1 The Smash-and-Grab Game*

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Abstract

The single-agent smash and grab problem takes the form $\text{Max}_x U(x) \cdot h(x)$. The agent selects a target value (how much to grab). The probability that he will receive (get away with) x is $h(x)$,.the cumulative probability distribution of the maximum achievable level of x. Generalizations are developed to two agents. Agent i $(i = 1, 2)$ maximizes $U^i(x_i) \cdot h(x_1, x_2)$. Significant special cases are seen when $U^i(x_i) = x_i$ (expected-return maximization); or when $h(x_1, x_2) = h(x_1 + x_2)$ (cutting slices from a cake of uncertain size). The players may choose their values independently (Nash game); in a predetermined sequence; or in a game of attrition. Simple examples exhibit second-mover advantage. which is discussed in detail.

JEL Classification

Keywords Game theory, Attrition, Nash equilibrium, Corruption

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1.1 The Smash-and-Grab Problem

In my hand I hold an envelope. Inside is a piece of paper on which a value in terms of money is enscribed. Please take part in a small experiment. Take a sheet of paper and write down a sum of money; sign the paper and pass it to me. One of these returns will be selected at random. If that is your return, and if the sum which you have written down is less than or equal to the sum enscribed inside the envelope, you will receive what you have written down. Otherwise you will get nothing. Plainly you need to know more to make a decision. To help you I will tell you that the sum written in the envelope is randomly generated by a process which has uniform density on the interval (£1m,£2m). Before you get too excited I had better admit at once that I am teasing you. I cannot perform the experiment because the ESRC shamefully turned down my application for a grant of £1m to fund it.

But wait a minute; why was my application for only £1m? That was because I take it for granted that my audience at the Royal Economic Society annual conference will consist exclusively of rational non-risk loving people. And in the above experiment a rational non-risk loving agent will always write down £1m. On why, more below.

The notional experiment is an instance of a *Smash-and-Grab* problem. Several interesting economic problems take this form:

$$\mathrm{Max}_x U(x) \cdot h(x) \tag{1}$$

This can be interpreted as maximizing expected utility when the agent can decide how much to go for, x, and the probability of success is a (declining) function $h(\cdot)$ of that target value. The title given to such a problem may be explained as follows. A criminal smashes the window of a jeweler's shop. For mathematical convenience assume that the shop window is furnished with a bounded or infinite supply of gold dust. Then the quantity taken can be considered a continuous variable. The thief can help himself to gold of the value of his choice, and he runs off with that amount. His only problem is that the more value he tries to carry away, the smaller is his chance of success. If captured he goes to jail and enjoys utility zero. His choice of how much value to try to carry away takes the form of (1).

A class of problems essentially the same as (1) is encountered when $U(x)$ decreases with x and $h(x)$ increases with x. An example of this type is provided by games of attrition. In the Bliss-Nalebuff ballroom dancing game, the players wait before individually providing a costly public good. Then x is the pre-determined maximum waiting time, and success takes the form of another player moving first. The longer the waiting time the lower utility on average; but the greater the chance of success.

Bliss and di Tella (1997) examine an imperfect information model in which the decision problem is that of a corrupt agent deciding how much graft to demand of a firm. If the firm cannot pay it exits and the agent gets nothing. The ability of firms to pay graft depends upon their overhead costs, which are independently drawn from a cumulative distribution which measures the probability that a particular firm will have overhead costs no greater than C:

$$F(C) \tag{2}$$

with $F(0) = 0$, $F(\infty) = 1$, and $F(\cdot)$ is an increasing function of C.

Each firm is in the territory of one corrupt agent. While the agent cannot observe the value of C for his firm, he does know the operating profit which all firms make, denoted P. Given P and the distribution $F(\cdot)$ he can decide how much graft to demand. The official demands a bribe: the firm either pays up or exits, and exit is irreversible.

If the corrupt official is only interested in the expected value of his return he will solve the following program:

$$Max_G \qquad G \cdot F\left(P - G\right) \tag{3}$$

The maximand is the product of the amount of graft demanded G and the probability of obtaining that amount. This is a smash-and-grab problem.

Examples can be multiplied easily. Consider courtship. The agent seeks a partner and follows the strategy of setting a target standard, and searching until a partner of at least that standard is found. The probability of success given a standard x is $h(x)$. Without success the agent is alone and enjoys utility zero. Ignoring the possibility of having a sliding standard; accepting partners of lower and lower standard as the search period nears completion, courtship will be seen to be a smash-and-grab problem.

