, the above inequalities are reasonable mainly because a strong differentiation, in favour of the high-tech sector, with regard to the intensive use of skilled labour, is almost an inescapable feature. Then the intensity of the usage of capital falls in between that for the two types of labour.

At this point we may refer to an issue that must await Chapter 12 for a thorough discussion. It has been seen how with a three-factor model a high wage rate for skilled labour can result in low factor prices for both capital and unskilled labour. Does that high factor price, for skilled labour, need to be the market cost of a standard input? Might w_3 stand for the additional cost imposed on producers by a poor economic environment or mal-functioning institutions? It is an intriguing idea. To make one obvious point, while deferring more extensive argument to Chapter 12, not all poor institutions, such as bad government, can be modelled as a high unit cost for a notional input. Suppose for example that property rights are insecure, and governing party thugs will help themselves to good share of any profit they can see. This is not a problem of elevated unit costs.

2.4. A toy model with skilled and unskilled labour

Despite its elegant construction, the Jones-Easton model is not easy apply because it often leads to complications and ambiguities. So a special case of Jones-Easton generates a toy which more readily provides definite conclusions, but may yet be rich enough to prove interesting. The model supposes a division of production between low-tech goods and high-tech goods, just as Wood assumes. One of the factors, skilled labour, is used exclusively in one sector. In the Jones-Easton model that is equivalent to assuming one of the parameters θ_{32} equal to zero. The model is:

$$c^1 [w_1, w_2, w_3] = p \tag{14}$$

$$c^2 [w_1, w_2] = 1 \tag{15}$$

$$y^1 \cdot \frac{\partial c^1 [w_1, w_2, w_3]}{\partial w_1} + y^2 \cdot \frac{\partial c^2 [w_1, w_2]}{\partial w_1} = K_0 \tag{16}$$

$$y^1 \cdot \frac{\partial c^1 [w_1, w_2, w_3]}{\partial w_2} + y^2 \cdot \frac{\partial c^2 [w_1, w_2]}{\partial w_2} = L_0 \tag{17}$$

$$y^1 \cdot \frac{\partial c^1 [w_1, w_2, w_3]}{\partial w_3} = S_0 \tag{18}$$

where $c^1 [w_1, w_2, w_3]$ and $c^2 [w_1, w_2]$ are respectively the unit cost functions for the high-tech and low tech sectors; w_1, w_2 and w_3 are the factor prices in terms of the low-tech good of respectively capital, unskilled labour and skilled labour; y^j is output in sector j $(j = 1, 2)$; and K_0, L_0, and S_0, are given factor supplies of respectively capital, unskilled labour and skilled labour.

Clearly the high-tech sector is skilled labour intensive relative to the low-tech sector, as it alone uses skilled labour. As the argument above has indicated, it is not completely obvious which sector will be intensive in the use of unskilled labour relative to capital. Above we favoured the assumption that the low-tech sector will use unskilled labour intensively relative to capital. The following exposition assumes that the high-tech sector has the higher capital/unskilled labour ratio. The opposite possibility has to be kept in mind and will be considered again below.

2.5. A separable version of the model

The model (14) to (18) is most easily understood from a special case. As often happens, once the special case has been exposited, it is not difficult to see what more general cases will look like. Suppose that the production function for high-tech output is a constant returns function of the form:

$$y^1 = \phi^1(n^h) \cdot f^h \left[k^h, \ell^h\right] \tag{19}$$

where k^h,ℓ^h and n^h are the inputs of respectively capital, unskilled labour and skilled labour into the high-tech sector. The production function is separable in the sense that the choice of the optimal amount of high-tech labour to use is independent of the other two inputs and their prices.

With the production function (19) we have a unit cost function:

$$c^1 [w_1, w_2, w_3] = c^{11} [w_1, w_2] \cdot c^{12} (w_3) \tag{20}$$

where the two $c^{1i} [\cdot]$ functions in (20) are distinct functions as is indicated by their different arguments. To get from the general to the special model (20) replaces (14) above.

