Robustness of Full Revelation in Multisender Cheap Talk*

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Abstract

This paper studies information transmission in a two-sender, multidimensional cheap talk setting where there are exogenous restrictions on the feasible set of policies for the receiver. Such restrictions are present in most applications, and by limiting the punishments available to the receiver, they can prevent the existence of fully revealing equilibria (FRE). We focus on equilibria that are robust to small mistakes by the senders, in that small differences between the senders' messages result in only small punishments by the receiver. For convex policy spaces in two dimensions, we provide a simple, local geometric condition, on the directions of the senders' bias vectors relative to the frontier of the policy space, that is necessary and sufficient for the existence of a robust FRE that is independent of the magnitudes of the biases. We also provide a specific policy rule for the receiver that supports a robust FRE whenever one exists. The same local geometric condition remains necessary and sufficient for existence even if we drop either the requirement of robustness or the requirement that the equilibrium be independent of the magnitudes of the biases (but not both). Our necessary and sufficient condition can be easily adapted if the receiver is uncertain about the directions of the biases and/or if the biases vary with the state of the world. Finally, we extend our characterization results for existence of robust FRE's to convex policy spaces in more than two dimensions and to non-convex two-dimensional policy spaces.

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1 Introduction

In sender-receiver games with cheap talk, the decision-maker (receiver) has imperfect information about the consequences of a policy and elicits reports from better-informed experts (the senders), whose preferences are not perfectly aligned with those of the decision-maker (i.e. the experts are "biased"). The advice transmitted by the senders is costless but unverifiable (hence, "cheap talk"), and the receiver cannot commit himself in advance to how he will respond to the senders' advice.[1] Cheap talk games with two biased experts have been used, for example, in organizational economics to analyze the interaction between the CEO of a multi-divisional firm and the division managers, and in political science to study the transmission of information from legislative committees to the legislature as a whole.[2] In both of these contexts, as well as in most other settings to which cheap-talk models have been applied, the decision-maker typically faces constraints on the set of feasible policies—these may stem from limited budgets, from physical restrictions on what is possible (within a given time frame), or from legal constraints.

Our objective in this paper is to provide simple geometric conditions, on the shape of the feasible set of policies relative to the directions of the senders' bias vectors, that are necessary and sufficient for the existence of equilibria that are not only fully revealing but have additional desirable properties.

In cheap-talk models in which the receiver can consult two equally well-informed senders, the receiver has the potential to extract all of the senders' information, by comparing the senders' messages and punishing any discrepancy between them. However, as Battaglini (2002) demonstrated, with a unidimensional policy space and senders whose preferences are biased, relative to the receiver's, in opposite directions, a fully revealing equilibrium exists if and only if the magnitudes of the senders' biases are small relative to the size of the policy space. Furthermore, those fully revealing equilibria are such that small deviations by the senders from their equilibrium strategies will result in large punishments by the receiver; in consequence, such equilibria fail to be 'robust' to small mistakes by the senders. For multidimensional policy spaces, Battaglini's equilibrium construction implies both that a fully revealing equilibrium exists no matter how large the magnitudes of the senders' biases (as long as the two vectors are linearly independent) and that small deviations or mistakes induce only small changes in the receiver's chosen policy.

The message of these contrasting results would appear to be that when the receiver's choice set expands from one dimension to more than one, full extraction of information from self-interested experts, even in a manner robust to small mistakes, becomes unproblematic. However, Battaglini's analysis of the multidimensional case assumes that both the state space and the receiver's policy space are the whole of $\mathbb{R}^d$; under this assumption, there are neither exogenous restrictions on the receiver's possible actions nor constraints stemming from the requirement of Perfect Bayesian Equilibrium that the receiver's action after

[1] For the seminal paper in this literature see Crawford and Sobel (1982).

[2] For the former application, see Alonso, Dessein, and Matouschek (2008) and for the latter, Gilligan and Krehbiel (1989) and Krishna and Morgan (2001a,b)

out-of-equilibrium messages be an optimal response to some beliefs.

Ambrus and Takahashi (2008) analyze the case of *compact* multidimensional state spaces. Since the receiver's out-of-equilibrium beliefs must in consequence be confined to the compact state space, the set of actions with which the receiver can punish any discrepancies between the senders' messages is therefore also confined to the (convex hull of the) compact state space. This limitation on the punishments available to the receiver can prevent the existence of fully revealing equilibria, and non-existence of such equilibria becomes more likely the larger the magnitudes of the senders' biases. Ambrus and Takahashi show that, when the magnitudes of the biases can be arbitrarily large, a fully revealing equilibrium exists if and only if the senders have a common least-preferred policy. While this characterization result is elegant, the equilibrium construction involves the receiver punishing any discrepancies between the senders' reports by choosing their common least-preferred policy. As they themselves acknowledge, the use of extreme punishments after even small deviations is unappealing, since such deviations could in practice arise from small mistakes by the senders. Their analysis leaves open the question of characterizing, for restricted state spaces or policy spaces, the conditions for existence of a fully revealing equilibrium that is robust to small mistakes.

This is the main question that we tackle in this paper. In our model, as in Ambrus and Takahashi (2008) and Battaglini (2002), the receiver and the senders all have quadratic utility functions, and sender i's ideal point differs from the receiver's by a vector, b_i, sender i's bias vector. We define an intuitive notion of robustness for a fully revealing equilibrium, that requires that the receiver responds to small discrepancies between the senders' messages with small punishments, that is, punishments that are close to the messages. For restricted policy spaces, and hence restricted punishment possibilities, we characterize the conditions under which there exists a fully revealing equilibrium that is robust to small mistakes by the senders and independent of the magnitudes of the biases.

We begin by focusing on convex policy spaces in an arbitrary number of dimensions. We prove that whenever there exists a fully revealing equilibrium (FRE) that is independent of the magnitudes of the biases, there also exists a robust FRE independent of these magnitudes. In other words, when biases can be arbitrarily large, if small deviations cannot be deterred with small punishments, then they cannot be deterred with any feasible punishments. Moreover, we show that for convex policy spaces that are two-dimensional or multidimensional and compact, it is sufficient for existence of a FRE (robust or not) that small deviations can be deterred (with small punishments). These preliminary results are extremely useful, because they show that a) robustness is, perhaps surprisingly, not a restrictive requirement on a FRE when biases can be arbitrarily large and the policy space is convex; and b) in the two-dimensional or compact multidimensional cases, we need only ensure that local deviations can be punished.

Section 3.1 then focuses on the case where the policy space is two-dimensional and convex. Proposition 4 identifies a simple, local geometric condition, on the directions of the senders' bias vectors relative to the frontier of the policy space, that is necessary and sufficient for the existence of a FRE (robust or not), independent of the magnitudes of the biases. The proposition also provides a specific policy rule for the receiver that supports a robust FRE whenever one exists. To describe this rule, observe that as the senders' biases become arbitrarily large, their indifference curves approach hyperplanes. Using the coordinate system defined by these limiting preferences of the senders, the policy rule specifies that, given any two reported states, the receiver chooses the component-wise minimum of these reports: this is the best policy for both senders that is also, for both of them, at least weakly inferior to both reported states. If this policy rule is feasible, it deters deviations from truthful reporting, independently of the magnitudes of the biases, and it does so in a manner robust to small mistakes. Proposition 4 shows that this policy rule is feasible, for all pairs of reports, if and only if the local geometric condition is satisfied.

To state this local geometric condition, define, for given bias vectors b_1 and b_2, the open convex cone $C(b_1, b_2)$, and the closed convex cone $\overline{C}(b_1, b_2)$, spanned by b_1 and b_2. The condition requires that at every smooth point on the frontier of the policy space, the inward-pointing normal vector to the frontier not lie in $C(b_1, b_2)$. This condition is easiest to interpret by examining its "strict" version: For a given smooth point θ on the frontier, the requirement that the inward-pointing normal vector not lie in $\overline{C}(b_1, b_2)$ holds if and only if, even for arbitrarily large biases, there exists a feasible policy for the receiver, close to θ, that would make both senders strictly worse off than if the receiver chose $y = \theta$. Now consider the boundary case of a smooth point θ on the frontier at which the inward normal vector coincides with the direction of one (or both) of the bias vectors. If the frontier is locally curved, so the inward normal vector is rotating, then the necessary and sufficient condition will be violated at a nearby θ; if instead, the frontier is linear in a neighborhood of θ, then there exist nearby policies on the frontier that would leave both senders weakly worse off than if the receiver chose the action $y = \theta$. If and only if the inward normal vector lies outside $C(b_1, b_2)$ for all smooth points on the frontier, all local deviations can be deterred with (possibly weak) local punishments.

For convex spaces in two dimensions, we also prove that the same condition remains necessary and sufficient for existence of a robust FRE even if the biases have known finite sizes. This is true because, when the receiver is constrained to use small punishments, whether or not the senders have incentives to deviate from truthtelling depends only on the orientations, not the magnitudes, of their bias vectors.

In Section 3.2, we extend our characterization to a convex state space of any dimension $d > 2$. The key observation here is that, for bias vectors that are linearly independent, the only directions of conflict between the senders and the receiver are the ones in the plane spanned by these vectors. Proposition 5 shows that, for existence of a FRE (robust or not) when the biases can be arbitrarily large, it is necessary and sufficient to look at the projection of the state space onto the subspace of conflict of interest and see whether a FRE can be constructed there. The reason is that, for arbitrarily large biases, no given shift of the receiver's action in a direction orthogonal to the plane of the biases can be certain to serve as a punishment for a deviating sender. Therefore, to be certain that he is actually punishing a deviation, the receiver needs to choose an action whose projection on the plane of the biases is worse for both senders. Such an action exists if and only if the projection of the state space onto the plane of the biases satisfies the necessary and sufficient condition identified in Proposition 4 for the two-dimensional case.

Since indivisibilities may cause the set of feasible actions for the receiver to be non-convex, it is important to address the question of the existence of a robust FRE for non-convex spaces. This we do in Section 4. We identify a pair of local geometric conditions, on the directions of the senders' biases relative to 1) the frontier of the convex hull of the policy space and 2) the frontier of the policy space itself, that together are necessary and sufficient for existence of a robust FRE, independent of the magnitudes of the biases. (When the policy space is convex, both conditions reduce to the condition identified in Proposition 4 in Section 3.1.) The second condition is necessary and sufficient for small deviations to be deterrable with small punishments, but for non-convex policy spaces, this is not sufficient for existence of a robust FRE: large deviations might not be deterrable even if small ones are. The first condition is necessary and sufficient for all deviations, including large ones, to be deterrable with feasible punishments, when the biases can be arbitrarily large.

Finally, in Section 5, we relax the assumptions that (i) the directions of the senders' biases are common knowledge and (ii) the biases are independent of the realization of the state. We prove that when the receiver does not know the actual biases but knows only the minimal closed cone in which they are certain to lie, and this minimal cone is the same for all states, then the necessary and sufficient condition for existence of a robust FRE for arbitrarily large biases is the same condition identified in Proposition 4 in Section 3.1, except that the known biases b_1 and b_2 there are replaced by the least aligned possible realizations of the biases.

2 The Model

The model we consider is the same as that of Ambrus and Takahashi (2008) and Battaglini (2002). The game is between two senders S_1, S_2 and a receiver R. Both senders perfectly observe $\theta \in \Theta \subseteq \mathbb{R}^d$ the realization of a random variable $\hat{\theta}$. We will refer to the realization θ as the state. The prior distribution of $\hat{\theta}$ is given by F and is commonly known. After observing θ, each sender S_i sends message $m_i \in \mathcal{M}_i$ to the receiver who then chooses a policy y from a set of feasible policies Y, a closed subset of Θ. We will refer to the pair (Θ, Y) as the *environment* of the game.

Given the state θ and the chosen policy y, the receiver's utility is $-|y - \theta|^2$, and each sender i's utility is $-|y - \theta - b_i|^2$. The vector $b_i \in \mathbb{R}^d$ is referred to as the bias vector of sender i. Given these utilities, an ideal policy for the receiver when the state is θ is $y^*(\theta) \in \arg\min_{y \in Y} |y - \theta|^2$. In the particular case in which $Y \equiv \Theta$, $y^*(\theta) = \theta$.[3]

Sender S_i's strategy will be denoted by $s_i : \Theta \longrightarrow \mathcal{M}_i$, and the receiver's strategy will be denoted by $y : \mathcal{M}_1 \times \mathcal{M}_2 \longrightarrow Y$. Given messages m_1, m_2, $\mu(m_1, m_2)$ denotes the receiver's belief about $y^*(\hat{\theta})$ after receiving m_1, m_2. We denote by $y^R(m_1, m_2) \in Y$ an optimal policy

[3]When the policy space Y is a strict subspace of Θ and Y is non-convex, the set $\arg\min_{y \in Y} |y - \theta|^2$ might not be a singleton. In such a case, we will focus on one particular ideal policy and label this policy $y^*(\theta)$; which ideal policy is singled out in this way is irrelevant.

for the receiver given belief $\mu(m_1, m_2)$, and $\mu(\cdot)$ will be referred to as the belief function of the receiver. Since the senders' payoffs depend on the receiver's choice of policy, it is more convenient to work directly with the receiver's beliefs over the ideal policy $y^*(\hat{\theta})$ than with his beliefs over $\hat{\theta}$. The equilibrium concept we use is *Perfect Bayesian Equilibrium*.