1.2 Smash and Grab and Monopoly/Oligopoly

The maxiimand in (1) is $U(x) \cdot h(x)$. When expected value is maximized, this becomes $x \cdot h(x)$, with $h(x)$ a decreasing function of x. The problem of a monopolist with zero marginal production problem is similar:

$$\mathrm{Max}_x x \cdot p(x) \tag{4}$$

where x is the quantity sold and $p(x)$ is the inverse demand function. When the Cournot-Nash equilibrium of oligopolists is at issue, the individual seller's maximand again takes a form which is similar to the smash and grab problem. Now it is:

$$\text{Max}_x x \cdot p(x + X) \tag{5}$$

where X is the total sold by other sellers.

It will serve the reader well to bear this analogy in mind when reading what follows. Some smash and grab results are in effect already familiar from IO theory. Later as the smash and grab problem is enriched, we enter new territory.

1.3 Multiple Solutions

In line with standard theory, we may assume $U(x)$ to be a strictly concave function. However $h(x)$ will often be non-convex, as would happen were x to be normally distributed. In such cases, even if the agent is risk neutral, multiple maximizing choices for x may be found. Figure 1 illustrates. The convex curve AA, with the appearance of an indifference curve, is a locus along which $x \cdot h(x)$ is constant. The height of the curve HH shows the value of $h(x)$. x_1 and x_2 are both maximizing values.

Or consider the case in which $U(x) = \sqrt{1 + x}$ and $h(x) = (1 + x)^{-\frac{1}{2}}$. Then (1) takes the value 1 for all positive x.

1.4 Two Examples

The Smash and Grab problem is not always well-defined. Consider the following example.

Example 1

$$U(x) = x^{\alpha} \tag{6}$$

$$h(x) = (1 + x)^{-\beta} \tag{7}$$

With $0 < \alpha \leq 1$ *and* $\beta > 0$. *The agent maximizes:*

$$x^{\alpha} (1 + x)^{-\beta} \tag{8}$$

If $\alpha = \beta = 1$ *the agent would maximize:*

$$\frac{x}{1 + x} \tag{9}$$

However (9) increases with x *on* $[0, \infty)$ *and there is no maximum. One might say that* $x = \infty$ *is the solution, but of course that is not a meaningful answer.*

In general, differentiating (8) with respect to x *yields:*

$$\alpha x^{\alpha-1} (1 + x)^{-\beta} - \beta x^{\alpha} (1 + x)^{-\beta-1} = 0 \tag{10}$$

$$\frac{x}{1+x} = \frac{\alpha}{\beta} \tag{11}$$

and it is evident why the case $\alpha = \beta = 1$ *causes problems.*
If $\alpha > \beta$*, (11) has no solution. From (10) the first derivative of the maximand is:*

$$x^{\alpha}(1+x)^{-\beta}\left[\frac{\alpha}{x} - \frac{\beta}{1+x}\right] \tag{12}$$

which is always positive with $\alpha > \beta$*.*
With $\alpha < \beta$*, the smash-and-grab problem has a solution. From (11):*

$$x = \frac{\alpha}{\beta - \alpha} \tag{13}$$

□

The case of no solution is parallel to a case in monopoly theory. If the inverse demand function is $(1+x)^{-1}$ the monopolist tries to maximize (8), and profit increases with extra sales however large are sales. In the monopoly instance we might be ready to say that the case is entirely unreasonable on grounds of realism, and to dismiss it. Can we argue similarly for the smash and grab problem? Some people who never married explain their history by saying: "I never met anyone who seemed good enough." One is tempted to reply that perhaps the individual concerned set the standard unrealistically high. But is that response correct? The smash and grab problem differs from monopoly in that an expected value is being maximized. Although psychologists may condemn it, it is not irrational from the strictly economic point of view to search for a partner so incredibly wonderful that the chance of finding that partner is vanishingly small.

The next Example is based on the uniform distribution, with expected-value maximization. It brings out the point that corner solutions may be of great importance, and incidentally justifies the claim which I made at the start of my presentation, that you should all have written down £1m.

Example 2 *The maximum value obtainable is uniformly distributed on* $[\beta, \beta + \alpha]$ *with* α *and* $\beta > 0$*. The agent solves:*

$$Max_x x \cdot \left[1 - \frac{x-\beta}{\alpha}\right] \tag{14}$$

This gives a first-order condition for an interior maximum:

$$\left[1 - \frac{x-\beta}{\alpha}\right] - \frac{x}{\alpha} = 0 \tag{15}$$

Hence:

$$x = \frac{\alpha + \beta}{2} \tag{16}$$

This solution is plainly incorrect if β is large and α small, as the agent could take β without risk. The general solution is:

$$x = Max\left[\frac{\alpha + \beta}{2}, \beta\right] \tag{17}$$

The value of the problem is:

$$Max\left[\frac{(\alpha + \beta)^2}{4\alpha}, \beta\right] \tag{18}$$

Figure 2 illustrates the solution to my experiment when the agent is an expected value maximizer. A risk-averse agent would be even less willing to put the certain £1m at risk by asking for more.

