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Incentives, Compensation, and Social Welfare

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Alternative wage structures under conditions of moral hazard are analysed from a social welfare standpoint. It is argued that ex post equity judgements in an uncertainty context should incorporate a preference for "positive correlation" of utilities of different individuals. In the design of compensation schemes, this may give rise to a conflict between ex post equity objectives and the need to provide effort incentives: relative performance clauses in compensation schemes that are useful for providing incentives are undesirable from an ex post equity standpoint. This is demonstrated by showing (a) in a context of independent production uncertainties, every rank-order tournament is welfare-dominated by a set of independent (randomized) contracts, and (b) welfare-optimal compensation schemes in general depend separately on an equity and an incentive component that tend to correlate agent compensations in different directions.

1. INTRODUCTION

A central question in welfare economics concerns the design of a system of rewards for productive effort that balances distributive considerations with the provision of effort incentives. This issue has usually been studied in the context of a model due to Mirrlees (1971) in which the conflict between distribution and incentives arises because the social planner lacks perfect information about the abilities of individuals. In this paper we analyse a model in which individual abilities are publicly known; a conflict between distribution and incentives arises instead from the existence of uncertainty in production processes and a moral hazard problem stemming from the unobservability of workers' effort.

Most analyses of moral hazard have concentrated on the tradeoff between insurance of agents against wage fluctuations and provision of effort incentives. The context is generally taken to be a capitalist firm where the self-interested "planner" has a residual claim on workers' output.[1] In other settings, such as public-sector or labour-managed firms, the planner may have preferences over the distribution of ex post welfare among the workers. This paper analyses how, with such preferences, effort incentives under moral hazard are constrained both by insurance and by distributive considerations.

In Section 2, we suggest that a concern for ex post equality of individual welfares can be expressed by a preference for positive correlation among welfare levels. We then consider what properties of the ex post welfare function incorporate this preference for correlation. Welfare functions that are additively separable across individuals are inappropriate, no matter how concave. With them, expected value depends only on the marginal distributions of individual welfare, regardless of correlation properties of the joint distribution. We propose instead that welfare functions satisfy a *complementarity* condition, so that the welfare benefit from increased utility of one individual is increasing in the income levels of other individuals. Whereas concavity properties of the welfare function may be sufficient for judging inequality in ex ante utility levels or in certainty contexts, complementarity properties are needed to judge ex post inequality when the distribution of ex post utilities is uncertain.

Section 3 analyses the implications of a preference for ex post equality for the welfare comparison of rank-order tournaments with independent contracts in settings involving moral hazard. Focusing on situations where the production shocks affecting different agents are independent, we identify two distinctive features of tournaments—their correlation properties and their risk properties. This is accomplished by constructing a reward scheme, called a *pseudo-tournament*, that relative to a tournament imposes the same risk on each agent (i.e. the same marginal distribution of reward) and provides the same effort incentives, but treats different agents independently. A tournament, by contrast, tends to correlate negatively the incomes of different agents. Under a fairly weak complementarity property of the ex post welfare function, we show that any tournament is welfare-dominated by a pseudo-tournament.

A pseudo-tournament is a set of *randomized* independent contracts that associate with each agent's output an independent lottery over the prizes. *Deterministic* independent contracts replace these lotteries by lump-sum payments yielding the same expected utility. These two schemes differ not in their correlation properties but in the riskiness of the marginal distributions of rewards. Comparisons of welfare between them are shown to hinge upon the level of the planner's risk aversion relative to that of individuals. Thus the pseudo-tournament allows us to link complementarity of the ex post welfare function with the undesirability of the correlation properties of tournaments and, by contrast, to link concavity with the undesirability of their risk properties.

Section 4 describes the implications of correlation preferences of the planner for the qualitative features of *welfare-optimal* compensation schemes in more general production environments. We show that with a complementary welfare function, the optimal compensation formula contains an equity component in addition to an incentive component. The equity component, which increases one agent's reward whenever another is paid more as a result of improved performance, is the feature of welfare-optimal reward schemes which differentiates them from those that are optimal in capitalist firms. Since the equity and incentive components tend to correlate wages of different agents in opposite directions, this demonstrates how normative preferences for ex post equality generate a conflict between distribution and incentives under moral hazard.

2. WELFARE JUDGEMENTS UNDER UNCERTAINTY

The ex ante approach to welfare economics under uncertainty makes social welfare a function of individuals' ex ante utilities. In contrast, the ex post approach focuses on the expected value of a social welfare function defined on individual ex post utilities in each state.[2] Hammond (1983) provides an axiomatic foundation for the use of expected ex post welfare as an objective function for a fully consequentialist planner.[3] While our formal analysis follows Hammond in adopting expected ex post welfare as an objective function, our analysis would still apply if we were to add to the ex post component an ex ante component which is the sum of an increasing concave function of individuals' ex ante utilities. Such an ex ante component would incorporate some non-consequentialist concerns, such as ex ante fairness (an issue discussed by Diamond (1967) and Broome (1984)).[4]

In a context without any uncertainty, attitudes towards inequality can be expressed through various forms of weakened concavity conditions on the welfare function.[5] We argue, however, that in the presence of uncertainty, while similar concavity conditions may be sufficient for expressing attitudes towards inequality in ex ante utility levels, *conditions on the nature of non-separability of the ex post welfare function are required for capturing aversion to inequality of ex post utility levels.* The structure of non-separability is crucial because the ex post inequality-aversion of the planner should imply a concern with the correlation between the ex post utilities of different individuals. Properties of the joint distribution of utilities, not reflected in the marginal distributions, should be relevant.

Specifically, letting $(\tilde{U}_1, \ldots, \tilde{U}_n)$ represent the random utility levels of individuals (induced by underlying sources of uncertainty in conjunction with chosen actions and the compensation scheme adopted), and $EW(\tilde{U}_1, \ldots, \tilde{U}_n)$ the objective of the planner, the question is what properties the welfare function W should possess for maximization of $EW(\tilde{U}_1, \ldots, \tilde{U}_n)$ to favour "equality of ex post utilities". The utility measures $\tilde{U}_i$ need not coincide with the von Neumann-Morgenstern utilities that express the ex ante risk-preferences of individuals, since the former are used by the planner to make equity judgements.[6] We impose the natural conditions that W is symmetric and is increasing in all its arguments.

Judgements about ex post inequality must reflect the fact that the distribution of ex post utilities (across individuals) is uncertain. This feature differentiates such judgements from those about inequality under perfect certainty or about ex ante inequality. The planner must express preferences over a conceptually distinct space of objects, namely, the probability distributions over interpersonal utility distributions, rather than the latter alone. In this context, consider the implications of additive separability of the welfare function. If W takes the form $\sum_i \phi(U_i)$ where ϕ is some increasing function, then the expectation of W with respect to the joint distribution of $(\tilde{U}_1, \ldots, \tilde{U}_n)$ reduces to $\sum_i E\phi(\tilde{U}_i)$. Each term in this expression involves an expectation taken with respect to the marginal distribution of utility of some individual. Thus, features of the joint distribution of $(\tilde{U}_1, \ldots, \tilde{U}_n)$ not reflected in the marginals become irrelevant to welfare judgements.

The following example suggests that this restriction can be ethically quite unappealing. Suppose there are just two individuals and each has marginal probability $\frac{1}{2}$ of being rich (utility U^H from consumption x^H) and probability $\frac{1}{2}$ of being poor (utility U^L from consumption x^L, where $U^L < U^H$ and $x^L < x^H$). Tables I and II show two different joint distributions of the utilities that are consistent with these marginal distributions. Under I, there is no ex post inequality at all, while under II one individual is rich and the other poor with probability $\frac{1}{2}$. No matter how one chooses to measure equality ex post, I generates a probability distribution for ex post equality that first-order stochastically

TABLE I

2 \ 1	U^L	U^H
U^H	0	$\frac{1}{2}$
U^L	$\frac{1}{2}$	0

TABLE II

2 \ 1	U^L	U^H
U^H	$\frac{1}{4}$	$\frac{1}{4}$
U^L	$\frac{1}{4}$	$\frac{1}{4}$

dominates the distribution from II. If the welfare function is additively separable, the planner is forced to treat I and II equivalently. I and II of course differ in another dimension as well, apart from the distributions over ex post equality levels. The distribution of average utility undergoes a mean-preserving increase in riskiness (in the sense of Rothschild and Stiglitz (1970)) as we move from II to I. However, this does not imply that either individual faces more risk, since they have identical marginal distributions in I and II. It is a consequence purely of the changed correlation between the utilities of the two agents. In this context the argument that aversion towards ex post inequality should dictate a preference for I over II seems fairly compelling.

