Pension systems in open economy

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Abstract: The paper focuses on the effects of the integration in a perfect world capital market of two large economies running respectively a pay-as-you-go and a fully funded pension system. Under the assumption that the pay-as-you-go system is in steady state and self financing, the integration causes changes in factor prices and divergent welfare effects both across countries and across generations. The steady state assumption is then removed and the focus is on the degree of maturity of the pay-as-you-go scheme and on how the latter changes the integration effects. Last, the paper focuses on the funding status of the pay-as-you-go system, removing the assumption that it is balanced. It allows for debt financing and it investigates how the timing of debt financing and the sources of the deficit in the scheme modify the two-country equilibrium.

Keywords: Overlapping generations, capital markets, pay-as-you-go, fully funded.

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1 Introduction

The creation of the Monetary Union in Europe and the process of globalisation involving different geographical areas have contributed to make the open economy aspects of pension systems and reforms a topical issue in the policy debate. Questions on the effects of the integration and competition in a world market of countries characterised by different models of welfare state, of which pensions are the largest expenditure program, are at the core of the policy discussion.

Differences in the *generosity* of the welfare state can have an impact on growth and competitiveness. This is often pointed out in the debate on the good economic performance of the US with respect to Europe, seen as two large economies competing in the world market. Differences in the *organisation* of pension schemes, even at the same level of generosity, can have an impact on the capital stock and therefore affect output and factors remuneration. The still ongoing debate in Britain on whether to join or not the single currency raises concerns on the impact that the *unfunded* pension liabilities of some continental European countries can have on British taxpayers, whose pensions derive largely from *funded* occupational pension schemes.

Economists have lately started investigating the open economy aspects of pension schemes. Some recent literature analyses the social security *coordination* problems arising from labour mobility [Homburg and Richter (1993)] and capital mobility [Beltrametti and Bonatti (1998)] and it shows that the contribution rate to pay-as-you-go social security schemes chosen by n uncoordinated identical small countries differs from the one a supernational benevolent authority would choose.

This paper focuses on a different open economy aspect of pension schemes. Starting from a closed economy framework, it investigates the effects on factor prices and welfare of the integration in an open economy with perfect capital markets of countries whose pension systems are organised according to different principles. Capital movements are in the paper the means through which pension policies in one country can have an impact on another country.

The paper aims at providing a simple framework where some of the issues raised in the current policy debate can be addressed: to this end, the paper introduces an overlapping generations model with two countries identical in preferences and technology and running respectively a pay-as-you-go and a fully funded pension system. While the questions addressed here are new, the tools for the analysis are mutuated from the traditional literature on overlapping generations: the model is based on Diamond (1965) and its open economy extensions by Buiter (1981) and Persson (1985).

I first develop a benchmark model where the pay-as-you-go system is in steady state balance and self-financing: this assumption allows to concentrate on the effects that the organisation of the pension system has on the *supply* side of the capital market and to distinguish them from those on the *demand* side. The policy debate on the unfunded pension liabilities issue and on the budgetary problems that they might generate limits the spillover effects across countries to the imbalance in the pay-as-you-go pension system. One can actually show that changes in factor prices and welfare take place when the economy is opened even though the pay-as-you-go pension scheme is balanced. The reason is known: a pay-as-you-go scheme affects consumption and saving decisions by agents who pay contributions and receive transfers under the system and therefore it influences the supply of capital. Though the result that the pay-as-you-go financing of a pension scheme depresses savings is a known one, I think it is important to relate it and include it into the framework of analysis. If one aims at discussing the implications of the interaction among countries with different social security systems, it is important to know in details the basic mechanisms at work. Moving to the welfare analysis, the paper points out that opening the capital markets has divergent welfare effects both across countries and across generations and therefore it is not trivial and straightforward to establish who gains and who loses out of the integration. This result goes against the general view according to which the country running a fully funded pension scheme is the loser.

I then extend the benchmark model to capture some real world features of public pension schemes. The main issues I address are the *maturity* of a public pension scheme and its *funding* status. Still assuming that the system is balanced, I first remove the assumption that the system is in steady state and investigate how the maturing of a pay-as-you-go pension system affects saving and consumption decisions by residents in the country with the pay-as-you-go system. I assess whether the effects on world factor prices and welfare caused by the integration in an international capital market are amplified or reduced by the pay-as-you-go pension system not being mature. It is shown that the effects on factor prices and welfare are larger when the integration takes place during the maturity of the pension scheme rather than during the transition to maturity. I then remove the assumption that the system is balanced in order to capture the pension liability issue appropriately. I focus on the financing means of the pension system and analyse explicitly debt financing to understand how the use of debt modifies the two-country equilibrium. Notice that the analysis of alternative financing means for the pay-as-you-go scheme adds a second dimension to the study of pension policies in open economies: namely, it involves the demand side of the capital market and it allows the joint study of the effects of alternative pension schemes on the supply and on the demand of capital. I concentrate on two specific questions: I first discuss whether the *timing* of the introduction of debt financing matters and affects the equilibrium level of per capita capital. The analysis throws some light on a relevant issue in the policy debate, i.e. whether the sharing among countries of the costs of adjustment of an unbalanced pension scheme reduces the welfare of the virtuous countries. I then investigate whether the *sources* of the imbalance in the pay-as-you-go scheme matter and have an impact on the same variable.

The paper is organised as follows: Section 2 presents the benchmark model and develops the welfare analysis. Section 3 extends the benchmark model introducing the degree of maturity of the pay-as-you-go scheme and debt financing. Section 4 concludes.

2 The benchmark model

2.1 Introduction

I consider two countries: Country A runs a pay-as-you-go pension system; Country B has a fully funded scheme. I assume that there are no other differences between the two countries. The lump-sum contributions paid by the working generation into the pay-as-you-go scheme in Country A are, by assumption, enough to cover the pensions promised to the old, i.e. the system is balanced. The governments in the two countries run the pension systems and do not have any other role.

I first assume that the economy is closed and then extend the analysis to an open economy environment: capital markets are perfectly integrated and capital can freely move between the two countries whilst labour is immobile. I investigate how *equilibrium prices* and *welfare* in the two countries are affected by the opening of the economy[1].

2.2 The closed economy

2.2.1 Consumers

In Country A the consumer's maximisation problem at time t, for each t is:

[1]Buiter (1981) studies the welfare effects of the participation in the world economy of two countries characterised by different rates of time preference; there is no government in his model. The government i spresent in Persson (1985), which investigates the effects on factor prices and welfare of an increase in public debt caused by a temporary decrease in the taxes levied on the young, comparing the closed, the small open and the two large economy cases. If one denotes by τ_1 per capita taxes/transfers in the first period of life, by τ_2 per capita taxes/transfers in the second period of life and by b the stock of debt per capita, the benchmark model I develop here focus on the case where $\tau_1 > 0$, $\tau_2 > 0$ and $b = 0$. The model in Persson is characterised by $\tau_1 > 0$, $\tau_2 = 0$ and $b > 0$. The general case where all the three variables are different from 0 is introduced in Section 3.

$$
\begin{gathered}
\max \log c_{1t}^{A} + \frac{1}{1+\vartheta} \log c_{2t+1}^{A} \\
s.t. \\
c_{1t}^{A} = w_t^{A} - D - s_t^{A} \\
c_{2t+1}^{A} = s_t^{A} \cdot \left(1 + r_{t+1}^{A}\right) + D \cdot (1+n)
\end{gathered} \tag{1}
$$

where D denotes the constant lump-sum contribution paid into the pay-as-you-go system and $D \cdot (1+n)$ denotes the pension received under the scheme. The return on the contributions into the pay-as-you-go scheme equals the population growth rate which is the same in both countries by assumption. c_1^i and c_2^i represent respectively consumption when young and old; w^i is the wage earned supplying one unit of labour inelastically and r^i is the return on savings.

Consumers in Country B solve the following maximisation problem:

$$
\begin{gathered}
\max \log c_{1t}^{B} + \frac{1}{1+\vartheta} \log c_{2t+1}^{B} \\
s.t. \\
c_{1t}^{B} = w_t^{B} - f - s_t^{B} \\
c_{2t+1}^{B} = \left(s_t^{\prime B} + f\right) \cdot \left(1 + r_{t+1}^{B}\right)
\end{gathered} \tag{2}
$$

where f indicates the lump-sum contributions paid into the fully funded scheme, $f = D$ by assumption and $s_t^{\prime B}$ denotes private savings.

Starting from Country A, the solution to the maximisation problem (1) gives the optimal levels of consumption:

$$
\begin{gathered}
c_{1t}^{A*} = \frac{1+\vartheta}{2+\vartheta} \cdot \left(w_t^{A} - D \cdot \frac{r_{t+1}^{A} - n}{1+r_{t+1}^{A}}\right) \\
c_{2t+1}^{A*} = \frac{1+r}{2+\vartheta} \cdot \left(w_t^{A} - D \cdot \frac{r_{t+1}^{A} - n}{1+r_{t+1}^{A}}\right)
\end{gathered} \tag{3}
$$

The optimal saving behaviour is:

$$s_t^{A*} = \frac{1}{2+\vartheta} \cdot \left[w_t^A - D \cdot \left(1 + \frac{(1+\vartheta)\cdot(1+n)}{1+r_{t+1}^A} \right) \right] \tag{4}$$

The indirect utility function associated to this problem reads:

$$V_t^A = H + \frac{2+\vartheta}{1+\vartheta} \log \left(w_t^A - D \cdot \frac{r_{t+1}^A - n}{1+r_{t+1}^A} \right) + \frac{1}{1+\vartheta} \log \left(1 + r_{t+1}^A \right) \tag{5}$$

where H is a constant.

For Country B the solution to problem (2) delivers:

$$\begin{aligned} c_{1t}^{B*} &= \tfrac{1+\vartheta}{2+\vartheta} \cdot w_t^B \\ c_{2t+1}^{B*} &= \tfrac{1+r}{2+\vartheta} \cdot w_t^B \end{aligned} \tag{6}$$

$$s_t^{B*} = s_t^{\prime B} + f = \frac{1}{2+\vartheta} \cdot w_t^B \tag{7}$$

Provided that all consumers choose to save an amount at least equal to the compulsory contribution required by the system, a fully funded scheme simply replaces private savings by an equivalent amount of public savings, leaving total savings unchanged.

The indirect utility function is[2]:

$$V_t^B = H + \frac{2+\vartheta}{1+\vartheta} \log w_t^B + \frac{1}{1+\vartheta} \log \left(1 + r_{t+1}^B \right) \tag{8}$$

2.2.2 Production

Both in Country A and B production is carried out according to a well-behaved, linearly homogeneous, neoclassical production function $Y_t^i = F(L_t^i, K_t^i)$

[2]These solutions apply both to the closed and the open economy case, provided the interest rate and the wage rate on which they depend are correctly defined.

with $i = A, B$ holding for every t, where L_t^i and K_t^i are respectively the labour force and the capital carried over from period $t-1$ in Country i. In per-capita terms one has $y_t^i = f(k_t^i)$ with k_t^i denoting capital per worker in Country i.