1.5 Smash-and-Grab solutions

With all functions differentiable a regular maximum solution to (1) requires:

$$U_1(x) \cdot h(x) + U(x) \cdot h_1(x) = 0 \tag{19}$$

and:

$$U_{11}(x) \cdot h(x) + 2U_1(x) \cdot h_1(x) + U(x) \cdot h_{11}(x) < 0 \tag{20}$$

where subscripts denote differentiation. While (20) is required for a regular maximum, natural restrictions on the functions do not ensure the condition (20). For a concave utility function, and $h(\cdot)$ a declining function, the first two terms of (20) will always be negative. However the third term is sign-ambiguous. If $h(\cdot)$ is derived from a cumulative probability distribution, for instance, its second derivative may well change sign. That would happen with the Normal Distribution. As a consequence of this sign ambiguity, the Smash-and-Grab problem may well have multiple local maximum solutions, and even multiple global maximum solutions. Bliss and di Tella (1997) illustrate such a case with a diagram.

When the agent is an expected value maximizer, $U(x) = x$, and (19) and (20) become:

$$h(x) + x \cdot h_1(x) = 0 \tag{21}$$

and:

$$2h_1(x) + x \cdot h_{11}(x) < 0 \tag{22}$$

Again (22) is in general sign ambiguous. However, despite the ambiguity, (21) and (22) lead directly to a useful result for the expected value maximizing smash and grab problem. Suppose that $h(x)$ is replaced by $h(\lambda x)$, where $\lambda < 1$.This corresponds to an increase in the scale of the loot available. Now $\frac{x}{\lambda}$ can be taken at the same risk of failure as previously applied to x.

Theorem 3 *The expected value maximizing smash and grab problem is homogeneous, in the sense that if it was optimal to take x before the change described above, it will now be optimal to take $\frac{x}{\lambda}$.*
Proof: Consider the class of problems to maximize:

$$x \cdot h(\lambda x) \tag{23}$$

The first-order condition is:

$$h(\lambda x) + x\lambda h_1(\lambda x) = 0 \tag{24}$$

Differentiating (24) totally with respect to λ gives:

$$\left[2\lambda h_1(\lambda x) + x\lambda^2 h_{11}(\lambda x)\right] \frac{dx}{d\lambda} + 2x h_1(\lambda x) + x^2 \lambda h_{11}(\lambda x) = 0 \tag{25}$$

$$\frac{dx}{d\lambda} = -\frac{x}{\lambda} \cdot \frac{2h_1(\lambda x) + x\lambda h_{11}(\lambda x)}{2h_1(\lambda x) + x\lambda h_{11}(\lambda x)} = -\frac{x}{\lambda} \tag{26}$$

Integrating (26) gives:

$$\log x = \log \frac{1}{\lambda} + \log c \tag{27}$$

$$x = \frac{x*}{\lambda} \tag{28}$$

where $x*$ is the solution for x when $\lambda = 1$.$\square$

Theorem 1 is an unsurprising result, because the shift from $h(x)$ to $h(\lambda x)$ corresponds to a change of the units in which x is measured. One would not expect that to affect the real level of optimizing x. In the general smash and grab problem (1) the substitution of $h(\lambda x)$ for $h(x)$ by itself does not correspond to a change of units. Furthermore, homogeneity will not apply, as the fact that there is no substitution of $U(\lambda x)$ for $U(x)$ breaks the simple scale effect.

The agent maximizes:

$$U(x) \cdot h(\lambda x) \tag{29}$$

The first-order condition is:

$$U_1(x) \cdot h(\lambda x) + U(x)\lambda h_1(\lambda x) = 0 \tag{30}$$

Differentiating (30) totally with respect to λ gives:

$$\begin{aligned} &\left[U_{11}(x)h(\lambda x) + 2U_1(x)\lambda h_1(\lambda x) + U(x)\lambda^2 h_{11}(\lambda x)\right] \frac{dx}{d\lambda} \\ &+U_1(x)xh_1(\lambda x) + U(x)h_1(\lambda x) + U(x)\lambda x h_{11}(\lambda x) = 0 \qquad (31) \\ &\frac{dx}{d\lambda} = \frac{U_1(x)xh_1(\lambda x) + U(x)h_1(\lambda x) + U(x)\lambda x h_{11}(\lambda x)}{U_{11}(x)h(\lambda x) + 2U_1(x)\lambda h_1(\lambda x) + U(x)\lambda^2 h_{11}(\lambda x)} \qquad (32) \end{aligned}$$

In Theorem 1 the sign ambiguity of $h_{11}(\lambda x)$ did not matter, as terms involving that value cancelled. Now they no longer cancel.

1.6 Two Players: Independent Move Nash Equilibrium

So far we have examined a single-player Smash-and-Grab Game. Later we will look at the same game with two players who play in a predetermined order.