Definition 1. The strategies (s_1, s_2, y) constitute a Perfect Bayesian Equilibrium if there exists a belief function $\mu(\cdot)$ such that:

(i) s_i is optimal given s_{-i} and y for $i \in \{1, 2\}$.

(ii) $y(m_1, m_2) = y^R(m_1, m_2)$ for each $(m_1, m_2) \in \mathcal{M}_1 \times \mathcal{M}_2$

(iii) If $s_1^{-1}(m_1) \cap s_2^{-1}(m_2) \neq \emptyset$ then $\mu(m_1, m_2)$ is derived from Bayes' rule.

2.1 Robust Fully Revealing Equilibrium

In what follows we will focus on a special kind of equilibria in which the receiver perfectly learns the ideal policy from the messages of the senders.

The strategies (s_1, s_2) are *fully revealing* if for all $\theta \in \Theta$ the conditional probability of the random variable $y^*(\hat{\theta})$ given messages $s_1(\theta)$ and $s_2(\theta)$ has mass one on $y^*(\theta)$. In particular $y^R(s_1(\theta), s_2(\theta)) = y^*(\theta)$. An equilibrium with fully revealing strategies is called a *fully revealing equilibrium* (FRE).

In a Perfect Bayesian Equilibrium, no restriction is imposed on beliefs in response to out-of- equilibrium messages, i.e. messages such that $s_1^{-1}(m_1) \cap s_2^{-1}(m_2) = \emptyset$. This implies that after any incompatible messages the receiver could choose any policy to punish the deviation. However, when the discrepancy between two incompatible messages is small, it might be reasonable to think that the receiver's chosen policy should be close to each of the messages, since small discrepancies might be due not to deliberate misrepresentation by the senders but rather to small mistakes. Battaglini (2002) was the first to raise this concern when analysing fully revealing equilibria in models with a *unidimensional* state space. He showed that in such models, when the senders were biased in opposite directions relative to the receiver, none of the fully revealing equilibria were robust to a perturbation of the game that allowed small mistakes in the senders' reports. This problem did not arise in his multidimensional analysis because, with no restrictions on the state or policy spaces and with his equilibrium construction, there were never any incompatible reports. However, as Ambrus and Takahashi (2008) pointed out, incompatible messages arise naturally, even with Battaglini's construction, when the state (or policy) space is restricted.

We now formulate a definition of robustness of an equilibrium to small mistakes that explicitly captures the requirement that when the discrepancy between two incompatible messages is small, the receiver's chosen policy should be close to each of the messages. In order to do so, we introduce some notation that will be used throughout the paper. Given $x \in \mathbb{R}^d$ and a scalar $\epsilon > 0$, $B(x, \epsilon) = \{y \in \mathbb{R}^d \mid |y - x| < \epsilon\}$ is the ball with centre x and radius ϵ. In particular $B(\theta + b, |b|)$ is the set of points that are closer to $\theta + b$ than θ. In other words, $B(\theta + b, |b|)$ is the set of points that a sender with bias b would prefer to θ.

Definition 2. Given some fully revealing strategies (s_1, s_2), a belief function $\mu(\cdot)$ *deters local deviations with local punishments* if for any $\theta \in \Theta$ and any $\epsilon > 0$, there exist $\delta > 0$ and $\delta' > 0$ such that if: (*i*) $\theta', \theta'' \in B(\theta, \delta) \cap \Theta$ and (*ii*) $y^*(\theta'), y^*(\theta'') \in B(y^*(\theta), \delta') \cap Y$, then

$$y^R(s_1(\theta'), s_2(\theta'')) \in B(y^*(\theta), \epsilon) \cap Y$$
$$|y^R(s_1(\theta'), s_2(\theta'')) - (\theta'' + b_1)| \geq |y^*(\theta'') - (\theta'' + b_1)|$$
$$|y^R(s_1(\theta'), s_2(\theta')) - (\theta' + b_2)| \geq |y^*(\theta') - (\theta' + b_2)|$$

A fully revealing equilibrium (s_1, s_2, y) supported by a belief function that deters local deviations with local punishments is called a *robust* fully revealing equilibrium.[4]

If Y is convex and/or $\Theta = Y$, then condition (*ii*) in Definition 2 is superfluous: in either case, whenever θ' and θ'' are close, $y^*(\theta')$ and $y^*(\theta'')$ are also close. Condition (*ii*) is relevant when Y is non-convex and $\Theta \supset Y$, because in this case, small changes in θ for $\theta \notin Y$ can result in large changes in the receiver's ideal policy $y^*(\theta)$.

There are two interpretations of the type of small mistakes to which Definition 2 requires the receiver to respond with only small punishments. That is, there are two interpretations for why, even if senders do not intend to mislead the receiver, the receiver might nevertheless receive incompatible reports. First, there might be some noise in the communication process, with the result that the receiver might not interpret the messages exactly as the senders intended. Second, even if the communication process were noiseless, the senders might not perceive the state perfectly accurately, and their errors might not be perfectly correlated. Under either interpretation of mistakes, our analysis would apply when the senders and the receiver were unaware that these mistakes might happen. Our robustness requirement ensures that as the size of the mistakes goes to zero, the outcome in the presence of mistakes approaches the outcome when mistakes never occur.

2.2 Preliminary Results

In a fully revealing equilibrium (FRE), the receiver perfectly learns the ideal policy from the pair of messages, and neither sender has an incentive to try to mislead the receiver by sending a different message. Using a similar argument to the Revelation Principle we can, without loss of generality, concentrate on equilibria in which each sender truthfully reports the ideal policy given his observation. The strategies (s_1, s_2) are *truthful* if $\mathcal{M}_1 = \mathcal{M}_2 = Y$ and $s_i(\theta) = y^*(\theta)$. An equilibrium with truthful strategies is called a *truthful equilibrium*.

Lemma 1 is an extension of Lemma 1 in Battaglini (2002) that incorporates our notion of robustness and simplifies our subsequent analysis.

Lemma 1. *For any (robust) fully revealing equilibrium there exists a (robust) truthful equilibrium that is outcome-equivalent to it.*

Proof: In the Appendix.

[4]Strategies (s_1, s_2) can together be fully revealing even if each sender's report by itself does not fully reveal the ideal policy. Battaglini's (2002) construction of a fully revealing equilibrium for an unrestricted multidimensional state space is an example of this possibility. We have stated Definition 2 in a way that allows for this possibility.

We are interested in robust fully revealing equilibria that persist even when the magnitudes of the senders' biases can be arbitrarily large. More precisely, if the senders' bias vectors are $(t_1 b_1, t_2 b_2)$, where the vectors b_1 and b_2 are commonly known and the positive scalars t_1 and t_2 can be arbitrarily large, we seek conditions under which there exists a robust fully revealing equilibrium that is independent of the values of t_1 and t_2. Such a robust FRE would remain a robust FRE even if the receiver were uncertain about the magnitudes of the senders' biases.[5]

Besides being invariant to the intensity of senders' preferences, such equilibria are also appealing because of the relative tractability of their characterization. As observed by Levy and Razin (2007), the indifference curves of a sender with a very large bias are very close to hyperplanes orthogonal to the bias vector. Furthermore, as the magnitude of the bias goes to infinity, a sender's ranking over policies becomes independent of the true state.

Proposition 1 and Proposition 2 below allow us to abstract from specifying particular belief functions when proving the existence and nonexistence of robust fully revealing equilibria that are independent of the magnitudes of the senders' biases.

Proposition 1 deals with the case in which the policy space coincides with the state space. The first part is essentially the same as in Ambrus and Takahashi (2008)'s Proposition 7, although it allows for non-compact state (and policy) spaces. It provides a necessary and sufficient condition for the existence of fully revealing equilibria that are independent of the magnitudes of the biases. The second part of Proposition 1 establishes a necessary and sufficient condition for the existence of a belief that deters local deviations with local punishments and is independent of the magnitudes of the biases. Finally, we show that the two conditions together are not only necessary but also sufficient for the existence of a robust FRE independent of the magnitudes of the biases.

Before proceeding we introduce two additional pieces of notation. Given a (bias) vector $b \in \mathbb{R}^d$ and a scalar $k \in \mathbb{R}$, we define $H(b, k) \equiv \{x \in \mathbb{R}^d \mid bx > k\}$. In other words, $H(b, k)$ is the half-space composed of all the points of $\mathbb{R}^d$ whose inner product with b is greater than k. In particular, for any point $\theta' \in H(b, b\theta)$, $b\theta' > b\theta$, and there exists a scalar $t > 0$ such that $\theta' \in B(\theta + tb, t|b|)$, that is, θ' is preferred to θ by a sender with bias tb. Finally, $h(b, k) \equiv \{x \in \mathbb{R}^d \mid bx = k)$ is the boundary of the half-space $H(b, k)$, that is, the set of points whose inner product with b is exactly k.

Proposition 1. *Suppose $Y \equiv \Theta \subseteq \mathbb{R}^d$. Given $b_1, b_2 \in \mathbb{R}^d$,*

(i) There exists a fully revealing equilibrium with biases $(t_1 b_1, t_2 b_2)$ for every $t_1, t_2 \geq 0$ if and only if[6]

$$\text{for any } \theta', \theta'' \in Y, \quad Y \not\subseteq H(b_1, b_1\theta'') \cup H(b_2, b_2\theta') \tag{1}$$

[5]Section 5 shows how our characterization results can be extended when the receiver is also, to some degree, uncertain about the directions of the senders' biases.

[6]In Ambrus and Takahashi (2008)'s Proposition 7, the condition states $co(\Theta) \not\subseteq H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$ for any $\theta', \theta'' \in \Theta$. The reason is that their policy space contains the convex hull of Θ and hence punishments might be taken from points that are in $co(\Theta) \setminus \Theta$. Since we want to allow for non-convex policy spaces our condition does not incorporate the convex hull of Θ.

(ii) There exists some fully revealing strategies and a belief function that deters local deviations with local punishments for biases $(t_1 b_1, t_2 b_2)$ for every $t_1, t_2 \geq 0$, if and only if

$$\text{for any } \theta \in Y \text{ and any } \epsilon > 0, \text{ there exists } \delta > 0 \text{ such that for any } \theta', \theta'' \in B(\theta, \delta) \cap Y$$
$$B(\theta, \epsilon) \cap Y \nsubseteq H(b_1, b_1\theta'') \cup H(b_2, b_2\theta') \tag{2}$$

(iii) Conditions (1) and (2) are necessary and sufficient for the existence of a robust fully revealing equilibrium.

Proof: In the Appendix.

When condition (1) holds, the receiver's policy rule $y^R(\theta', \theta'')$ in a truthful fully revealing equilibrium will satisfy $y^R(\theta', \theta'') = \theta'$ if $\theta' = \theta''$ and $y^R(\theta', \theta'') \in Y \backslash (H(b_1, b_1\theta'') \cup H(b_2, b_2\theta'))$ if $\theta' \neq \theta''$. Such a rule is feasible and ensures that sender 1 (resp., 2) has no incentive to deviate to a report of θ' (resp., θ'') when the true state is θ'' (resp., θ'), even for arbitrarily large magnitudes of the biases.

Proposition 2 deals with the case in which $Y \subseteq \Theta$, that is, the policy space might be a strict subset of the state space. It states that there exists a (robust) fully revealing equilibrium for $Y \subseteq \Theta$, if and only if there exist a (robust) fully revealing equilibrium when the space state is reduced to coincide with the policy space. In other words, we can ignore those states that cannot be implemented as a policy.

Proposition 2. *Given $Y \subseteq \Theta \subseteq \mathbb{R}^d$ and $b_1, b_2 \in \mathbb{R}^d$, the following two statements are equivalent:*

(i) For the environment (Θ, Y), there exists a (robust) fully revealing equilibrium for all biases $(t_1 b_1, t_2 b_2)$, $t_1, t_2 \geq 0$.

(ii) For the environment (Y, Y), there exists a (robust) fully revealing equilibrium for all biases $(t_1 b_1, t_2 b_2)$, $t_1, t_2 \geq 0$.

Proof: $(i) \Rightarrow (ii)$: Trivial

$(ii) \Rightarrow (i)$: Suppose there exists a (robust) FRE in (Y,Y). By Lemma 1, there exists a truthful (robust) equilibrium outcome-equivalent to it. Denote the truthful equilibrium by (s_1, s_2, y^R) where for all $\theta \in Y$, $s_i(\theta) = y^*(\theta) = \theta$. For $\theta \in \Theta$ we define the following strategies: $\tilde{s}_i(\theta) = y^*(\theta)$. We claim that $(\tilde{s}_1, \tilde{s}_2, y^R)$ is a (robust) FRE in (Θ, Y).

