Similarity issues in cointegration analysis

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Abstract:

Usually cointegration models involve a dynamic, stochastic component as well as deterministic components. This paper identifies relevant cointegration models in terms of interpretability and similarity with respect to parameters of deterministic components. Similarity implies that inference on cointegration rank or common trends can be separated from inference on parameters of deterministic components. The idea is that the functional form and thereby the interpretation of deterministic components is not questioned in connection with the rank test, but it can be tested subsequently. The paper focuses on likelihood based inference in vector autoregressive models.

1. Introduction

Cointegration analysis in vector autoregressive models is viewed as addressing two separate questions. The first concerns the number of cointegrating relations in vector time series or equivalently the number of common trends. The second is related to analysis of coefficients such as those of the cointegrating vector and of deterministic components. In terms of inference, determination of the number of cointegrating relations is based on non-standard asymptotic distributions, while remaining inference is based on standard χ^2-distributions.

One way to separate the analysis of these questions is to find similar tests. Although cointegration analysis is rather complicated it is indeed possible to find some similarity properties. This paper demonstrates that some of the likelihood based tests suggested in the literature are more convenient in this sense than others. As it happens, "similar" tests also allow for more attractive interpretations. The idea is to test for cointegration rank in a model where for instance deterministic components of the process are not affected by the cointegration rank. Then the test for cointegration rank is asymptotically similar with respect to the parameters of deterministic components. Thereby at least a partial separation of the two questions is obtained.

Cointegration tests are based on asymptotic approximations of the test distribution. It is well-known that such approximations are often very poor. Similarity seems to be an important issue in stabilising the likelihood ratio test. As an example the test for no cointegration in a first order model is similar and the distribution approximation to the likelihood ratio test is unusually good (Nielsen, 1997,1998). However, in the general models which are used in applications the cointegration tests are not similar: there are problems with the possible number of unit roots and with deterministic trend parameters. The considerations of this paper eliminates the latter problem.

Consider in general statistical models with a multivariate parameter $\theta = (\psi, \lambda, \tau)$ where it is of interest to test the hypothesis $\psi = 0$. If the rejection frequency for the critical region, CR, does not depend on λ, τ, that is

$$P_{\lambda,\tau}(CR) = \alpha$$

then the test is said to be similar (Cox and Hinkley, 1974, p. 134). If the rejection probability depends on λ but not τ we say the test is similar with respect to τ. Finally, if the rejection probability depends on the sample length, but

$$P^T_{\lambda,\tau}(CR) \to \alpha_\lambda \qquad \text{for} \qquad T \to \infty$$

we say the test is asymptotically similar with respect to τ as opposed to exactly similar as above. Asymptotic similarity is of course not as attractive as exact similarity. However, in reasonably specified models, where parameter estimates are not too extreme, asymptotic similarity seems sufficient for the desired separation of questions of interest.

Among many types of cointegration analysis this paper focuses on the maximum likelihood based analysis of cointegration rank in vector autoregressive models suggested by Johansen (1988,1995a). The test for cointegration rank is based on the canonical correlation analysis given by Hotelling

(1936) and Bartlett (1938), see also Anderson (1984, Chapter 12). It turns out that similarity properties of cointegration are closely linked with invariance properties of canonical analysis which will be discussed in Section 2.

In Section 3 similarity properties are discussed for cointegration rank tests in full system I(1) analysis. The test for no cointegration in a first order autoregressive model is similar. For more general models asymptotic similarity is found with respect to deterministic trend parameters. This holds subject to certain conditions on the cointegration rank and the I(1) properties of the process. Note, that asymptotic similarity could be improved to an exact property if relevant initial variables are given stationary distributions. However, the statistical analysis is conditional on these variables and we therefore consider the usage of stationary initial distributions as an asymptotic approximation as well.

The similarity mentioned above is contingent on I(1) properties of the system. This assumption is relaxed in I(2) analysis which is discussed in Section 4. The techniques which are used in the full system analyses are also useful in partial systems where certain stochastic regressors are introduced. A brief overview is given in Section 5. Finally, Section 6 summarises recommendations for a general approach to cointegration analysis in practice.