In a Nash equilibrium the two players simultaneously maximize their respective objectives, given the other player's optimizing value. Thus Player I chooses a value of x_1 which maximizes:

$$U^1(x_1) \cdot h(x_1, x_2) \tag{33}$$

while Player II chooses a value of x_2 which maximizes:

$$U^2(x_2) \cdot h(x_1, x_2) \tag{34}$$

We can learn a lot about the two-player smash and grab game by computing the Nash solution for the two examples above, with the sum of the two choices replacing the choice of a single player.

In the case of the first example the players solve:

$$\text{Max}_{x_1} x_1^{\alpha} \cdot [1 + x_1 + x_2]^{-\beta} \tag{35}$$

and:

$$\text{Max}_{x_2} x_2^{\alpha} \cdot [1 + x_1 + x_2]^{-\beta} \tag{36}$$

The first-order conditions are:

$$\alpha x_1^{\alpha-1} \cdot [1 + x_1 + x_2]^{-\beta} - \beta x_1^{\alpha} [1 + x_1 + x_2]^{-\beta-1} = 0 \tag{37}$$

$$\frac{\alpha}{x_1} = \frac{\beta}{1 + x_1 + x_2} \tag{38}$$

The symmetrical condition holds for x_2. So the solution is symmetrical with $x_1 = x_2 = x$ and:

$$x = \frac{\alpha}{\beta - 2\alpha} \tag{44}$$

With the uniform distribution example the players solve:

$$\text{Max}_{x_1} x_1 \cdot \left[1 - \frac{x_1 + x_2 - \beta}{\alpha}\right] \tag{45}$$

and:

$$\text{Max}_{x_2} x_2 \cdot \left[1 - \frac{x_1 + x_2 - \beta}{\alpha}\right] \tag{36}$$

This gives first-order conditions:

$$\left[1 - \frac{x_1 + x_2 - \beta}{\alpha}\right] - \frac{x_1}{\alpha} = 0 \tag{37}$$

and:

$$\left[1 - \frac{x_1 + x_2 - \beta}{\alpha}\right] - \frac{x_2}{\alpha} = 0 \tag{38}$$

From which the solution is:

$$x_1 = x_2 = \frac{\alpha + \beta}{3} \tag{39}$$

As the previous solution has already shown, a mechnically-derived interior solution may not be sensible. If α is small and β large, two players following the rule (39) would together opt for less than β, in which case they could each aim for something more with certainty of obtaining it. However if the two players together opt for the safe bet of just β, we have a standard case of indeterminacy of the Nash solution, since many divisions of the total β could be Nash solutions.

1.7 Two Agents: A pre-determined move order

Now consider a simple extension of the Smash-and-Grab problem. Each agent chooses an x-value. Each will only obtain their chosen x-value if they jointly pass the same functional test. If they coordinated they would solve the same program and divide the payoff according to some rule. But suppose that they do not coordinate at all. Each chooses an x-value independently. Suppose that one player moves first, the other second, the order being determined in advance.

This case needs careful interpretation. It is easy to imagine that the first mover takes an amount observed by the second-mover, who then optimizes given that level. This cannot be required strictly, because in a common-knowledge situation the second-mover can calculate what the first mover will take, and has no need to observe it. In fact move order as such is irrelevant. When we assume it, and model it as will be done in this section, what we are doing is to make one player (the first mover) into a Stackelberg leader who optimizes given the reaction function of the other player (the second-mover).

This game is easily solved. Assume that the first mover has decided on the value x^1. Then the second-mover solves:

$$Max_{x^2} \qquad x^2 \cdot h\left(x^1, x^2\right) \tag{45}$$

Denote the solution to (45) by $x^2(x^1)$. The first mover will solve:

$$Max_{x^1} \qquad x^1 \cdot h\left(x^1, x^2(x^1)\right) \tag{46}$$

Take the case in which $h(x^1, x^2) = x^1 + x^2$. Consider in particular the model of Bliss and di Tella (1997). Suppose that two corrupt agents take bribes in a pre-determined order, with exit probability determined by the sum of their demands. From the point of view of the second-mover agent, the bribe taken by the first-mover is equivalent to a decrease in firm profitability. Bliss and di Tella (1997) show that an increase in profitability cannot increase the graft taken by a single agent by more than the said increase. It could however decrease the total graft taken. The agent could take out the benefits of higher profitability as a higher probability of obtaining the amount demanded, albeit for a lower demand. Translated to the two agent model, this means that the first agent, who plainly decreases the profit remaining, causes the second agent to take less. The two together may take more or less in total compared with the amount taken by a single maximizing agent.