Consider the out-of-equilibrium messages (y', y'') where $y' \neq y''$ and denote by $x = y^R(y', y'')$ the receiver's policy after the report (y', y''). By Proposition 1,

$$b_1(y'' - x) \geq 0, \qquad b_2(y' - x) \geq 0. \tag{3}$$

For sender S_1, we need to show that for any $\theta \in \Theta$ such that $y^*(\theta) = y''$, $|\theta + tb_1 - y''| \leq |\theta + tb_1 - x|$ for all $t > 0$. For any $\theta \in \Theta$ with $y^*(\theta) = y''$, y'' is the closest point in Y to θ. Hence, $|\theta - y''| \leq |\theta - x|$. Define z as the midpoint of the segment $[x, y'']$. Then

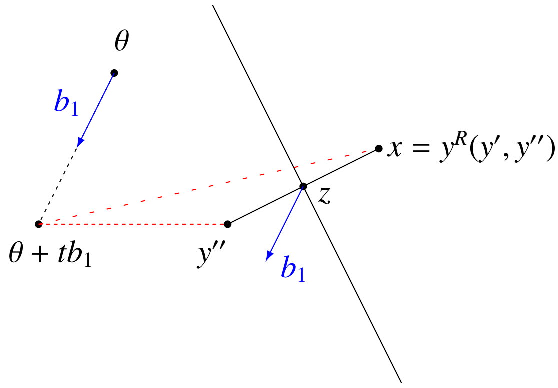

Figure 1

$\theta(y'' - x) \geq z(y'' - x)$, and by (3), $(\theta + tb_1)(y'' - x) \geq z(y'' - x)$ for all $t > 0$, or in other words $|\theta + tb_1 - y''| \leq |\theta + tb_1 - x|$ for all $t > 0$. A similar argument for S_2 shows that for any $\theta \in \Theta$ such that $y^*(\theta) = y'$, $|\theta + tb_2 - y'| \leq |\theta + tb_2 - x|$ for all $t > 0$. Therefore $(\tilde{s}_1, \tilde{s}_2, y^R)$ is a FRE in (Θ, Y). (See Figure 1.)

Finally if we further assume that the initial equilibrium (s_1, s_2, y^R) is robust, then by Proposition 1, for any $y \in Y$ and $\epsilon > 0$, there exists $\delta > 0$ such that for all $y', y'' \in B(y, \delta) \cap Y$, $y^R(y', y'') \in B(y, \epsilon)$. In particular, for any $\theta \in \Theta$ and $\theta', \theta'' \in \Theta$ such that $y^*(\theta'), y^*(\theta'') \in B(y^*(\theta), \delta)$, we have that $y^R(y^*(\theta'), y^*(\theta'')) \in B(y^*(\theta), \epsilon)$. □

Given Proposition 2, the shape of the state space Θ is irrelevant (as long as $Y \subseteq \Theta$), and all that matters for the existence of a (robust) FRE is the shape of the policy space, relative to the bias vectors of the senders. For the rest of the paper, we can therefore focus, when proving existence results for (robust) FRE's, on the case in which $\Theta \equiv Y$. Proposition 1, which is stated for the case $\Theta \equiv Y$, will be our primary tool.

Finally, we discuss two special cases where, for any number of dimensions and any shape of Y, it is straightforward to draw conclusions about the existence of a robust fully revealing equilibrium when the biases can be arbitrarily large. First, if the senders' bias vectors are in exactly the same direction (i.e. $b_1 = tb_2$ for some strictly positive scalar t), then for arbitrarily large magnitudes there always exists a robust FRE. In it, the receiver responds to any discrepancy between the messages by choosing whichever of the two reported states leads to a smaller inner product with (each of) the bias vectors. In other words, the receiver's chosen policy coincides with whichever of the reported states would be less preferred, if biases were infinitely large, by both senders. Such a strategy for the receiver ensures that neither sender can strictly gain by deviating from truthful reporting, and since the receiver's chosen policy always coincides with one of the senders' messages, this FRE satisfies our definition of robustness.

Second, if the biases are exactly opposite (i.e. $b_1 = tb_2$ for some strictly negative scalar t), then it follows from part (*i*) of Proposition 1 and Proposition 2 that a FRE exists for arbitrarily large biases "if and only if Y is included in a lower dimensional hyperspace that is orthogonal to the direction of the biases" (Ambrus and Takahashi (2008, p. 13)). In addition, it follows from part (*ii*) that when a FRE exists in this case, a robust FRE exists as well: a (truthful) robust FRE is supported by a response function for the receiver such that $y(\theta', \theta'') = \lambda\theta' + (1-\lambda)\theta''$, for $\lambda \in [0, 1]$.

For the remainder of the paper, we will exclude these two special cases and assume that b_1 and b_2 are linearly independent.

3 Convex Policy Space

We begin by focusing on convex state spaces in an arbitrary number of dimensions. Proposition 3 below shows that, when biases can be arbitrarily large, whenever there exists a fully revealing equilibrium (FRE), there also exists a robust FRE. In other words, if small deviations cannot be deterred with small punishments, then they cannot be deterred with any feasible punishments. Moreover, we show that for convex state spaces that are two-dimensional or multidimensional and compact, it is sufficient for existence of a FRE (robust or not) that small deviations can be deterred (with small punishments). These preliminary results are extremely useful, because they show that a) robustness is, perhaps surprisingly, not a restrictive requirement on a FRE when biases can be arbitrarily large and the state space is convex; and b) in the two-dimensional or compact multidimensional cases, we need only to ensure that local deviations can be punished.

Proposition 3. *Given $Y \subseteq \mathbb{R}^d$ convex and $b_1, b_2 \in \mathbb{R}^d$ linearly independent, the following statements are equivalent:*

(i) There exists a fully revealing equilibrium for all biases (t_1b_1, t_2b_2) with $t_1, t_2 \geq 0$

(ii) There exists a robust fully revealing equilibrium for all biases (t_1b_1, t_2b_2) with $t_1, t_2 \geq 0$.

When we further assume that a) $Y \subseteq \mathbb{R}^2$ or that b) $Y \subseteq \mathbb{R}^d$ and Y is compact, then the following statement is also equivalent to the previous two:

(iii) Local deviations can be deterred for all biases (t_1b_1, t_2b_2) with $t_1, t_2 \geq 0$.

Proof: By Proposition 2, we can restrict attention to the case in which $\Theta \equiv Y$.
$(ii) \Rightarrow (i)$ is trivial.
$(i) \Rightarrow (ii)$ We argue in two steps. First, we prove that if local deviations from $\theta \in Y$ cannot be deterred with a local punishment, then there exists $\epsilon > 0$ such that

$$B(\theta, \epsilon) \cap Y \subseteq \overline{H}(b_1, b_1\theta) \cup \overline{H}(b_2, b_2\theta)$$

where $\overline{S}$ denotes the closure of S. Note that this statement is independent of whether Y is convex or not. Second, we use the first result and the convexity of Y to show that if a local deviation cannot be deterred with a local punishment, it cannot be deterred with any punishment and hence a fully revealing equilibrium does not exist.

STEP 1: If local deviations from $\theta \in Y$ cannot be deterred with local actions then by Proposition 1 there exists $\epsilon > 0$ such that for every $\delta > 0$ there exist $\theta'_\delta, \theta''_\delta \in B(\theta, \delta) \cap Y$ such that $B(\theta, \epsilon) \cap Y \subseteq H(b_1, b_1\theta''_\delta) \cup H(b_2, b_2\theta'_\delta)$. We show that for that same ϵ, $B(\theta, \epsilon) \cap Y \subseteq$

$\overline{H}(b_1, b_1\theta) \cup \overline{H}(b_2, b_2\theta)$. Suppose that $B(\theta, \epsilon) \cap Y \not\subseteq \overline{H}(b_1, b_1\theta) \cup \overline{H}(b_2, b_2\theta)$. Then there exists $\tilde{\theta} \in B(\theta, \epsilon) \cap Y$, such that $b_1\tilde{\theta} < b_1\theta$ and $b_2\tilde{\theta} < b_2\theta$. Define $\tilde{\delta} = min\{\frac{|b_1(\theta-\tilde{\theta})|}{|b_1|}, \frac{|b_2(\theta-\tilde{\theta})|}{|b_2|}\}$ and denote $\tilde{\theta}', \tilde{\theta}'' \in B(\theta, \tilde{\delta})$ the corresponding $\theta'_{\tilde{\delta}}$ and $\theta''_{\tilde{\delta}}$ such that $B(\theta, \epsilon) \cap Y \subseteq H(b_1, b_1\tilde{\theta}'') \cup H(b_2, b_2\tilde{\theta}')$. But by the definition of $\tilde{\delta}$, $b_1\tilde{\theta} < b_1\tilde{\theta}''$ and $b_2\tilde{\theta} < b_2\tilde{\theta}'$ and hence $\tilde{\theta} \in B(\theta, \epsilon) \cap Y \setminus \left(H(b_1, b_1\tilde{\theta}'') \cup H(b_2, b_2\tilde{\theta}')\right)$ which is a contradiction. See Figure 2.

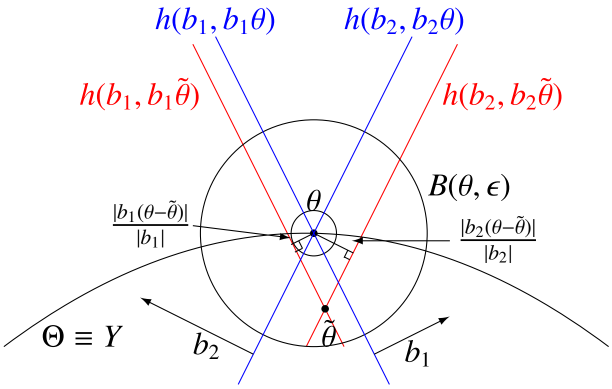

Figure 2

STEP 2: Suppose that local deviations from $\theta \in Y$ cannot be deterred with a local punishment. By Step 1 there exists $\epsilon > 0$ such that $B(\theta, \epsilon) \cap Y \subseteq \overline{H}(b_1, b_1\theta) \cup \overline{H}(b_2, b_2\theta)$. Define $\theta' = \arg\min\{b_2\tilde{\theta} \mid \tilde{\theta} \in \overline{B}(\theta, \epsilon) \cap Y\}$ and $\theta'' = \arg\min\{b_1\tilde{\theta} \mid \tilde{\theta} \in \overline{B}(\theta, \epsilon) \cap Y\}$. Clearly $B(\theta, \epsilon) \cap Y \subset H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$ and hence either $b_1\theta > b_1\theta''$ or $b_2\theta > b_2\theta'$. Without loss of generality assume that $b_1\theta > b_1\theta''$. We show now that the deviation $\{\theta', \theta''\}$ cannot be deterred in Y. (See Figure 3.)

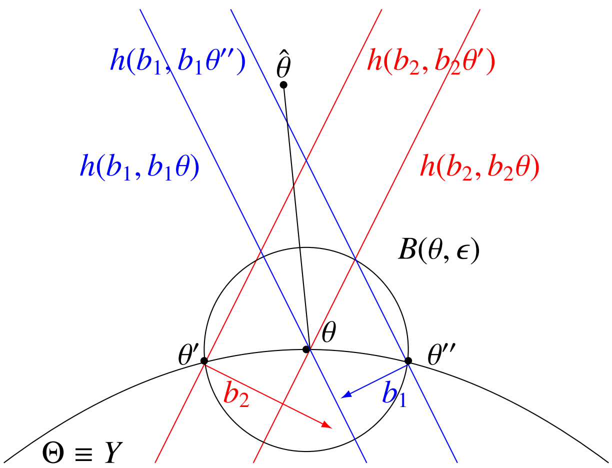

Figure 3: Every fully revealing equilibrium is robust: if a local deviation cannot be deterred with a local punishment, it cannot be deterred with any punishment.

Suppose there exists $\hat{\theta} \in Y$ such that $b_1\hat{\theta} \leq b_1\theta'' < b_1\theta$ and $b_2\hat{\theta} \leq b_2\theta' \leq b_2\theta$. We claim that $b_2\hat{\theta} < b_2\theta$ and hence by the convexity of Y there exists a $\lambda \in (0, 1)$ such that $\lambda\hat{\theta} + (1 - \lambda)\theta \in B(\theta, \epsilon) \cap Y$ and $b_1(\lambda\hat{\theta} + (1 - \lambda)\theta) < b_1\theta$, $b_2(\lambda\hat{\theta} + (1 - \lambda)\theta) < b_2\theta$ which contradicts that $B(\theta,\epsilon)\cap Y \subseteq \overline{H}(b_1,b_1\theta)\cup\overline{H}(b_2,b_2\theta)$.

To see that $b_2\hat{\theta} < b_2\theta$, suppose $b_2\hat{\theta} = b_2\theta = b_2\theta' = \min\{b_2\tilde{\theta} \mid \tilde{\theta} \in B(\theta,\epsilon)\cap Y\}$ (See Figure 4). Since local deviations from θ cannot be deterred, by Proposition 1 there exists an $\tilde{\epsilon} > 0$ such that for any $\delta > 0$ there exists $\theta'_\delta, \theta''_\delta \in B(\theta,\delta)\cap Y$ such that $B(\theta,\tilde{\epsilon})\cap Y \subset H(b_1,b_1\theta''_\delta)\cup H(b_2,b_2\theta'_\delta)$. Consider $\delta < \min\{\epsilon, \tilde{\epsilon}\frac{|b_1t_2|}{|b_1|}\}$, where t_2 is the unit normal vector to b_2 such that $b_1t_2 < 0$. In particular, since $\delta < \tilde{\epsilon}\frac{|b_1t_2|}{|b_1|}$, $b_1\theta''_\delta > b_1(\theta+\tilde{\epsilon}t_2)$ and hence there exists $\mu \in (0,1)$ such that $\mu\hat{\theta}+(1-\mu)\theta \in B(\theta,\tilde{\epsilon})\cap Y$ and $b_1\theta''_\delta > b_1(\mu\hat{\theta}+(1-\mu)\theta)$. Moreover, since $\delta < \epsilon$ and $b_2\hat{\theta} = b_2\theta = b_2\theta' = \min\{b_2\tilde{\theta} \mid \tilde{\theta} \in B(\theta,\epsilon)\cap Y\}$, $b_2(\mu\hat{\theta}+(1-\mu)\theta) \le b_2\theta'_\delta$. But this contradicts that $B(\theta,\tilde{\epsilon})\cap Y \subset H(b_1,b_1\theta''_\delta)\cup H(b_2,b_2\theta'_\delta)$.