Theorem 6.1 *In the special model factor-price equalization does not necessarily result. If there is any substitutability between unskilled labour and capital, Stolper-Samuelson magnification is a feature of the model, but its impact is moderate relative to the two-factor HOS model.*

Proof: Given relative product price p, unit-cost price equality requires:

$$c^{11}[w_1, w_2] \cdot c^{12}(w_3) = p \tag{21}$$

$$c^2[w_1, w_2] = 1 \tag{22}$$

Given the factor intensity assumption for capital and unskilled labour, if two countries produce both products and share the same value of $c^{12}(w_3)$, factor-price equalization follows from (21) and (22). It is as if the price of the high-tech product in terms of the low-tech product were $p/c^{12}(w_3)$ in a standard HOS model. Equally if the two countries do not share the same value of $c^{12}(w_3)$, it is as if they faced different output prices, and factor-price equalization will not feature.

With Stolper-Samuelson magnification suppose a rise in p, and imagine that the changes which result do not include any alteration in the value of $c^{12}(w_3)$. Then the usual conclusions of the HOS model apply, and the real wage of unskilled labour in terms of either product will fall. However if there is any substitutability between the capital and unskilled labour inputs, the high-tech sector will increase in size; the marginal product of skilled labour w_3 will rise; and the high-tech sector will experience a rise of its unit cost function in terms of w_1 and w_2. The final effect is that p has risen, but so has w_3. Then p/w_3 has gone up but by less than the rise in p. Note that w_3 increases only because p/w_3 has risen, so the rise in w_3 cannot in a greater percentage than the rise in p. It is as if the rise in p were more moderate than its true arithmetic value. Because magnification does not depend upon the size of the increase in p, magnification will still follow.□

2.6. The model without separability

The discussion of the separable model, and the proof of the theorem makes clear why greater complications will be encountered is separability is not assumed. Suppose one tries to push through a similar argument to the theorem using the non-separable cost-price-equality equations (14) and (15), rather than (21) and (22). So long as w_3 is not altered, everything is as standard HOS reasoning. With the assumption that the high-tech sector is capital intensive, an increase in p will cause w_1 to rise, and with some substitution the high-tech sector will expand.

The expansion of the high-tech sector increases the demand for skilled labour, and for that reason w_3 will rise to cut back demand to equality with the fixed supply. Now, without separability the change in w_3 has a differential effect on the marginal attractiveness of the other two inputs, capital and unskilled labour. Then Stolper and Samuelson's "complicated patterns of complementarity and competitiveness" make themselves felt.

Thus suppose the unlikely case that unskilled and skilled labour are close substitutes. So long as w_3 is constant the model will be essentially the familiar 2X2 HOS model. A rise in p will expand the high-tech sector, and w_3 will rise, just as in the argument above. Now the rise in w_3 will cause the substitution of unskilled for skilled labour and w_3 will fall back somewhat. Taking all effects into account the conslusion is that w_3 will rise by less than in the separable case. That is a sufficient weakening of the previous argument to upset the conclusion that magnification will be present although moderated.

2.7. The capital intensity assumption reversed

The argument above has throughout employed the assumption that the high-tech sector has the higher capital/unskilled labour ratio. The opposite assumption is possible if not wholly convincing. There is no need however to rehearse the entire analysis again with the assumption reversed. The method has been demonstrated and the interested reader can work out what happens with a different assumption.

That is not to say that the assumption is inconsequential for the qualitative conclusions.of the model. Take the basic instance of magnification in the separable model. Now the low-tech sector has the higher capital/unskilled labour ratio. Suppose again a rise in p, the relative price of the high-tech good. With some substitutability the high-tech sector will expand, but in this case the Stolper-Samuelson effect is a magnified fall in the return to capital and a rise in the unskilled wage rate. Even so the expansion of the high-tech sector will increase the demand for skilled labour. Once more this is equivalent to a moderation of the price rise, hence a moderation, but not the cancellation of magnification.