Theorem 4 *Two uncoordinated agents cannot take the same total graft as coordinating agents (who take $x\hat{}$).*

Proof: Suppose not. The first agent takes x^1, the second takes x^2, and:

$$x^1 + x^2 = x^\wedge \tag{47}$$

Then the second agent solves:

$$Max_{x^2} \qquad x^2 \cdot h\left(x^1 + x^2\right) \tag{48}$$

for which the first-order condition given (47) is:

$$h\left(x^\wedge\right) + x^2 \cdot h_1\left(x^\wedge\right) = 0 \tag{49}$$

However to maximize for one agent $x^\wedge$ must satisfy:

$$h\left(x^\wedge\right) - x^\wedge \cdot h_1\left(x^\wedge\right) = 0 \tag{50}$$

and (49 and (50) are inconsistent.$\square$

Inspection of equations (49) and (50) shows why we cannot go further and say whether two agents take more or less in total. What happens to $h_1\left(x^\wedge\right)$ when the size of $x^\wedge$ varies is ambiguous. For certain, the corrupt agents obtain less expected profit, because they do not coordinate. The conclusion is the same as Shleifer and Vishny [1993], who argue from what is, in effect, a simultaneous moves model. But whether society is better or worse off when corrupt agents coordinate their strategies cannot be decided in general.

1.8 Solving Examples for the Two Agent Case

In this section explicit solutions are obtained for cases involving simple functional forms. In each case agents are expected value maximizers. First consider the model of Example 1 above, now with two players and a pre-determined move order. We look first at the decision problem of the second-mover when the first-mover has taken x_1. Then the second-mover solves:

$$\text{Max}_{x_2} x_2^\alpha \cdot [1 + x_1 + x_2]^{-\beta} \tag{51}$$

the first-order condition is:

$$\alpha x_2^{\alpha-1} \cdot [1 + x_1 + x_2]^{-\beta} - \beta x_2^\alpha \cdot [1 + x_1 + x_2]^{-\beta-1} = 0 \tag{52}$$

$$\frac{\alpha}{x_2} = \frac{\beta}{1 + x_1 + x_2} \tag{53}$$

$$x_2 = \frac{\alpha}{\beta - \alpha}\left(1 + x_1\right) \tag{54}$$

As with the single-agent problem, we require$\beta > \alpha$. If $\beta > 2\alpha$, we have an example of a general proposition, that there can be a *second-mover advantage* (SMA).

1.9 Second-Mover Advantage

Discussing my research with colleagues and students has convinced me that some find the concept of second-mover advantage to be counter-intuitive. It should not be. If it is, that may be because different cases easily become confused. It is trivial to construct asymmetric games in which the second mover does better than the first mover. Therefore consider a symmetric game. Denote various outcomes as follows (C,C); (N,N), (S,N). These are respectively the symmetric cooperative solution; the symmetric Nash solution, and the solution in which the first player plays the optimal Stackelberg move, and the other player plays the best reply to that move. Then VC, VN, VS and VSS are values of various solutions to the players concerned. VC is the value of the cooperative solution; VN is the value of the symmetric Nash solution; VS is the value of being the Stackelberg leader; and VSS is the value of being the follower in the (S,N) outcome.

Definition 5 *A symmetric game exhibits Second-Mover Advantage if VSS> VS.*

Notice that the requirement of the definition is stronger than VSS>VN. When that happens the passive player is helped rather than harmed by the other player becoming a Stackelberg leader, which is a commonplace result. The definition says that the passive player is helped so much by the other player becoming a Stackelberg leader that he ends up better off than that Stackelberg leader. As the Stackelberg player can always play the Nash move, it is obvious that VS≥VN. For that reason the definition implies VSS≥VN; but is much stronger. All this is long familiar from the higher reaches of the IO literature See in particular Bulow, Geanakoplos and Klemperer (1985). For second mover advantage to arise, the players' strategies must be strategic complements- reaction curves slope upwards.

The first-mover solves:

$$\mathrm{Max}_{x_1} x_1^{\alpha} \cdot \left[1 + x_1 + \frac{\alpha}{\beta - \alpha}\left(1 + x_1\right)\right]^{-\beta} \tag{55}$$

This is equivalent to:

$$\mathrm{Max}_{x_1} x_1^{\alpha} \cdot \left(\frac{\beta}{\beta - \alpha}\right)^{-\beta} \left[1 + x_1\right]^{-\beta} \tag{56}$$

which is the same as the single-mover problem. Therefore the solution is:

$$x_1 = \frac{\alpha}{\beta - \alpha} \tag{57}$$

The first mover takes exactly what a single player would take. Then the second-mover takes his own bite; possibly less than the first-mover takes; possibly even more.