This preference leads to the following condition on W.[7]

Weak Complementarity (with two individuals)

$$W(U^H, U^H) + W(U^L, U^L) - W(U^H, U^L) - W(U^L, U^H) > 0 \quad \text{whenever } U^L < U^H. \tag{1}$$

Note that for twice-differentiable welfare functions this requires

$$\frac{\partial^2 W}{\partial U_1 \partial U_2}(x, x) > 0 \quad \text{for all } x. \tag{2}$$

The rest of this section considers how this condition can be strengthened or extended to more general situations. For this an alternative characterization of (1) is convenient. Call a transformation of the following form an *elementary transformation on identical intervals* (ETI).[8] Given U^L, U^H, the probability of (U^H, U^H) and the probability of (U^L, U^L) are increased by $d\,(d > 0)$, while the probability of (U^H, U^L) and the probability of (U^L, U^H) are decreased by d. ETI's leave the marginal distributions of individuals' utilities unchanged, but they increase the likelihood of ex post equal interpersonal utility distributions and reduce the likelihood of unequal ones. Then W reflects a preference for ETI's if and only if it satisfies weak complementarity.

Let us now consider a generalization of an ETI. Given $U_1^L < U_1^H$ and $U_2^L < U_2^H$, with U_1^L and U_2^L (similarly U_1^H and U_2^H) possibly unequal, we define an *elementary transformation on non-identical intervals* (ETN) to be the following. The probability of (U_1^H, U_2^H) and the probability of (U_1^L, U_2^L) are increased by $d\,(d > 0)$, while the probability of (U_1^H, U_2^L) and the probability of (U_1^L, U_2^H) are decreased by d. Though leaving the marginal distributions unchanged, ETN's increase the likelihood of both agents doing (relatively) well or badly together and reduce the likelihood of one doing well and the other badly. The following condition on W is necessary and sufficient for any ETN to increase expected social welfare:

Strong Complementarity (with two individuals)

$$W(U_1^H, U_2^H) + W(U_1^L, U_2^L) - W(U_1^H, U_2^L) - W(U_1^L, U_2^H) > 0 \tag{3}$$

whenever $U_1^L < U_1^H$, $U_2^L < U_2^H$

This condition is stronger than condition (1). For twice-differentiable welfare functions, it requires

$$\frac{\partial^2 W}{\partial U_1 \partial U_2}(x, y) > 0 \quad \text{for all } (x, y). \tag{4}$$

The attractiveness of an ETN, and hence the appeal of strong complementarity, may be felt to vary with the relative positions of the intervals (U_1^L, U_1^H) and (U_2^L, U_2^H). Suppose we focus on how an ETN affects the distribution of ex post equality, as measured by $-|U_1 - U_2|$. If one interval is a subset of the other, then the distribution of $-|U_1 - U_2|$ produced by the ETN first-order stochastically dominates the original distribution. On the other hand, if the intervals are disjoint, the ETN produces a mean-preserving reduction in the variability of $-|U_1 - U_2|$, i.e. a second-order stochastic improvement in ex post equality. In the intermediate case in which the intervals partially overlap, the ETN always raises the mean of $-|U_1 - U_2|$ and, depending on the degree of overlap, the distribution of $-|U_1 - U_2|$ resulting from the ETN will either first-order or second-order stochastically dominate the original distribution. The case for preferences to be based on second-order stochastic dominance of ex post equality levels is considerably weaker than that based on first-order dominance. Moreover, one could argue that ex post equality could alternatively be measured by $-|U_1 - U_2|/(U_1 + U_2)$, in which case the result that ETN's lead to first- or second-order stochastic dominance no longer holds. In view of all this, strong complementarity is not so compelling as its weaker version. Hence for the remainder of the paper we shall employ only the latter condition.

When there are more than two individuals, assessing the ex post equality of joint probability distributions over utilities is more complicated. The difficulties are analogous to those faced in developing a suitable notion of correlation for multivariate distributions.[9] A simple approach in this context is to assess the equality of the joint distribution of U_i and U_j for each pair of individuals i and j independently of the correlation of these utilities with the utilities of others. It is shown in Meyer (1985) that for expected ex post welfare to depend only on the pairwise joint distributions, it is necessary and sufficient that W be pairwise separable, i.e. be of the form[10]

$$W \equiv \sum_{i=1}^{n} \sum_{j=1}^{n} V(U_i, U_j). \tag{5}$$

A pairwise-separable welfare function is appealing not only for its simplicity but also because the Gini coefficient for the distribution of utilities is based on a welfare function of this form (see Sen (1973, p. 31) and Sheshinski (1972)):

$$W = \frac{1}{n^2} \sum_i \sum_j \text{Min}\,(U_i, U_j) = \bar{U}(1 - G)$$

where

$$G = \frac{1}{2n^2 \bar{U}} \sum_{i=1}^{n} \sum_{j=1}^{n} |U_i - U_j|.$$

In this formula, $\bar{U}$ represents average utility and G the Gini coefficient. As Sen (1973), Yitzhaki (1979), and Hey and Lambert (1980) have noted, $n^2 \bar{U} G$ can be viewed as the aggregate level (over all pairs of individuals) of "relative deprivation", where the relative deprivation of individual i with respect to j is Max $(0, U_j - U_i)$.[11] By generalizing the definition of relative deprivation, and aggregating over all pairs, one can derive other pairwise-separable measures of total welfare.

If we adopt an ex post welfare function of the form (5), we will naturally want the function V to favour ex post equality of the joint distribution of each pair of utilities. Corresponding to the two-person case, we will say that W is *pairwise separable and weakly (resp. strongly) complementary* if it is has the form (5) and if V satisfies condition (1) or (2) (resp. (3) or (4)).[12]

For welfare functions that are not pairwise separable, the following generalizations of ETI's and ETN's are helpful. Let $\bar{U}_{-i,-j}$ denote a given configuration of utility levels for individuals other than i and j and $(U_i, U_j; \bar{U}_{-i,-j})$ the configuration where i and j have utilities U_i and U_j and the remaining $(n-2)$ individuals have $\bar{U}_{-i,-j}$. Given $U_i^L < U_i^H$, $U_j^L < U_j^H$, and $\bar{U}_{-i,-j}$, we define a *generalized elementary transformation on non-identical intervals* (GETN) to be the following. The probabilities of $(U_i^H, U_j^H; \bar{U}_{-i,-j})$ and of $(U_i^L, U_j^L; \bar{U}_{-i,-j})$ are increased by $d(d>0)$, while the probabilities of $(U_i^H, U_j^L; \bar{U}_{-i,-j})$ and of $(U_i^L, U_j^H; \bar{U}_{-i,-j})$ are decreased by d. A *generalized elementary transformation on identical intervals* (GETI) takes the same form, except that we require $U_i^L = U_j^L$ and $U_i^H = U_j^H$. As is the case for ETI's and ETN's, GETI's and GETN's leave all marginal distributions unchanged. For an arbitrary set of utility levels for $(n-2)$ individuals, GETI's and GETN's increase the likelihood of the remaining two agents doing relatively well or badly together while reducing the likelihood of one of them doing relatively well and the other badly. Any GETN increases expected social welfare if and only if W satisfies:

$$W(U_i^H, U_j^H; \bar{U}_{-i,-j}) + W(U_i^L, U_j^L; \bar{U}_{-i,-j}) > W(U_i^H, U_j^L; \bar{U}_{-i,-j}) + W(U_i^L, U_j^H; \bar{U}_{-i,-j}) \tag{6}$$

for any i, j in $\{1, \ldots, n\}$, any $\bar{U}_{-i,-j}$, provided $U_i^L < U_i^H$, $U_j^L < U_j^H$.