2.8. The separable model depicts North-South trade

Our model is a toy because it lacks mathematical generality. The term is not intended to indicate that the model is not to be taken seriously. On the contrary, it will be argued that the model is useful for the analysis of trade between North and South in a globalized world. In particular this toy model performs better than either the standard HOS model, or Adrian Wood's adaptation of that model.

Here the North is a rich country well endowed with capital in both its forms. That is to say the North is relatively well-endowed with both physical and human capital, where the latter is identified with skilled labour. The South, on the other hand, is poorly endowed with each kind of capital, and therefore relatively well-endowed with unskilled labour. Other assumptions are standard and the same as in the HOS model. Thus technology is the same in both countries; all factors are internationally immobile; but goods are perfectly mobile, so that the relative price p is the same in both countries. Finally, for the sake of the present argument, the high-tech sector has the larger ratio of capital to unskilled labour at all factor prices.

Suppose a liberalization of trade that equalizes p in both countries. Previously p would have taken a higher value in the South, because the local factor endowments entail a scarcity of the high-tech good, and trade restrictions inhibited imports. Similarly p would previously have taken a lower value in the North, because the local factor endowments entail a relative scarcity of the low-tech good, and trade restrictions inhibited imports.

Now Theorem 6.1 above tells the whole story. First, and crucially, factor price equalization does not apply. This is a welcome and agreeable conclusion. It is always a cause of embarrassment that a model such as the 2X2 HOS model, which starts with assumptions that are by no means absurd, should arrive at conclusions that are patently unrealistic. Chief among these unrealistic conclusions is the equalization of real wages in all trading countries. Of course no-one would take the HOS assumptions to be exactly correct. And in Chapter 5 above we have noted numerous instances in which factor price equalization may not apply even given HOS assumptions, or their generalizations. Even so, real wages vary hugely between trading nations, and even between trading nations that are apparently diversified in their productions. A model that depicts such an outcome as only natural given varied endowments of two types of capital is appealing.

Next note that Theorem 6.1 describes what happens when p changes, either up or down. What it shows is magnification just as with the Stolper-Samuelson result in the 2X2 HOS model, but moderated in its extent. To be specific, consider the North as trade expands. In the North p rises, which is the same as saying that the relative price of the low-tech good falls. The table shows what happens to each of the factor prices, all measured in terms of the low-tech good. The directions of the price changes are indicated by arrows. An arrow pointing upwards indicates an increased factor price. An arrow pointing downwards indicates a lower factor price.

Factor	Capital	Skilled labour	Unskilled labour
Factor Price change	↑	↑	↓

Table 6.1 Effects on real factor prices of a rise in p.

2.9. Trade and Inequality

Table 6.1 is drawn up from the point of view of the North, following a rise in p. It can be used to tell the story of the South, where the opening up of trade will cause p to fall. In that case obviously the same table applies with the direction of each arrow reversed. If we ignore the column headed Capital, the model tells essentially the same story as that related by Adrian Wood. In the North the wage rate for skilled labour rises; the wage rate for unskilled labour falls. If those were the only changes they would amount to an unambiguous increase in inequality, not unlike the changes that have been observed in northern countries (notably the USA) in the last two decades. Now consider how that account of the change in inequality caused by the opening up of trade has to be modified by taking into consideration the arrow on the left of the table, which shows the change in the return to capital. That return rises.

It is always the case that the implications of trade for income distribution are determined by two separate facts. First, how are factor prices altered? Secondly, what is the pattern of factor ownership between the various households in the economy? For an empirical analysis of the effect of trade accounting for both these influences, see Bourgignon (1989). Where the particular present instance is concerned, trade liberalization and the North, the issue seems to be plain. While the ownership of capital may be widely dispersed, it is unequally distributed, and large holdings of physical capital are highly correlated with substantial holdings of human capital. Human capital is the same as skilled labour in the present model. Therefore it follows that the rise in the return to capital will only accentuate the increase in inequality in the North already detailed by Wood.