It is important to note that there are many other types of cointegration analysis beside those considered in this paper. As an example analysis based on regression rather than canonical correlations is given in Phillips (1991). Similarity with respect to deterministic trend parameters can often be obtained using detrending strategies as discussed by for instance Dickey and Fuller (1979), Kiviet and Phillips (1992), Lütkepohl and Saikkonen (1997) and Oya and Toda (1998). The point of this paper is to make the concept of similarity precise in a likelihood based analysis of vector autoregressive models.

In the mathematical notation details about for instance dimensions are usually omitted. The symbol $|I + \Pi| < 1$ means that the eigenvalues of the matrix $I + \Pi$ are smaller than one in absolute value. Further, $\alpha_\perp$ denotes the orthogonal complement to a matrix α and $\overline{\alpha} = \alpha\left(\alpha'\alpha\right)^{-1}$.

2. Some invariance properties of canonical correlations

Cointegration is analysed using regression and canonical correlation techniques. Some of the properties of canonical correlation analysis are inherited in cointegration analysis and will be discussed briefly.

The sample canonical correlations of two sets of random vector, $X_1, \ldots, X_T$ and $Y_1, \ldots, Y_T$ are found as solutions of the eigenvalue problem

$$\left| \rho^2 S_{XX} - S_{XY} S_{YY}^{-1} S_{YX} \right| = 0,$$

where for instance the product moment matrix $S_{XY} = \sum_{t=1}^{T} X_t Y_t'$. A convenient notation for these solutions is $\mathrm{CanCor}(X_t, Y_t)$. The eigenvalue problem is invariant with respect to non-singular linear transformations of the two blocks of data so that

$$\mathrm{CanCor}\left(X_t, Y_t\right) = \mathrm{CanCor}\left(AX_t, BY_t\right) \tag{1}$$

for non-singular, square matrices A, B.

If the vector Y_t has two components V_t, W_t it follows from (1) that

$$\mathrm{CanCor}\left\{X_t, \begin{pmatrix} V_t \\ W_t \end{pmatrix}\right\} = \mathrm{CanCor}\left\{X_t, \begin{pmatrix} V_t | W_t \\ W_t \end{pmatrix}\right\}$$

where the least squares correction of V_t for W_t is obtained using

$$B = \begin{pmatrix} I & -S_{VW} S_{WW}^{-1} \\ 0 & I \end{pmatrix}.$$

Often cointegration analysis involves the canonical correlations of two variables X_t and Y_t which are both corrected for a third variable Z_t by least squares, denoted by

$$\mathrm{CanCor}\left(X_t, Y_t | Z_t\right).$$

3. Full system I(1) analysis

The similarity properties of I(1) models depend on the cointegration rank of interest as well as the lag length. Initially, the test of non-stationarity against stationarity in a first order model is discussed for various specifications of trend and some exact similarity properties are found. Next, the more general test of reduced cointegration rank and the inclusion of lags is considered. In these situations only asymptotic similarity is found.

3.1. A simple hypothesis in a first order model

The first order vector autoregressive model is given by the equation

$$\Delta X_t = \Pi X_{t-1} + \varepsilon_t \qquad \text{for } t = 1, \ldots, T,$$

where X_0 is fixed, the innovations are independently, identically normal distributed with zero mean and variance Ω and, finally, the parameters Π and Ω vary freely.

A process of this model is a random walk with level X_0 if $\Pi = 0$,

$$X_t = \sum_{i=1}^{t} \varepsilon_i + X_0. \tag{2}$$

For $|I + \Pi| < 1$ the process can be interpreted as stable with a zero level since by simple recursion,

$$X_t = \sum_{i=0}^{t-1} (I + \Pi)^i \varepsilon_{t-i} + (I + \Pi)^t X_0, \tag{3}$$

and X_0 can be a given an initial distribution so that X_t is stationary with expectation zero. In order to distinguish these properties the hypothesis $\Pi = 0$ is considered.