If W is twice-differentiable this amounts to requiring

$$\frac{\partial^2 W}{\partial U_i \partial U_j}(U_i, U_j; \bar{U}_{-i,-j}) > 0 \quad \text{for all } (U_i, U_j; \bar{U}_{-i,-j}). \tag{6'}$$

The condition on W expressed by (6) or (6′) can then be termed *generalized strong complementarity*. However, if we insist on the planner expressing preferences only for the less-controversial GETI's, we obtain the condition of *generalized weak complementarity* where (6) applies only if $U_i^L = U_j^L$ and $U_i^H = U_j^H$, and correspondingly (6′) only if $U_i = U_j$.

We have discussed several assumptions, of varying degrees of strength, that incorporate a preference for ex post equality into the ex post welfare function. In the following two sections we examine the implications of these assumptions for the welfare evaluation of tournaments and for the form of welfare-optimal reward schemes.

3. THE WELFARE EVALUATION OF TOURNAMENTS

When many risk-averse workers are producing under conditions of moral hazard, an incentive scheme that bases rewards to any agent only on the rank order of that agent's output is called a tournament. Tournaments have been shown to be attractive reward schemes if the uncertainties associated with the production of different agents are highly correlated, because they reduce the dependence of rewards on the common component of agents' shocks. Relative to individualistic contracts, which base agents' payments only on their own output, the benefits of the tournament's ability to filter out the common shock outweigh the costs of making agents' payments fluctuate with the idiosyncratic shocks of others as well as with their own.[13] When production uncertainties are independent, the benefits of tournaments disappear, and only the costs of the extra risk imposed remain.

Here we show that when evaluated in terms of ex post equality, tournaments possess another undesirable feature—they lead to "negative correlation" among the rewards of different agents. This feature emerges most clearly when production uncertainties are independent: for the rest of this section, we make that assumption. Given any tournament, we construct an incentive scheme, called a *pseudo-tournament*, that imposes exactly the same amount of risk on agents and entails the same expected costs for the planner, but rewards agents independently. Requiring only that the ex post welfare function satisfies generalized weak complementarity, we demonstrate that expected social welfare under any tournament is strictly less than under the corresponding pseudo-tournament.

Suppose there are n identical agents, indexed $k = 1, \ldots, n$, involved in separate production processes. Each agent k's production function is $q_k = q(a_k, \varepsilon_k)$, where q_k represents k's output, a_k represents k's effort, and ε_k represents an exogenous random variable. Assume $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ are mutually independent. Suppose each agent has the following von Neumann-Morgenstern preferences over reward I^k and effort a_k:

$$U(I^k, a_k) = X(I^k) - G(a_k),$$

where X is strictly increasing, continuous, and concave. The principal observes neither agents' efforts nor the realization of any ε_k, but only the vector of outputs $(q_1, \ldots, q_n)$. Thus rewards are based solely on outputs. The profit or surplus associated with production vector $(q_1, \ldots, q_n)$ and reward vector $(I^1, \ldots, I^n)$ is

$$\textstyle\sum_{k=1}^{n} (q_k - I^k).$$

A compensation system is a function that determines a lottery over rewards to each agent, given a particular output vector $(q_1, \ldots, q_n)$. A rank-order tournament is a compensation system specifying a set of n prizes $(I^1, \ldots, I^n)$ where I^l, the lth prize, is the deterministic reward to the agent whose output occupies lth rank from the top. If t_l denotes $X(I^l)$, the tournament is equivalently specified by the n prizes in utility units $(t_1, \ldots, t_n)$.[14]

Suppose that the tournament specified by $(t_1, \ldots, t_n)$ implements the symmetric effort vector $(a, a, \ldots, a)$ as a Nash equilibrium in the agents' strategies. We now describe how to construct a pseudo-tournament corresponding to this tournament. The probability $\alpha_l(q)$ that an agent will obtain the lth rank in the tournament, conditional on having produced output q and given that all other agents are choosing effort a, is:

$$\alpha_l(q) = \frac{(n-1)!}{(n-l)!(l-1)!}[F(q, a)]^{n-l}[1 - F(q, a)]^{l-1} \tag{10}$$

where $F(\cdot, a)$ is the distribution function for output for any agent, given action a. Note that the unconditional probability that the agent will obtain lth prize upon choosing effort a is $1/n$; this is obtained by taking the expectation of (10) over q with respect to the density $f(q, a)$ (corresponding to $F(q, a)$). Using the probabilities $\alpha_l(q)$, we construct the pseudo-tournament as follows. Each agent's compensation is determined by a lottery that is a function only of his own output. Given output q, an agent receives a payment of t_l (in utility units) with probability $\alpha_l(q)$, $l = 1, \ldots, n$; the lotteries used for different agents are conducted independently. Note the contrast with the non-independent lotteries that the tournament automatically imposes on agents. In the tournament, if payments to $(n-1)$ agents are determined, then so is the payment to the n-th agent.[15]

From the perspective of each agent, the pseudo-tournament looks identical to the tournament. It offers exactly the same pattern of rewards for each level of output, and therefore each agent will choose effort a.[16] Since total expected wage costs are the sum of expected wage costs across different agents, the two schemes generate the same expected profit. This is because the unconditional marginal distributions of agents' rewards are identical under the pseudo-tournament and the tournament: given symmetric effort levels, the probability of receiving any one of the n prizes is $1/n$. This also implies that if the ex post welfare function is additively separable across agents, expected social welfare is equal under the pseudo-tournament and the tournament.

The following proposition presents the expected social welfare comparison when W satisfies generalized weak complementarity.

Proposition 1. *Assume that the ex post welfare function W is symmetric and satisfies generalized weak complementarity. Then, given any tournament implementing symmetric effort levels, expected social welfare is strictly less under the tournament than under the corresponding pseudo-tournament.*

Proof. Since the tournament assigns each prize to one and only one agent and since W is symmetric, social welfare under the tournament is deterministic and equal to $W(t_1, \ldots, t_n)$.

Under the pseudo-tournament, each agent's prize is drawn independently from a distribution that is uniform over the n prizes. Thus each of the n^n possible configurations of prizes is equally likely, so expected social welfare under the pseudo-tournament is

$$\frac{1}{n^n}\sum_{i_1=1}^{n} \cdots \sum_{i_n=1}^{n} W(t_{i_1}, \ldots, t_{i_n}).$$

The proof is completed by the following lemma, which is proved in the Appendix. ||

Lemma 1. *Assume that W is symmetric and satisfies generalized weak complementarity. Then, given any integer n and any vector* $(t_1, \ldots, t_n)$, *where it is not the case that* $t_1 = \cdots = t_n$,

$$W(t_1, \ldots, t_n) < \frac{1}{n^n}\sum_{i_1=1}^{n} \cdots \sum_{i_n=1}^{n} W(t_{i_1}, \ldots, t_{i_n}).$$

The basis of the proof of Lemma 1 is the demonstration that the joint distribution of agents' utilities under the pseudo-tournament can be derived from the joint distribution under a tournament by a finite sequence of GETI's. Each GETI increases expected social welfare since W satisfies generalized weak complementarity. Intuitively, the result can be explained as follows. The pseudo-tournament and the tournament differ only in the correlation among agents' utilities that they generate. The former scheme treats agents independently, whereas the latter yields negative correlation of their rewards. In a tournament, if one agent's output improves and he obtains a higher rank and reward, other agents automatically get a lower rank and reward. The property of ex post welfare functions that incorporates a preference for independence over negative correlation, given identical marginal distributions, is precisely weak complementarity.[17]

For the special case in which W is pairwise separable and weakly complementary, the proposition can be proved in a particularly simple and illuminating manner. For W of the form $\sum_{i \neq j} V(U_i, U_j)$, with V weakly complementary,

$$EW(U_1, \ldots, U_n) = \sum_{i \neq j} EV(U_i, U_j)$$

where the expectation in each term in the sum is taken only with respect to the pairwise joint distribution of U_i and U_j.[18] Since the tournament induces symmetric effort levels, the pairwise joint distributions are the same for all pairs, and the same is true of the pseudo-tournament. Hence we need focus on only one (i, j) pair. Under the tournament, the probability that agents i and j both receive the same prize t_l is zero for all l. For any pair of prizes (t_l, t_m) with $l \neq m$, the probability that i receives t_l and j receives t_m is $1/n(n-1)$, since all such pairs of prizes are equally likely. Under the pseudo-tournament, every pair of prizes (t_l, t_m) is equally likely, so each arises with probability $1/n^2$. Tables III and IV show the joint distribution of U_i and U_j under the tournament and the pseudo-tournament, respectively. The pseudo-tournament makes it possible for i and j to receive the same prize and makes any unequal pair of prizes less likely than the tournament does.