Turning to the South, the story is again just the reverse of what happens in the North. The relative price p falls rather than rising. And the consequences can be read from Table 6.1 when all the arrows have their directions reversed. In the South, as in the North, large-scale ownership of physical capital and large-scale ownership of skilled labour are highly correlated. For this reason the changes that bring increased inequality to the North are reversed in the South, where they bring reduced inequality. The picture seems to be quite similar to the conclusions of Wood's model.

On closer inspection there are differences between Wood's two-factor HOS-style model and the present three-factor separable model. First the magnification which features in any HOS model, and hence in Woods version of the same model, is present in the separable model, but is moderated in its magnitude. More important, possibly, the separable model allows more scope for different conclusions as its assumptions are varied. For instance, we have concentrated on the assumption that the high-tech sector has the higher ratio of capital to unskilled labour. The opposite assumption cannot be dismissed as absurd. What happens if we take that case? It is equivalent to re-labelling two of the factors: capital becomes unskilled labour; and unskilled labour becomes capital. Then Table 6.1 is replaced by Table 6.2 below.

Factor	Capital	Skilled labour	Unskilled labour
Factor Price change	$\downarrow$	$\uparrow$	$\uparrow$

Table 6.2 More effects on real factor prices of a rise in p.

Suddenly the clear picture of the change in income distribution is replaced by a fog. In the North unskilled labour is now better off. What about the rich household that owns both capital and skilled labour? It loses and it gains. It loses because the return to the physical capital that it owns goes down. But it gains because the wage rate of its skilled labour goes up. The final implications for the income of any particular household depend upon the numbers concerned, on precise quantities and exact changes in factor prices. Over the whole income distribution anything is possible. It could happen, for example, that rich rentiers, owning mainly physical capital, will lose, while middle-class professionals, owning mainly skilled labour, will gain.

As long as we stay with the root assumption that only the high-tech sector uses skilled labour, and skilled labour enters separably into production, the above is as far as varying assumptions in the model can take us. However matters become even more complicated if the assumption of separability is relaxed. Then we let loose Stolper and Samuelson's complicated patterns of complementarity and competitiveness. There is no need to dive into these murky waters. It is enough to conclude that in general the consequences of trade liberalization in a high dimension world allow for a rich variety of possibilities.

2.10. Labour migration in the separable model

It is claimed above that it is a definite advantage of the separable three-factor model that it does not give us factor-price equalization. That is because factor-price equalization looks distinctly unrealistic. Also unrealistic, possibly, is the Rybczynski theorem which is feature of the HOS model. The Rybczynski theorem leads to the conclusion that migration of factors is unnecessary, because we have factor-price equalization. But should it happen it is innocuous. The outputs of the two sectors in the country receiving a factor inflow adjust to absorb the increased supply at constant factor prices, and existing residents are unharmed.

This comforting picture does not hold with the separable three-factor model. Take a migration into a small country of skilled labour. If the result were to be as the Rybczynski account, there would be no effect on factor prices; certainly no effect on w_1 and w_2. Suppose, to simplify the argument, that there are fixed factor proportions in both sectors where capital and unskilled labour are concerned. Then the cost-price equality equations take the form:

$$a_k^1 w_1 + a_u^1 w_2 + a_s^1 w_3 = p \tag{23}$$

$$a_k^2 w_1 + a_u^2 w_2 = 1 \tag{24}$$

where $a_j^{1 \text{ or } 2}$ is the unit input of factor j ($j = k$, u or s) into the sector indicated by the superscript. The fixed factor proportions assumption means that of all the a coefficients in (23) and (24) only a_s^1 can vary. The consequence will be that with fixed coefficients, and so long as production remains diversified, the full-employment conditions for the two factors capital and unskilled labour determine the outputs of the two sectors, and these cannot be influenced by the supply of skilled labour. For that reason the size of the high tech-sector will not change. Therefore a_s^1 must rise so that the high-tech sector can absorb the increased supply of skilled labour. That can only happen because of a fall in w_3, the price of skilled labour services. A rise in a_s^1 and a fall in w_3 has an ambiguous implication for the product $a_s^1 w_3$, it depends upon the elasticity of demand for skilled labour.