Profit maximising behaviour determines the equilibrium factor prices:

$$\begin{aligned} r_{t+1}^i &= f^{'}(k_{t+1}^i) \\ w_t^i &= f(k_t^i) - k_t^i f^{'}(k_t^i) \end{aligned} \tag{9}$$

2.2.3 Equilibrium in the closed economy

Given k_t, w_t and r_t, consumption, savings and investment plans of consumers and firms must be consistent in the goods and capital markets. It is enough to look at one clearing condition. Focusing on capital markets, the savings of the young must finance the stock of capital for the next period. In per capita terms the equilibrium condition for both countries is:

$$(1+n) \cdot k_{t+1}^i = (1+n) \cdot a_{t+1}^i = s_t^{i*} \tag{10}$$

where a_{t+1}^i denotes private wealth per worker at the beginning of period $t+1$ in Country i.[3].

The equilibrium of the two closed economies is represented by the solutions to problem (1) and (2); by factor prices (9) and by the capital market equilibrium condition (10).

[3]As long as capital is the only investment opportunity, there is no distinction between wealth and capital and the capital market equilibrium condition can be written both in terms of k and a. When in the final section debt is introduced, $a \neq k$ and a is the relevant variable to define the capital market equilibrium condition.

When the economies are closed the steady state in Country A and B is described by the following equations:

$$\frac{c_2^i}{c_1^i} = \frac{1 + r^i}{1 + \vartheta} \tag{11}$$

$$w^A - D \cdot \frac{r^A - n}{1 + r^A} - c_1^A = \frac{c_2^A}{1 + r^A} \tag{12}$$

$$w^B - c_1^B = \frac{c_2^B}{1 + r^B} \tag{13}$$

$$w^i = f(k^i) - k^i f^{'}(k^i) \tag{14}$$

$$r^i = f^{'}(k^i) \tag{15}$$

$$w^A - D - c_1^{A*} = k^A \cdot (1 + n) \tag{16}$$

$$w^B - c_1^{B*} = k^B \cdot (1 + n) \tag{17}$$

I now compare the steady states in the two countries.

2.2.4 Comparing the two steady states

It is well known that the relationship between the interest rate and the population growth rate is crucial in the analysis of the effects of pay-as-you-go versus fully funded pension systems.

Individual consumption plans are unaffected by the pay-as-you-go pension scheme only if $r = n$. If $r > n$ ($r < n$), consumption in both periods is *lower* (*higher*) than it would be were the pay-as-you-go pension system absent. Savings are lower when a pay-as-you-go pension scheme is in place independently of r being higher or lower than n[4].

Assuming dynamic efficiency in Country A, i.e. $r^A > n$, and applying the above results in the model developed here, from the comparison between consumption levels both of the young and the old in the two countries, the conclusion is that $c_1^{A*} < c_1^{B*}$ and $c_2^{A*} < c_2^{B*}$. The opposite results holds if there is dynamic inefficiency, i.e. $r^A < n$. Total savings in Country A are lower than in Country B independently of r^A being higher or lower than n.

Concluding, the capital/labour ratio in Country A is lower, the interest rate higher and the wage rate lower than in Country B.

Following Diamond (1965), the equilibrium in the two economies can be represented in the (w, r) space. From the equilibrium conditions in production one obtains the factor price frontier $w_t = \phi(r_t)$. Given equation (9), the properties of the factor price frontier are:

$$\begin{aligned} \frac{dw}{dr} &= \phi'(r) = -k \\ \frac{d^2w}{dr^2} &= \phi''(r) = -\frac{1}{f''(k)} \end{aligned} \tag{18}$$

To determine the equilibrium in Country A and B, the factor price frontier is combined with the capital market equilibrium condition. Taking the equilibrium interest rate r_{t+1} and substituting for k_{t+1} from equation (16) for

[4]See Azariadis (1993), Theorem 18.1, page 292.

Country A and (17) for Country B, a relationship between r_{t+1} and w_t for each country can be obtained. I denote the capital market equilibrium condition for Country A by $r^A_{t+1} = \psi^A(w^A_t)$ and for Country B by $r^B_{t+1} = \psi^B(w^B_t)$. If Walrasian stability is assumed, $\frac{dr^i_{t+1}}{dw^i_t} = \psi'^i < 0$.

The functions ϕ and ψ^i can be plotted in the (w, r) space. In order to have a unique and stable equilibrium point in both economies, both ψ^A and ψ^B must cut ϕ from above. Given that savings in Country A are always lower than savings in Country B, ψ^A always lies above ψ^B. The distance between the two curves depends on r^A. Figure 1 represents the equilibria in the two closed economies.

If capital markets are closed, the following relationships between interest rates and between wages can be observed:

$$
\begin{aligned}
r^A &> r^B > n \\
r^A &> n > r^B \\
n &> r^A > r^B
\end{aligned}
\tag{19}
$$

$$
w^A < w^B \tag{20}
$$

Economy A always has a higher rate of return on capital and a lower wage than Economy B. These relationships follow from the different methods of finance of the pension schemes. Incidentally, dynamic efficiency is not guaranteed as one can see considering the relationship between interest rates and growth rates. From now onwards I concentrate only on the case of dynamic efficiency in both countries.

2.3 The world economy with perfect capital mobility

From time t onwards Countries A and B are linked together in a perfect world capital market. I analyse what the consequences of the integration are, taking into account that the only difference between the two countries is the organisation of the pension system.

Moving from a closed economy to an integrated world economy, the young still maximise (1) in Country A and (2) in Country B. The factor market equilibrium conditions are unchanged but, given perfect capital mobility there is a unique interest rate r_t^w and given the assumption on production, a unique capital/labour ratio $k_t^w$and wage rate w_t^w.

To complete the description of the single-period equilibrium in the world economy I need to consider the world capital market equilibrium condition and the balance of payments accounts. The general expression for the world capital market equilibrium condition is:

$$\left(w_t^A - D - c_{1t}^{A*}\right) \cdot N_t^A + \left(w_t^B - c_{1t}^{B*}\right) \cdot N_t^B = K_{t+1}^A + K_{t+1}^B \tag{21}$$

Using $\frac{N_t^A}{N_t^A + N_t^B} = \frac{1}{2}$, defining $k_{t+1}^w = \frac{K_{t+1}^A + K_{t+1}^B}{N_{t+1}^A + N_{t+1}^B}$, and recalling that $w^A = w^B$ equation (21) can be expressed in per capita terms:

$$w_t^w - \frac{D}{2} - \frac{1}{2} \cdot \left(c_{1t}^{A*} + c_{1t}^{B*}\right) = (1+n) \cdot k_{t+1}^w \tag{22}$$

As to the balance of payments accounts, to derive both the current account deficit and the trade balance deficit, foreign debt per worker is defined:

$$e_t^i = k_t^w - a_t^i \tag{23}$$

where $i = A, B$.

One can define the current account *deficit* q_t as the increase in foreign debt, that is:

$$q_t^i = \left[(1+n)k_{t+1}^w - k_t^w\right] - \left[(1+n)a_{t+1}^i - a_t^i\right] \tag{24}$$

The trade balance *deficit* b_t is the difference between the current account deficit and interest payments abroad.

$$b_t^i = q_t^i - r_t^w e_t^i = (1+n)e_{t+1}^i - (1+r_t^w)e_t^i \tag{25}$$

From the world market equilibrium condition one knows that world savings must be equal to world investment, therefore $q_t^A + q_t^B = 0$. Capital accounts in the two countries must sum to 0 in equilibrium.

The steady state equilibrium in the two country model with perfect capital mobility is described by equation (11) plus equations (12), (13), (14) and (15) where w^i and r^i are replaced by w^w and r^w.

The world capital market equilibrium condition is:

$$w^w - \frac{1}{2}D - \frac{1}{2}\left(c_1^{A*} + c_1^{B*}\right) = (1+n)\cdot k^w \tag{26}$$

Country A and B's net worths are:

$$a^A = \frac{w^w - D - c_1^{A*}}{1+n} \tag{27}$$

$$a^B = \frac{w^w - c_1^{B*}}{1+n} \tag{28}$$

Finally,

$$q^i = ne^i = n(k^w - a^i) \tag{29}$$

is Country i's steady state current account balance. The steady state trade balance is:

$$b^i = (n - r^w) \cdot e^i \tag{30}$$

In the closed economy equilibrium, wages and savings are higher and interest rates lower in Country B. If capital is perfectly mobile from time t onwards, investing in Country A where interest rates are higher is a profitable opportunity for residents of Country B. If residents in Country B invest abroad, interest rates in Country A decrease with respect to the closed economy level, while interest rates in Country B go up. Given that wealth is higher in Country B than in Country A in the world steady state, i.e. $a^B > a^A$[5], Country B exports capital abroad and runs a current account

[5]Consider the steady state equations (27) and (28): if $r^w = n$, then $c_1^{A*} = c_1^{B*}$ and $a^A < a^B$ by D. If r^w increases above n, c_1^{A*} decreases and becomes lower than c_1^{B*}. a^A is still lower than a^B but by an amount which is less than D. The higher the difference between the world interest rate and the population growth rate, the lower consumption in Country A is. The lower consumption in Country A cannot compensate for the effect that the contribution D has on total wealth in Country A. This can be shown calculating the partial derivative of total wealth in Country A with respect to the lump sum contribution D.

$$\frac{\partial a^A}{\partial D} = \frac{1}{1+n} \cdot \left(-1 + c_{11}^{*A} \cdot \frac{r^w - n}{1 + r^w}\right)$$

where c_{11}^{*A} represents the derivative of first period consumption in Country A with respect to its first argument , i.e. net lifetime income $w^w - D\frac{r^w - n}{1+r^w}$. If consumption is a normal good so that $0 < c_{11}^{*A} < 1$ and the population growth rate is positive as assumed surplus. Conversely, Country A imports capital from Country B and runs a current account deficit.

As far as the trade balance is concerned, if $r^w > n$ Country B runs a steady state trade balance deficit and Country A has a steady state trade balance surplus. If the opposite relation between the population growth rate and the world interest rate holds, then Country B runs a steady state trade balance surplus and Country A a steady state trade balance deficit as it is clear from (30).

The per capita capital stock in the world economy lies between the two autarchy levels, if the model is *stable*. This can be shown by contradiction[6].

Given that the per capita capital stock in the two country economy is higher than the closed economy level for the country having a pay-as-you-go system and lower for the country running a fully funded scheme, wages in the first country are higher and in the second country lower in the new steady state. As regards interest rates, integration triggers capital movements from Country B where the interest rate is lower to Country A where the interest rate is higher. The common world interest rate lies between the two autarchy levels.