To show formally that $EV(U_i, U_j)$ is strictly higher under the pseudo-tournament, we shall show that the distribution in Table IV can be derived from that in Table III by a finite sequence of ETI's. The sequence consists of

$$\binom{n}{2} = \frac{n(n-1)}{2}$$

ETI's, each corresponding to a different (l, m) pair (with $l < m$). For each (l, m), the ETI increases the probability of (t_l, t_l) and (t_m, t_m) by

$$\frac{1}{n(n-1)} - \frac{1}{n^2} = \frac{1}{n^2(n-1)}$$

and reduces the probability of (t_l, t_m) and (t_m, t_l) by the same amount.[19]

TABLE III

j \ i	t_1	t_2	$\cdots$	t_n
t_n	$\frac{1}{n(n-1)}$	$\frac{1}{n(n-1)}$	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
t_2	$\frac{1}{n(n-1)}$	0	$\cdots$	$\frac{1}{n(n-1)}$
t_1	0	$\frac{1}{n(n-1)}$	$\cdots$	$\frac{1}{n(n-1)}$

TABLE IV

j \ i	t_1	t_2	$\cdots$	t_n
t_n	$\frac{1}{n^2}$	$\frac{1}{n^2}$	$\cdots$	$\frac{1}{n^2}$
$\vdots$	$\vdots$	$\vdots$	o	$\vdots$
t_2	$\frac{1}{n^2}$	$\frac{1}{n^2}$	$\cdots$	$\frac{1}{n^2}$
t_1	$\frac{1}{N^2}$	$\frac{1}{n^2}$	$\cdots$	$\frac{1}{n^2}$

While the pseudo-tournament differs from the tournament in the correlation among utilities that it induces, it mimics the tournament in subjecting each individual to a lottery over the prizes, given his output. Since the pseudo-tournament is a system of randomized independent contracts, it is interesting to compare it to the deterministic independent contracts scheme which rewards an agent whose output is q with a lump-sum payment of $\sum_{l=1}^{n} \alpha_l(q)t_l$ in utility terms. Both schemes induce the same effort levels, and the deterministic scheme, by removing some of the risk from the agents, lowers expected wage costs and hence raises expected profits. The welfare comparison of these schemes hinges upon the level of risk aversion of the planner relative to that of individuals. As long as the planner is at least as averse to the riskiness of any agent's consumption as the agent concerned (i.e. as long as social welfare, expressed as a function of individuals' von Neumann-Morgenstern utilities, is weakly concave in each argument alone), a pseudo-tournament is welfare-dominated by deterministic independent contracts.[20]

To summarize, tournaments differ in two distinct dimensions—correlation and risk—from deterministic independent contracts, while two distinct properties of the ex post welfare function—complementarity and concavity in individual arguments—affect the welfare comparison between these schemes. Construction of the pseudo-tournament, a system of randomized independent contracts, enables one to link generalized weak complementarity with a preference for independence over negative correlation of rewards, and weak concavity with a preference for deterministic over randomized independent contracts.

4. WELFARE-OPTIMAL COMPENSATION SCHEMES

This section derives qualitative characteristics of ex post welfare-optimal compensation schemes in fairly general production environments. Our purpose is to show how a social preference for positive correlation between welfares of different agents can (i) create a conflict between the distributive objective of ex post equality and the "efficiency" objective of providing adequate effort incentives and (ii) generate qualitative divergences between welfare-optimal compensation schemes and those that are optimal for a capitalist firm.

The main ideas can be presented most simply in a two-agent context, where the two agents are (ex ante) identical in all respects. We consider a set-up similar to that analysed in Mookherjee (1984), thus weakening the assumptions about the production and monitoring technology made in the previous section. For each agent k, a performance signal q^k ($k=1,2$) is observed by the principal. The set $Q=\{q_1,\ldots,q_m\}$ of possible realizations of this signal is finite. Each agent chooses from a finite set A of possible actions. Any pair (a_1, a_2) of actions chosen by the agents generates a joint probability distribution for their performance signals; $\pi_{ij}(a_1, a_2)$ denotes the consequent probability that agent 1's performance signal takes the value q_i and agent 2's q_j.

In the context of a capitalist firm, assume that the principal is risk-neutral, while for the socially-owned firm assume that the planner is constrained only to choose compensation schemes that break even on average.[21] Let Y_{ij} denote expected output of the firm when (q_i, q_j) is the vector of observed performances. Assume that the principal pays a deterministic amount I_{ij}^k to agent k in this case.[22] The expected profit of the firm is then

$$\sum_{i,j} \pi_{ij}(a_1, a_2)[Y_{ij} - I_{ij}^1 - I_{ij}^2]. \tag{11}$$

In a capitalist firm, the risk-neutral principal's objective is to maximize (11) subject to voluntary participation and incentive compatibility constraints, as analyzed in Mookherjee (1984). In the socially-owned firm the objective is instead the maximization of expected social welfare[23]

$$\sum_{i,j} \pi_{ij}(a_1, a_2) W(X(I^1_{ij}), X(I^2_{ij})) \tag{12}$$

subject to the constraint that expected surplus (11) is nonnegative, in addition to the same incentive (and also perhaps the participation) constraints. However, as pointed out by Grossman and Hart (1983), the qualitative characteristics of optimal compensation schemes can be discerned from the analysis of cost-minimizing schemes. Letting (a_1, a_2) denote the action pair to be implemented and letting $v^k_{ij} = X(I^k_{ij})$, the compensations in utility units, be the control variables, the relevant problem is the choice of a scheme $\{v^1_{ij}, v^2_{ij}\}$ to minimize expected wage costs (where h is the inverse of X):

$$\sum_{i,j} \pi_{ij}(a_1, a_2)[h(v^1_{ij}) + h(v^2_{ij})] \tag{13}$$

subject to the incentive constraints:

$$\sum_{i,j} \pi_{ij}(a_1, a_2)v^1_{ij} - G(a_1) \geqq \sum_{i,j} \pi_{ij}(a, a_2)v^1_{ij} - G(a) \quad \text{for all } a \in A \tag{14}$$

$$\sum_{i,j} \pi_{ij}(a_1, a_2)v^2_{ij} - G(a_2) \geqq \sum_{i,j} \pi_{ij}(a_1, a)v^2_{ij} - G(a) \quad \text{for all } a \in A \tag{15}$$

the participation constraints:

$$\sum_{i,j} \pi_{ij}(a_1, a_2)v^k_{ij} - G(a_k) \geqq \underline{U} \quad \text{for } k = 1, 2 \tag{16}$$

and a minimum welfare constraint for the socially-owned firm:

$$\sum_{i,j} \pi_{ij}(a_1, a_2) W(v^1_{ij}, v^2_{ij}) \geqq \underline{W}. \tag{17}$$

Provided the welfare function W is continuous, cost-minimizing schemes exist.[24] Assuming that a suitable constraint qualification holds and that h and W are twice continuously differentiable, optimal compensation schemes satisfy the following Kuhn–Tucker conditions:

There exist scalars λ_1, λ_2 and nonnegative multipliers λ, $\alpha_1(a)$, and $\alpha_2(a)$ such that

$$h'(v^1_{ij}) = \lambda W_1(v^1_{ij}, v^2_{ij}) + \lambda_1 - \sum_{a \in A} \alpha_1(a) \frac{\pi_{ij}(a, a_2)}{\pi_{ij}(a_1, a_2)} \tag{18}$$

$$h'(v^2_{ij}) = \lambda W_2(v^1_{ij}, v^2_{ij}) + \lambda_2 - \sum_{a \in A} \alpha_2(a) \frac{\pi_{ij}(a_1, a)}{\pi_{ij}(a_1, a_2)} \tag{19}$$

for all i, j such that $\pi_{ij}(a_1, a_2) > 0$.[25] Since the welfare constraint is absent for a capitalist firm, the first term on the right-hand sides of (18) and (19) does not arise. To explain the role of this term, we will represent the first-order conditions (18) and (19) in a more convenient form.