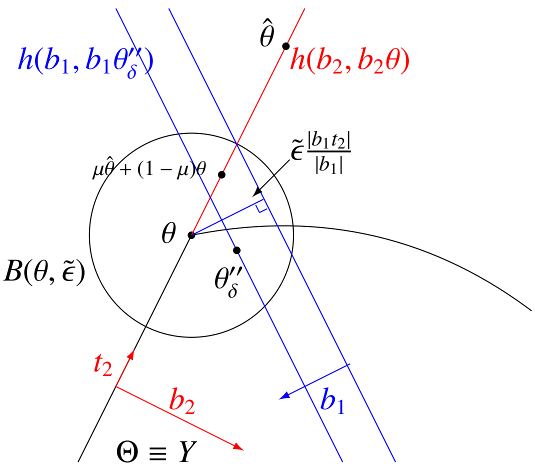

Figure 4

$(ii) \Rightarrow (iii)$ is trivial.

$(iii) \Rightarrow (i)$ Suppose there exists $\theta',\theta'' \in Y$ such that $Y \subseteq H(b_1,b_1\theta'')\cup H(b_2,b_2\theta')$. Then $h(b_1,b_1\theta'')\cap h(b_2,b_2\theta')\cap Y = \emptyset$ and for Y compact or $Y \subseteq \mathbb{R}^2$, there exist[7]

$$\tilde{\theta}' \in \arg\min_y\{b_1y \mid y \in Y, b_2y = b_2\theta'\} \text{ and}$$
$$\tilde{\theta}'' \in \arg\min_y\{b_2y \mid y \in Y, b_1y = b_1\theta''\}.$$

Note that $H(b_1,b_1\tilde{\theta}'')\cup H(b_2,b_2\tilde{\theta}') = H(b_1,b_1\theta'')\cup H(b_2,b_2\theta')$ and hence $Y \subseteq H(b_1,b_1\tilde{\theta}'')\cup H(b_2,b_2\tilde{\theta}')$. We show that for any $\lambda \in (0,1)$, $Y \subseteq H(b_1,b_1(\lambda\tilde{\theta}'+(1-\lambda)\tilde{\theta}''))\cup H(b_2,b_2\tilde{\theta}')$ and therefore given $\tilde{\theta}'$, for every $\delta > 0$ there exists a $\lambda \in (0,1)$ such that $\lambda\tilde{\theta}'+(1-\lambda)\tilde{\theta}'' \in B(\tilde{\theta}',\delta)\cap Y$ and

$$Y \subseteq H(b_1,b_1(\lambda\tilde{\theta}'+(1-\lambda)\tilde{\theta}''))\cup H(b_2,b_2\tilde{\theta}')$$

and hence a local deviation from $\tilde{\theta}'$ cannot be deterred[8]. See Figure 5.

Suppose that there exists $\lambda \in (0,1)$ and $\hat{\theta} \in Y$ such that $\hat{\theta} \notin H(b_1,b_1(\lambda\tilde{\theta}'+(1-\lambda)\tilde{\theta}''))\cup$

[7]If Y is compact then the minimum is reached within the set. This is also the case if $Y \subseteq \mathbb{R}^2$ because $h(b_1,b_1\theta'')\cap h(b_2,b_2\theta')\cap Y = \emptyset$ implies that the sets $Y\cap h(b_2,b_2\theta')$ and $Y\cap h(b_1,b_1\theta'')$ are closed, bounded (from below) half-lines and hence they have a minimum. For general $Y \subseteq \mathbb{R}^d$, even if $Y\cap h(b_i,b_i\theta)$ is closed and bounded from below, it might be the case that the minimum is never reached.

[8]Note that for any $\epsilon > 0$, $B(\tilde{\theta}',\epsilon)\cap Y \subseteq Y \subseteq H(b_1,b_1(\lambda\tilde{\theta}'+(1-\lambda)\tilde{\theta}''))\cup H(b_2,b_2\tilde{\theta}')$, and hence local deviations from $\tilde{\theta}'$ cannot be deterred with local punishments.

$H(b_2, b_2\tilde{\theta}')$. Since $\hat{\theta} \in H(b_1, b_1\tilde{\theta}'') \cup H(b_2, b_2\tilde{\theta}')$,

$$\begin{aligned} &\hat{\theta} \in H(b_1, b_1\tilde{\theta}'') \\ &\hat{\theta} \notin H(b_2, b_2\tilde{\theta}') \\ &\hat{\theta} \notin H(b_1, b_1(\lambda\tilde{\theta}' + (1-\lambda)\tilde{\theta}'')) \end{aligned}$$

Moreover since $\tilde{\theta}', \tilde{\theta}'' \in Y \subseteq H(b_1, b_1\tilde{\theta}'') \cup H(b_2, b_2\tilde{\theta}')$, $b_2\tilde{\theta}' < b_2\tilde{\theta}''$ and $b_1\tilde{\theta}'' < b_1\tilde{\theta}'$. Putting all the inequalities together we find that

$$b_2\hat{\theta} \leq b_2\tilde{\theta}' < b_2\tilde{\theta}'' \tag{4}$$

$$b_1\tilde{\theta}'' < b_1\hat{\theta} < b_1\tilde{\theta}' \tag{5}$$

By (4) there exists $\mu \in (0, 1]$ such that $b_2(\mu\hat{\theta} + (1-\mu)\tilde{\theta}'') = b_2\tilde{\theta}'$ and by convexity $\mu\hat{\theta} + (1-\mu)\tilde{\theta}'' \in Y$. But by (5), $b_1(\mu\hat{\theta} + (1-\mu)\tilde{\theta}'') < b_1\hat{\theta} < b_1\tilde{\theta}'$ which contradicts the definition of $\tilde{\theta}'$. □

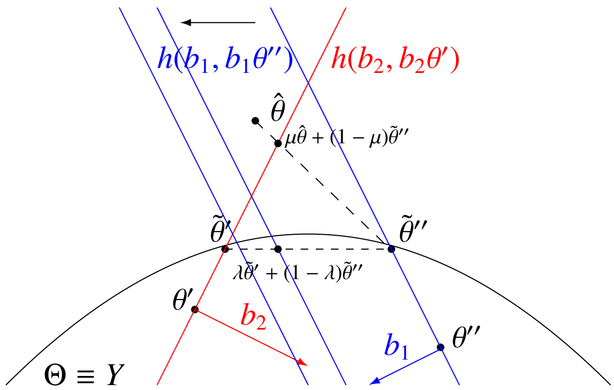

Figure 5: In $\mathbb{R}^2$, if a deviation cannot be deterred, there is a local deviation that cannot be deterred with local actions.

We next turn to a detailed analysis of the case of two-dimensional state spaces. Section 3.2 then shows how we can extend the results for the two-dimensional case to higher dimensions, building on the fact that the biases of the senders span a two-dimensional space outside of which there is no conflict of interest between them and the receiver.

3.1 Policy Space a Subset of $\mathbb{R}^2$

We begin by defining some notation that will be used for the rest of the paper.

Given $S \subset \mathbb{R}^2$ closed and convex, we denote the frontier of S by $Fr(S)$. We say that a point $s \in Fr(S)$ is *smooth* if there exists a unique tangent hyperplane to $Fr(S)$ at s. Any point in $Fr(S)$ that is not smooth will be called a *kink*. The set of smooth points in the frontier is denoted by $\widetilde{Fr}(S)$. For any $s \in \widetilde{Fr}(S)$, we denote by $n_S(s)$ the unit normal vector to $Fr(S)$ at s in the *inward* direction to S. In particular, for S convex, $n_S(s)$ is the unique vector that satisfies that $n_S(s)s' \geq n_S(s)s$ for all $s' \in S$.

Given $b_1, b_2 \in \mathbb{R}^2$, $C(b_1, b_2) = \{\alpha b_1 + \beta b_2 \mid \alpha, \beta > 0\}$ is the open convex cone, and $\overline{C}(b_1, b_2) = \{\alpha b_1 + \beta b_2 \mid \alpha, \beta \geq 0\}$ the closed convex cone, spanned by the vectors b_1, b_2.

Given $b_1, b_2 \in \mathbb{R}^2$ linearly independent, we denote by $n_1, n_2 \in \mathbb{R}^2$, the normal vectors to b_1, b_2 respectively such that $n_1 b_2 = 1$ and $n_2 b_1 = 1$. The pair (n_2, n_1) forms a basis for $\mathbb{R}^2$. Note that for any $\theta \in \mathbb{R}^2$, the coordinates of θ with respect to the basis (n_2, n_1) are $(b_1\theta, b_2\theta)$. For any $\theta', \theta'' \in \mathbb{R}^2$ we denote by $\theta' \wedge_{\{b_1,b_2\}} \theta''$ the component-wise minimum of θ' and θ'' in the basis (n_2, n_1). In other words, $\theta' \wedge_{\{b_1,b_2\}} \theta''$ is the point in $\mathbb{R}^2$ whose coordinates with respect to the basis (n_2, n_1) are $(\min\{b_1\theta', b_1\theta''\}, \min\{b_2\theta', b_2\theta''\})$. Equivalently, $b_1(\theta' \wedge_{\{b_1,b_2\}} \theta'') = \min\{b_1\theta', b_1\theta''\}$ and $b_2(\theta' \wedge_{\{b_1,b_2\}} \theta'') = \min\{b_2\theta', b_2\theta''\}$.

Proposition 4 below provides a geometric condition that is easy to check and that determines whether or not a robust FRE exists. The proposition also provides a specific policy rule for the receiver that supports a robust FRE whenever one exists.

Proposition 4. *Given $Y \subseteq \mathbb{R}^2$ convex and $b_1, b_2 \in \mathbb{R}^2$ linearly independent, the following statements are equivalent:*

(i) There exists a robust fully revealing equilibrium for biases (b_1, b_2).

(ii) There exists a (robust) fully revealing equilibrium for all biases $(t_1 b_1, t_2 b_2)$ with $t_1, t_2 \geq 0$.

(iii) For every $\theta \in \widetilde{Fr}(Y)$, $\quad n_Y(\theta) \notin C(b_1, b_2)$.

(iv) For every $\theta', \theta'' \in Y$, $\quad \theta' \wedge_{\{b_1,b_2\}} \theta'' \in Y$

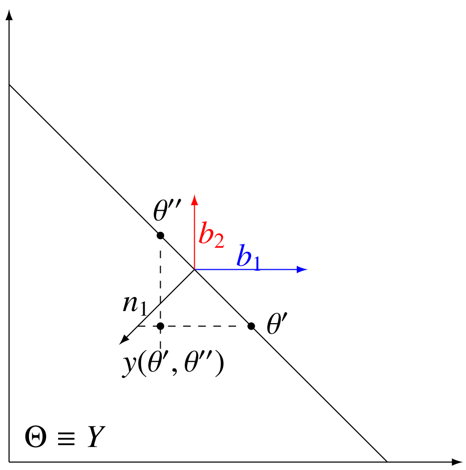

Figure 6

Before presenting the proof, we make some observations. As Proposition 3 already showed, when the policy space is convex, requiring robustness of a FRE when the biases can be arbitrarily large does not restrict the circumstances under which it exists. Proposition 4 shows that when robustness is required, then existence becomes no more likely when we drop the assumption that the biases can be arbitrarily large. The reason is that when the receiver is constrained to use small punishments, then whether the senders have incentives to deviate from truthtelling depends only on the orientations, not the magnitudes, of their bias vectors.

Condition (*iii*) in the proposition is a simple, local geometric condition, on the directions of the senders' bias vectors relative to the frontier of the policy space. This condition is easiest to interpret by examining its "strict" version: For a given $\theta \in \widetilde{Fr}(Y)$, $n_Y(\theta) \notin \overline{C}(b_1, b_2)$ holds if and only if there exists a feasible policy for the receiver, close to θ, that would make both senders strictly worse off than if the receiver chose $y = \theta$. Now consider the boundary case of a $\theta \in \widetilde{Fr}(Y)$ at which $n_Y(\theta)$ coincides with the direction of one of the bias vectors, say b_1. If $\widetilde{Fr}(Y)$ is locally curved at θ, so $n_Y(\theta)$ is rotating, then condition (*iii*) will be violated at a nearby θ'. If, instead, $\widetilde{Fr}(Y)$ is linear in a neighborhood of θ, then there exists a nearby policy on the frontier that would leave sender 2 strictly worse off and sender 1 no better off than if the receiver chose $y = \theta$.

Condition (*iv*) provides a policy rule for the receiver, *the min-rule*, that deters deviations in a robust way whenever a FRE exists. Whenever the reports (θ', θ'') of the senders do not agree, the receiver's action is rationalized by a belief that allocates mass one to $\theta' \wedge_{\{b_1, b_2\}} \theta'' \in Y$.