In the second example the maximum value of pay-off obtainable is uniformly distributed on $[\beta, \beta + \alpha]$. A single agent takes x chosen to maximize:

$$\text{Max}_x \; x \left[1 - \frac{x - \beta}{\alpha} \right] \tag{58}$$

We have seen above that the solution is:

$$x = \text{Max} \left[\beta, \frac{\alpha + \beta}{2} \right] \tag{59}$$

and the value will be:

$$\text{Max} \left[\beta, \frac{(\alpha + \beta)^2}{4\alpha} \right] \tag{60}$$

Making use of this single-player solution, we derive the solution to the two-stage game. The first-mover takes y, which has the effect of presenting the second mover with a one-stage problem in which β has been reduced to $\beta - y$. Consider first the case in which the second-mover will take $\frac{\alpha+\beta-y}{2}$. That is equivalent to saying that the second-mover will be placed in a position in which his interior tangent solution is maximal. Then the first-mover chooses y to maximize:

$$y \left[1 - \frac{y - \beta + \frac{\alpha+\beta-y}{2}}{\alpha} \right] \tag{61}$$

The first-order conditions for the maximization of (61) are:

$$\alpha - y + \beta - \frac{\alpha + \beta - y}{2} - y - 0 \tag{62}$$

$$y = \frac{3}{5} (\alpha + \beta) \tag{63}$$

And the second-mover takes:

$$\frac{1}{5} (\alpha + \beta) \tag{64}$$

There is a first-mover advantage. The value of this solution to the first-mover is:

$$\frac{3}{5}(\alpha+\beta)\left[1-\frac{\frac{4}{5}(\alpha+\beta)-\beta}{\alpha}\right] \tag{65}$$

Which is equal to:

$$\frac{3}{25}(\alpha+\beta)\left[1-\frac{\beta}{\alpha}\right] \tag{66}$$

This solution only make sense if $\alpha > \beta$.

If $\beta \geq \alpha$, the first-mover should take $\beta - \alpha$, leaving the second-mover to choose x to maximize:

$$x\left[1-\frac{x-\alpha}{\alpha}\right] \tag{67}$$

The solution is:

$$x = \alpha \tag{68}$$

And the value to the first-mover is:

$$\beta - \alpha \tag{69}$$

1.10 Non-Linear-Additive Cases

Most of the argument above has concerned itself with the case in which:

$$h[x_1, x_2] = h[x_1 + x_2] \tag{70}$$

Only the sum of the two player's demands count for success or failure of the project. In many interesting cases (70) is not satisfied. Staying with examples, imagine two dishonest officials working for the same Ministry or company, and able to divert funds to their private accounts. For one dishonest official the probability of detection will increase with the amount stolen, and we have a classic smash-and-grab problem. When there are two dishonest officials, what determines the probability that both will be found out? This assumes that either neither fraud is detected or that both are detected. The smash-and-grab problem requires that feature. It is not unreasonable, however, as it may well be that the detection of one fraud always triggers an investigation that detects all frauds.

The condition (69) requires that the total sum dishonestly taken determines the probablity of detection. Equally plausible for this example is:

$$h\left[x_1, x_2\right] = h\left[\mathrm{Max}\left(x_1, x_2\right)\right] \tag{71}$$

when the largest sum taken by fraud determines the probability of detection.

Similarly consider two players joining their efforts to achieve some objective. Their utilities decrease with that effort, but the probability of success rises with those efforts. Then (70) says that only the sum of their efforts matters. This implies that their separate efforts are perfect substitutes.Equally plausible for this example is:

$$h\left[x_1, x_2\right] = h\left[\mathrm{Min}\left(x_1, x_2\right)\right] \tag{72}$$

The probability that a team will win is a function of the lowest effort proided by any member. We can confine attention to (71), as (72) is its mirror image. Bearing in mind the example of two dishonest officials within the same organiation should help to motivate the following discussion. If the two players have the same utility functions, the objective function of official $i\,(i = 1, 2)$ is:

$$u(x_i) \cdot h\left[\mathrm{Max}\left(x_1, x_2\right)\right] \tag{73}$$

If the two players were to coordinate they would select $x_i = x_1 = x_2$ to maximize (74). Denote the value of x which achieves that maximization by $x*$. Then the best reply for either player to the choice of a value y by the other is:

$$x = x* \text{ if } y \leq x* \tag{74}$$

$$x = y \text{ if } y > x* \tag{75}$$

If the two players choose their x-values independently, there are infinitely many Nash equilibria. Both players choosing the same value of $x \geq x*$ is a Nash equilibrium. If $x > x*$, the equilbrium is Pareto-dominated by the equilibrium with $x = x*$. Plainly the natural focal solution among Nash equilibria is $x = x*$.

