

Micro-econometrics: Notes on Week 6 Exercise

Julia Giese

JULIA.GIESE@NUFFIELD.OX.AC.UK

Notes on suggested solutions:

$$y_i = \alpha_i \mathbf{e} + \mathbf{X}_i \beta + \varepsilon_i \quad (1)$$

- a) Make clear what the problem in our model (1) is, i.e. we have unobservable individual effects α_i , and if these are correlated with \mathbf{X}_i , i.e. we have fixed effects, OLS estimates would be biased. Hence, we need to eliminate them from the equation. This can be done by pre-multiplying with a suitable matrix \mathbf{J} .
- e) For consistency of an estimator, we need to show that it converges to its true values and that its variance tends to zero asymptotically. Since $\hat{\beta}$ is unbiased as we showed in b), it will converge to its true value. Since α_i are eliminated from the regression, its variance will not feature in $Var\hat{\beta}$. For the case where \mathbf{J} is idempotent and $Var(\varepsilon_{it}) = \sigma^2 I$,

$$Var\hat{\beta} = \sigma^2 \left(\sum_{i=1}^N \mathbf{X}'_i \mathbf{J}' \mathbf{J} \mathbf{X}_i \right)^{-1}.$$

It is easy to see that the denominator – a scalar – goes to infinity as $N \rightarrow \infty$, so $Var\hat{\beta} \rightarrow 0$.

For efficiency, we need to check whether there might be an estimator with smaller variance. Intuitively, since we eliminated α_i without having to do so (remember that it is now random, i.e. not correlated with \mathbf{X}_i), we are omitting information that we should be exploiting. Furthermore, $E(\alpha_i \varepsilon_{it})$ may not equal zero, in which case we may also be able to exploit this.

- f) Multiplying out the matrices $\mathbf{J}y_i$, one arrives at the same formulation of the fixed effects regression as in last week's problem set, i.e. y_{it} in deviation from each individual's mean over time.
- g) Make sure you come up with a truly different transformation where $\mathbf{J}\mathbf{e} = 0$ still holds, i.e. we still eliminate the fixed effects. One example would be to choose \mathbf{J} such that one arrives at a model in differences over time. You lose the information from one time period, which may be problematic if T is small, but you eliminate α_i .