Summarising, the relationships between r, k, w under autarchy and under perfect capital mobility can be stated as follows:

$$\begin{aligned} k^A &< k^w < k^B \\ r^B &< r^w < r^A \\ w^A &< w^w < w^B \end{aligned} \tag{31}$$

throughout the paper, $\frac{\partial a^A}{\partial D} < 0$.

[6]See Part A of the Appendix for a formal proof.

Wages decrease in the economy with the fully funded scheme and interest rates increase because capital has a more profitable investment opportunity in open economy. The opposite results on wages and interest rates hold for the country with the pay-as-you-go scheme.

Using ϕ and ψ^i defined in the earlier section, in the open economy there is a unique (world) capital market equilibrium condition ψ^w (Figure 2). The new ψ^w function lies between ψ^A and ψ^B. w and r move along the factor price frontier until the new equilibrium (w^w, r^w) is reached.

As observed in the introduction, once the two countries are integrated, the simple fact that they differ in the organisation of the pension system affects the equilibrium. The interest rate in the country with a fully funded scheme goes up and the wage rate down *but that happens independently of the pay-as-you-go scheme in the other country being balanced or not*. Given the impact that the pay-as-you-go structure of the pension scheme has on savings, pension policies in Country A affect Country B through the supply side of the capital market.

2.4 Welfare analysis

This section is devoted to the analysis of who gains and who loses out of the integration between the two countries with the two different pension systems. I first consider the generations alive when the economy is opened up and then turn to the future generations.

2.4.1 The old at time t

Integration comes unexpectedly at the beginning of time t. The young at time t in both countries choose their consumption and saving levels knowing that they are no longer in a closed economy. The old at time t decide how much to consume and save at time $t-1$ before the integration takes place and not knowing or expecting that at time t Country A and B will be integrated. They cannot reoptimise their objective functions and therefore change their consumption and saving decisions. They are nonetheless affected by the opening of the economy because k_t, the stock of capital existing at the beginning of period t and resulting from their savings s_{t-1}, can be shifted, given the perfect capital mobility assumption. The interest rate r_t faced by those who are old at time t in both countries is different from the one they expected and based their saving/consumption decisions on. If k_t can move, the integration at time t implies the *equalisation* of interest rates, per capita capital and wages *at time* t[7].

Looking at the old in Country A and B, they are affected by the integration in a world capital market only via the *change in the interest rate.* Their *wages* are *unaffected.* The return the old in Country A earn on their savings decided at time $t-1$ is less than expected as $r^w < r^A$. The opening of the economy causes them a windfall loss. The old in Country B receive a higher return on their savings as $r^w < r^B$. They experience a welfare gain following the integration.

The old in the country with a pay-as-you-go pension system are worse off

[7] For an alternative representation of the timing of the opening of the economies, see Buiter (1981).

in an integrated economy. The old in the country with a fully funded pension scheme are better off. A high interest rate is therefore not necessarily a bad thing, as generally thought. If those whose indirect utility function depends only on the interest rate are considered and if they are all lenders, a higher interest rate implies higher welfare.

2.4.2 The young at time t and future generations

In Country A wages go up and interest rates down with respect to the closed economy level. Wages at time t depend on k_t^w, which is higher than k_t^A, the stock of capital existing in Country A had the economy been still closed. Interest rates at time $t+1$ are lower than their autarchy level because capital flows from Country B to Country A and because the increase in wage determines an expansion in savings.

In Country B wages go down with respect to the closed economy level because k_t^w is lower than k_t^B, the stock of capital in Country B under autarchy. Interest rates increase both because residents invest abroad (capital outflows) and because savings are reduced, given the decrease in wages.

To analyse how these changes in factor prices affect the utility of the young at time t, I transform the indirect utility functions (5) and (8) for the young in both countries as follows:

$$V^A = \left(w_t^A - D \cdot \frac{r_{t+1}^A - n}{1 + r_{t+1}^A} \right)^{\frac{2+\vartheta}{1+\vartheta}} \cdot \left(1 + r_{t+1}^A\right)^{\frac{1}{1+\vartheta}} \tag{32}$$

$$V^B = {w_t^B}^{\frac{2+\vartheta}{1+\vartheta}} \cdot \left(1 + r_{t+1}^B\right)^{\frac{1}{1+\vartheta}} \tag{33}$$

To assess how the opening of the economy affects the welfare of the young in both countries, I compare the slopes of the indirect utility functions with the slope of the factor price frontier and take into account the movements along the factor price frontier caused by perfect capital mobility and established in the last section.

For the young at time t in Country A:

$$\left.\frac{dw}{dr}\right|_{V^A} = -\frac{\frac{\partial V^A}{\partial r}}{\frac{\partial V^A}{\partial w}} = -\frac{1}{1+r^A_{t+1}} \cdot s^{*A}_t = -\frac{1}{1+n} \cdot \frac{k^A_{t+1}}{1+f'(k^A_{t+1})} \tag{34}$$

and for Country B:

$$\left.\frac{dw}{dr}\right|_{V^B} = -\frac{\frac{\partial V^B}{\partial r}}{\frac{\partial V^B}{\partial w}} = -\frac{1}{1+r^B_{t+1}} \cdot s^{*B}_t = -\frac{1}{1+n} \cdot \frac{k^B_{t+1}}{1+f'(k^B_{t+1})} \tag{35}$$

If $r^A = n$, from equation (32) it is clear that $V^A = V^B$, i.e. the two indirect utility functions coincide. This happens because current and future consumption which enter the indirect utility function are the same in both countries. Given that $s^{*A}_t < s^{*B}_t$ always and therefore $k^A_{t+1} < k^B_{t+1}$, one concludes that:

$$\left| \left.\frac{dw}{dr}\right|_{V^B} \right| > \left| \left.\frac{dw}{dr}\right|_{V^A} \right| \tag{36}$$

This is represented in Figure 3. When $r^A = n$, the two indirect utility functions intersect. For a given interest rate level, the wage in Country A needs to be higher than the wage in Country B in order to guarantee the consumers the same utility level.

The slope of the factor price frontier ϕ is[8]:

[8] See equation (18).

$$\left.\frac{dw}{dr}\right|_{FPF} = -k_t \tag{37}$$

Equation (37) can be compared to equations (35) and (34), assuming steady state values. I find:

$$\begin{aligned} \left|\left.\frac{dw}{dr}\right|_{FPF}\right| &> \left|\left.\frac{dw}{dr}\right|_{V^B}\right| \\ \left|\left.\frac{dw}{dr}\right|_{FPF}\right| &> \left|\left.\frac{dw}{dr}\right|_{V^A}\right| \end{aligned} \tag{38}$$

I represent the relative slopes of the relevant curves in Figure 4. If $r^A > r^B > n$, as assumed, movements along the factor price frontier towards higher wages and lower interest rates represent welfare improvements (higher indifference curves). Movements along the factor price frontier towards lower wages and higher interest rates indicate a decrease in welfare (lower indifference curves).

According to this analysis, *the young in Country A are better off in the open economy equilibrium; the young in Country B are worse off.* In Country A wages are higher and interest rates lower in the world economy and therefore the young can reach higher indifference curves thanks to the opening of the economy. In Country B wages are lower and interest rates higher in the world economy. The young are on lower indifference curves owing to the opening of the economy.

I point out that this result holds not only for the young at time t but for all the future generations. Given my assumptions on the timing of the integration, its effects fully operate on the generation born at time t and therefore, though the long run equilibrium is not reached immediately at time t, it is enough to focus on this generation to analyse the welfare effects of the integration on all future generations.

The reduction in welfare for generation t and all future generations in Country B and the increase in welfare for generation t and all future generations in Country A can be explained as follows.

If $r^A > r^w > r^B > n$, as it is the case in this paper, then $k^A < k^w < k^B < k^{GR}$ where k^{GR} identifies the golden rule level of capital. Both the autarchy equilibria and the open economy one are characterised by underaccumulation of capital.

In Country A the stationary utility level is higher under openness than under autarchy and the total positive effect on welfare is determined by two partial effects having the same positive sign: first, Country A imports capital from abroad and the country runs a steady state current account deficit. Given $r^w < r^A$, Country A is a net borrower at a lower rate. Output net of what they have to repay to the lenders is higher than output under autarchy and therefore the total amount of resources available for consumption increases. Second, the inefficiency of the pay-as-you-go system decreases: $(r^w - n) < \left(r^A - n\right)$.

Country B exports capital abroad. The capital-labour ratio moves further away from the golden rule level. This effect tends to decrease welfare. At the same time, the country has a steady state current account surplus: it is a net lender at a rate higher than the closed economy one. This effect tends to increase welfare. Under the assumptions made in the paper, the first negative effect more than counterbalances the second positive effect.

2.4.3 Total welfare effects

The assessment of the total welfare effects of the integration is non trivial.

If transfers are excluded, the investigation above clarifies that opening the capital markets has divergent welfare effects across the young and the old *alive at time* t and across countries: in Country A the young and future generations are positively affected by capital market mobility whilst the old are negatively hit. In Country B the young and future generations are negatively affected by the opening of the economy whilst the old are positively influenced.

The assessment of the total welfare impact of the integration between the two countries with the two different pension schemes requires the introduction of a social welfare function: depending upon the weight given to the welfare of present and future generations, different conclusions on the welfare impact of the opening of the economy are reached. This result goes against a common view in the policy debate according to which the country whose pension scheme is fully funded loses out of the integration. This view disregards the distributional impact of the integration within each country which, on the contrary, needs to be taken into account.

3 Extensions of the benchmark model

In this section I extend the benchmark model in two directions: first, I address the issue of the *maturity* of the pay-as-you-go pension scheme, maintaining the assumption that it is balanced in each period. Then I focus on its *funding* status to address explicitly the unfunded pension liabilities issue and to complement the analysis of the supply side of the capital market performed in the previous section with that of the demand.

3.1 Introducing maturity in the two period OLG model

Social security expenditure and revenues are connected via the social security budget equation. A pension system is *balanced* when contributions collected at time t are enough to finance pensions at time t. A pension system is *mature*, i.e. in steady state, when its revenues and expenditure do not change over time except for the constant rate of population growth.

The observed frequency of adjustment to social security schemes almost everywhere in the world can be interpreted as a sign that the systems are not mature yet and the different histories characterising pension schemes across various countries may suggest that each of them is marked by a different degree of maturity. The fact that a pension system has not reached yet the stage of full maturity can have implications for international capital markets and capital flows at the time of the integration. This is the issue I focus my attention on in the next sections.

I analyse the maturing of the pay-as-you-go system focusing on two alternative frameworks: the maturing can either take place via changes on the benefit side or via changes in the coverage rate characterising the scheme. I discuss these in turn, focusing on how they affect the social security budget constraint.