Defining for given positive scalar λ and action pair (a_1, a_2)

$$r(v^1, v^2) \equiv h'(v^1) - \lambda W_1(v^1, v^2) \tag{20}$$

and using K^1_{ij} to denote $[\lambda_1 - \sum_{a \in A} \alpha_1(a)[\pi_{ij}(a, a_2)/\pi_{ij}(a_1, a_2)]]$ and K^2_{ij} analogously, the first-order conditions can be written as

$$r(v^1_{ij}, v^2_{ij}) = K^1_{ij} \tag{21}$$

$$r(v^2_{ij}, v^1_{ij}) = K^2_{ij}. \tag{22}$$

In (21) and (22) the same r function is used for both agents, by virtue of the symmetry of the welfare function and the inessential assumption that $a_1 = a_2$. Note that K^1_{ij} and K^2_{ij} can be interpreted as "performance evaluations" of the two agents, each being an average likelihood ratio that the agent in question deviated from the stipulated action to an alternative one. The left-hand sides of (21) and (22) represent the difference between the marginal wage cost to the principal and the marginal welfare benefit of providing the relevant agent with an additional unit of utility in state (i, j). It is easy to check that the local second-order conditions for the cost-minimization problem imply that r is increasing in its first argument. The dependence of r on its second argument depends on the complementarity properties of the welfare function. Under weak complementarity (2), r is decreasing at points where $v^1 = v^2$.

Equation (22) can be inverted to yield a solution for v^2_{ij} at points where r is strictly increasing in its first argument:

$$v^2_{ij} = s(v^1_{ij}, K^2_{ij}). \tag{23}$$

The function s is continuously differentiable, increasing in its second argument and also in its first argument at points where the welfare function is complementary. Now substitute (23) in (21) to obtain

$$r(v^1_{ij}, s(v^1_{ij}, K^2_{ij})) \equiv l(v^1_{ij}, K^2_{ij}) = K^1_{ij}. \tag{24}$$

Clearly l is continuously differentiable and decreasing in K^2_{ij} at points of complementarity of W. The local second-order conditions for the cost-minimization problem also imply that l is increasing in its first argument in a neighbourhood of the optimum. Inverting (24) at points where l is strictly increasing in v^1_{ij} we obtain a "reduced form" solution for v^1_{ij} (and analogously for v^2_{ij}):

$$v^1_{ij} = m(K^1_{ij}, K^2_{ij}) \tag{25}$$

$$v^2_{ij} = m(K^2_{ij}, K^1_{ij}). \tag{26}$$

At points where m is well-defined, it is strictly increasing in its first argument. We restrict ourselves hereafter to such points. Then the payment to each agent increases with an improvement in that agent's performance evaluation. This feature is shared by compensation schemes that are optimal in a capitalist firm.[26] The dependence of m on its first argument represents a tradeoff between the provision of appropriate incentives and the insurance of agents against wage fluctuations. It represents the *incentive component* in optimal compensation schemes.

The second argument of m is more interesting. It arises nontrivially only at points (v^1_{ij}, v^2_{ij}) where the welfare function is not locally separable.[27] At points of complementarity of W, m is increasing in this argument. Thus if 2's performance evaluation K^2 improves, not only should agent 2 be paid more by virtue of the incentive component in 2's compensation, but also agent 1 should be paid more. This arises solely from the planner's preference for positive correlation between the utilities of the two agents, and can be interpreted as an *equity component* in welfare-optimal compensation schemes. It is complementarity of the welfare function (not concavity or quasiconcavity) that gives rise to this component. Further, it is this equity component that differentiates the welfare-optimal compensation scheme from that which is optimal in a capitalist firm.

The incentive and equity components have contrasting effects in the optimal compensation formula. When the production processes of the two agents are interdependent, the incentive component incorporates elements of relative performance, which tend to correlate agent utilities negatively. Thus in general whenever agent 1's performance q_i "improves", in the sense of an increase in K^1_{ij}, agent 1's wage rises but agent 2's wage falls because its relative performance assessment K^2_{ij} decreases. On the other hand, the equity component tends to correlate agent utilities positively. Thus with interdependent production, the incentive and equity components work opposite to each other in the correlation properties they generate. This may be viewed as a conflict between distribution and incentives in the design of compensation schemes under moral hazard.

The following proposition compares the relative sizes of the incentive and equity effects for the special case of independent production.

Proposition 2. *Suppose that* $\pi_{ij}(a_1, a_2) = \pi_i(a_1)\pi_j(a_2)$, *i.e. production and related uncertainties are independent for the two agents, and that both agents are required to choose the same action a (which is not the least costly action for them). Suppose that the planner is risk-neutral, and that the welfare function W is weakly complementary.*

(i) *Independent contracts are not welfare-optimal.*
(ii) *If agent* 1*'s performance evaluation is identical to that of* 2, *they are paid the same wage. Starting from this position, if* 2*'s evaluation improves marginally while* 1*'s evaluation remains the same, then both* 1 *and* 2 *are paid more but* 2*'s increment exceeds* 1*'s.*

The proof is presented in the Appendix. The proposition shows that neither the capitalist wage system, which in this setting comprises a set of independent contracts, nor equal profit-sharing, under which all agents' wages increase by the same amount when one individual's performance improves, is optimal. Independent contracts are suboptimal because they lack an equity component, while equal profit sharing places insufficient weight on the incentive component.[28,29]

5. CONCLUDING COMMENTS

We have proposed that welfare functions should satisfy a complementarity condition, in order to express aversion to ex post inequality. We have explored the implications of such a condition for the evaluation of compensation schemes under moral hazard. As noted in Section 2, the assessment of ex post inequality is significantly more complicated in the multi-person case than in the two-person case.[30] Nevertheless, for the setting analyzed in Section 3, the weakest of the complementarity conditions discussed, namely generalized weak complementarity, was sufficient for proving that tournaments are welfare-dominated by pseudo-tournaments. In other settings requiring an assessment of multivariate correlation, it may be desirable to strengthen this condition. The motivation for, and implications of, such extensions of the complementarity property are explored in Meyer (1985).

Our analysis of the welfare implications of tournaments and other incentive schemes was also restricted to a single-period framework. In a multi-period setting, issues of stratification (long-run inequality or lack of mobility) in organizations can be examined. Meyer (1986) studies the consequences for stratification of the optimal design of multi-period incentive schemes in a capitalist firm. A welfare analysis of such schemes is a topic for future research.

APPENDIX

Proof of Lemma 1. The method of proof is induction. It is obvious that the required inequality holds for $n = 2$. So assume it holds for $(n-1)$ and fewer agents and consider the case of n agents. Let the set of all tuples

$$S = \{(t_{i_1}, \ldots, t_{i_n}) \mid i_j = 1, \ldots, n \text{ all } j = 1, \ldots, n\}$$

be partitioned into the following classes:

(a) Class S_1 consisting of all tuples where the same prize t_i occurs n times

(b) Class S_2 of all tuples where no prize appears only once or all n times
(c) Class S_3 of all tuples where some prize t_i occurs only once

We shall show that if the required inequality holds for all $m \leqq (n-1)$ then for $m=n$:

$$\sum_{s \in S_i} W(s) > (\# S_i) W(t_1, \ldots, t_n) \quad \text{for all } i = 1, 2, 3 \tag{27}$$

where $(\# S_i)$ denotes the cardinality of S_i.

(a) Class S_1:

In this case we proceed in three steps.

(i) Given any two prizes a and b we first show that given W defined on R^n,

$$W(a, a, \ldots, a) + W(b, b, \ldots, b) > W(a, a, \ldots, a, b) + W(b, b, \ldots, b, a) \tag{28}$$

for any value of $n = 1, 2, \ldots$.