The maximum likelihood test is based on the canonical correlations

$$\text{CanCor}\left(\Delta X_t, X_{t-1}\right).$$

Under the hypothesis the process in (2) depends only on the initial value X_0 and the scaling matrix $\Omega^{1/2}$. By the invariance property (1) it follows that the distribution of the canonical correlations and therefore of the test statistic itself only depends on Ω and X_0 through $\Omega^{-1/2} X_0$. The test is therefore similar if it is assumed that $X_0 = 0$. This corresponds to the interpretation that the level of the process is zero under the alternative but non-zero under the hypothesis unless $X_0 = 0$. For general X_0 the test is asymptotically similar, see Johansen (1995a, Theorem 6.1).

3.2. A model with a constant level

Consider the model given by the equation

$$\Delta X_t = (\Pi, \Pi_c) \begin{pmatrix} X_{t-1} \\ 1 \end{pmatrix} + \varepsilon_t \qquad \text{for } t = 1, \ldots, T.$$

As above the process is a random walk with level X_0 if $(\Pi, \Pi_c) = 0$. By including the constant Π_c in the innovations of the representation (3) it follows that the process is stable around a level proportional to Π_c if $|I + \Pi| < 1$. Consequently, the deterministic component of the process is the same in these two cases of rather different behaviour of the stochastic component.

The hypothesis $(\Pi, \Pi_c) = 0$ is analysed in terms of

$$\mathrm{CanCor}\left\{\Delta X_t, \begin{pmatrix} X_{t-1} \\ 1 \end{pmatrix}\right\}.$$

Under the hypothesis the distribution of these correlations initially seems to depend on Ω and X_0 through $\Omega^{-1}X_0$ as argued above. However, using (1) it follows that the canonical correlations equal

$$\mathrm{CanCor}\left\{\Delta X_t, \begin{pmatrix} X_{t-1} - \sum_{t=1}^{T} X_{t-1}/T \\ 1 \end{pmatrix}\right\}$$

and therefore do not depend on $\Omega^{-1}X_0$. Hence the likelihood ratio test for the hypothesis is similar. This is not the case, not even asymptotically, in the related analysis where the hypothesis of interest is that Π is restricted, but Π_c is left unrestricted, see Johansen (1995a, Theorem 6.1)

3.3. A model with a linear trend

Consider the model given by the equation

$$\Delta X_t = (\Pi, \Pi_l) \begin{pmatrix} X_{t-1} \\ t \end{pmatrix} + \mu + \varepsilon_t \qquad \text{for } t = 1, \ldots, T. \tag{4}$$

This is related to the case of a non-zero level. If $(\Pi, \Pi_l) = 0$ the process is a random walk around a linear trend with slope μ and if $|I + \Pi| < 1$ the process is stable around a linear trend with a slope proportional to Π_l. Again the test of the hypothesis $(\Pi, \Pi_l) = 0$ is exactly similar, that is, under the hypothesis the distribution of

$$\mathrm{CanCor}\left\{\Delta X_t, \left(\begin{array}{c} X_{t-1} \\ t \end{array}\right)\middle| 1\right\} = \mathrm{CanCor}\left\{\Delta X_t, \left(\begin{array}{c} X_{t-1}| \, t \\ t \end{array}\right)\middle| 1\right\}$$

does not depend on Ω, μ, X_0. Note, that the statistical analysis is based on demeaning of the first differences and detrending of the levels of the time series.

3.4. Determination of rank

In the model given by (4) cointegration is analysed using a reduced rank hypothesis of the type $\mathrm{rank}(\Pi, \Pi_l) \leq r$, where r is smaller than the dimension of the time series. Such a hypothesis can be parametrised as

$$(\Pi, \Pi_l) = \alpha\,(\beta', \beta_l')$$

where α,β_l are $(\mathrm{dim}X \times r)$-matrices, β_l' is an r-vector and all parameters vary freely.

Representation and corresponding interpretations for processes of this model is given by Granger's representation theorem in Johansen (1995a, Chapter 4). Under the hypothesis $\alpha_\perp' X_t$ is a random walk around a linear trend with slope μ. If, in addition, $\beta'\alpha$, or equivalently $\alpha_\perp'\beta_\perp$, has full rank then $\beta' X_t$ is stable around a linear trend with slope proportional to Π_l.