Consider the example depicted in Figure 6. The feasible set $Y \equiv \Theta$ is the set of non-negative y_1,y_2 such that $y_1 + y_2 \leq k$, for some $k > 0$, representing a setting where the receiver has to allocate funds from a budget of k to two different potential uses. Sender 1 is biased towards one use of funds, while Sender 2 is biased towards the other use. For the bias vectors illustrated, condition (*iii*) in Proposition 4 is satisfied, and therefore there exists a robust FRE, even for arbitrarily large biases. At smooth points along the segment of the frontier where $y_1 + y_2 = k$, $n_Y(\theta) \notin \overline{C}(b_1, b_2)$, and there exists a feasible policy for the receiver, close to θ, that would make both senders strictly worse off than if the receiver chose $y = \theta$. Condition (*iv*) in Proposition 4 provides such a rule. For messages (θ', θ'') that are both smooth points along this segment of the frontier, let the receiver's belief be such that $y^R(\theta', \theta'') = (\min\{\theta'_1, \theta''_1\}, \min\{\theta'_2, \theta''_2\})$. Such a belief satisfies both conditions (1) and (2) in Proposition 1. Slightly changing the belief so that $y^R_i < \min\{\theta'_i, \theta''_i\}$ would allow the receiver to deter local deviations along this segment of the frontier with strict local punishments, no matter how large the magnitudes of the biases.

At smooth points along the segments of the frontier where $y_1 = 0$ or where $y_2 = 0$, the directions of $n_Y(\theta)$ and one of the senders' biases coincide. For messages (θ', θ'') that are both smooth points along the same such segment, a belief for R such that $y^R_i = \min\{\theta'_i, \theta''_i\}$ for $i = 1, 2$ again deters local deviations with local punishments. However, along each of these segments, at most one sender can be punished strictly for deviating, since when the magnitudes of the biases are extremely large, each of these segments essentially lies along an indifference curve of one of the senders.

Proposition 4 is valid even for non-compact state spaces. In the special case where $Y \equiv \Theta$ is compact, the condition that Ambrus and Takahashi (2008, Proposition 8) showed to be necessary and sufficient for existence of a FRE for arbitrarily large biases can be shown to be equivalent to our condition (*iii*).[9] However, their result, in contrast to ours, is not valid for unbounded state spaces.

[9]In Ambrus and Takahashi's model, Y is assumed to contain the convex hull of Θ, so all policies in $(co(\Theta))$ are feasible, and hence the effective policy space in their model is convex.

In our setting with $Y \equiv \Theta$, Ambrus and Takahashi's result can be expressed in the following way. First recall that as the magnitude of the bias goes to infinity, a sender's ranking over policies becomes independent of the true state. Thus, when Θ is compact and biases are arbitrarily large, each sender's least-preferred point(s) in Θ is(are) independent of the true state. For compact Θ and arbitrarily large biases, Ambrus and Takahashi's result is that a FRE exists if and only if for sufficiently large magnitudes of the biases the senders have a common least-preferred policy in Θ. With no robustness requirement imposed on the equilibrium, it is clear that the common least-preferred point can be used by the receiver to punish any discrepancies in the senders' reports and thereby deter deviations from truthtelling.[10] The equivalence, for compact, convex $Y \equiv \Theta$, between our condition (*iii*) in Proposition 4 and Ambrus and Takahashi's condition, shows that existence of a common least-preferred point for the senders is sufficient for existence not only of a FRE but also for a *robust* FRE, when the biases can be arbitrarily large.

Proof of Proposition 4: By Proposition 2, we can restrict attention to the case $\Theta \equiv Y$.
(*i*) $\Rightarrow$ (*iii*): Suppose there exists $\theta \in \widetilde{Fr}(Y)$ such that $n_Y(\theta) \in C(b_1, b_2)$. Since Y is convex $Y \subseteq \overline{H}(n_Y(\theta), n_Y(\theta)\theta)$. We can find $\epsilon > 0$ such that

$$B(\theta, \epsilon) \cap \overline{H}(n_Y(\theta), n_Y(\theta)\theta) \subset \overline{B}(\theta + b_1/2, |b_1|/2) \cup \overline{B}(\theta + b_2/2, |b_2|/2) \tag{6}$$

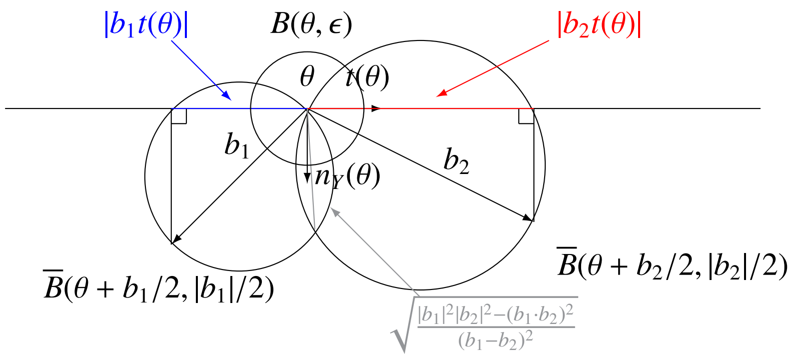

Figure 7

More precisely, if we denote by $t(\theta)$ a unit normal vector to $n_Y(\theta)$, any $0 < \epsilon \leq \min\{|b_1 t(\theta)|, |b_2 t(\theta)|, \sqrt{\frac{|b_1|^2|b_2|^2-(b_1 \cdot b_2)^2}{(b_1-b_2)^2}}\}$ will satisfy (6).[11] See Figure 7.
Moreover, for any $\delta > 0$,

$$B(\theta - b_1/2, |b_1|/2) \cap Y \cap B(\theta, \delta) \neq \emptyset$$
$$B(\theta - b_2/2, |b_2|/2) \cap Y \cap B(\theta, \delta) \neq \emptyset$$

[10] The common least-preferred policy (when the magnitudes are large) serves as a punishment even to senders with small biases. Denote by $\underline{\theta}$ the common least-preferred policy when the magnitudes of the biases are sufficiently large. In particular $\underline{\theta}$ satisfies that $b_i\underline{\theta} \leq b_i\theta$ for $i = 1, 2$ and any $\theta \in \Theta$. This implies that for any realization of the ideal policy θ, $|\theta + b_i - \underline{\theta}|^2 = |b_i|^2 + |\theta - \underline{\theta}|^2 + 2b_i(\theta - \underline{\theta}) > |b_i|^2 = |\theta + b_i - \theta|^2$. In other words, θ is closer to $\theta + b_i$ than $\underline{\theta}$ and hence θ is preferred to $\underline{\theta}$.

[11] The last number in this minimum corresponds to the length of the common chord of the two balls. It is derived using standard trigonometry.

Consider $\tilde{\epsilon} = \min\{\epsilon, |b_1|/2, |b_2|/2\}$. Then for any $\delta > 0$ consider θ' an arbitrary element of $B(\theta - b_2/2, |b_2|/2) \cap Y \cap B(\theta, \delta)$ and θ'' an arbitrary element of $B(\theta - b_1/2, |b_1|/2) \cap Y \cap B(\theta, \delta)$. We show that $B(\theta, \tilde{\epsilon}) \cap Y \subset B(\theta'' + b_1, |b_1|) \cup B(\theta' + b_2, |b_2|)$ and hence the equilibrium is not robust.
Consider $\tilde{\theta} \in B(\theta, \tilde{\epsilon}) \cap Y$, then since $\tilde{\epsilon} \leq \epsilon$, $\tilde{\theta} \in \overline{B}(\theta + b_1/2, |b_1|/2) \cup \overline{B}(\theta + b_2/2, |b_2|/2)$. Suppose $\tilde{\theta} \in \overline{B}(\theta + b_1/2, |b_1|/2)$, then

$$|\tilde{\theta} - (\theta'' + b_1)| \leq |\tilde{\theta} - (\theta + \frac{b_1}{2})| + |\theta - \frac{b_1}{2} - \theta''| < \frac{|b_1|}{2} + \frac{|b_1|}{2} = |b_1|$$

which implies that $\tilde{\theta} \in B(\theta'' + b_1, |b_1|)$. The case $\tilde{\theta} \in \overline{B}(\theta + b_2/2, |b_2|/2)$ is analogous.

$(iii) \Rightarrow (ii)$: By Proposition 3, it is enough to show the existence of a fully revealing equilibrium for arbitrarily large biases. We argue by contradiction. By Proposition 1, suppose that there exist $\theta', \theta'' \in Y$ such that $Y \subseteq H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$. Denote by $x = h(b_1, b_1\theta'') \cap h(b_2, b_2\theta') \notin Y$ and consider any $\tilde{\theta} \in \widetilde{Fr}(Y)$[12] that lies in the interior of the triangle formed by θ', θ'' and x (See Figure 8). In particular, since Y is convex, $h(n_Y(\tilde{\theta}), n_Y(\tilde{\theta})\tilde{\theta})$ is a separating hyperplane to Y and:

$$n_Y(\tilde{\theta})(\theta' - \tilde{\theta}) \geq 0 \tag{7}$$

$$n_Y(\tilde{\theta})(\theta'' - \tilde{\theta}) \geq 0 \tag{8}$$

$$n_Y(\tilde{\theta})(x - \tilde{\theta}) < 0 \tag{9}$$

Moreover, since b_1, b_2 span $\mathbb{R}^2$ there exists $\alpha, \beta \in \mathbb{R}$ such that $n_Y(\tilde{\theta}) = \alpha b_1 + \beta b_2$. Substituting this into equations (7), (8), (9), and then substracting (9) from (7) and (8), we obtain

$$0 < \alpha b_1(\theta' - x) - \beta b_2(\theta' - x) = \alpha b_1(\theta' - \theta'') \tag{10}$$

$$0 < \alpha b_1(\theta'' - x) - \beta b_2(\theta'' - x) = \beta b_2(\theta'' - \theta') \tag{11}$$

where the equalities follow by the definition of x. And given that $b_1\theta' > b_1\theta''$ and $b_2\theta' < b_2\theta''$, (10) and (11) imply $\alpha > 0$ and $\beta > 0$ respectively. Hence $n_Y(\tilde{\theta}) \in C(b_1, b_2)$ which contradicts (iii).

$(ii) \Rightarrow (i)$: Trivial.

$(iv) \Rightarrow (ii)$: Suppose that for all $\theta', \theta'' \in Y$ $\theta \equiv \theta' \wedge_{\{b_1, b_2\}} \theta'' \in Y$. By the definition of $\theta' \wedge_{\{b_1, b_2\}} \theta''$, $b_1\theta \leq b_1\theta''$ and $b_2\theta \leq b_2\theta'$. Therefore $\theta \notin H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$ and hence $Y \not\subseteq H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$ and by Proposition 1 there exists a FRE for arbitrarily large biases.

[12]Note that $Fr(Y)$ has at most a countable number of kinks. Since Y is convex, $Fr(Y)$ is locally the graph of a concave (convex) function and hence the derivative of this function is monotonic, and it has at most a countable number of jumps.

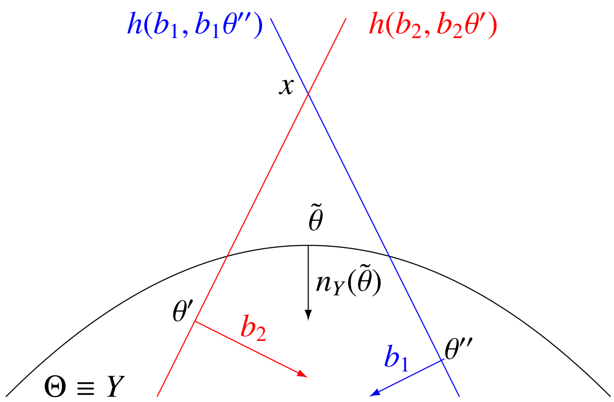

Figure 8

$(ii) \Rightarrow (iv)$: Consider $\theta', \theta'' \in Y$. If $b_1\theta' \leq b_1\theta''$ and $b_2\theta' \leq b_2\theta''$ then $\theta' \wedge_{\{b_1,b_2\}} \theta'' = \theta' \in Y$. Analogously, if $b_1\theta' \geq b_1\theta''$ and $b_2\theta' \geq b_2\theta''$, then $\theta' \wedge_{\{b_1,b_2\}} \theta'' = \theta'' \in Y$. Suppose then that $b_1\theta' > b_1\theta''$ and $b_2\theta' < b_2\theta''$. By Proposition 1, $Y \not\subseteq H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$. Consider $y \in Y$ such that $b_1 y \leq b_1\theta''$ and $b_2 y \leq b_2\theta'$. Then

$$b_1 y \leq b_1(\theta' \wedge_{\{b_1,b_2\}} \theta'') = b_1\theta'' < b_1\theta' \tag{12}$$

$$b_2 y \leq b_2(\theta' \wedge_{\{b_1,b_2\}} \theta'') = b_2\theta' < b_2\theta'' \tag{13}$$

By (12), there exists $\alpha \in [0, 1)$ such that $b_1(\alpha y + (1 - \alpha)\theta') = b_1(\theta' \wedge_{\{b_1,b_2\}} \theta'')$. Denote $y' = \alpha y + (1-\alpha)\theta'$. By the convexity of Y, $y' \in Y$. By (13), $b_2 y' \leq b_2\theta' = b_2(\theta' \wedge_{\{b_1,b_2\}} \theta'') < b_2\theta''$, and hence there exists $\beta \in [0, 1)$ such that $b_2(\beta y' + (1 - \beta)\theta'') = b_2(\theta' \wedge_{\{b_1,b_2\}} \theta'')$. Denote $y'' = \beta y' + (1 - \beta)\theta''$. By convexity, $y'' \in Y$. Finally, $b_1 y'' = b_1(\theta' \wedge_{\{b_1,b_2\}} \theta'')$, $b_2 y'' = b_2(\theta' \wedge_{\{b_1,b_2\}} \theta'')$, and the linear independence of b_1 and b_2 together imply $\theta' \wedge_{\{b_1,b_2\}} \theta'' = y'' \in Y$.
Finally, the case $b_1\theta' < b_1\theta''$ and $b_2\theta' > b_2\theta''$ is handled in a symmetric fashion, using the deviation (θ'', θ') instead of (θ', θ''). □

3.2 Multidimensional Spaces

The results for arbitrarily large biases derived in the previous section extend easily to higher dimensions. For b_1, b_2 linearly independent, the only directions of conflict between the senders and the receiver are the ones in the plane spanned by these two vectors. Thus, senders will not have incentives to deviate by misreporting dimensions of the state orthogonal to this plane. On the other hand, the receiver could potentially utilize these dimensions of no conflict to punish inconsistent messages. However, this strategy cannot be guaranteed to work for the receiver if the senders' biases can be arbitrarily large. Proposition 5 shows that it is necessary and sufficient to project the state space onto the plane of the bias vectors and to check whether condition (iii) in Proposition 4 is satisfied by this two-dimensional projection.