3.1.1 Increasing benefits

Maturing can be represented by an increase in the benefits successive generations of retirees are entitled to.

The longer the system has been in place, the longer the period participants spend paying contributions before they reach the retirement age. In a defined contribution system, longer contribution periods imply higher pension rights. In a defined benefit scheme, the longer the system has been in place, the more likely it is that the participants satisfy all the requirements set to get full benefits. If one observes how expenditure on social security has evolved over time, one notices that it has been constantly increasing not only because the number of retirees and the time spent in retirement have been increasing, but also because the amount of each pension paid has been going up.

I denote by $\overline{p}$ the full right pension. A retiree is entitled to the pension $\overline{p}$ only when he fulfils all the requirements set by the law: these in general differ from country to country and therefore cannot be specified in detail. According to the arguments put forward above, I assume that a retiree is more likely to satisfy these requirements, however specified, at late rather than early stages of the system's life. Analytically, this assumption can be expressed as follows: at t all the working population contributes to the system, so that the population coverage is complete, but all the retired population at time t is entitled to only a fraction $\gamma_t < 1$ of the full right pension $\overline{p}$. γ_t denotes the degree of maturity of the pay-as-you-go system: it is non decreasing in time[9] and when the system is mature $\gamma_t = 1$[10].

A change in maturity is represented by marginal changes in γ_t. To match the change in γ_t, the government in Country A can either increase the equilibrium contribution rate or start borrowing. Here, I discuss the first policy option and leave the discussion on alternative financing means to the next section.

At time t the social security budget constraint is:

$$\tau_t N_t = \gamma_t \overline{p} N_{t-1}$$

which is equivalent to:

$$\tau_t = \gamma_t \overline{p} \cdot \frac{1}{1+n} \tag{39}$$

Equation (39) says that, when γ_t increases and a new level of benefits is

[9] A non decreasing γ is consistent with the assumption that a retiree is more likely to be entitled to a full pension at late stages of the pension scheme. The assumption is reasonable if the requirements to get full benefits are fixed when the system is introduced and are not subject to any change during the system's life. One could take an alternative view and consider that, as the system grows older, some changes in the original set of requirements to get full benefits are necessary. Increasing retirement age is an example. In this case, rules may become tighter and γ may go down. This would imply that γ is non monotonic and one can observe decreases in the single amount of pensions paid as time goes by.

[10] γ_t can also be interpreted as the fraction of period t the retiree is alive. An increase in γ_t is therefore to be interpreted as an improvement in life expectancy. In this sense, the model may also be suitable to discuss the effects of population ageing for a given participation period in the labour force, once uncertainty is introduced and people form expectations on how long they will live after retirement.

set, the lump-sum contributions adjust to meet them, so that the equilibrium between revenues and expenditure is maintained at each point in time.

At time t, the agents'maximisation problem is a revised version of (1):

$$\begin{gathered}\max \log c_{1t}^A + \tfrac{1}{1+\vartheta} \log c_{2t+1}^A \\ s.t. \\ c_{1t}^A = w_t^A - \tau_t - s_t^A \\ c_{2t+1}^A = s_t^A \cdot \left(1 + r_{t+1}^A\right) + \gamma_{t+1}\overline{p}\end{gathered} \tag{40}$$

Note that, while contributions paid at time t are proportional to maturity γ_t, pension benefits received at time $t+1$ are proportional to maturity γ_{t+1}. Therefore, though the system is balanced in each period, each individual contributes by less than he gets in terms of benefits and the rate of return on contributions does not depend only on the constant rate of population growth but it is increasing in $\frac{\gamma_{t+i}}{\gamma_{t+i-1}}$[11]. Certainty is assumed: agents perfectly know the degree of maturity of the system when formulating their maximising programs.

Problem (40) can be solved for the optimal levels of consumption and savings. Focusing on the latter and substituting for the social security budget constraint, optimal savings at time t are:

$$s_t^{*A} = \frac{1}{2+\vartheta} \cdot w_t^A - \frac{1}{2+\vartheta} \cdot \frac{\gamma_t \overline{p}}{1+n} \cdot \left[1 + \frac{(1+\vartheta)\cdot(1+n)}{1+r_{t+1}^A} \cdot \frac{\gamma_{t+1}}{\gamma_t}\right] \tag{41}$$

The first term in the above expression is the level of savings that would

[11]Notice that in this case the relationship between r and n no longer identifies the dynamic efficiency/inefficiency region. In order to have dynamic efficiency during the transition $\frac{\gamma_{t+i}}{\gamma_{t+i-1}} \cdot (1+n) < 1+r$ must hold.

prevail under a fully funded system. Being the pension system in Country A unfunded, the second term identifies the reduction in savings induced by the pay-as-you-go system.

When $\gamma_t = \gamma_{t+1}$, the social security constraint (39) holds and $\tau_t = D$, the expression for savings in (41) coincides with (4), except for the γ_t in front of the square brackets.

If $\gamma_t = \gamma_{t+1} < 1$, taxes are lower and savings are higher than in the case where the pay-as-you-go system is mature. The lower γ_t, the higher savings are. If $\gamma_t = \gamma_{t+1} = 1$, the level of savings is the same, as one would expect given that in the formulation used the equality $\gamma_t = \gamma_{t+1} = 1$ implies the maturity of the scheme: in this case, the difference between the level of savings in a country with a fully funded system and in a country with a pay-as-you-go scheme only depends on the lump-sum contribution and on the difference between r and n.

When $\gamma_t < \gamma_{t+1} < 1$, so that the system is maturing, one finds that:

$$\frac{\partial s_t^{*A}}{\partial \frac{\gamma_{t+1}}{\gamma_t}} < 0 \tag{42}$$

The result is that the maturing of the pension system, as described in the model, has a negative impact on savings. If benefits are changing period after period and the contributions adjust to maintain the system in financial equilibrium, savings decrease during the build-up of the system, they reach their minimum level when the system is mature and then they are constant.

Given that savings are lower when the system is mature, the effects of integration on factor prices are larger if the integration takes place during the maturity period rather than during the transition[12].

The *positive* effect on savings of the smaller scale of the system more than offsets the *negative* effect on savings of the higher rate of return gained on contributions during the maturing of the scheme. This finding implies that the impact on factor prices and welfare of the integration between a country running a fully funded system and a country having a pay-as-you-go one is larger if it takes place at a *late* stage in the life of the pay-as-you-go system.

3.1.2 Increasing coverage

An increase in coverage captures another aspect of the maturing of a pay-as-you-go system. The purpose of this extension of the model is to investigate whether this alternative representation of the maturing of the pension system has different implications on total savings and therefore factor prices at the time of the integration.

At time t, a share γ_t of the total population is covered by the pension scheme and the remaining share $1-\gamma_t$ is not covered by it. As in the previous subsection, maturing is represented via marginal increases in γ_t which are expected and known by the agents[13]. Contributions to the system

[12]This is clear if we compare (41) with its steady state version:

$$s_t^{*A} = \frac{1}{2+\vartheta} \cdot w_t^A - \frac{1}{2+\vartheta} \cdot \frac{\overline{p}}{1+n} \cdot \left[1 + \frac{(1+\vartheta)\cdot(1+n)}{1+r_{t+1}^A}\right]$$

Given that γ_t and γ_{t+1} are both less than 1 during the transition, the reduction in savings associated with the pay-as-you-go scheme is smaller when the system is not mature.

[13]If compared with the increasing benefit framework, notice that here the assumption that γ_t is non decreasing in time is the only plausible one, given that once a group enters the scheme, it cannot leave it.

are assumed to be fixed and an increase in coverage translates into higher benefits[14]. The social security budget constraint at time t is:

$$\overline{\tau}\gamma_t N_t = p_t \gamma_{t-1} N_{t-1}$$

or, given that pensions are in this case the dependent variable:

$$p_t = \frac{\gamma_t}{\gamma_{t-1}} \cdot (1+n) \cdot \overline{\tau} \tag{43}$$

Recalling that there are two groups in this economy, total savings are given by the sum of the savings of the two groups, i.e.,

$$s_t^A = \gamma_t \cdot s_t^{A\gamma} + (1-\gamma_t) \cdot s_t^{A1-\gamma} \tag{44}$$

Maintaining the assumption of a logarithmic utility function, savings for the two groups are:

$$s_t^{A\gamma} = \frac{1}{2+\vartheta} \cdot \left(w_t^A - \overline{\tau}\right) - \frac{1+\vartheta}{2+\vartheta} \cdot \frac{(1+n)\cdot\overline{\tau}}{1+r_{t+1}^A} \cdot \frac{\gamma_{t+1}}{\gamma_t} \tag{45}$$

and:

$$s_t^{A1-\gamma} = \frac{1}{2+\vartheta} \cdot w_t^A \tag{46}$$

Combining (45) and (46), total savings are:

[14]Given this assumption, increasing coverage implies increasing benefits, exactly the case discussed in the previous subsection. The increase in benefits is here determined by the higher rate of return on given contributions guaranteed by the inclusion in the scheme of a larger group of people. In the previous extension of the model higher benefits are paid by higher per capita lump-sum contributions.

$$s_t^A = \frac{1}{2+\vartheta} \cdot w_t^A - \frac{1}{2+\vartheta} \cdot \overline{\tau}\gamma_t \cdot \left(1 + \frac{(1+\vartheta)\cdot(1+n)}{1+r_{t+1}^A} \cdot \frac{\gamma_{t+1}}{\gamma_t}\right) \qquad (47)$$

If $\gamma_t = \gamma_{t+1} < 1$, savings are higher than the level prevailing under maturity because, though the covered group saves as much as it would save under maturity, there is still a group which is not covered by the pension scheme and its savings are higher.

If the system is mature, $\gamma_t = \gamma_{t+1} = 1$ and (47) reduces to the standard expression for savings under a pay-as-you-go scheme.

When $\gamma_t < \gamma_{t+1} < 1$, (42) holds and an increase in coverage reduces total savings at time t because the fraction of the population already covered save less given the increase in the rate of return of their fixed contributions. It also decreases savings at time $t+1$ because it reduces the size of the high saving group. This effect is not observed in the increasing benefits framework.

As shown in the previous subsection, the *positive* effect on savings due to the presence of an uncovered group more than counterbalances the *negative* impact on savings associated with the higher rate of return the covered group benefits from during the transition. The implication is that savings are lower when the system is mature, i.e. when all the population groups are covered, than during the maturing of the system.

3.1.3 Maturity as increasing benefits or coverage: a comparison

The result that savings are lower during the steady state rather than during the maturing of the scheme does not depend on the way maturing is represented. Both changing the number of people involved in the system and the scope of the matured rights one finds that a mature system is associated with a lower level of savings. The reason why one reaches in both cases the same conclusion, though starting from two logically distinct frameworks, is that in both cases one observes marginal increases in pension outlays matched by marginal increases in contributions. In the first framework, *individual* pensions go up because the system is maturing and therefore *individual* contributions increase to keep the system balanced. In the second framework, *total* contributions go up because there are more contributors to the system owing to the maturing of the scheme and therefore *total* pensions increase to keep the system balanced (instead of in surplus). In both cases, the effects on factor prices and welfare are therefore larger when the integration takes place during the maturity of the pay-as-you-go scheme.