The likelihood ratio test of the hypothesis is given by the $(\mathrm{dim}X - r)$ smallest canonical correlations,

$$\mathrm{CanCor}\left\{\Delta X_t, \left(\begin{array}{c} X_{t-1} \\ 1 \end{array}\right)\right\}.$$

If $\mathrm{rank}(\Pi, \Pi_l) = r$ and $\beta'\alpha \neq 0$ these can be rewritten as

$$\mathrm{CanCor}\left\{\Delta X_t, \left(\begin{array}{c} \beta' X_{t-1} \\ \alpha_\perp' X_{t-1} \\ t \end{array}\right)\middle| 1\right\} = \mathrm{CanCor}\left\{\Delta X_t, \left(\begin{array}{c} \beta' X_{t-1} \\ \alpha_\perp' X_{t-1}|t \\ t \end{array}\right)\middle| 1\right\}.$$

Therefore it can be argued as above that the distribution of these correlations do not depend on the parameters for trend, level and innovation variance for the component $\alpha_\perp' X_t$. The test is therefore similar with respect to these parameters, given by $\alpha_\perp' \mu, \alpha_\perp' X_0, \alpha_\perp' \Omega \alpha_\perp$. Further, as a consequence of Johansen (1995a, Theorem 6.2) the test is asymptotically similar with respect to the remaining parameters for trend, level and innovation variance. In combination these results imply asymptotic similarity with respect to $\beta_l, \mu, X_0, \Omega$.

Unfortunately the distribution of the test depends drastically on α, β. If the actual rank of (Π, Π_l) is smaller than the one, r, which is tested for, new asymptotic distributions arise just as in standard canonical correlation analysis. Further, if $\text{rank}(\Pi, \Pi_l) = r$, but $\beta' \alpha$ has reduced rank then the process is said to be integrated of higher order and a different asymptotic distribution is obtained. This is discussed in Section 4 below.

3.5. Inclusion of lags

In applications models usually include further lagged variables to allow for a better description of the short term dynamics of the data. Consider,

$$\Delta X_t = (\Pi, \Pi_l) \begin{pmatrix} X_{t-1} \\ t \end{pmatrix} + \sum_{j=1}^{k-1} \Gamma_j \Delta X_{t-j} + \mu + \varepsilon_t \qquad \text{for } t = 1, \ldots, T. \quad (5)$$

The results are nearly as above.

Under the hypothesis $\text{rank}(\Pi, \Pi_l) \leq r$ and certain conditions the process can be given the representation

$$X_t = C \sum_{i=1}^{t} \varepsilon_t + Y_t + \tau_c + \tau_l t$$

where Y_t is stationary linear process, correlated with the random walk component and with exponentially decreasing coefficients. The common trends parameter C and the level and trend parameters, τ_0, τ_1 are given in terms of the matrix $\Psi = 1 - \sum_{j=1}^{k-1} \Gamma_j$ as

$$\begin{aligned} C &= \beta_\perp \left(\alpha_\perp' \Psi \beta_\perp \right)^{-1} \alpha_\perp', \\ \beta' \tau_c &= \overline{\alpha}' \left(\Psi C - I \right) \mu + \left(\overline{\alpha}' \Psi C \Psi - I - \overline{\alpha}' \Psi \overline{\beta} \right) \beta_l', \\ \tau_l &= \left(C\Psi - I \right) \overline{\beta} \beta_l' + C\mu. \end{aligned}$$

Note, that $\beta_\perp' \tau_c$ depends on the initial values, $X_0, \ldots, X_{1-k}$, and is not identified, see Johansen (1995a, Section 5.7) and Rahbek (1997). The relevant assumptions to the process are

1. $\mathrm{rank}(\Pi, \Pi_l) = r$
2. the I(1) condition, that the matrix $\alpha_{\perp}' \Psi \beta_{\perp}$ has full rank.
3. the processes $\beta' X_t$ and ΔX_t can be given stationary initial distributions.