1.11 Second-Mover Ambiguity

Among the many issues that may be examined in terms of a general model, what may be called Second-Mover Ambiguity (SMAMB) is particularly intriguing. Take a game in which the players' demands enter the $h(\cdot)$ function additively, in which both players are expected value maximizers, and in which the players move in a pre-determined order. After the first player has taken x_1, the second-mover solves:

$$Max_{x_2} \qquad x_2 \cdot h(x_1 + x_2) \tag{76}$$

As $h(\cdot)$ may well be non-concave, it appears that the first-mover could find it optimal to place the second-mover in a position in which he is indifferent between a set of moves with more than one element. That outcome would be unsatisfactory to the first-mover. Suppose that the second-mover is indifferent between x_2^1 and x_2^2, with $x_2^1 > x_2^2$. Then:

$$h\left(x_1 + x_2^1\right) < h\left(x_1 + x_2^2\right) \tag{77}$$

and the first-mover strictly prefers the second-mover to play x_2^2. But he cannot guarantee that outcome. We might assume for instance that the second-mover will randomize between equal value moves. As optimal move multiplicity cases are non-generic it would usually pay the first-mover to slightly shade his move away from the value that would be optimal if he could decide the second-mover's choice for him, selecting only moves from the second-mover's optimal set. Then , strictly speaking, there may exist no optimal move for the first-mover, although an upper bound for the value of the game to the first-mover can be approached as closely as desired.

1.12 Conclusions

This paper reports work in progress, and it is not even evident to me in which direction the argument will be, or ought to be, developed. Focussing on corruption, which was the original motiivation of the study, the some useful observations may be gleaned from results already obtained.

Definition 6 *An anti-corruption drive is an intervention that shifts $h(\cdot)$ so that it takes a lower value at each point on $(0, \infty)$.*

Now note the following points:

- An anti-corruption drive must lower the value of (1). Therefore if the corrupt agents are expected-value maximizers, an anti-corruption drive will lower the amount taken corruptly. That need not be so when corrupt agents are expected-utility maximizers. In any case, lowering the probability that agents will get away with small graft may encourage them to "go for bust" - taking large bribes despite the relatively smaller chance of carrying off the big stolen prize.

- When two (or more) corrupt agents find themselves in situations with early (or late) mover advantage, the result may be either competitive dashes to the loot, or wars of attrition when they wait for the other to move first.
- With such complex reactions, an anti-corruption drive needs to define its objectives clearly, and to be optimally designed to achieve those objectives.