Given $b_1, b_2 \in \mathbb{R}^d$ linearly independent, denote by $\Pi_b \subset \mathbb{R}^d$ the plane spanned by these two vectors. Denote by $Proj_b : \mathbb{R}^d \longrightarrow \Pi_b$ the orthogonal projection onto Π_b. We will denote by x_b a generic element of Π_b and by B_b and H_b the two-dimensional balls and half-spaces in the plane Π_b. Finally θ_b will denote a generic element of $Y_b \equiv Proj_b(Y)$.

Proposition 5. *Given $Y \subseteq \mathbb{R}^d$ compact and convex and $b_1, b_2 \in \mathbb{R}^d$ linearly independent, the following statements are equivalent:*

(i) There exists a (robust) fully revealing equilibrium for all biases $(t_1 b_1, t_2 b_2)$ with $t_1, t_2 \geq 0$

(ii) For every $\theta_b \in \widetilde{Fr}(Y_b)$, $n_{Y_b}(\theta_b) \notin C(b_1, b_2)$

Proof of Proposition 5: By Proposition 3, it is enough to show the equivalence for fully revealing equilibria. Given Proposition 2 we can focus on the case $\Theta \equiv Y$. We show that for $Y \subset R^d$, a fully revealing equilibrium exists for all biases $(t_1 b_1, t_2, b_2)$ with $t_1, t_2 \geq 0$ if and only if, for the two-dimensional state space Y_b, a fully revealing equilibrium exists for arbitrarily large magnitudes of the biases, where now the biases are regarded as two-dimensional vectors in Π_b. The equivalence claimed in the proposition then follows from Proposition 4.

Given $\tilde{\theta} \in \mathbb{R}^d$, define $\tilde{\theta}_b \equiv Proj_b(\tilde{\theta})$. Then

$$\tilde{\theta} \in H(b, b\theta) \Longleftrightarrow b\tilde{\theta} > b\theta \Longleftrightarrow b\tilde{\theta}_b > b\theta_b \Longleftrightarrow \tilde{\theta}_b \in H_b(b, b\theta_b). \tag{14}$$

Suppose there does not exist a fully revealing equilibrium for all biases $(t_1 b_1, t_2 b_2)$ with $t_1, t_2 \geq 0$. By Proposition 1, $Y \subseteq H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$ for some $\theta', \theta'' \in Y$. Define $\theta'_b \equiv Proj_b(\theta')$ and $\theta''_b \equiv Proj_b(\theta'')$. Then it follows from (14) that $Y_b \subseteq H_b(b_1, b_1\theta''_b) \cup H_b(b_2, b_2\theta'_b)$. The reverse implication is proved analogously, again using (14). □

Proposition 5 implies that for the existence in high-dimensional spaces of a FRE (robust or not) that is independent of the sizes of the biases, it is necessary and sufficient to look at the projection of the policy space onto the subspace of conflict of interest and see whether a FRE can be constructed there.[13] The reason is that when the equilibrium is required to exist regardless of the magnitudes of the biases, then no given shift of the receiver's action in a direction orthogonal to the plane of the biases can be certain to serve as a punishment for a deviating sender. Therefore, to be certain that he is actually punishing a deviation, the receiver needs to choose an action whose projection on the plane of the biases is worse for both senders. Such an action exists if and only if the projection of the state space onto the plane of the biases satisfies (*iii*) in Proposition 4.

If the biases have known finite magnitudes, then condition (*ii*) in Proposition 5 is sufficient for existence of a robust FRE but not necessary. In this case, the receiver might be

[13] The assumption in the proposition that Y is compact ensures that its projection onto the plane spanned by b_1 and b_2 is closed. We could relax the assumption of compactness as long as Y were such that its projection onto Π_b was closed.

able to exploit the dimensions orthogonal to the biases for punishments. In fact, if the state space were unrestricted in one dimension orthogonal to the plane of the biases, a robust FRE would always exist.

4 Non-Convex Policy Spaces

This section considers the case where the policy space is non-convex, for example because of indivisibilities. Before presenting our result, we need to generalize our definition of an *inward* normal vector to a smooth point on the frontier.[14] Consider an arbitrary set S and a smooth point s on its frontier $Fr(S)$. The *inward* normal vector, $n_S(s)$, to $Fr(S)$ at s is the normal vector that satisfies the condition that there exists an $\epsilon > 0$ such that for any $0 < \delta < \epsilon$, $s + \delta n_S(s) \in S$.

We also need to define a specific type of kink that might be particularly perverse when we are dealing with non-convex sets. We say that a kink point θ is *non-convex* if Y is locally not convex at θ, that is, for all $\epsilon > 0$, there exist $\theta', \theta'' \in B(\theta, \epsilon) \cap Fr(Y)$ such that for all $\lambda \in (0,1)$, $\lambda\theta' + (1-\lambda)\theta'' \notin Y$. A kink point is *linear with normal vectors* $\{n_1, n_2\}$ if $Fr(Y)$ is locally linear to both sides of θ and the inward normal vectors to these locally linear segments of $Fr(Y)$ are n_1 and n_2.

Proposition 6. *Suppose $Y \subseteq \mathbb{R}^2$ is compact and $Fr(Y)$ has finitely many kinks. Given $b_1, b_2 \in \mathbb{R}^2$ linearly independent, the following statements are equivalent:*

(i) There exists a robust fully revealing equilibrium for all biases $(t_1 b_1, t_2 b_2)$ with $t_1, t_2 \geq 0$

(ii) 1. For every $\theta \in \widetilde{Fr}(co(Y))$, $n_{co(Y)}(\theta) \notin C(b_1, b_2)$, and

2. For every $\theta \in \widetilde{Fr}(Y)$, $n_Y(\theta) \notin C(b_1, b_2)$, and there does not exist a non-convex kink in $Fr(Y)$ that is linear with normal vectors $\{b_1, b_2\}$.

Proof of Proposition 6: In the Appendix.

Condition (*ii*-2) is necessary and sufficient for small deviations to be deterrable with small punishments, whether the biases have known finite magnitudes or whether they can be arbitrarily large. (When only local punishments are considered, senders' incentives to deviate from truthtelling depend only on the orientations, not the magnitudes, of their biases.) When Y is convex, this condition is necessary and sufficient for existence of a robust FRE, as Proposition 4 shows. (For Y convex, non-convex kinks never exist.) When Y is non-convex, however, deterrence of small deviations with small punishments is no longer sufficient for existence of a robust FRE. Condition (*ii*-1) is necessary and sufficient for all deviations, including large ones, to be deterrable with feasible punishments, when the biases can be arbitrarily large. Since for non-convex Y, deterrability of large deviations depends in general on the magnitudes as well as the orientations of the biases, existence of

[14] Recall that for a convex set S we defined an *inward* normal vector to $s \in \widetilde{Fr}(S)$, as the only vector $n_S(s)$ such that $n_S(s)s' \geq n_S(s)s$ for all $s \in S$. This definition does not apply to non-convex sets.

a robust FRE for arbitrarily large biases implies, but is not in general implied by, existence of a robust FRE for biases of known finite magnitudes. We illustrate these points with two examples, displayed in Figures 9 and 10.

Figure 9 presents an example in which all local deviations can be deterred with local actions, because condition (*ii*-2) is satisfied. However, at points along the dashed line connecting A to C, which is part of the frontier of $co(Y)$, condition (*ii*-1) is violated. To see the consequence of this violation, observe that if sender 1 reports C and sender 2 reports A, and the magnitudes of the biases are very large, then there is no feasible response for the receiver that would suffice to deter S_1, in state A, from deviating to a report of C, and that would also deter S_2, in state C, from deviating to a report of A—any response that would deter both of these deviations would have to lie northeast of both the line through AD and the line through CD. Hence, a fully revealing equilibrium does not exist.

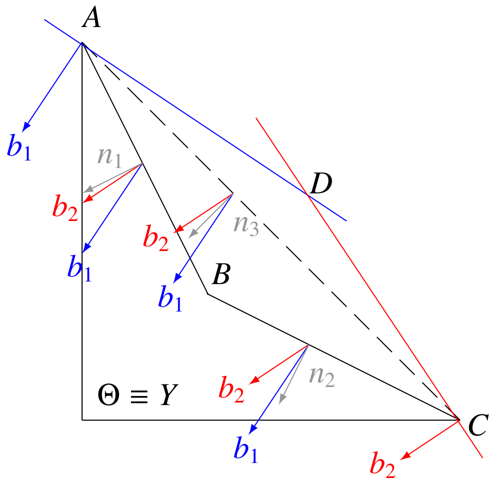

Figure 9

In Figure 9, if the northeast boundary of Y had been the segment AC instead of the segments AB and BC, so Y had been convex, then it would have been necessary, for existence of a robust FRE, that at states θ along AC, local deviations be deterrable with local punishments. Had this condition been satisfied (for the biases shown, it would not have been), this would have implied that for any pair of incompatible reports, both of which lay along AC, there existed a feasible punishment—it would not have been necessary to consider explicitly the global deviation represented by the pair of reports (C,A). It is because of the non-convexity of Y in Figure 9—A$\in Y$ and C$\in Y$ but segment AC$\not\subseteq Y$— that deterrence, for all $\theta \in Y$, of small deviations with small punishments does not guarantee that large deviations such as that represented by (C,A) can be deterred. Figure 9 thus shows that, in Proposition 3, condition (*iii*) no longer implies condition (*i*) if the assumption of convexity of Y is dropped.

Figure 10 displays an example in which there exists a fully revealing equilibrium for arbitrarily large biases, because condition (*ii*-1) is satisfied. For very large magnitudes of the biases, point C is the least-preferred point in $Y \equiv \Theta$ for both senders, so it can be used by the receiver to punish any discrepancies in the senders' reports.[15] However, no robust FRE exists. To see why, note that along segment AB on the frontier of Y, condition (ii-2) is violated; as a consequence, it is not possible to deter local deviations along segment AB with local punishments. This example shows that in Proposition 3, condition (i) no longer implies condition (iii) if the assumption of convexity is dropped.

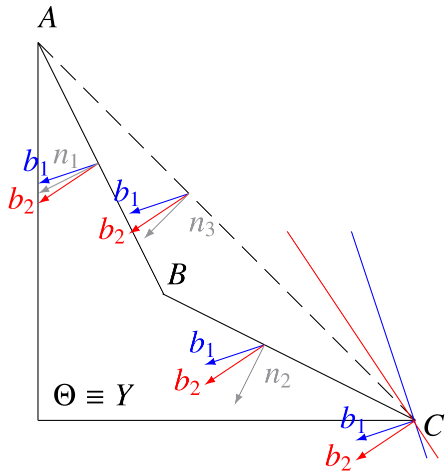

Figure 10

Finally, Figure 11 illustrates the case in which $Fr(Y)$ has a non-convex linear kink with normal vectors $\{b_1, b_2\}$ at point D. On each side of this point, $Fr(Y)$ is locally linear, and the normal vectors coincide with the two bias vectors. As a consequence, at point D, there are no small punishments available to the receiver in response to small mistakes by both senders, so a robust fully revealing equilibrium does not exist. This is so despite the facts that a) condition (ii)-1 is satisfied, so a (non-robust) fully revealing equilibrium exists for arbitrarily large biases, and b) for every $\theta \in \widetilde{Fr}(Y)$, $n_Y(\theta) \notin C(b_1, b_2)$, so for all points other than D, small deviations are deterrable with small punishments.

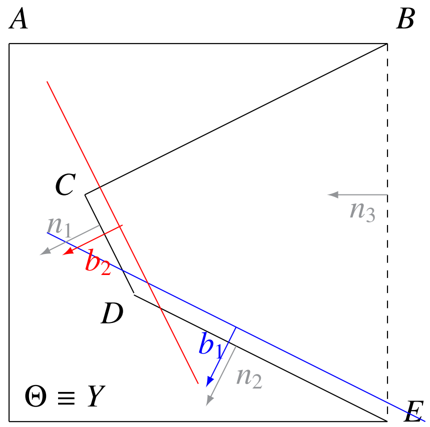

Figure 11

[15] See footnote 10

5 Uncertain Biases

In this section, we relax the assumptions that the directions (b_1, b_2) of the senders' bias vectors are (i) common knowledge and (ii) independent of the realization of the state.