Take first the case when the demand for skilled labour is inelastic. Then when the supply of skilled labour increases $a_s^1 w_3$ falls. This, as inspection of equation (23) shows, is equivalent in its effects on the prices w_1 and w_2 to a rise in p. Then with the high-tech sector capital intensive the Stolper-Samuelson theorem implies that w_1 will rise and w_2 will fall. From the point of view of both kinds of labour in-migration of skilled labour is not innocuous. It entails a decline in real wages. Plainly the other case, when the demand for skilled labour is price elastic, are in part the mirror image of the inelastic case. The effects on the prices w_1 and w_2 are equivalent to a fall in p. Now w_1 will fall and w_2 will rise. Migration of skilled labour into the country is bad for capital and good for unskilled labour. A nice case is when the demand function for skilled labour has unit elasticity. Then skilled labour is harmed, because w_3 falls, but there are no knock-on effects on the other factors.

As often happens, the fixed-coefficient case gives a good idea of what to expect in a more general model with factor substitution. With the above analysis consider what will occur if the ratio of capital to unskilled labour can vary in each sector according to just the two prices w_1 and w_2. So we retain the separability assumption, and w_3 does not affect the optimal choice of the capital to unskilled labour ratio. Now an increase in S_0 will cause w_3 to fall, and $p - a_s^1 w_3$ will change according to the elasticity of demand for skilled labour. Suppose an inelastic demand for skilled labour, so that the effect on the other factor prices is equivalent to a rise in p. So w_1 rises and w_2 falls. The high-tech sector expands and both sectors become less capital intensive. The substitution of unskilled labour for capital releases just enough capital to balance the increased demand for capital caused by the expansion of the high-tech sector. All this is as standard HOS analysis. The implications for income distribution are the same as with fixed coefficients. An inflow of skilled labour harms both kinds of labour and is good for capital owners.

What happens if migration takes the form of an inflow of unskilled labour? If the supply of skilled labour was not a problem we would be back to standard Rybczynski theory. There would be no effect on factor prices because the low-tech sector would expand, and the high-tech sector would contract to absorb the increased supply of unskilled labour. In the general model a similar story applies. But now the contraction of the high-tech sector causes a fall in the demand for skilled labour, the price of which falls. What that implies depends again upon the elasticity of demand for skilled labour. If, as seems likely, the demand is inelastic, then $a_s^1 w_3$ will fall. That, as we have seen, is equivalent to a rise in p. One could say that the Rybczynski effect induces a Stolper-Samuelson effect. A rise in p raises w_1, the return to capital, and lowers the unskilled wage rate w_2. An inflow of unskilled labour harms all workers and is good for capital.

2.11. Concluding remarks

Our point of departure for the present chapter is the numerous difficulties with the simple 2X2 HOS model. That model has dominated trade theory for the last 60 years. Yet its realism must be seriously questioned. It can yield unrealistic results, such as factor price equalization, or the Rybczynski account of factor migration. A model with only two goods evidently involves a high level of aggregation, but that may serve well where trade between two countries, or types of countries is the focus of analysis. Having only two factors of production is more problematic. It is not clear which those factors should be. and having three factors, but allowing perfect mobility for one of them, as Wood does, may not be realistic.

An unrestrained launch into high-dimension trade theory however may not bring us applicable results. Those models yield few theorems with easy insights attached. And the reader may be choked by a complex taxonomy of cases depending upon the exact specification of the patterns of complementarity and competitiveness to which Stolper and Samuelson made reference long ago. The chapter has shown how a very limited step in the direction of generality; just one extra factor, and that employed separably in one sector, gives a model with many attractive features. In particular it looks more suitable for depicting trade between strongly different regions. It does not need specialization or factor intensity reversals to depict economic inequality. It replicates the magnification result of HOS theory but shows how that effect is moderated by a price change for the third factor. It allows a richer menu for the possibilities that arise when trade between North and South is liberalized without making that analysis too horribly complicated.