3.2 Underfunding

Part of the previous section was devoted to the analysis of the maturing of the pay-as-you-go system in terms of marginal increases in pension outlays, matched by increases in lump-sum taxes, until maturity is reached. The analysis was based on the assumption that the government is willing to increase taxes to finance social security. This is not necessarily the case. The current level of contribution rates is already perceived as being high. This happens for at least two reasons: on one side, social security seems to offer a bad deal, because the implicit rate of return on contribution is not competitive if compared with market interest rates. People are not keen on giving up larger shares of their salaries, knowing they can be invested at higher returns. On the other side, the distortionary effects of increasing taxation have to be taken into account[15]. For these reasons, increasing contributions currently ranks quite low in the list of reform proposals to social security.

In this section I remove the assumption that the pay-as-you-go scheme is balanced and I investigate a situation where the pay-as-you-go system is *underfunded*: contributions collected at, say, time t are not enough to pay pensions at time t and therefore debt financing is involved.

Underfunding and debt financing are introduced in the model in two different ways to capture different changes pension schemes may be subject to: I first consider a case where the pension path is given and constant and I analyse the effects of a drop in contributions with respect to the given level of benefits to be financed. For a given level of benefits, a drop in payroll taxes can be interpreted as the inability or the unwillingness of the government to maintain them at their previous level. One can also think of a drop in payroll taxes as the sign that the contribution rate chosen to finance the system is not the right one and other sources of revenues are necessary to keep the scheme balanced. If the system is *underfunded*, debt financing is required.

As an alternative framework to study the effects of underfunding, I then analyse the impact of a one-shot, unexpected increase in benefits, combined with constant lump-sum contributions. The increase in benefits can be interpreted as a sign of the maturing of the system, as discussed in the previous section, except that in this case the change in benefits is *unexpected* and the government does not want to change contributions in the first place and

[15] The model developed here is based on the assumption that contributions are lump sum and the labour supply is inelastic, therefore the distortionary effect of social security financing cannot be tackled.

therefore debt covers the deficit in the system.

In both the cases just sketched, contributions are then adjusted to keep debt per capita constant.

In the analysis I pay attention to two specific issues: I first focus on the *timing* of underfunding and subsequent adjustment (closed vs open economy) and on how it affects the world equilibrium. The analysis captures a relevant policy issue: namely, it throws some light on the costs of taking part into the adjustment process of an unbalanced pay-as-you-go scheme. I then look at the *sources* of underfunding and compare the effects on the world equilibrium generated by a reduction in payroll taxes and by an increase in benefits to assess whether the sources of the imbalance in the pay-as-you-go scheme matter or not.

3.2.1 Underfunding caused by a drop in contributions

For a given and constant benefits' path, I examine the impact of a decrease in contributions matched by an increase in debt and followed by an adjustment in contributions to keep the debt per capita constant. I distinguish two cases on the basis of the *timing* of the drop in payroll taxes. In one case (Case α) the drop took place in the past, well before the integration period. The country adjusts to the shock when the economy is still closed. In the other case (Case β), it takes place right before the integration in a world capital market and the adjustment process involves both countries.

The analysis aims at studying how the use of debt as a financing means of the pension scheme affects the capital stock of Country A and whether the timing of the use of debt is of any relevance.

Case α I assume that at time $t-2$ there is a drop in collected taxes so that lump-sum contributions are not enough to pay pension benefits at time $t-2$. The social security budget constraint in per capita terms is[16]:

$$\tau_{t-2}^{\alpha} = \frac{\overline{p}}{1+n} - b_{t-1}^{\alpha} \cdot (1+n) \tag{48}$$

where b_{t-1}^{α} is the level of debt per capita in Case α at the beginning of period $t-1$. The government starts borrowing in order to cover the social security deficit. As anticipated, I assume that from time $t-1$ onwards lump-sum contributions are set to keep the debt per capita constant, i.e. $b_{t-1}^{\alpha} = b_t^{\alpha} = \overline{b}^{\alpha}$. At time $t-1$ and for all the following periods the social security budget constraint is:

$$\tau_{t-1}^{\alpha} = \frac{\overline{p}}{1+n} + b_{t-1}^{\alpha} \cdot (r_{t-1} - n) \tag{49}$$

Case β Differently from the previous case, at time $t-2$ the lump-sum contributions are still enough to cover the pension benefits. The system is

[16] In absolute terms, if the system is balanced, the social security budget constraint at time $t-2$ is $T_{t-2} = P_{t-2}$. If there is a deficit which is covered by debt the next period, the constraint becomes $P_{t-2} - T_{t-2} = B_{t-1}$.

Using lower case letters for variables expressed in per capita terms one has:

$$p_{t-2} \cdot N_{t-3} - \tau_{t-2} \cdot N_{t-2} = b_{t-1} \cdot N_{t-1}$$

Dividing by N_{t-2} one finds:

$$\frac{p_{t-2}}{1+n} - \tau_{t-2} = b_{t-1} \cdot (1+n)$$

Rearranging terms, introducing the superscript α to denote the case under examination and recalling that the pension path is constant one has the constraint (48).

hit by a drop in payroll taxes at time $t-1$. In this case, the relevant social security budget constraints in per capita terms are the following:

$$\tau_{t-2}^{\beta} = \frac{\overline{p}}{1+n} \tag{50}$$

and $b_{t-1}^{\beta} = 0$.

$$\tau_{t-1}^{\beta} = \frac{\overline{p}}{1+n} - b_t^{\beta} \cdot (1+n) \tag{51}$$

at the time of the payroll tax drop. Finally, assuming $b_t^{\beta} = b_{t+1}^{\beta} = \overline{b}^{\beta}$, for time t and all the following periods the constraint is:

$$\tau_t^{\beta} = \frac{\overline{p}}{1+n} + b_t^{\beta} \cdot (r_t - n) \tag{52}$$

The effects of the drop in contributions I am interested in studying how the shock and its timing affect the stock of capital of the country with a pay-as-you-go pension system when the economy is opened. To this end, it is enough to assume that the drop in contributions in Case α takes place one period earlier than in Case β, provided that in the latter case the system adjusts to the change in taxes *when* the integration takes place, whilst in the former case it adjusts *before* the opening of the economy. I also constrain the two payroll tax cuts to be equal, i.e. $\tau_{t-2}^{\alpha} = \tau_{t-1}^{\beta}$, so that $b_{t-1}^{\alpha} = b_t^{\beta}$.

Taking into account that debt per capita is stabilised one period after the drop in contributions, the last equality can be extended to the following ones:

$$b_{t-1}^{\alpha} = b_t^{\alpha} = b_t^{\beta} = b_{t+1}^{\beta} = \overline{b}^{i} \tag{53}$$

Given that the level of benefits is the same in both cases, the assumption of equal payroll tax cuts ensures that the size of the scheme is the same both before and after the cut in contributions: differences in the capital stock at time t when the economy is opened can only arise from the adjustment taking place at an earlier period, i.e. from the system being financed partly by debt for more periods.

I start analysing the effect of the cut in contributions in Case α: if τ_{t-2}^{α} decreases, the net wage $w_{t-2} - \tau_{t-2}^{\alpha}$ goes up. The gross wage w_{t-2} and the interest rate r_{t-2} are unaffected. The capital market equilibrium condition changes: on the demand side, savings have now to finance both the capital stock and the debt. The new equilibrium condition is:

$$s_{t-2}^{\alpha} = (1+n) \cdot \left(k_{t-1}^{\alpha} + b_{t-1}^{\alpha}\right) \tag{54}$$

Though higher net wages have a positive impact on savings, the increase in public debt determines an excess demand in the capital market and therefore a reduction in the stock of per capita capital k_{t-1}^{α}[17]. Given that the cuts in contributions are the same, given equation (53) and assuming that the economy is still closed, the changes now described will be observed in Case β at time $t-1$. The relevant conclusion is that:

$$k_{t-1}^{\alpha} = k_{t}^{\beta} \tag{55}$$

It now remains to be established how k_{t-1}^{α} and k_{t}^{α} compare[18], in order to draw some conclusions on how the change in financing caused by the drop

[17] This is shown in Part B of the Appendix.

[18] The relationship between k_t^{α} and k_t^{β} can be inferred from this one.

in contributions influences the effect of the integration on factor prices and welfare.

The decrease in k^{α}_{t-1} causes a fall in w^{α}_{t-1} and a rise in r^{α}_{t-1}. In order to keep the level of debt per capita constant, lump-sum contributions must increase: both the gross and the net wage at time $t-1$ are lower than they were at time $t-2$. It can be shown[19] that the fall in the gross (and the net) wage determines a further reduction in the stock of capital: agents at time $t-1$ are poorer than they were at time $t-2$ but they have to absorb the same level of debt per capita. Though the rise in the interest rate has a positive effect on savings, this effect is more than counterbalanced by the reduction in gross and net wages. The conclusion is that:

$$k^{\beta}_{t} = k^{\alpha}_{t-1} > k^{\alpha}_{t} \tag{56}$$

3.2.2 Underfunding caused by an increase in benefits

In this section I analyse the effects of a one shot increase in benefits. Once benefits go up, they are kept constant at the higher level. Contributions are not immediately adjusted to meet the higher benefits and this generates a deficit which is covered issuing debt. In the following period contributions increase to maintain the stock of debt per capita constant. I assume that the increase in benefits is *unexpected*, so that the savings of the old generation when the shock takes place are unaffected, while their second period consumption increases. While in the framework presented in the previous section the old generation at the time of the shock was not affected by the

[19] See Part B of the Appendix for a formal proof.

drop in contributions, in this case its welfare goes up. As I did before, I distinguish two cases: in Case α the increase in benefits takes place at time $t-2$ and the adjustment is over before the two countries integrate. In Case β the increase in benefits happens at time $t-1$ and the adjustment and the integration take place at the same time.

Case α At time $t-2$ pensions paid to the old generation go up. Contributions are fixed. The social security budget constraint is:

$$\frac{p_{t-2}^{\alpha}}{1+n} = \overline{\tau}^{\alpha} + b_{t-1}^{\alpha} \cdot (1+n) \tag{57}$$

At time $t-1$ we have:

$$\tau_{t-1}^{\alpha} = \frac{\overline{p}}{1+n} + b_{t-1}^{\alpha} \cdot (r_{t-1} - n) \tag{58}$$

Once pensions increase at time $t-2$, they are constant at the higher level for all the following periods.

