# On Testing Overidentifying Restrictions in

# Dynamic Panel Data Models

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#### Abstract

The conventional Sargan (1958) / Hansen (1982) test of overidentifying restrictions and the Tilting Parameter test of Imbens, Spady and Johnson (1998) are compared in the context of the AR(1) dynamic panel data model using Monte Carlo experiments. Interestingly, the size properties of the former are found to be superior in this setting. Nevertheless, the Sargan / Hansen test is found to have no power in panels of dimensions that are commonly encountered in empirical work. A simple procedure for reducing the number of moment conditions tested is shown to improve both the size and power properties of the conventional test. A useful diagnostic procedure is also suggested.

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Keywords: Dynamic panel data; tests of overidentifying restrictions; exponential tilting;

GMM.

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### 1 Introduction

Dynamic panel data (DPD) models estimated using the Generalised Method of Moments (GMM) have become an important tool in the empirical analysis of microeconomic panels with a large number of individual units and relatively short time series. An important baseline case is the first order autoregressive (AR(1)) model with unobserved individual-specific effects considered by Arellano and Bond (1991). We write this as

$$y_{it} = \alpha y_{i,t-1} + \eta_i + v_{it} \tag{1}$$

where i = 1, ..., N; t = 2, ..., T;  $T \ge 3$  and  $|\alpha| < 1$ .

Adopting what are now standard assumptions concerning the error components and initial conditions process (notably that the error terms  $v_{it}$  are not autocorrelated; for a convenient summary see Blundell and Bond (1998), p. 118), Arellano and Bond (1991) noted the validity of the following set of moment conditions

$$E[y_{i,t-s}(\Delta y_{it} - \alpha \Delta y_{i,t-1})] = 0 \text{ for } t = 3, .., T \text{ and } s = 2, .., (t-1)$$
(2)

where  $\Delta$  is the first difference operator. Since these involve the use of lagged levels of  $y_{it}$  as instruments for the first *dif* ferenced equations we follow Blundell and Bond (1998) – hereafter BB – by referring to them as the DIF moment conditions. They constitute all of the secondorder linear moment conditions that are available under the maintained assumptions of Arellano and Bond (1991). Under the additional assumption that the deviation of the initial conditions from  $\eta_i/(1-\alpha)$  be uncorrelated with the level of  $\eta_i/(1-\alpha)$  itself, BB establish that

$$E[(y_{it} - \alpha y_{i,t-1}) \triangle y_{i,t-1}] = 0 \text{ for } t = 3, 4, .., T$$
(3)

BB provide simulation evidence that the use of these additional moment conditions yields substantial gains in terms of the properties of the 2-step GMM estimators (especially in the 'weak instrument' case which occurs for values of  $\alpha$  approaching 1). The ability to test their validity reliably is therefore of some importance. Again following BB, we refer to the combined set of moment conditions given by (2) and (3) as the SYS moment conditions. This paper is concerned with how to test the validity of the DIF and SYS moment conditions given a particular data set.

The validity of the moment conditions implied by DPD models is commonly tested using the conventional GMM test of overidentifying restrictions associated with Sargan (1958) and Hansen (1982).<sup>1</sup> (For a description of the testing procedure in the DPD setting see Arellano and Bond (1991).) Several studies of moment condition models in other contexts have found that the Sargan test has poor size properties for samples of the size commonly encountered in econometric practice (see for example Altonji and Segal (1996), and Burnside and Eichenbaum (1996)). The work of Imbens, Spady and Johnson (1998) highlights this problem and proposes a number of alternative 'Tilting Parameter' tests of overidentifying restrictions. These are motivated by the exponential tilting approach to GMM estimation discussed in Imbens (1997). The authors find that the size properties of the robust Tilting Parameter test based on the conventional GMM estimator (referred to here as the TP test) are superior to those of the conventional Sargan test in the particular moment condition models that they examine.<sup>2</sup> Section 3.1 below uses Monte Carlo experimentation to examine whether this finding also holds in the case of the AR(1) DPD model and whether the size properties of the Sargan test are a cause for concern in this context.

The tests of overidentifying restrictions considered in section 3.1 test the validity of the entire set of DIF moment conditions given in (2). These moments involve the use of *all* available levels of  $y_{it}$  that are valid instruments for the first differenced equations under the maintained assumptions of Arellano and Bond (1991). The use of this 'full instrument set' results in the

number of moment conditions tested growing rapidly as T increases. The question of how many moments to use for a given sample size has been addressed in the context of the properties of GMM estimators by Anderson and Sørenson (1996) and Koenker and Machado (1999). However, the effect of the dimensionality of the moment conditions tested on the finite sample properties of tests of overidentifying restrictions is less well studied and is one of the contributions of this paper. The increasing availability of microeconomic panels with moderately large values of T raises the question of how test statistics (and estimators) based on the full instrument set perform in this setting.

To anticipate the main finding of section 3.1, our results suggest that the size properties of the Sargan test are less 'sensitive' to the number of moment conditions becoming large (for a given cross-sectional sample size, N) than are those of the TP test. Nevertheless the asymptotic approximation to the null distribution of the Sargan test is found to become very poor at values of (N,T) that are empirically relevant. Section 3.2 therefore investigates the effects on the size and power properties of the Sargan test of a simple and commonly used procedure for reducing the dimensionality of the moment conditions. This procedure omits instruments with long lag lengths and can be readily implemented using software such as DPD for Ox (see Doornik, Arellano and Bond (1999)). Our results give guidance as to when the use of tests based on the full instrument set is problematic and show that restricting the instrument set in the way described here can offer substantial gains in terms of power even when the test based on the full instrument set remains correctly sized.