Over sixty years ago Samuleson and Stolper wondered why labour supports protection when economic theory seems to say that it is harmful. Much more recently many economists have wondered why migration of labour into industrial countries can evoke fierce popular resistance, although much analysis seems to suggest that it is beneficial in its economic effects. One answer serves for all such examples: reality is more complicated than simple models. Such was Stolper and Samuelson's analysis. They argued that views on protection are not determined solely by judgements of its efficiency. It is essential that its consequences for income distribution be taken into account. Migration in all its implications is hugely complicated. It involves disparate economic effects, and it has powerful cultural and political implications.

That said, our separable three-factor model already shows how migration may have very different consequences for factor interests than the usual Rybczynski analysis would indicate. Depending upon the precise assumptions, factors already resident in the country can lose when labour migrates in. It is no surprise that labour of the same type as the migrants always loses. The other type of labour may lose or gain, as may capital. Here the relatively simple model confirms what intuition would suggest for more complicated cases. Few would doubt that the considerable flow of violinists from the ex-Soviet-Union to the West has made life tougher for western violinists. That particular group represents a negligible political weight. Where mass migration is concerned the political effects can be huge. In France they toppled a left-leaning government. Of course while economic arguments of the "stealing our jobs" variety figured in the debates of the time, they were diluted considerably by claims to do with national identity, not to speak of chauvanism and racialism.

References

[1] Barro, Robert J. G. Mankiw and X. Sala-i-Martin (1995), "Capital Mobility and Economic Growth", *American Economic Review*, 85, 1, March, 103-115.

[2] Bliss, C. (1998) "The Corn Laws and the CAP", in Gary Cook, ed., *The Economics and Politics of International Trade*, Vol. 2, Freedom and Trade, Routledge, 1998, pp.148-165

[3] Bliss, C. (2003) "A Specific Factors Model with Historical Application", *Review of International Economics*, Volume 11, Issue 2, May

[4] Bourgignon, Francois and Morrisson, Christian (1989), *External Trade and Income Distribution*, OECD, Paris

[5] Ethier, Wilfred (1984), "Higher dimensional issues in trade theory", in R.W. Jones and P.B. Kenen, eds., *Handbook of International Economics*, Volume I, North-Holland

[6] Jones, R.W. (1971),"A three-factor model in theory, trade and history", in Bhagwati, J.N. et als, eds., *Trade, Balance of payments and growth*, North-Holland Publishing Company

[7] Jones, Ronald and Easton, Stephen T. 1983, "Factor intensities and factor substitution in general equilibrium", *Journal of International Economics*, 15, 65-99

[8] Jones, R.W. and J.P. Neary 1984, “Positive Theory of International Trade”, in R.W. Jones and P.B. Kenen, eds., *Handbook of International Economics*, Volume I, North-Holland

[9] Paul Krugman and Robert Lawrence (1993) "Trade, Jobs, and Wages," NBER Working Papers 4478, National Bureau of Economic Research

[10] Ruffin, Roy J. 1981, “Trade and factor movements with three factors and two goods”, *Economics Letters*, 7, 177-182

[11] Ruffin, Roy and Jones, Ronald 1977, “Protection and real wages: the neo-classical ambiguity”, Journal of Economic Theory, **14**, 337-348

[12] Rybczynski, T.M. (1955), ‘Factor endowments and relative commodity prices’, *;ica,* 22:336-341

[13] Samuelson, Paul, A. (1971) “Ohlin was Right”, *Scandinavian Journal of Economics*, Vol. 73, No.4, 1071, 365-384

[14] Stolper, W.F. and P.A. Samuelson 1941-2, “Protection and Real Wages”, *Review of Economic Studies,* Volume IX, 58-73

[15] Temin, Peter (1966),“Labour Scarcity and the Problem of American Industrial Efficiency in the 1850s”, *The Journal of Economic History*, September