The likelihood ratio test for the hypothesis $\mathrm{rank}(\Pi, \Pi_l) \leq r$ is based on

$$\mathrm{CanCor}\left\{ \Delta X_t, \left(\begin{array}{c} X_{t-1} \\ t \end{array} \right) \middle| 1, (\Delta X_{t-j})_{j=1,\ldots,k-1} \right\}$$

and as above it is asymptotically similar with respect to the parameters Ω, β_l, μ and initial values $X_0, \ldots, X_{1-k}$ under the assumptions listed. Note, that the third assumption might be redundant for this result to hold. This is at least the case for a univariate second order model and a bivariate first order model (Nielsen, 1998).

The results in Section 3.1, 3.2 concerning models without linear trends can be generalised correspondingly. Note, that also in the model with a constant level the level of the common trends, $\beta_{\perp}' \tau_c$ is not identified.

4. Full system I(2) analysis

The I(1) condition that the matrix $\alpha_{\perp}' \Psi \beta_{\perp}$ has full rank is necessary for the interpretation and to ensure that only one asymptotic distribution is involved in rank testing. If the condition is not satisfied the process could be I(2) and a different asymptotic distribution applies. In applications the parameters are often so that the I(1) condition nearly is violated in the sense that some characteristic roots are close to be unit roots. Then the interpretation of the results and the asymptotic distribution approximation are likely to fail. An I(2) analysis imposes additional unit roots. This is likely to reduce similarity problems and therefore finite sample problems as well as new interpretations of the data may arise.

The I(2) analysis can be formulated as a sub-model of the I(1) model (5) with two reduced rank problems. It can be formulated so that the similarity with respect to the parameters for the deterministic trend is preserved. The first reduced rank problem is as before

$$(\Pi, \Pi_l) = \alpha \left(\beta', \beta_l' \right).$$

A reformulation of the model gives that this corresponds to a reduced rank relation between the second differences $\Delta^2 X_t$ and the vector (X'_{t-1}, t). The second reduced rank problem addresses $\alpha'_\perp \Psi \beta_\perp$ and arise in relation to the pair $\alpha'_\perp \Delta^2 X_t$ and $(\Delta X'_{t-1}, 1)$. It is parametrised in terms of ξ, η, η_l as

$$\begin{aligned} \alpha'_\perp \Psi \beta_\perp &= \xi \eta' \\ -\alpha'_\perp \mu &= \xi \eta'_l + \alpha'_\perp \Psi \overline{\beta} \beta'_l \end{aligned}$$

This formulation allows for a linear trend in all linear combinations of the process. The multi-cointegrating relation has a trend with slope proportional to β_l and the cointegrating, but not multi-cointegrating, relation has slope proportional to η_l. The slope of the pure I(2) component is not identified just as the level of the common trends is not identified in the I(1) model. A full analysis is given in Rahbek, Kongsted and Jørgensen (1998) based on the two stage procedure of Johansen (1995b).

In correspondence, an I(2) model with a constant level of the cointegration vector is treated in Paroulo (1996).

5. Analysis with Stochastic Regressors

Inclusion of stochastic regressors in the cointegration analysis may be viewed as an analysis of some of the variables, denoted "endogenous", conditional on the remaining weakly exogeneous variables in the vector autoregressive model given by (5). This is referred to as partial or conditional analysis. A different approach is to include additional explanatory regressors in the analysis of (5) and discuss cointegration subject to specification of the explanatory variables. Both approaches have asymptotic similarity properties which resemble the previous results.

5.1. Partial Analysis

For partial analysis divide the process as $X_t = (Y'_t, Z'_t)'$. The hypothesis of reduced cointegration rank, $\text{rank}(\Pi, \Pi_l) \leq r$, in the I(1) model (5) could be analysed in a partial analysis of Y_t given Z_t in terms of the canonical correlations,

$$\text{CanCor}\left\{ \Delta Y_t, \left(\begin{array}{c} \Delta X_{t-1} \\ t \end{array} \right) \middle| \Delta Z_t, 1, (\Delta X_{t-j})_{j=1,..,k-1} \right\}.$$

There are two important points in connection with this kind of analysis. First, if the conditioning process is weakly exogeneous for the cointegrating vector then the conditional likelihood analysis for cointegration is equivalent to the full likelihood system analysis. The weak exogeneity property is satisfied if

$$(0, I_{\dim Z})\,(\Pi, \Pi_l) = 0. \tag{6}$$

Second, the asymptotic distribution depends on the joint distribution of Y_t and Z_t. Harbo, Johansen, Nielsen and Rahbek (1998) analyse the situation where the full system is I(1) and (6) holds, and find that this analysis is asymptotically similar with respect to the parameters of the deterministic trend and the regressor ΔZ_t.