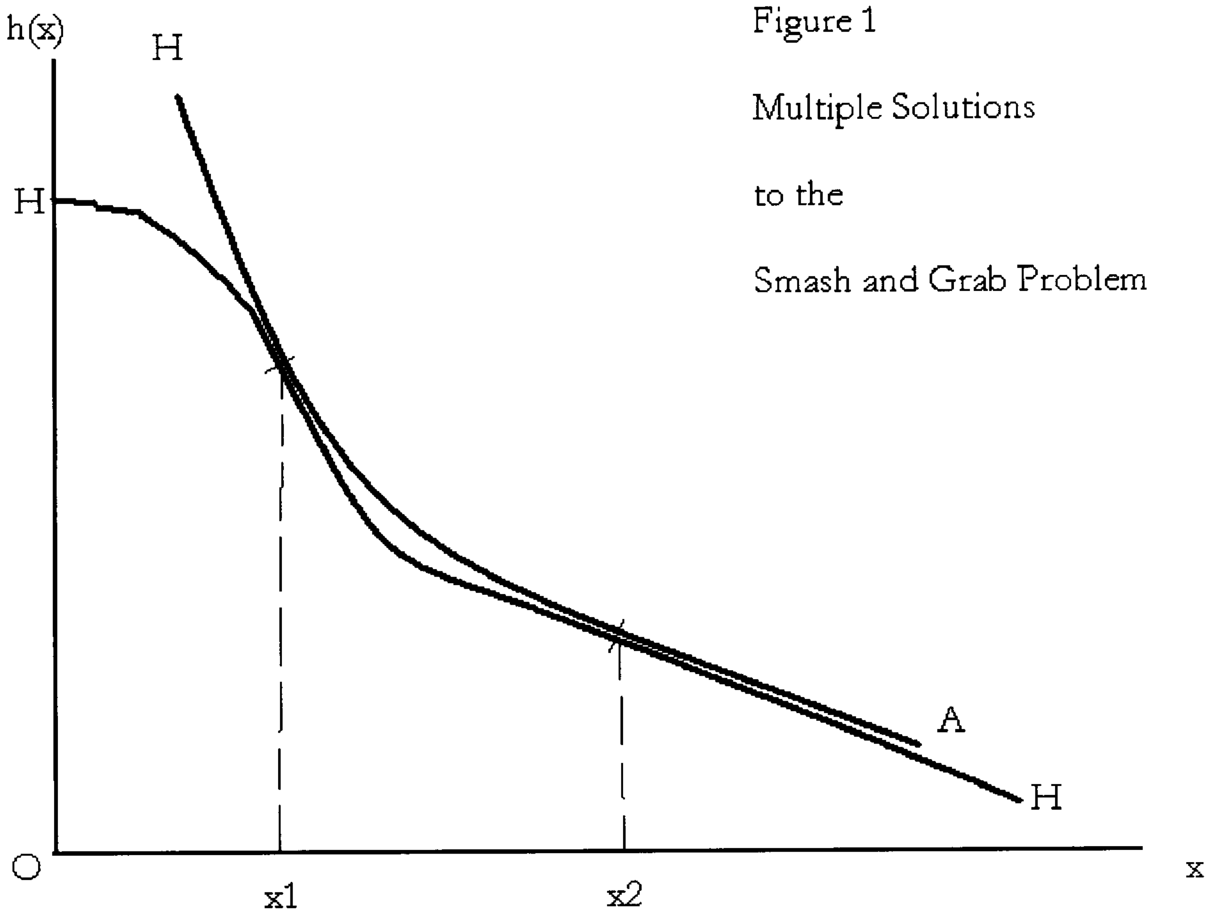


Figure 1

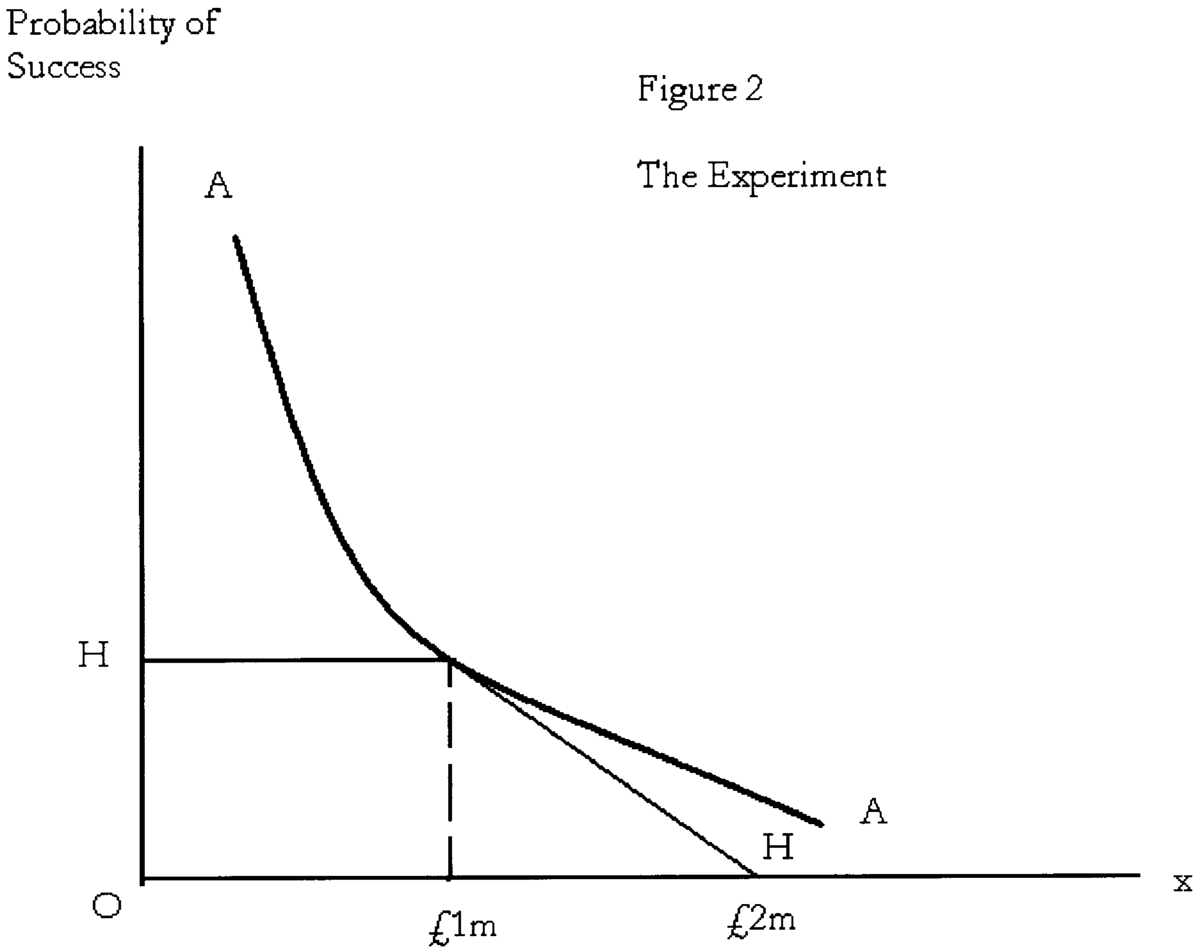
Multiple Solutions

to the

Smash and Grab Problem

Figure 2

The Experiment



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