Micro-econometrics: Notes on Week 5 Exercise

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Notes on suggested solutions:

a) Note that the OLS estimator for β in Victoria's solutions looks different to the one most of you derived (and the one derived in class). However, they are equivalent which is shown by:

$$\begin{aligned}\sum_{t=1}^{T}\sum_{i=1}^{N}(x_{it}-\bar{x})(y_{it}-\bar{y}) &= \sum_{t=1}^{T}\sum_{i=1}^{N}(x_{it}y_{it}-x_{it}\bar{y}-\bar{x}y_{it}+\bar{x}\bar{y}) \\ &= \sum_{t=1}^{T}\sum_{i=1}^{N}(x_{it}y_{it})-NT\bar{x}\bar{y} \\ &= \sum_{t=1}^{T}\sum_{i=1}^{N}x_{it}y_{it}-\bar{x}\sum_{t=1}^{T}\sum_{i=1}^{N}y_{it} \\ &= \sum_{t=1}^{T}\sum_{i=1}^{N}((x_{it}-\bar{x})y_{it})\end{aligned}$$

b) Given our slightly different notation of the OLS estimator, we will need to prove the second result in b) for a different expression, i.e. we need to show:

$$\sum_{t=1}^{T}\sum_{i=1}^{N}(x_{it}-\bar{x})(y_{it}-\bar{y}) = \sum_{t=1}^{T}\sum_{i=1}^{N}(x_{it}-\bar{x}_i)(y_{it}-\bar{y}_i)+T\sum_{t=1}^{T}\sum_{i=1}^{N}(\bar{x}_i-\bar{x})(\bar{y}_i-\bar{y})$$

This is done in a very similar way to the other proofs in b), we just need to add and subtract an additional term:

$$\begin{aligned}\sum_{t=1}^{T}\sum_{i=1}^{N}(x_{it}-\bar{x})(y_{it}-\bar{y}) &= \sum_{t=1}^{T}\sum_{i=1}^{N}x_{it}y_{it}-\bar{y}\sum_{t=1}^{T}\sum_{i=1}^{N}x_{it}-\bar{x}\sum_{t=1}^{T}\sum_{i=1}^{N}y_{it}+\sum_{t=1}^{T}\sum_{i=1}^{N}\bar{x}\bar{y} \\ &\quad +2T\sum_{i=1}^{N}\bar{x}_i\bar{y}_i-2T\sum_{i=1}^{N}\bar{x}_i\bar{y}_i \\ &= \sum_{t=1}^{T}\sum_{i=1}^{N}(x_{it}-\bar{x}_i)y_{it}-\sum_{t=1}^{T}\sum_{i=1}^{N}(x_{it}-\bar{x}_i)\bar{y}_i \\ &\quad +T\sum_{it}^{N}(\bar{x}_i-\bar{x})\bar{y}_i-T\sum_{it}^{N}(\bar{x}_i-\bar{x})\bar{y} \\ &= \sum_{t=1}^{T}\sum_{i=1}^{N}(x_{it}-\bar{x}_i)(y_{it}-\bar{y}_i)+T\sum_{t=1}^{T}\sum_{i=1}^{N}(\bar{x}_i-\bar{x})(\bar{y}_i-\bar{y})\end{aligned}$$

The prove in c) then follows easily from the results in b).