In the partial I(2) analysis weak exogeneity is contingent on the additional restriction

$$(0, I_{\dim Z})'\Psi = 0$$

since the I(2) model involves two reduced rank problems, see Paruolo and Rahbek (1996).

5.2. Explanatory Regressors

In general, it is of interest to extend the model (5) with a stochastic regressor V_t without modelling the full system of X_t and V_t. A discussion of this issue based on the ideas concerning similarity is given by Mosconi and Rahbek (1998).

If V_t is a linear process which cumulates to an I(0) process, cointegrating properties of X_t can be tested using

$$\mathrm{CanCor}\left\{ \Delta X_t, \left(\begin{array}{c} X_{t-1} \\ t \end{array} \right) \middle| V_t, 1, (\Delta X_{t-j})_{j=1,..,k-1} \right\}. \tag{7}$$

The asymptotic distribution of the test is the same as in the full system I(1) analysis. It does therefore not depend on the coefficient of the deterministic trend and the regressor V_t.

On the other hand, if V_t cumulates to an I(1) process then the analysis based on (7) involves the coefficient of V_t as a nuisance parameter. This is avoided in an analysis based on

$$\mathrm{CanCor}\left\{ \Delta X_t, \left(\begin{array}{c} X_{t-1} \\ t \\ \Sigma_{i=1}^{t-1} V_i \end{array} \right) \middle| V_t, 1, (\Delta X_{t-j})_{j=1,..,k-1} \right\}$$

Such a test has the same asymptotic distribution as that of the partial system discussed above and in fact this kind of analysis may be viewed as a special case of the partial analysis above under certain conditions. However, it is only if there is no feed-back from the endogenous process in the exogeneous process, that the I(1) regularity conditions can be formulated in terms of the partial system alone.

6. Conclusion

In this paper classes of models have been presented in which it is possible to give some separation of the questions related to cointegration rank and specific coefficients, both in terms of interpretation and similarity properties. Cointegration can then be analysed in two steps: first the cointegration rank is determined using one of the presented models and next restrictions on the coefficients can be tested using χ^2 inference.

The rank tests gives bounds for ranks and estimates for the rank can therefore be found using these tests sequentially. More specifically, for the I(1) models the idea is to test the hypotheses

$$\text{rank} = 0,\ \text{rank} \leq 1,\ \text{rank} \leq 2, \ldots$$

against a general alternative and estimate the rank from the first accepted hypothesis. Consistency properties of this procedure can be formulated as in Johansen (1995a, Section 12.1). Note, that it is not necessary to consider the more complicated procedures given in Johansen (1995a, Section 12.2,3). Correspondingly, for I(2) models the procedure given in Rahbek et al. (1997) is consistent. The relevant asymptotic distributions for rank tests are tabulated in the literature:

I(1) model with linear trend	Johansen (1995a),Table 15.4
I(1) model with constant level	Johansen (1995a),Table 15.2
I(1) model with zero level	Johansen (1995a),Table 15.1
I(2) model with linear trend	Rahbek et al. (1998)
I(2) model with constant level	Paroulo (1996), Table 5
partial I(1) model	Harbo et al. (1998)

Slightly more accurate tables for most of the models are given by Doornik (1997)

Given the rank is known, restrictions of the coefficients can be tested using the χ^2-distribution. In a model with a linear trend in all components of the process it could for instance be relevant to test that the cointegrating relation does not have a linear trend using the restriction $\beta_l = 0$.

A vast amount of applications of cointegration procedures exists in the literature. An application based on the central I(1) model with a linear trend is for instance described in the monograph Hendry (1995, Chapter 16). Procedures are implemented in various computer programs such as PcFiml by Doornik and Hendry (1997) and CATS by Hansen and Juselius (1995)

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