Let $X_n(m)$ denote the value of W where a occurs $(n-m)$ times and b occurs m times (where $0 \leqq m \leqq n$). Then we have to show that

$$X_n(0) + X_n(n) > X_n(1) + X_n(n-1) \tag{29}$$

holds for all $n = 1, 2, \ldots$. Note first that given any positive integer q, (29) implies

$$X_{n+q}(0) + X_{n+q}(n) > X_{n+q}(1) + X_{n+q}(n-1) \tag{30}$$

$$X_{n+q}(q) + X_{n+q}(n+q) > X_{n+q}(1+q) + X_{n+q}(n+q-1). \tag{31}$$

In (30) and (31), W is $(n+q)$ dimensional; the number of arguments is now q greater. In (30) all the new arguments are constrained to be a's, and in (31) they are all b's.

To prove (29) holds for all n, note that it is true for $n=2$. Suppose it holds for all values of $m \leqq (n-1)$; we shall show it holds for $m=n$.

Consider first the case where n is odd. Since (29) holds for $(n+1)/2$, (30) implies (taking $q=(n-1)/2$):

$$X_n(0) + X_n\left(\frac{n+1}{2}\right) > X_n(1) + X_n\left(\frac{n-1}{2}\right) \tag{32}$$

whereas (31) implies

$$X_n\left(\frac{n-1}{2}\right) + X_n(n) > X_n\left(\frac{n+1}{2}\right) + X_n(n-1). \tag{33}$$

Adding (32) and (33) it is clear that (29) holds for $m=n$.

Now suppose n is even. Then (29) holds for $(n/2)$; taking $q=n/2$, (30) implies

$$X_n(0) + X_n\left(\frac{n}{2}\right) > X_n(1) + X_n\left(\frac{n}{2}-1\right) \tag{34}$$

$$X_n\left(\frac{n}{2}\right) + X_n(n) > X_n\left(\frac{n}{2}+1\right) + X_n(n-1). \tag{35}$$

Further, since (29) holds for $n=2$

$$X_2(0) + X_2(2) > X_2(1) + X_2(1)$$

and by taking $q=(n/2-1)$, (31) implies

$$X_{n/2+1}\left(\frac{n}{2}-1\right) + X_{n/2+1}\left(\frac{n}{2}+1\right) > X_{n/2+1}\left(\frac{n}{2}\right) + X_{n/2+1}\left(\frac{n}{2}\right). \tag{36}$$

Now take $q=n/2-1$ and apply (30) to (36) to obtain

$$X_n\left(\frac{n}{2}-1\right) + X_n\left(\frac{n}{2}+1\right) > X_n\left(\frac{n}{2}\right) + X_n\left(\frac{n}{2}\right), \tag{37}$$

and the result follows from (34), (35) and (37).

(ii) In the second step, note that (28) implies that for any pair t_i, t_j of distinct prizes:

$$W(t_i, \ldots, t_i) + W(t_j, \ldots, t_j) > W(t_i, \ldots, t_i, t_j) + W(t_j, \ldots, t_j, t_i) \tag{38}$$

Adding (38) over every pair (t_i, t_j) of distinct prizes in $\{t_1, \ldots, t_n\}$ we get

$$(n-1)\sum_{i=1}^{n} W(t_i, \ldots, t_i) > \sum_{i=1}^{n} \left[\sum_{j=1, j \neq i}^{n} W(t_j, t_j, \ldots, t_j, t_i)\right]. \tag{39}$$

(iii) In the third step, we show that

$$\sum_{s \in S_1} W(s) = \sum_{i=1}^{n} W(t_i, \ldots, t_i) > nW(t_1, \ldots, t_n) \tag{40}$$

for all $n = 1, 2, \ldots$ Clearly (40) holds for $n = 2$. Suppose it holds for all $m \leqq (n-1)$. Define $\bar{W}_{t_i}(x_1, \ldots, x_{n-1}) \equiv W(x_1, \ldots, x_{n-1}, t_i)$ on R^{n-1}; then $\bar{W}_{t_i}$ is complementary and symmetric. Since (40) holds for $m = (n-1)$

$$\sum_{j=1, j \neq i}^{n} W(t_j, t_j, \ldots, t_j, t_i) = \sum_{j=1, j \neq i}^{n} \bar{W}_{t_i}(t_j, \ldots, t_j) > (n-1)\bar{W}_{t_i}(t_{-i}) = (n-1)W(t_1, \ldots, t_n) \tag{41}$$

where t_{-i} denotes a permutation of $T \backslash \{t_i\}$, $T = \{t_1, \ldots, t_n\}$. From (39) and (41), it is clear that (40) holds for n.

(b) Class S_2

A typical element of S_2 is one with m distinct prizes $t(1), \ldots, t(m)$, where $t(i)$ occurs K_i times, with $K_i \geqq 2$, $m < n$, and $\sum_{i=1}^{m} K_i = n$. Let such an element be denoted by $s^* = (t(1), K_1, t(2), K_2; \ldots, ; t(m), K_m)$, and let the associated value of W at s^* be denoted by $\tilde{W}(t(1), K_1; t(2), K_2; \ldots, t(m), K_m)$. Given any such element, define $T^* = T \backslash \{t(1), \ldots, t(m)\}$, the set of prizes that do not appear in s^*.

Take any partition of T^* into m groups, where group i contains $(K_i - 1)$ distinct prizes (this is possible since T^* has $(n-m)$ elements and $\sum_{i=1}^{m} (K_i - 1) = n - m$). Then define T_i^* to be prize $t(i)$ combined with prizes in the i-th group of the chosen partition, so T_i^* has K_i distinct prizes.

Given $t^i \in T_i^*$, let $(t^1, K_1, t^2, K_2; \ldots, t^m, K_m)$ denote the element of S_2 where t^i appears K_i times. Then we show that

$$\sum_{t^1 \in T_1^*} \sum_{t^2 \in T_2^*} \cdots \sum_{t^m \in T_m^*} W(t^1, K_1;\ t^2, K_2; \ldots; t^m, K_m) > K_1 \cdot K_2 \cdots K_m W(t_1, \ldots, t_n) \tag{42}$$

To prove this, define (for any integer p between 1 and m) $\hat{W}(T_1^*, T_2^*, \ldots, T_p^*; t^{p+1}, K_{p+1}; \ldots t^m, K_m)$ to be the value of W at the element of S_2 where any group $i \leqq p$ is represented by a permutation of elements in T_i^*, whereas any group $j > p$ is represented by a repetition of t^j. Then given the result on Class S_1 sequences, i.e., (40) holds for all n, it follows that for all p:

$$\begin{aligned}\sum_{t^{p+1} \in T_{p+1}^*} & \hat{W}(T_1^*, T_2^*, \cdot\cdot, T_p^*; t^{p+1}, K_{p+1}; t^{p+2}, K_{p+2}; \ldots; t^m, K_m) \\ & > K_{p+1} \hat{W}(T_1^*, T_2^*, \ldots, T_p^*, T_{p+1}^*; t^{p+2}, K_{p+2}; \ldots, ; t^m, K_m)\end{aligned} \tag{43}$$

Applying (43) successively with $p = 1, 2, \ldots$, we obtain (42).

Repeat this procedure with different starting points corresponding to the same $(K_1, \ldots, K_m)$, i.e., the same number of common elements in each group (but differing common prizes $t(i)$ in any group i), and over different partitions of the set T^* of prizes not appearing in the starting point (for each starting point). Then we shall obtain on adding (42) over all such repetitions

$$r \cdot \sum_{s \in S_2(K_1, \ldots, K_m)} W(s) > r[\# S_2(K_1, \ldots, K_m)] W(t_1, \ldots, t_n) \tag{44}$$

where $S_2(K_1, \ldots, K_m)$ denotes the subset of S_2 where the same prize occupies the first K_1 positions, another prize the next K_2 positions, and so on, $\# S_2(K_1, \ldots, K_m)$ denotes the cardinality of this set (and equals $n!/(n-m)!$), and r is some positive integer representing the number of times a particular element in $S_2(K_1, \ldots, K_m)$ appears in the process of repeating the procedure. Note that since the repetition process treats all n prizes symmetrically, r is uniform for different elements of $S_2(K_1, \ldots, K_m)$. The right hand side of (42) assigns coefficient $r[\# S_2[K_1, \ldots, K_m)]$ to $W(t_1, \ldots, t_n)$ since the coefficient on the right hand side of (42) equals $K_1 \cdot K_2 \cdots K_m$, the number of terms appearing on the left hand side of (42).