Case β At time $t-2$ contributions are still enough to cover the benefits paid:

$$\tau_{t-2}^{\beta} = \frac{\overline{p}}{1+n} \tag{59}$$

and $b_{t-1}^{\beta} = 0$.

At time $t-1$, pension benefits go up:

$$\frac{p_{t-1}^{\beta}}{1+n} = \overline{\tau}^{\beta} + b_{t}^{\beta} \cdot (1+n) \tag{60}$$

Assuming $b_t^\beta = b_{t+1}^\beta = \overline{b}^\beta$, for time t and all the following periods the constraint is:

$$\tau_t^\beta = \frac{\overline{p}}{1+n} + b_t^\beta \cdot (r_t - n) \tag{61}$$

The effects of an increase in benefits To evaluate the effects of underfunding caused by an increase in benefits and to assess the relevance of the timing of the shock, I follow the same procedure introduced to discuss the effects of underfunding generated by a drop in contributions. I assume that the increase in pension benefits is the same in both cases, i.e. $p_{t-2}^\alpha = p_{t-1}^\beta$, to ensure that the size of the pension scheme is the same not only before but also after the shock has hit the system. It follows that also in this case $b_{t-1}^\alpha = b_t^\beta$ and given the assumption that the debt per capita must be kept constant after the first period increase, the equality (53) holds.

The increase in pensions at time $t-2$ benefits the old generation that can enjoy a higher level of consumption. Given that the change in benefits is unexpected, the capital stock at time $t-2$ is given and unaffected by the rise in pensions. Wages and taxes for the young at time $t-2$ do not change but their savings decrease because they take into account that the pension they will receive is higher than the one their parents were expecting. Moreover, their lower savings must satisfy a higher demand of capital, given that the government has to sell its debt. The per capita stock of capital goes down.

I now have to analyse what happens to k_t^α and what the relationship between k_{t-1}^α and k_t^α is. Given that after the increase in benefits, contributions are adjusted to keep debt per capita constant, I have here the same policy experiment analysed in Section 3.2.1 and the conclusions stated there apply. Namely, the result in (56) is confirmed[20].

3.2.3 Underfunding and integration

At the time of the opening of the economy, the country with a pay-as-you-go scheme has a higher stock of capital if underfunding and the consequent change in financing take place right before the integration rather than well before it. This result holds whatever the source of underfunding: in both the cases analysed the capital/labour ratio is higher at the time of the shock than after the adjustment. The implication is that, considering the effects on factor prices at the time of the integration, one expects a larger impact on them if the system adjusted in the past when the economy was still closed. The shorter the period since the system was discovered to be underfunded, the higher the accumulated per capita capital stock.

This result, though obtained in a very simplified model, can throw some light on the debate on the riskiness of the pay-as-you-go systems of some continental European countries. The major worries stem from their future liabilities or their implicit debt. One way to interpret the future liabilities issue is to relate it to the analysis pursued in these sections and consider that the benefits promised cannot be sustained at the current contribution levels.

The model says that the impact on the capital stock and on factor prices of the integration between a country with a fully funded scheme and a country with a pay-as-you-go one is actually smaller when the pay-as-you-go system is still undergoing the adjustment process than if it has already concluded

[20]For a formal proof of the results stated in this section, see Part C of the Appendix.

it. This implies that the country running a fully funded pension scheme is subject to a *smaller* change in factor prices *if it does take part into the adjustment process* of the pay-as-you-go scheme rather than if it does not.

Recalling the welfare effects of the integration, the old at the time of the integration in the country with the fully funded scheme, if the latter participates into the adjustment process, have a return on their contribution which is lower than the one they get if the other country has already completed the adjustment process. The young do not suffer from it in terms of lower wages as much as they do if the integration takes place after the adjustment is over. Depending upon the weight that the young and the old and future generations have in the welfare function of Country B, the model points out that, if the integration takes place, *the opportunity that Country A has to export its system imbalance is not necessarily welfare reducing for Country B.*

Considering the country with the pay-as-you-go pension scheme, the young benefits from the opening of the economy taking place at the same time as the adjustment of the pension system, because the increase in taxes to keep debt per capita constant is smaller, being the world interest rate lower than the autarchy one. The savings of the young are not reduced as much as they are when the economy is closed and the young in Country A are the only ones to bear the adjustment burden. While the young and future generations are better off if the system adjusts when the economy is open, the old are worse off because their savings are compensated with a rate of return which is lower than the one prevailing under the assumption that the adjustment is over.

3.2.4 Underfunding caused by a reduction in contributions or an increase in benefits: a comparison

In the previous sections I focused on the effects of two given shocks *over time.* In this section I compare the effects of the two shocks described above *at a given point in time.*

Starting from the first period effects, it is possible to show[21] that the increase in interest rates caused by a drop in contributions is *smaller* than the increase in interest rates caused by a rise in benefits. The decrease in the capital/labour ratio is therefore larger when pensions increase rather than when contributions go down. In this sense, *the sources of underfunding matter,* because the stock of per capita capital prevailing in the economy depends on the cause of the deficit.

An intuitive explanation for this is the following: when pensions increase, agents have extra income in the second period of their life. They can transfer part of this increase in purchasing power to the first period of life by decreasing savings. When payroll taxes go down, current income goes up and consumption increases. To transfer part of the increase in current income to the second period of life, agents save more (or decrease their savings by less).

Moving to the second period effects, I have already pointed out that in the second period the policy change is the same in both cases. Given that the first period level of the capital/labour ratio is lower when pensions increase than when contributions decrease and that the change the ratios are subject to is the same in both cases, in the second period and in the long run the

[21] See Part D of the Appendix.

stock of per capita capital is lower when the shock hits the benefit side than when it hits the contribution side.

4 Conclusions

In an open economy *national* pension policies can have an *international* dimension via their impact on labour and capital markets. This paper studies some international aspects of pension policies focusing on the capital market. Namely, it develops a positive analysis of the effects of the integration in a world capital market of two countries whose pension systems are organised according to different financing methods, namely pay-as-you-go and fully funded. The paper shows that factor prices and welfare change following the integration between the two countries also when the pay-as-you-go system is balanced. The pay-as-you-go financing of the pension scheme affects the supply of capital by reducing savings and this explains the result. The paper shows that the change in factor prices causes divergent welfare effects both across countries and across generations. The analysis is then extended to include the degree of maturity of the scheme and its funding status. The first extension offers further insights on how pension systems affect the supply side of the capital market. Namely, it shows that the effects on factor prices and welfare are reduced by the system not being mature. The second extension draws the attention on the financing means of the pay-as-you-go pension system and it introduces debt financing explicitly. Debt financing depresses the capital/labour ratio in the country running the pay-as-you-go system even further. The paper shows that, given the imbalance in the pay-as-you-go, the country running the fully funded system is subject to a smaller change in factor prices if it does take part into the adjustment process rather than if it does not. Moreover, it shows that the stock of per capita capital prevailing in the economy depends on the sources of the imbalance.

Though the model is stylised, it helps to clarify the basic mechanisms at work and it provides some answers to the pressing questions present in the policy debate. Only when one is familiar with the basic mechanisms, one can have a better understanding of the role, scope and objectives of a social security system in a highly integrated economic area and have a sound basis on which to found reform proposals.

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5 Appendix

5.1 Part A

Assume that $k^w < k^A$. If the economy is closed, at a level of per capita capital equal to k^w excess savings in the capital market of Country A would be observed:

$$k^w < a^A = w^w - D - c_1^{A*} \tag{62}$$

In the condition above if $\left(D + c_1^{A*}\right)$, which measures consumption in Country A[22], is replaced by $\left[\frac{1}{2}\left(c_1^{A*} + c_1^{B*}\right) + \frac{1}{2}D\right]$, which measures world consumption, the disequilibrium in the capital market is reinforced, if the model is stable, as:

$$D + c_1^{A*} > \frac{1}{2}\left(c_1^{A*} + c_1^{B*}\right) + \frac{1}{2}D^{23} \tag{63}$$

Capital market equilibrium requires $k^w > k^A$. Following the same reasoning, the condition $k^w < k^B$ can be established.

[22] As the contributions to the pay-as-you-go pension scheme decrease savings, here I consider them as a consumption component.

[23] Condition (63) can be rewritten as follows:

$\frac{1}{2} \cdot \left(c_1^{A*} + D\right) + \frac{1}{2} \cdot \left(c_1^{A*} + D\right) > \frac{1}{2} \cdot \left(c_1^{A*} + D\right) + \frac{1}{2}c_1^{B*}$

Simplifying I get:

$c_1^{A*} + D > c_1^{B*}$

Given that this inequality holds, condition (63) follows.

5.2 Part B

Maintaining the assumption of a logarithmic utility function and taking into account that pensions are fixed whilst contributions are variable, savings at time $t-2$ are:

$$s_{t-2} = \frac{1}{2+\vartheta} \cdot (w_{t-2} - \tau_{t-2}) - \frac{1+\vartheta}{2+\vartheta} \cdot \frac{\overline{p}}{1+r_{t-1}} \tag{64}$$

I first calculate $\frac{dr_{t-1}}{db}$[24] in order to study the short run effects of the drop in payroll taxes at time $t-2$ combined with an increase in debt at time $t-1$.

Without specifying a production function, the interest rate at time $t-1$ can be expressed as a function of the capital stock at time $t-1$ according to equation (9).

Total wealth per capita at time $t-1$ is:

$$a_{t-1} = k_{t-1} + b \tag{65}$$

and the capital market equilibrium condition requires:

$$a_{t-1} = \frac{s_{t-2}}{1+n} \tag{66}$$

Given the above equations, the change in interest rates generated by the shock here analysed can be expressed as:

$$\frac{dr_{t-1}}{db} = f^{''} \left(\frac{da_{t-1}}{db} - 1 \right) \tag{67}$$

Taking into account the social security budget constraint at time $t-2$ one knows that:

[24] Given that $b_{t-1} = b_t = \overline{b}$, one can drop the time subscripts on b.

$$d\tau_{t-2} = -db \cdot (1+n) \tag{68}$$

Using (68) and (64) one can calculate:

$$\frac{da_{t-1}}{db} = \frac{1}{2+\vartheta} + \frac{1+\vartheta}{2+\vartheta} \cdot \frac{\overline{p}}{1+n} \cdot \frac{1}{(1+r_{t-1})^2} \cdot \frac{dr_{t-1}}{db} \tag{69}$$

Plugging (69) in (67) and rearranging terms one finds:

$$\frac{dr_{t-1}}{db} \cdot \left[1 - \frac{f''}{(1+r_{t-1})^2} \cdot \frac{1+\vartheta}{2+\vartheta} \cdot \frac{\overline{p}}{1+n}\right] = -f'' \cdot \left[1 - \frac{1}{2+\vartheta}\right] \tag{70}$$

or:

$$\frac{dr_{t-1}}{db} = \frac{-f'' \cdot \frac{1}{2+\vartheta} \cdot (1+n) \cdot (1+\vartheta)}{\left[(1+n) - f'' \cdot \frac{1+\vartheta}{2+\vartheta} \cdot \frac{\overline{p}}{(1+r_{t-1})^2}\right]} > 0 \tag{71}$$

given that $f'' < 0$.