Suppose that the players have a common prior joint distribution G over (θ, b_1, b_2). Each sender observes θ and his own bias vector, while the receiver does not observe any of these realizations. The definition of a fully revealing equilibrium remains unchanged.

Proposition 7. *Given $Y \subseteq \mathbb{R}^2$ convex, suppose that there exists a closed convex cone $\overline{C}(\underline{b}, \overline{b}) = \{\alpha \underline{b} + \beta \overline{b} \mid \alpha, \beta \geq 0\}$, such that for all $\theta \in \Theta$, the supports of the conditional distributions of the bias directions b_1 and of b_2 given θ are both contained in $\overline{C}(\underline{b}, \overline{b})$. Then conditions (i) and (ii) are equivalent and imply (iii):*

(i) For all $\theta \in \widetilde{Fr}(Y)$, $\quad n_Y(\theta) \notin C(\underline{b}, \overline{b})$.

(ii) For all $\theta', \theta'' \in Y$, $\quad \theta' \wedge_{\{\underline{b}, \overline{b}\}} \theta'' \in Y$.

(iii) There exists a (robust) fully revealing equilibrium for arbitrarily large magnitudes of the biases.

Moreover, if the conditional distribution of the bias directions (b_1, b_2) given θ assigns positive density to $(\underline{b}, \overline{b})$ for all $\theta \in Y$, then (iii) implies (i) and (ii).

Proof of Proposition 7: In the Appendix.

Proposition 7 says that when the receiver does not know the actual biases but knows only the minimal closed cone in which they are certain to lie, and this minimal cone is the same for all states, then the necessary and sufficient condition for existence of a robust FRE for arbitrarily large biases is condition (iii) in Proposition 4, with the known biases b_1 and b_2 replaced by the least aligned possible realizations, $\underline{b}$ and $\overline{b}$. Condition (i) in Proposition 7 ensures that for all true states on the frontier of Y, the receiver can find local punishments that would deter local deviations, whether the realized values of (b_1, b_2) were $(\underline{b}, \overline{b})$ or $(\overline{b}, \underline{b})$. This in turn implies that for any more closely aligned realizations of the biases, local deviations would continue to be deterrable by local punishments.

A Appendix

In order to prove Lemma 1 we show the equivalence between our concept of robustness and the concept of *continuity on the diagonal* introduced by Ambrus and Takahashi (2008), which requires that whenever a sequence of reports converges to a pair of compatible messages, then the sequence of induced actions should converge to the action induced by the limiting pair of (compatible) messages. Formally the definition is as follows[16]:

Definition 3 (Ambrus and Takahashi (2008))**.** A fully revealing equilibrium (s_1, s_2, y) is *continuous on the diagonal* if

$$\lim_{n\to\infty} y(s_1(\theta_1^n), s_2(\theta_2^n)) = y^*(\theta)$$

for any sequence $\{(\theta_1^n, \theta_2^n)\}_{n\in\mathbb{N}}$ of pairs of states such that $\lim_{n\to\infty} y^*(\theta_1^n) = \lim_{n\to\infty} y^*(\theta_2^n) = y^*(\theta)$.

Proposition 8. *A fully revealing equilibrium (s_1, s_2, y) is robust if and only if it is continuous on the diagonal.*

Proof. $\Rightarrow$) Consider any pair of sequences $\{(\theta_1^n, \theta_2^n)\}_{n\in\mathbb{N}} \subset \Theta$ such that $\lim_{n\to\infty} y^*(\theta_1^n) = \lim_{n\to\infty} y^*(\theta_2^n) = y^*(\theta)$. Since μ deters local deviations with local actions, for every $\epsilon > 0$ there exists a $\delta > 0$ such that for all $y^*(\theta'), y^*(\theta'') \in B(y^*(\theta), \delta) \cap Y$, $y(s_1(\theta'), s_2(\theta'')) \in B(y^*(\theta), \epsilon)$. Now, $\lim_{n\to\infty} y^*(\theta_1^n) = \lim_{n\to\infty} y^*(\theta_2^n) = y^*(\theta)$ implies that for that $\delta > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $y^*(\theta_1^n), y^*(\theta_2^n) \in B(y^*(\theta), \delta) \cap Y$, which implies that $y(s_1(\theta_1^n), s_2(\theta_2^n)) \in B(y^*(\theta), \epsilon)$ and hence the equilibrium is continuous on the diagonal.
$\Leftarrow$) We argue by contradiction. Suppose that μ does not deter local deviations with local actions. Then there exists $\theta \in \Theta$ and $\epsilon > 0$ such that for all $n \in \mathbb{N}$ there exists θ_1^n, θ_2^n such that $y^*(\theta_1^n), y^*(\theta_2^n) \in B(y^*(\theta), \frac{1}{n}) \cap Y$ with

$$y(s_1(\theta_1^n), s_2(\theta_2^n)) \notin B(y^*(\theta), \epsilon) \setminus \left(B(\theta_1^n + b_2, |b_2|) \cup B(\theta_2^n + b_1, |b_1|)\right).$$

Note that for any n such that $\frac{1}{n} < \epsilon$, $\theta_1^n \neq \theta_2^n$, because if $\theta_1^n = \theta_2^n$, $y(s_1(\theta_1^n), s_2(\theta_2^n)) = y^*(\theta_1^n) \in B(y^*(\theta), \epsilon) \setminus \left(B(\theta_1^n + b_2, |b_2|) \cup B(\theta_2^n + b_1, |b_1|)\right)$. Since (s_1, s_2, y) is an equilibrium, $y(s_1(\theta_1^n), s_2(\theta_2^n)) \notin B(\theta_1^n + b_2, |b_2|) \cup B(\theta_2^n + b_1, |b_1|)$, otherwise either sender 1 would have an incentive to deviate to $s_1(\theta_1^n)$ when θ_2^n is realized, or sender 2 would have an incentive to deviate to $s_2(\theta_2^n)$ when θ_1^n is realized. Hence $y(s_1(\theta_1^n), s_2(\theta_2^n)) \notin B(\theta, \epsilon)$, which contradicts the diagonal continuity of the equilibrium. □

Proof of Lemma 1 Consider a robust fully revealing equilibrium (s_1, s_2, y^R) supported by the belief function $\mu(\cdot)$ and consider the following strategies: $\tilde{s}_i : \Theta \longrightarrow Y$, such that $\tilde{s}_i(\theta) = y^*(\theta)$; $\tilde{y} : \Theta \times \Theta \longrightarrow Y$, such that $\tilde{y}(y, y') = y(s_1(y), s_2(y'))$ and the belief function $\tilde{\mu}(\theta, \theta') = \mu(s_1(y), s_2(y'))$. Since (s_1, s_2, y) is robust, it is continuous on the diagonal

[16]Both diagonal continuity and our robustness concept can be defined for arbitrary equilibria/strategies. However, we will use those concepts only for fully revealing equilibria. For convenience, therefore, we have stated the definitions only in the context of fully revealing equilibria/strategies.

and hence, given $\{\theta_1^n\}, \{\theta_2^n\}$ with $\lim y^*(\theta_1^n) = \lim y^*(\theta_2^n) = y^*(\theta)$, $\lim \tilde{y}(y^*(\theta_1^n), y^*(\theta_2^n)) = \lim y(s_1(y^*(\theta_1^n)), s_2(y^*(\theta_2^n))) = y^*(\theta)$. Therefore $(\tilde{s}_1, \tilde{s}_2, \tilde{y})$ is continuous on the diagonal and thus robust. □

Proof of Proposition 1 *(i):* $\Rightarrow$) Suppose there exist $\theta', \theta'' \in Y$ such that $Y \subseteq H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$. Then $y(s_1(\theta'), s_2(\theta'')) \in H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$. In particular, denoting $y \equiv y(s_1(\theta'), s_2(\theta''))$, either $b_1(y - \theta'') > 0$ or $b_2(y - \theta') > 0$. Suppose that $b_1(y - \theta'') > 0$ and consider $t_1 > \frac{|y-\theta''|^2}{2b_1 \cdot (y-\theta'')}$. Then $y(s_1(\theta'), s_2(\theta'')) \in B(\theta'' + t_1 b_1, t_1|b_1|)$ which implies that for the sender 1with bias $t_1 b_1$ has an incentive to deviate to $s_1(\theta')$ given θ''. The symmetric argument could be made if $b_2(y - \theta') > 0$ with $t_2 > \frac{|y-\theta'|^2}{2b_2 \cdot (y-\theta')}$.

$\Leftarrow$) Consider truthful strategies and the following belief function $\mu(\cdot)$ such that $\mu(\theta, \theta)$ allocates mass one on θ and $\mu(\theta', \theta'')$ with $\theta' \neq \theta'' \in Y$, puts mass one in an element of $Y \setminus H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$. Given a report (θ', θ''), $\mu(\theta', \theta'') \notin H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$ so in particular $\mu(\theta', \theta'') \notin B(\theta'' + t_1 b_1, t_1|b_1|)$ and $\mu(\theta', \theta'') \notin B(\theta' + t_2 b_2, t_2|b_2|)$. So none of the two senders has an incentive to deviate.

(ii): $\Rightarrow$) Suppose there exist some fully revealing strategies (s_1, s_2) and a belief function $\mu(\cdot)$ that deters local deviation with local punishments, then for any $\theta \in Y$ and any $\epsilon > 0$ there exists $\delta > 0$ such that for every $\theta', \theta'' \in B(\theta, \delta)$, $\bar{\mu}(s_1(\theta'), s_2(\theta'')) \in B(\theta, \epsilon) \cap Y \setminus (B(\theta' + t_1 b_1, t_1|b_1|) \cup B(\theta + t_2 b_2, t_2|b_2|))$ for any $t_1, t_2 \geq 0$. Hence $B(\theta, \epsilon) \cap Y \nsubseteq H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$.

$\Leftarrow$) By the argument used in the proof of Lemma 1 we can focus on truthful strategies. For any $\theta \in \Theta$ define $\mu(\theta, \theta)$ a belief that allocates mass one to θ. If $\theta \neq \theta' \in \Theta$ define $\mu(\theta, \theta')$ a belief that allocates mass one to an element of $\arg\min_{s \in Y \setminus (H(b_1, b_1\theta') \cup H(b_2, b_2\theta))} |s - \theta|$, if $Y \nsubseteq H(b_1, b_1\theta') \cup H(b_2, b_2\theta)$, and any arbitrary belief if $Y \subseteq H(b_1, b_1\theta') \cup H(b_2, b_2\theta)$. To see that this belief function deters local deviation with local punishments consider any $\theta \in \Theta$ and any $\epsilon > 0$, by hypothesis, for $\tilde{\epsilon} = \epsilon/3$ there exists $0 < \delta < \tilde{\epsilon}$ such that for all $\theta', \theta'' \in B(\theta, \delta) \cap Y$, $B(\theta, \tilde{\epsilon}) \cap Y \nsubseteq H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$. Consider any $\hat{\theta} \in B(\theta, \tilde{\epsilon}) \cap Y \nsubseteq H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$. And $|\overline{\mu}(\theta', \theta'') - \theta| \leq |\mu(\theta', \theta'') - \theta'| + |\theta' - \theta| \leq |\hat{\theta} - \theta'| + |\theta' - \theta| \leq |\hat{\theta} - \theta| + 2|\theta' - \theta| < 3\tilde{\epsilon} = \epsilon$, hence $\overline{\mu}(\theta', \theta'') \in B(\theta, \epsilon) \setminus (H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')) \subset B(\theta, \epsilon) \setminus (B(\theta'' + t_1 b_1, t_1|b_1|) \cup B(\theta' + t_2 b_2, t_2|b_2|)$. □

(iii): The necessity is given by parts *(i)* and *(ii)*. To see the sufficiency, consider truthful strategies and the belief specified in the previous paragraph. Note that given condition (2), $Y \setminus (H(b_1, b_1\theta') \cup H(b_2, b_2\theta)) \neq \emptyset$ for any $\theta \neq \theta' \in Y$. □

Proof of Proposition 6: By Proposition 2, we can restrict attention to the case in which $\Theta \equiv Y$. By Proposition 1, there exists a robust fully revealing equilibrium if and only if both conditions (*i*) and (*ii*) of Proposition 1 are satisfied. We will show that in fact condition (*ii*-1) of Proposition 6 is equivalent to condition (*i*) of Proposition 1 and condition (*ii*-2) of Proposition 6 is equivalent to part (*ii*) of Proposition 1.