From (44) it follows upon adding across all possible values of m and $(K_1, \ldots, K_m)$ satisfying $m < n$, $K_i \geqq 2$, $\sum_{i=1}^{m} K_i = n$, that

$$\sum_{s \in S_2} W(s) > (\# S_2) W(t_1, \ldots, t_n).$$

(c) Class S_3

First consider the subset S_{33} of S_3 where $i > j$ implies person i receives a prize no higher than j's prize. Thus a vector $(t_1, \ldots, t_n)$ of prizes in S_{33} has the property that $t_i \geqq t_{i-1}$, for all $i \geqq 2$. Note that any element of S_3 is a permutation of some element in S_{33}; so if $P(s)$ denotes the s^et of vectors that are permutations of s then $\{P(s) \mid s \in S_{33}\}$ is a partition of S_3.

Next take any subset $\hat{S}$, say, of $(n-k)$ prizes $(t_{i_1}, \ldots, t_{i_{n-k}})$ where $i_l \geqq i_{l-1}$ for all $l = 2, \ldots, (n-k)$. Let S^c denote the complement of S. Let S^* denote the subset of S_{33} where each of these $(n-k)$ prizes, and only these, is awarded to exactly one person (so any prize in $\hat{S}^c$ is awarded to none or at least two persons). Given $\hat{S}$, any element in $\hat{S}^*$ can be characterized by $\{t_{j_1}, K_1; t_{j_2}, K_2; \ldots t_{j_m}, K_m\}$ where $m \leqq k$, $j_l \leqq j_{l+1}$, $K_l \geqq 2$ and $\sum_{l=1}^{m} K_l = k$. Prize t_{j_l} here is the l-th highest among those from $\hat{S}^c$ represented in this element, and is paid to K_l persons. For any given integer vector $(K_1, \ldots, K_m)$ satisfying $K_l \geqq 2$ and $\sum_{l=1}^{m} K_l = k$, it is clear that the number of vectors that are permutations of the vector characterized by a particular choice $(t_{j_1}, t_{j_2}, \ldots, t_{j_m})$ from

$\hat{S}^c$ for the m slots of repetitive prizes, is the same for all such choices. Hence applying the results for Class S_1, and S_2 sequences (corresponding to each choice of $(K_1, \ldots, K_m)$ satisfying the appropriate conditions, and then adding over all choices) we obtain

$$\sum_{s\in C(\hat{S})} W(s) > \# C(\hat{S}) W(t_1, t_2, \ldots, t_n) \tag{45}$$

where $C(\hat{S}) \equiv \bigcup_{s \in \hat{S}^*} P(S)$.

Adding (45) over all nonempty subsets $\hat{S}$ of $\{t_1, \ldots, t_n\}$, we obtain the desired result. ||

Proof of Proposition 2. From (25) and (26) it is clear that 1 and 2 must be paid the same amount if their evaluations are identical. The welfare function is complementary at this point, so m is strictly increasing in its second argument. Hence an improvement in 2's evaluation raises agent 1's wage. Therefore independent contracts are not optimal. To show that 2's increment exceeds 1's, we have to show that

$$m_1(K^*, K^*) \geqq m_2(K^*, K^*) \tag{46}$$

where K^* is the common evaluation of the two agents. Differentiating $l(m(K^1, K^2), K^2) = K^1$ with respect to K^1 and K^2 respectively we obtain

$$m_1(K^1, K^2) = [l_1(m(K^1, K^2), K^2)]^{-1} \quad \text{and} \quad m_2(K^1, K^2) = -\frac{l_2(m(K^1, K^2), K^2)}{l_1(m(K^1, K^2), K^2}$$

so (46) holds as long as

$$1 + l_2(m(K^*, K^*), K^*) \geqq 0,$$

which reduces to

$$r_1(s(v, K^*), v) + r_2(v, s(v, K^*)) \geqq 0$$

upon differentiation of $l(v, K) = r(v, s(v, K))$. Now when $v = m(K^*, K^*)$, $s(v, K^*) = m(K^*, K^*)$ and thus it suffices to show that for $v^* = m(K^*, K^*)$:

$$r_1(v^*, v^*) + r_2(v^*, v^*) \geqq 0$$

or

$$h''(v^*) - \lambda W_{11}(v^*, v^*) - \lambda W_{12}(v^*, v^*) \geqq 0 \tag{47}$$

From the second-order conditions of the cost-minimization problem it is clear that (47) holds. ||

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NOTES

1. See, for example, Mirrlees (1976), Shavell (1979), Holmström (1979, 1982), Grossman and Hart (1983), Green and Stokey (1983), Nalebuff and Stiglitz (1983), and Mookherjee (1984).
2. The contrast between the ex ante and the ex post approaches has been examined by, among others, Diamond (1967), Starr (1973), Mirrlees (1974), Harris and Olewiler (1979), Myerson (1981), Hammond (1981, 1982, 1983), and Broome (1984).
3. Full consequentialism is, however, seen as unappealing by some moral philosophers, such as Williams (1973, 1982), Nagel (1980), and Parfit (1984).
4. It can readily be verified that the results of Section 3 would be completely unaffected by using welfare functions that depend on ex ante and ex post utilities in a much more general way. The analysis of welfare-optimal compensation schemes in Section 4 would, however, be considerably more complicated, though the spirit of our main points would be unaffected.
5. See Dasgupta, Sen, and Starrett (1973) and Sen (1973).
6. See Harsanyi (1955) and Arrow (1963) for different views on whether risk-preferences should be used as a basis for interpersonal utility comparisons.
7. The term "complementarity" is adapted from the context of utility functions. We use "weak" to indicate that the inequality need hold only for identical pairs (U^L, U^H) of utility levels for the two individuals..
8. "Identical intervals" refers to the fact that the values U^H and U^L are the same for the two individuals.
9. See Meyer (1985) for a detailed analysis of this issue.
10. The same V function applies to all pairs because the welfare function W is symmetric.
11. The term "relative deprivation" is used by Runciman (1966), and the papers of Yitzhaki and of Hey and Lambert are presented as formalizations of Runciman's ideas.

12. Note that $W = -n^2\bar{U}G$ is pairwise separable and weakly complementary but not strongly complementary.

13. See Green and Stokey (1983).

14. Henceforth $(t_1, \ldots, t_n)$ are used as arguments of the welfare function, despite the fact that the utilities entering the welfare function are not necessarily von Neumann-Morgenstern utilities. This eases the notation, and is harmless as long as the utility functions employed for interpersonal comparisons are strictly increasing in compensation levels (for further details see footnote 23).

15. The calculation of the quantities $\alpha_l(q)$, and hence the implementation of the pseudo-tournament, depends crucially on the independence of the production shocks $\varepsilon_1, \varepsilon_2 \ldots, \varepsilon_n$. Suppose that in addition to the independent shocks $\{\varepsilon_k\}$, there is a common shock η, so that $q_k = f(a_k, \varepsilon_k, \eta)$. For each η, the tournament automatically imposes lotteries described by the quantities $\alpha_l(q, \eta)$, which are equal to the right-hand side of (10) with $F(q, a, \eta)$ replacing $F(q, a)$. Yet, unless the planner observes the value of η, he cannot compute these quantities. Consequently, he cannot construct a pseudo-tournament that imposes exactly the same amount of risk on each individual as the tournament and differs only in the correlation among rewards that it induces.

16. The tournament and the pseudo-tournament induce equivalent games among agents in the choice of effort levels. Hence even if the solution concept is some appropriate refinement of Nash equilibrium (say to incorporate the possibility of collusion among agents), both compensation schemes will induce the same effort decisions.