The result described in Section 3.2.1 is confirmed: a cut in taxes at time $t-2$ combined with an increase in debt puts pressure on the demand side of the capital market and drives interest rates up.

I now calculate the second period effect $\frac{dr_t}{db}$, associated with the increase in contributions necessary to maintain the stock of debt per capita constant. Calculating and comparing the first period effect on interest rates $\frac{dr_{t-1}}{db}$ with the second period effect $\frac{dr_t}{db}$ one obtains information on the stock of per capita capital at time $t-1$ and t: on the basis of this information one can draw some conclusions on the relationship between the timing of the shock and world factor prices after the integration.

Following the same procedure developed above, one finds:

$$\frac{da_t}{db} = \frac{1}{(1+n)} \cdot \left[\frac{1}{(2+\vartheta)} \cdot \frac{d\widetilde{w}_{t-1}}{db} + \frac{1+\vartheta}{2+\vartheta} \cdot \frac{\overline{p}}{(1+r_t)^2} \cdot \frac{dr_t}{db} \right] \tag{72}$$

where $\widetilde{w}_{t-1}$ denotes net wages at time $t-1$. Net wages change because both gross wages and taxes change at time $t-1$: gross wages are reduced because the capital stock at time $t-1$ is lower, as I have just showed, and taxes increase to keep the stock of debt per capita constant. The total effect is that net wages at time $t-1$ decrease. Formally:

$$d\widetilde{w}_{t-1} = -(k_{t-1}+b)dr_{t-1} - (r_{t-1}-n)db < 0$$

Substituting (72) in (67) expressed at time t and rearranging terms one gets:

$$\frac{dr_t}{db} \cdot \left[1 - \frac{f''}{(1+r_t)^2} \cdot \frac{1+\vartheta}{2+\vartheta} \cdot \frac{\overline{p}}{1+n} \right] = f'' \cdot \left[\frac{1}{(2+\vartheta)\cdot(1+n)} \cdot \frac{d\widetilde{w}_{t-1}}{db} - 1 \right] \tag{73}$$

or:

$$\frac{dr_t}{db} = \frac{-f'' \cdot \left[-\frac{1}{2+\vartheta} \cdot \frac{d\widetilde{w}_{t-1}}{db} + (1+n) \right]}{\left[(1+n) - f'' \cdot \frac{1+\vartheta}{2+\vartheta} \cdot \frac{\overline{p}}{(1+r_t)^2} \right]} > 0 \tag{74}$$

Also the second period effect of the policy change on interest rates is positive: the capital/labour ratio decreases even further. One can therefore conclude that:

$$k_{t-1}^{\alpha} = k_t^{\beta} > k_t^{\alpha} \tag{75}$$

Comparing (71) with (74) and taking into account that $\frac{d\widetilde{w}_{t-1}}{db} < 0$, one can add a further result to the above ones, i.e.:

$$\frac{dr_t}{db} > \frac{dr_{t-1}}{db} \tag{76}$$

The second period impact is larger than the short run one.

5.3 Part C

I calculate $\frac{dr_{t-1}}{db}$ and $\frac{dr_t}{db}$ for the increase in benefits case. I proceed exactly as in Part B, taking into account that the change the social security budget constraint is subject to in the first period is different. Namely, at time $t-2$:

$$dp_{t-2} = db \cdot (1+n)^2 \tag{77}$$

Notice that both the increase in pensions and in debt per capita remain for all future periods and that contributions at time $t-1$ adjust to keep debt per capita constant.

The first period effect of changes in benefits and financing means on interest rates is:

$$\frac{dr_{t-1}}{db} \cdot \left[1 - \frac{f''}{1+n} \cdot \frac{1+\vartheta}{2+\vartheta} \cdot \frac{\overline{p}}{(1+r_{t-1})^2}\right] = -f'' \cdot \left[1 + \frac{1+\vartheta}{2+\vartheta} \cdot \frac{1+n}{1+r_{t-1}}\right] > 0 \tag{78}$$

or:

$$\frac{dr_{t-1}}{db} = \frac{-f''(1+n) \cdot \left[1 + \frac{1+n}{1+r_{t-1}} \cdot \frac{1+\vartheta}{2+\vartheta}\right] \cdot}{\left[(1+n) - f'' \cdot \frac{1+\vartheta}{2+\vartheta} \cdot \frac{\overline{p}}{(1+r_{t-1})^2}\right]} > 0 \tag{79}$$

The increase in benefits matched by an increase in debt raises interest rates and it decreases the stock of capital at time $t-1$.

I now calculate $\frac{dr_t}{db}$. Given that the second period policy change is the same in both the scenarios analysed in the paper (taxes increase to keep the debt per capita constant), condition (74) holds. Therefore also in this case $\frac{dr_t}{db} > 0$ and the capital/labour ratio goes further down.

5.4 Part D

As regards the short run effect, comparing (70) and (78) or (71) and (79) one can conclude that the increase in interest rates caused by a drop in contributions is *smaller* than the increase in interest rates caused by a rise in benefits. Both shocks determine a deficit in the system, but the sources of *underfunding* matter and the stock of per capita capital prevailing in the economy depends on the cause of the deficit.

Given that k_{t-1} when contributions drop is higher than k_{t-1} when benefits go up and given that the second period effect on interest rates is represented in both cases by the same differential, also in the second period (and in the long run) the capital/labour ratio is lower when pensions increase than when payroll taxes are cut.