Prop.1-(i) $\Rightarrow$ *Prop.6-(ii-1)*: Suppose there exists $\theta \in \widetilde{Fr}(co(Y))$ such that $n_{co(Y)}(\theta) \in$

$C(b_1, b_2)$. Then there exists $\alpha > 0$, $\beta > 0$ such that $n_{co(Y)}(\theta) = \alpha b_1 + \beta b_2$. Moreover, since $\theta \in \widetilde{Fr}(co(Y))$, $h(n_{co(Y)}(\theta), n_{co(Y)}(\theta)\theta)$ is the unique separating hyperplane to $co(Y)$ at θ. Hence neither $h(b_1, b_1\theta)$ nor $h(b_2, b_2\theta)$ are separating hyperplanes of $co(Y)$. In particular, there exist $\theta', \theta'' \in Y$ such that:

$$b_1\theta'' < b_1\theta < b_1\theta' \qquad b_2\theta' < b_2\theta < b_2\theta''$$

We now show that $Y \subseteq H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$, which contradicts Prop.1-(i). Suppose there exists $\tilde{\theta} \in Y$ such that $\tilde{\theta} \notin H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$, then $b_1\tilde{\theta} \leq b_1\theta'' < b_1\theta$ and $b_2\tilde{\theta} \leq b_2\theta' < b_2\theta$. And hence $n_{co(Y)}(\theta)\tilde{\theta} = \alpha b_1\tilde{\theta} + \beta b_2\tilde{\theta} < \alpha b_1\theta + \beta b_2\theta = n_{co(Y)}(\theta)\theta$ which contradicts the definition of $n_{co(Y)}(\theta)$.

Prop.6-(ii-1) $\Rightarrow$ Prop.1-(i): Suppose there exist $\theta', \theta'' \in Y$ such that $Y \subseteq H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$. Since Y is compact, consider $\tilde{\theta}' \in \arg\min\{b_2 y \mid y \in Y\}$ and $\tilde{\theta}'' \in \arg\min\{b_1 y \mid y \in Y\}$. Since $b_1\tilde{\theta}'' \leq b_1\theta''$ and $b_2\tilde{\theta}' \leq b_2\theta'$, $Y \subseteq H(b_1, b_1\tilde{\theta}'') \cup H(b_2, b_2\tilde{\theta}')$. In particular $b_1\tilde{\theta}' > b_1\tilde{\theta}''$, $b_2\tilde{\theta}'' > b_2\tilde{\theta}'$ and $x = h(b_1, b_1\tilde{\theta}'') \cap h(b_2, b_2\tilde{\theta}') \notin Y$. Moreover, by the definition of $\tilde{\theta}', \tilde{\theta}''$, $Y \subset \overline{H}(b_1, b_1\tilde{\theta}'') \cap \overline{H}(b_2, b_2\tilde{\theta}')$ and x cannot be written as a convex combination of points in Y ($x \notin co(Y)$). Now choose any point $\tilde{\theta} \in \widetilde{Fr}(co(Y))$ such that $\tilde{\theta}$ belongs to the triangle formed by $x, \tilde{\theta}'$ and $\tilde{\theta}''$. Then denoting $n = n_{co(Y)}(\tilde{\theta})$ we have that $n(\tilde{\theta}' - \tilde{\theta}) \geq 0$, $n(\tilde{\theta}'' - \tilde{\theta}) \geq 0$, $n(x - \tilde{\theta}) < 0$ which implies that $n(\tilde{\theta}' - x) > 0$ and $n(\tilde{\theta}'' - x) > 0$. Using $\{b_1, b_2\}$ as a base for $\mathbb{R}^2$ we can write $n = \alpha b_1 + \beta b_2$ and hence $\alpha b_1(\tilde{\theta}' - \tilde{\theta}'') > 0$ and $\beta b_2(\tilde{\theta}'' - \tilde{\theta}') > 0$ which implies $\alpha, \beta > 0$ and therefore $n \in C(b_1, b_2)$ which contradicts Prop.5-(ii-2).

Prop.1-(ii) $\Rightarrow$ Prop.6-(ii-2): Suppose there exists $\theta \in \widetilde{Fr}(Y)$ such that $n_Y(\theta) \in C(b_1, b_2)$. Then for any $\delta > 0$, both $B(\theta, \delta) \cap Y \cap H(b_i, b_i\theta) \neq \emptyset$ and $B(\theta, \delta) \cap Y \cap \{y \in \mathbb{R}^2 \mid b_i y < b_i\theta\} \neq \emptyset$ for $i = 1, 2$. Moreover there exists $\epsilon > 0$ such that

$$B(\theta, \epsilon) \cap Y \subseteq \overline{H}(b_1, b_1\theta) \cup \overline{H}(b_2, b_2\theta) \tag{15}$$

For any $\delta > 0$ consider $\theta' \in B(\theta, \delta) \cap Y \cap \{x \in \mathbb{R}^2 \mid b_2 x < b_2\theta\}$ and $\theta'' \in B(\theta, \delta) \cap Y \cap \{x \in \mathbb{R}^2 \mid b_1 x < b_1\theta\}$. Then $B(\theta, \epsilon) \cap Y \subseteq H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$. To see this, consider $\tilde{\theta} \in B(\theta, \epsilon) \cap Y$. By (15), $\tilde{\theta} \in \overline{H}(b_1, b_1\theta) \cup \overline{H}(b_2, b_2\theta)$. Suppose $\tilde{\theta} \in \overline{H}(b_1, b_1\theta)$, then $b_1\tilde{\theta} \geq b_1\theta > b_1\theta''$ so $\tilde{\theta} \in H(b_2, b_2\theta')$. Similarly, if $\tilde{\theta} \in \overline{H}(b_2, b_2\theta)$, then $\tilde{\theta} \in H(b_2, b_2\theta')$. Hence $\tilde{\theta} \in H(b_1, b_1\theta'') \cup H(b_2, b_2\theta')$.

Prop.6-(ii-2) $\Rightarrow$ Prop.1-(ii): Suppose that local deviations from $\theta \in Y$ cannot be deterred. By Step 1 of Proposition 3, there exists an $\epsilon > 0$ such that $B(\theta, \epsilon) \cap Y \subseteq \overline{H}(b_1, b_1\theta) \cup \overline{H}(b_2, b_2\theta)$. Moreover for all $\delta > 0$, $B(\theta, \delta) \cap Y \not\subseteq \overline{H}(b_1, b_1\theta) \cap \overline{H}(b_2, b_2\theta)$ because if not θ would be locally the worst point for both senders and a local deviation could be deterred by choosing θ. Moreover if $Fr(Y)$ does not have a non-convex kink at θ that is linear with normal vectors $\{b_1, b_2\}$, and has a finite number of kinks, then there exits either $\theta' \in H(b_1, b_1\theta) \setminus \overline{H}(b_2, b_2\theta)$ or $\theta'' \in H(b_2, b_2\theta) \setminus \overline{H}(b_1, b_1\theta)$ such that $Fr(Y)$ is differentiable in (θ, θ') (alternatively differentiable in (θ, θ'')). Assume we are in the first case, then and by the mean value theorem there exists $\tilde{\theta} \in (\theta, \theta')$ such that $t(\tilde{\theta}) = \gamma(\theta' - \theta)$, where $t(\tilde{\theta})$ is the tangent vector to $Fr(Y)$ at $\tilde{\theta}$. Using b_1, b_2 as a base of $\mathbb{R}^2$, we have that $n_Y(\tilde{\theta}) = \alpha b_1 + \beta b_2$

and hence, $0 = n_Y(\tilde{\theta})(\theta' - \theta) = \alpha b_1(\theta' - \theta) + \beta b_2(\theta' - \theta)$. And since $b_1(\theta' - \theta) > 0$ and $b_2(\theta' - \theta) < 0$, we have that both α and β have the same sign. Moreover since $n_Y(\tilde{\theta})$ is the inward normal vector and $B(\theta, \epsilon) \cap Y \subseteq \overline{H}(b_1, b_1\theta) \cup \overline{H}(b_2, b_2\theta)$, it has to be that both $\alpha, \beta > 0$, and hence $n_Y(\tilde{\theta}) \in C(b_1, b_2)$ □

Proof of Proposition 7: $(i) \Leftrightarrow (ii)$: This follows from Proposition 4.

$(ii) \Rightarrow (iii)$: Consider first the case $\Theta = Y$. We show that if for any pair of reports (θ', θ'') in Y such that $\theta' \neq \theta''$, the receiver responds by choosing $y^R(\theta', \theta'') = \theta' \wedge_{\{\underline{b},\overline{b}\}} \theta''$, this response deters both senders from deviating, whatever the realizations of $b_1, b_2 \in \overline{C}(\underline{b}, \overline{b})$, and therefore the truthful strategies (s_1, s_2) together with y^R constitute a robust FRE.
Since $b_1, b_2 \in \overline{C}(\underline{b}, \overline{b})$, there exist $\alpha_1, \alpha_2, \beta_1, \beta_2 \geq 0$ such that, $b_i = \alpha_i \underline{b} + \beta_i \overline{b}$, for $i = 1, 2$. Denote by $\hat{\theta} = \theta' \wedge_{\{\underline{b},\overline{b}\}} \theta''$.

$$\begin{aligned} b_1\hat{\theta} &= (\alpha_1 \underline{b} + \beta_1 \overline{b})\hat{\theta} = \alpha_1 \underline{b}\hat{\theta} + \beta_1 \overline{b}\hat{\theta} \\ &= \alpha_1 \min\{\underline{b}\theta', \underline{b}\theta''\} + \beta_1 \min\{\overline{b}\theta', \overline{b}\theta''\} \\ &= \min\{b_1\theta', b_1\theta'', \alpha_1 \underline{b}\theta' + \beta_1, \overline{b}\theta'', \alpha_1 \underline{b}\theta'' + \beta_1 \overline{b}\theta'\} \\ &\leq \min\{b_1\theta', b_1\theta''\} \end{aligned} \tag{16}$$

Analogously, $b_2\hat{\theta} \leq \min\{b_2\theta', b_2\theta''\}$, and therefore the action $\hat{\theta}$ deters the two senders with biases (b_1, b_2) from deviating. Note that whenever θ', θ'' converge to θ, $\theta' \wedge_{\{\underline{b},\overline{b}\}} \theta''$ also converges to θ, and hence $\theta' \wedge_{\{\underline{b},\overline{b}\}} \theta''$ deters local deviations with local actions. Furthermore, observe that the inequality (16) and the analogous inequality for b_2 hold for any $b_1, b_2 \in \overline{C}(\underline{b}, \overline{b})$ independently of whether those values of the biases belong to the support of the conditional distribution of the biases given the realization of the state.
Consider now $Y \subsetneq \Theta$, and for any $\theta \in \Theta$ define $\tilde{s}_i(\theta) = y^*(\theta)$. We show that for any realisation of the biases (b_1, b_2), $(\tilde{s}_1, \tilde{s}_2, y^R)$ is a robust FRE in (Θ, Y) for arbitrarily large biases.
Given $y', y'' \in Y$ denote by $x = y^R(y', y'') = y' \wedge_{\{\underline{b},\overline{b}\}} y''$. For sender S_1 we need to show that for any $\theta \in \Theta$ such that $y^*(\theta) = y''$, $|\theta + tb_1 - y''| \leq |\theta + tb_1 - x|$ for all $t > 0$ and for all $b_1 \in \overline{C}(\underline{b}, \overline{b})$. Consider any such $\theta \in \Theta$ with $y^*(\theta) = y''$, that is, y'' is the closest point in Y to θ. In particular $|\theta - y''| \leq |\theta - x|$. Define z as the midpoint of the segment $[x, y'']$. Then $\theta(y'' - x) \geq z(y'' - x)$, and by (16), $(\theta + tb_1)(y'' - x) \geq z(y'' - x)$ for all $t > 0$ and all $b_1 \in \overline{C}(\underline{b}, \overline{b})$, or in other words $|\theta + tb_1 - y''| \leq |\theta + tb_1 - x|$ for all $t > 0$ and all $b_1 \in \overline{C}(\underline{b}, \overline{b})$. A similar argument for S_2 shows that for any $\theta \in \Theta$ such that $y^*(\theta) = y'$, $|\theta + tb_2 - y'| \leq |\theta + tb_2 - x|$ for all $t > 0$ and all $b_2 \in \overline{C}(\underline{b}, \overline{b})$. Therefore $(\tilde{s}_1, \tilde{s}_2, y^R)$ is a FRE in (Θ, Y).
$(iii) \Rightarrow (i)$: Given Y, if for $\Theta \supsetneq Y$ there exists a robust FRE then for $\Theta = Y$ there exists a robust FRE. Given that for all $\theta \in Y$, the realization of biases $(\underline{b}, \overline{b})$ has positive probability, then Proposition 4 implies that condition (i) must hold. □

References

Alonso, R., W. Dessein, and N. Matouschek (2008), "When Does Coordination Require Centralization?", *American Economic Review*, 98(1), 145-79.

Ambrus, A., and S. Takahashi (2008), "Multi-Sender Cheap Talk with Restricted State Spaces", *Theoretical Economics*, 3(1), 1-27.

Battaglini, M. (2002), "Multiple Referrals and Multidimensional Cheap Talk", Econometrica, 70(4), 1379-1401.

Crawford, V.P., and J. Sobel (1982), "Strategic Information Transmission", *Econoometrica*, 50(6), 1431-51.

Gilligan, T.W. and K. Krehbiel (1989), "Asymmetric Information and Legislative Rules with a Heterogeneous Committee", *American Journal of Political Science*, 33, 459-490.

Krishna, V., and R.J. Morgan (2001a), "Asymmetric Information and Legislative Rules: Some Amendmanets", *American Political Science Review*, 95, 435-452.

Krishna, V., and R.J. Morgan (2001b), "A Model of Expertise", *Quarterly Journal of Economics*, 116, 747-775.

Levy, G. and R. Razin (2007), "On the Limits of Communication in Multidimensional Cheap Talk: A Comment", *Econometrica*, 75, 885-893.