

	D ₁	D ₂	D ₃
English	1	0	0
Welsh	0	1	0
Scottish	0	0	1

Interpretation

For English: D₁=1, D₂=0
 $Y = a + b_1 + \dots$

For Welsh: D₁=0, D₂=1
 $Y = a + b_2 + \dots$

For Scots: D₁=0, D₂=0
 $Y = a + \dots$

Three-category 'parent' \Rightarrow two dummies, since

$$D_3 = 1 - D_1 - D_2$$

estimate:

$$Y = a + b_1D_1 + b_2D_2 + \dots$$

Effects relative to omitted category.

Pattern of effects is **not** affected by choice of omitted category.

Returned significance contrasts are

(above: whether English & Welsh significantly different from Scots, not from each other)

Interactions

Consider X_3

Assume its effect differs by country

Make

$$Z_4 = D_1 \times X_3$$

$$Z_5 = D_2 \times X_3$$

Estimate

$$Y = a + b_1 D_1 + b_2 D_2 + b_3 X_3 + b_4 Z_4 + b_5 Z_5 + \dots$$

Consider

$$Y = \dots + b_3 X_3 + b_4 Z_4 + b_5 Z_5 + \dots$$

$$Y = \dots + b_3 X_3 + b_4 D_1 X_3 + b_5 D_2 X_3 + \dots$$

For English: $D_1=1, D_2=0$

$$Y = \dots + b_3 X_3 + b_4 X_3 + \dots$$

For Welsh: $D_1=0, D_2=1$

$$Y = \dots + b_3 X_3 + b_5 X_3 + \dots$$

For Scots: $D_1=0, D_2=0$

$$Y = \dots + b_3 X_3 + \dots$$

Recap:

$$Z_4 = D_1 \times X_3$$

$$Z_5 = D_2 \times X_3$$

$$Y = a + b_1 D_1 + b_2 D_2 + b_3 X_3 + \\ b_4 Z_4 + b_5 Z_5 + \dots$$

Interpretation:

For English, unit change in X_3 gives
expected change in Y of $(b_3 + b_4)$

For Welsh, unit change in X_3 gives
expected change in Y of $(b_3 + b_5)$

For Scots, unit change in X_3 gives
expected change in Y of (b_3)

Multiplicative interactions

Consider X_1 and X_2

Make $Z_3 = X_1 \times X_2$

estimate

$$Y = a + b_1 X_1 + b_2 X_2 + b_3 Z_3 + \dots$$

$$Y = a + b_1 X_1 + b_2 X_2 + b_3 X_1 X_2 + \dots$$

$$Y = a + b_1 X_1 + b_2 X_2 + (b_3 X_1) X_2$$

$$\partial Y / \partial X_2 = b_2 + (b_3 X_1)$$