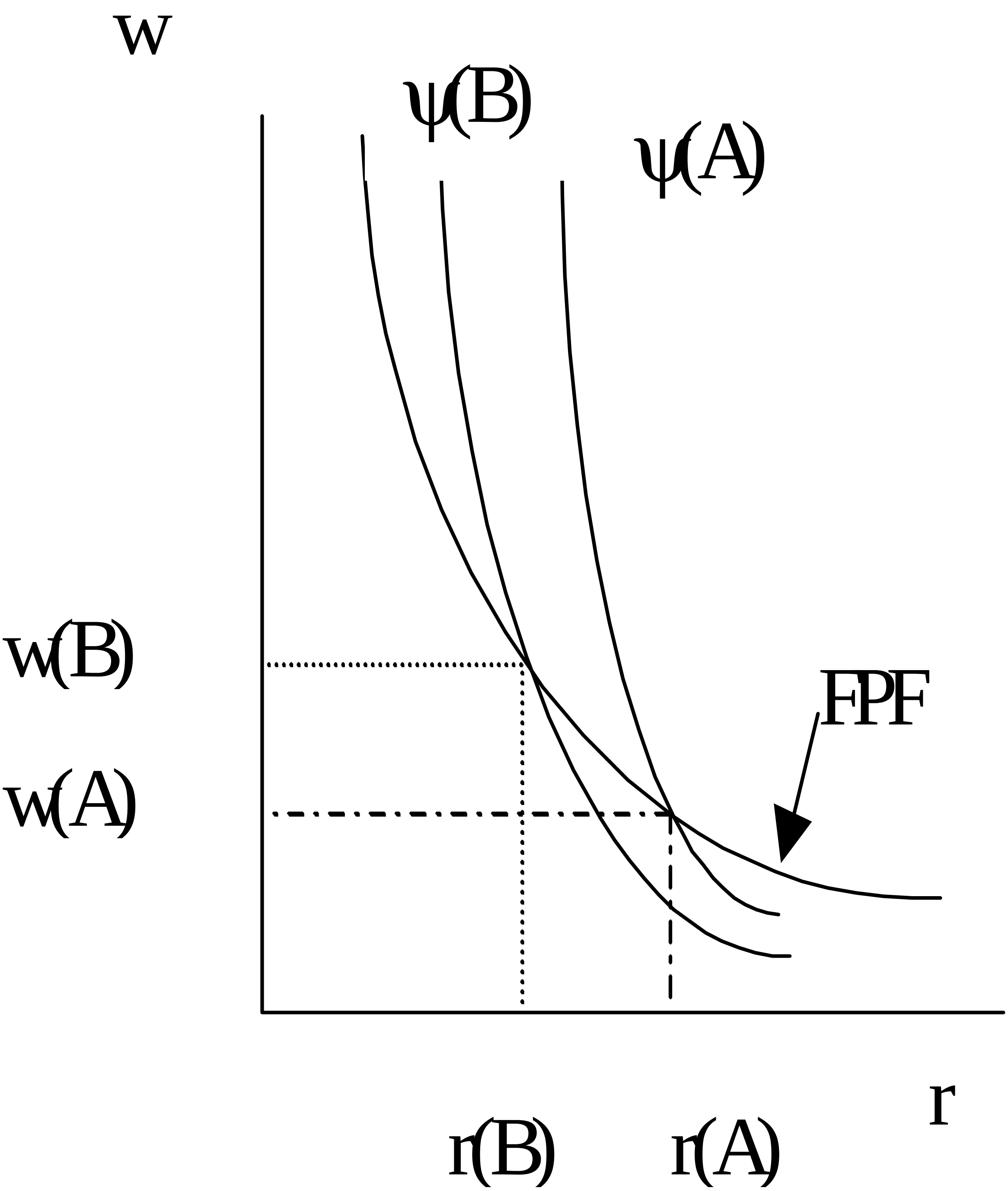

Figure 1

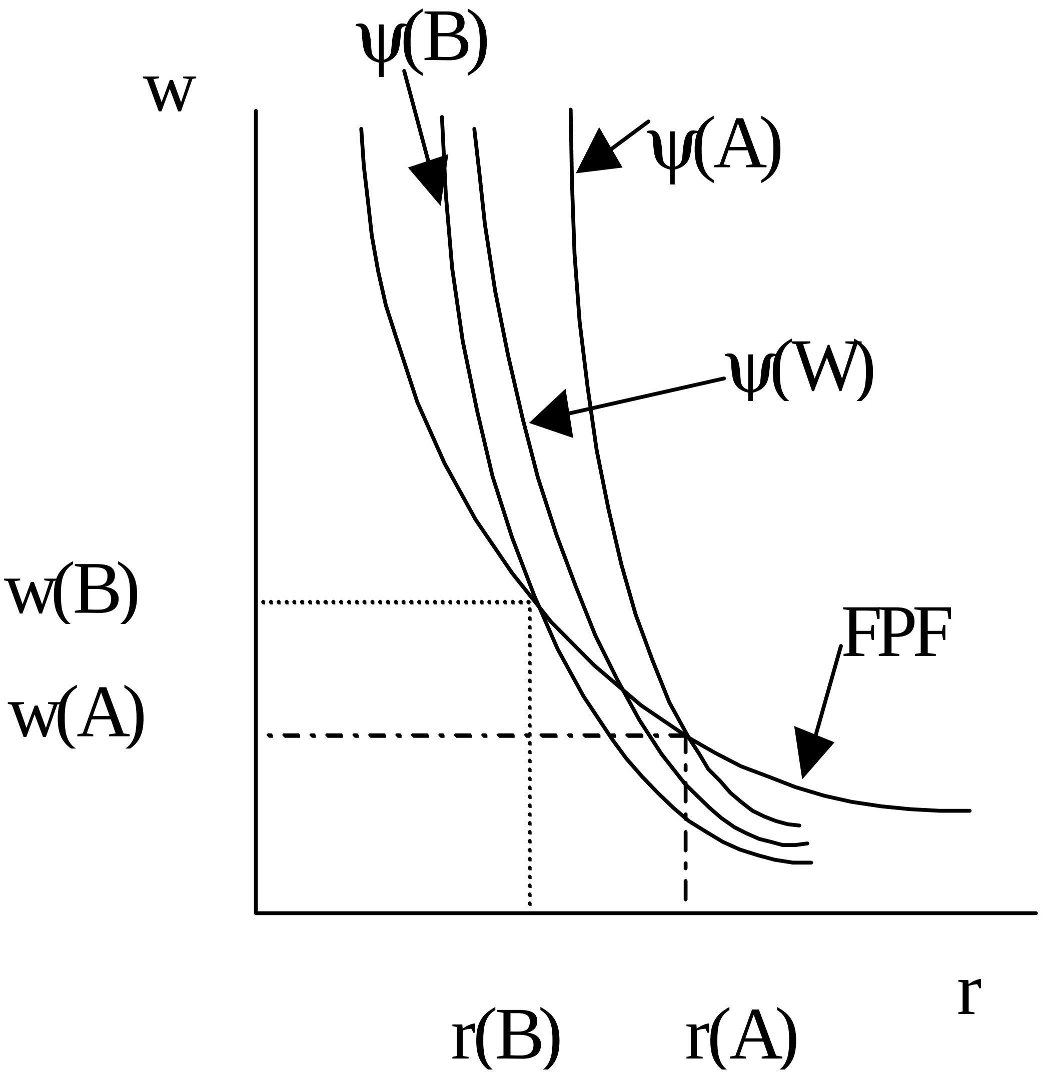

Figure 2

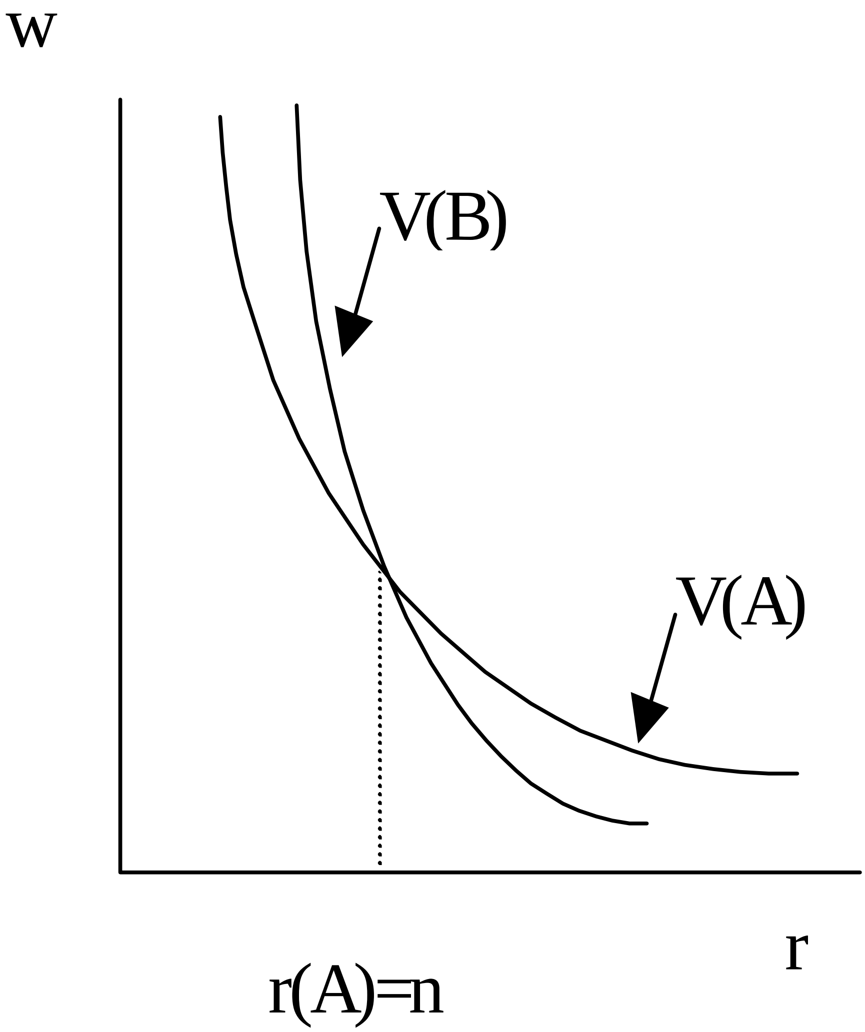

Figure 3

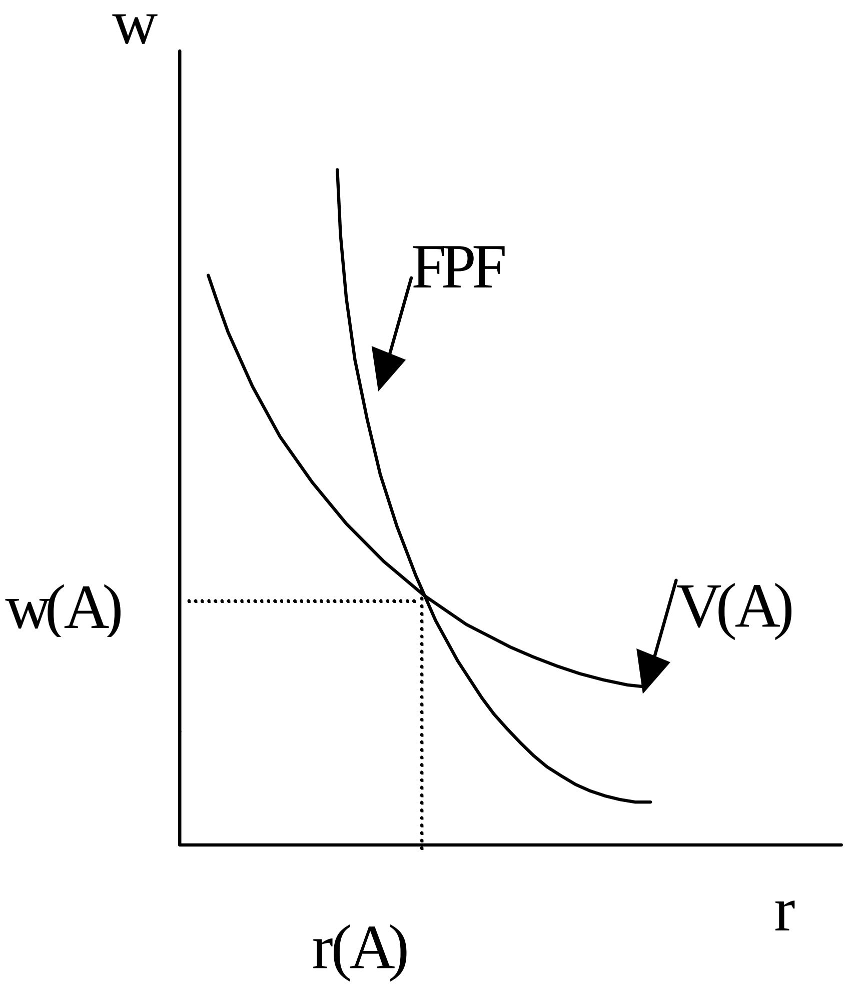

Figure 4a

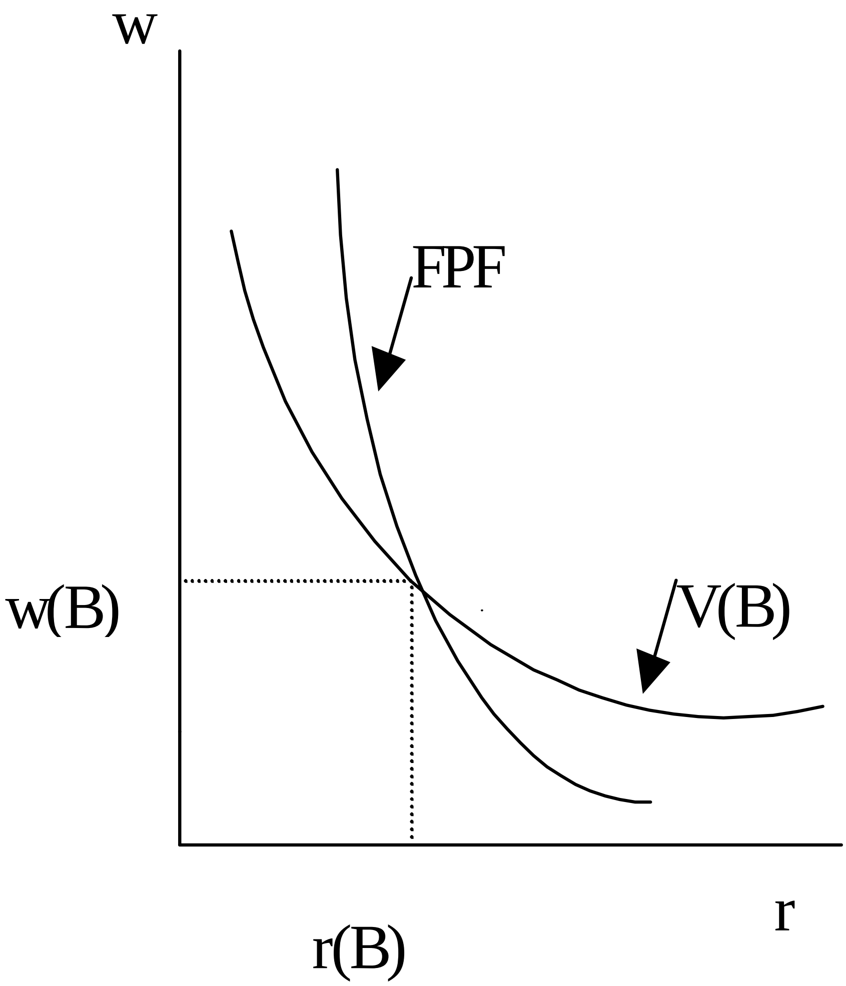

Figure 4b