17. Note that with a continuum of agents, the ex post distributions under the tournament and the pseudo-tournament are identical. However, with a countable infinity of agents this is not generally the case. For example, consider the tournament analysed by Nalebuff and Stiglitz (1983) where the agent with the worst performance suffers a large penalty, whereas all others get the same wage. With n agents, the ex ante probability of being penalized is $1/n$ for each of them. The distribution of the number of agents penalized under the corresponding pseudo-tournament is binomial, with n trials and failure probability $1/n$. As n goes to infinity, this distribution converges to a Poisson, whereas under the tournament exactly one agent will be penalized.

18. We are omitting from the summation the terms for which $i = j$, since the expectations of these terms depend only on the marginal distributions and are thus equal under the pseudo-tournament and the tournament.

19. It is clear that this sequence converts all of the off-diagonal elements in Table III into the corresponding ones in Table IV. That the diagonal elements are equalized follows from the fact that ETI's leave the marginal distributions, and hence the row and column sums, unchanged.

20. For a formal proof, see Meyer and Mookherjee (1984, p. 11).

21. The consequences of dropping this assumption are analyzed in Meyer and Mookherjee (1984, Section 5).

22. If the welfare function is concave then the principal will never prefer to randomize wages corresponding to a given pair of performance levels.

23. Here again we have expressed ex post welfare as a function of individual von Neumann-Morgenstern utilities, for convenience of notation. This is legitimate if the utility function employed by the planner to make equity judgments is strictly increasing in the level of compensation. If $U(I_i, a_i)$ denotes the utility assigned by the planner to an agent with compensation I_i and effort level a_i, and the "true" ex post welfare function is

$$W^*(U(I^1, a_1), U(I^2, a_2))$$

we can define, given the effort levels (a_1, a_2), the following function

$$W(v^1, v^2) \equiv W^*(U(X^{-1}(v^1), a_1), U(X^{-1}(v^2), a_2))$$

where v^i is a von Neumann-Morgenstern utility level for i. The important point to note is that W inherits the complementarity properties of W^*, as long as the two individuals are identical in all respects.

24. This follows from the fact that the feasible set for the socially-owned firm is a subset of that for the capitalist firm and thus by the argument in Proposition 1 in Mookherjee (1984) can be bounded without loss of generality.

25. For those performance outcomes that cannot occur under (a_1, a_2) the principal can pay a suitably low amount to both agents.

26. Compare with Mookherjee (1984, equation (5)).

27. If $W_{12} = 0$, r is locally independent of its second argument and hence so is m.

28. In Section 3, we showed that rank-order tournaments are welfare-dominated by independent contracts. The latter in turn are dominated by schemes involving positively correlated wage payments.

29. As production shocks become more correlated, the relative importance of the incentive component grows, and the qualitative divergence between the welfare-optimal and the capitalist compensation schemes shrinks, as shown by Propositions 6 and 7 in Meyer and Mookherjee (1984).

30. See Epstein and Tanny (1980) for the latter.

REFERENCES

ARROW, K. (1963) *Social Choice and Individual Values*, 2nd edition (New Haven: Yale University Press).

BROOME, J. (1984), "Uncertainty and Fairness", *Economic Journal*, **94**, 624-632.

DASGUPTA, P., SEN, A. and STARRETT, D. (1973), "Notes On the Measurement of Inequality", *Journal of Economic Theory*, **6**, 180-187.

DIAMOND, P. (1967), "Cardinal Welfare, Individualistic Ethics and Interpersonal Comparisons of Utility: A Comment", *Journal of Political Economy*, **75**, 765-766.

EPSTEIN, L. and TANNY, S. (1980), "Increasing Generalized Correlation: A Definition and Some Economic Consequences", *Canadian Journal of Economics*, **13**, 16-34.

GREEN, J. and STOKEY, N. (1983), "A Comparison of Tournaments and Contracts", *Journal of Political Economy*, **91**, 349-364.

GROSSMAN, S. and HART, O. (1983), "An Analysis of the Principal Agent Problem", *Econometrica*, **51**, 7-46.

HAMMOND, P. (1981), "Ex-Ante and Ex-Post Welfare Optimality Under Uncertainty", *Economica*, **48**, 235-250.

HAMMOND, P. (1982), "Utilitarianism, Uncertainty and Information", in Sen, A. K. and Williams, B. (eds.) *Utilitarianism and Beyond* (Cambridge: Cambridge University Press).

HAMMOND, P. (1983), "Ex-post Optimality as a Dynamically Consistent Objective for Collective Choice under Uncertainty", in Pattanaik, P. K. and Salles, M. (eds.) *Social Choice and Welfare* (Amsterdam: North Holland Publishing Company).

HARRIS, R. and OLEWILER, N. (1979), "The Welfare Economics of Ex-post Optimality", *Economica*, **46**, 137-147.

HARSANYI, J. (1955), "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons", *Journal of Political Economy*, **63**, 309-321.

HEY, J. D. and LAMBERT, P. J. (1980), "Relative Deprivation and the Gini Coefficient: Comment", *Quarterly Journal of Economics*, **95**, 567-573.

HOLMSTRÖM, B. (1979), "Moral Hazard and Observability", *Bell Journal of Economics*, **10**, 79-91.

HOLMSTRÖM, B. (1982), "Moral Hazard in Teams", *Bell Journal of Economics*, **13**, 324-340.

MEYER, M. (1985), "Multivariate Correlation and the Measurement of Ex Post Inequality under Uncertainty" (mimeo, St. John's Colllege, Oxford).

MEYER, M. (1986), "Multiperiod Incentive Schemes and Stratification" (mimeo, St. John's College, Oxford).

MEYER, M. and MOOKHERJEE, D. (1984), "Incentives, Compensation and Social Welfare" (Working Paper 780, Stanford Business School, Stanford University).

MIRRLEES, J. (1971). "An Exploration in the Theory of Optimal Income Taxation", *Review of Economic Studies*, **38**, 175-208.

MIRRLEES, J. (1974), "Notes on Welfare Economics, Information and Uncertainty", in Balch, M., McFadden, D. and Wu, S. (eds.) *Essays on Economic Behavior Under Uncertainty* (Amsterdam: North Holland) 243-258.

MIRRLEES, J. (1976), "The Optimal Structure of Incentives and Authority Within an Organization", *Bell Journal of Economics*, 7, 105-131.

MOOKHERJEE, D. (1984), "Optimal Incentive Schemes with Many Agents", *Review of Economic Studies*, **51**, 433-446.

MYERSON, R. (1981), "Utilitarianism, Egalitarianism, and The Timing Effect in Social Choice Problems", *Econometrica*, **49**, 883-897.

NAGEL, T. (1980), "The Limits of Objectivity", in McMurrin, S. *Tanner Lectures on Human Values* (Cambridge: Cambridge University Press).

NALEBUFF, B. and STIGLITZ, J. (1983), "Prizes and Incentives: Towards a General Theory of Compensation and Competition", *Bell Journal of Economics*, **14**, 21-43.

PARFIT, D. (1984) *Reasons and Persons* (Oxford: Clarendon Press).

ROTHSCHILD, M. and STIGLITZ, J. (1970), "Increasing Risk: A Definition", *Journal of Economic Theory*, **2**, 225-243.

RUNCIMAN, W. G. (1966) *Relative Deprivation and Social Justice* (London: Routledge).

SEN, A. (1973) *On Economic Inequality* (Oxford: Clarendon Press).

SHAVELL, S. (1979), "Risk Sharing and Incentives in the Principal and Agent Relationship", *Bell Journal of Economics*, **10**, 55-73.

SHESHINSKI, E. (1972), "Relation between a Social Welfare Function and the Gini Index of Inequality", *Journal of Economic Theory*, 4, 98-100.

STARR, R. (1973), "Optimal Production and Allocation under Uncertainty", *Quarterly Journal of Economics*, **87**, 81-95.

WILLIAMS, B. (1973), "A Critique of Utilitarianism", in Smart, J. and Williams, B. (eds.) *Utilitarianism: For and Against* (Cambridge: Cambridge University Press).

WILLIAMS, B. (1982) *Moral Luck* (Cambridge: Cambridge University Press).

YITZHAKI, S. (1979), "Relative Deprivation and the Gini Coefficient", *Quarterly Journal of Economics*, **93**, 321-324.