The structure of the paper is as follows: section 2 describes the design of the Monte Carlo experiments; section 3 reports our results; and section 4 concludes.

## 2 Simulation Design

We consider the AR(1) DPD model given by

$$y_{it} = \alpha y_{i,t-1} + \eta_i + v_{it}$$

$$v_{it} = e_{it} + \gamma e_{it-1}$$

$$(4)$$

where i = 1, ..., N; t = 2, ..., T;  $T \ge 3$ ;  $|\alpha| < 1$  and  $\eta_i$  is an unobserved individual-specific effect. All of the experiments reported in section 3 below employed a special case of (4) as the data generating process (DGP), with the error components specified as

$$(e_{i1}, ..., e_{iT}, \eta_i)' \sim iid N(0, I)$$
 (5)

where *iid* stands for independent and identically distributed. The initial conditions were generated by

$$y_{i1} = \frac{\eta_i}{1 - \alpha} + u_i \tag{6}$$

where  $u_i$  was *iid*  $N(0, \frac{1}{1-\alpha^2})$ , and independent of  $\eta_i$  and  $e_{it}$  for t = 1, ..., T.<sup>3</sup> Each reported result is based on the generation of 5000 artificial panels.

Given our specification of the error components and initial conditions process, setting  $\gamma = 0$ in (4) implies the validity of both the DIF and SYS moment conditions.<sup>4</sup> If  $\gamma \neq 0$  in (4) then both (2) and (3) are invalid moment conditions.

It is worth considering (2) in some detail. Letting  $m_T$  denote the total number of DIF moment conditions when all of the available instruments are utilised, we see that  $m_T = 1 + 2 + 3 + ... + (T-2) = 0.5(T-1)(T-2)$ . As was noted above, this results in the number of moment conditions increasing rapidly as the time series dimension of the panel increases: for example,  $m_T = 6,28,66$  for T = 5,9,13 respectively. Equation (2) can also be written as  $E(Z'_i \triangle v_i) = 0$ where  $\triangle v_i = (\triangle v_{i3}, \triangle v_{i4,...}, \triangle v_{iT})'$  and  $Z_i$  is the  $(T-2) \ge m_T$  matrix of instruments given by

We refer below to the non-zero entries in this  $Z_i$  matrix as the full instrument set for the first differenced equations.

#### 3 Results

#### 3.1 Comparison of the Sargan and TP Tests

Recall that under the null hypothesis of moment condition validity, both the Sargan and TP tests have an asymptotic  $\chi^2$  distribution with degrees of freedom equal to the number of overidentifying restrictions. Table 1 shows the effects of varying the time series dimension (*T*) of the panel on the null rejection frequencies of the Sargan and TP tests of DIF and SYS moment conditions. The parameters of the DGP were  $\alpha = 0.4, \gamma = 0$  and the cross-sectional sample size was N = 100. *T* was increased from 5 to 7 and finally to 9. The full instrument set for the first differenced equations was used in each case. The results are also shown as a set of QQ plots in Figure 1.<sup>5</sup>

Whilst the null rejection frequencies for the Sargan tests are well approximated by the nominal sizes for all three values of T, the TP tests become increasingly 'oversized' as T increases. The nominal critical values of the TP test are very unreliable when testing the SYS moments for a panel with N = 100, T = 7 and when testing both sets of moment conditions for a panel with N = 100, T = 9. Figure 1 gives a graphical demonstration of the fact that the null distributions of the TP tests are poorly approximated by the relevant asymptotic  $\chi^2$  distributions in these cases. This is worrisome given the common occurrence in applied microeconomic work of panels with dimensions similar to these. In contrast, the Sargan test appears to be relatively robust to the number of moment conditions becoming larger for a given value of N. Nevertheless, the QQ plots reveal that its asymptotic approximation is beginning to worsen when T = 9. Repetitions of the above experiment for different values of  $\alpha$  added little to our conclusions and so are not reported here. In particular, the results for the weak instrument case with  $\alpha = 0.9$  were very similar to those shown in Table 1.<sup>6</sup>

Holding N fixed at 100 and increasing T again so that T = 11, 13, 15 resulted in null rejection frequencies for the Sargan test of the DIF moments equal to 0.08, 0.02 and 0.00 respectively (when the 10% nominal critical values were used and  $\alpha = 0.4$  as in Table 1). The striking finding that the Sargan test based on the full instrument set essentially never rejects when T (and hence the number of moment conditions) becomes too large for a given value of N was a general one. With  $\alpha = 0.4$ , zero null rejection frequencies (using the 10% nominal critical value) were also associated with the following (N, T) pairs: (125, 16), (85, 13), (70, 12), and (40, 10). Section 3.2 below investigates the effects on the size and power of the Sargan test of reducing the number of moment conditions tested while the dimensions of the panel are held constant.

#### 3.2 Reducing the Number of Moment Conditions

Consider testing the validity of a subset of the DIF moment conditions given in (2) by exploiting only available instruments with a lag length less than or equal to l for each differenced equation. Formally, we consider the Sargan test  $(S_l)$  of the moment conditions

$$E[y_{i,t-s}(\triangle y_{it} - \alpha \triangle y_{i,t-1})] = 0 \text{ for } t = 3, .., T \text{ and } s = 2, .., r$$
(8)

where  $r = \min(l, t - 1)$ . For example, when l = 3 and T = 6, the instrument matrix in (7) is replaced by

$$\begin{pmatrix}
y_{i1} & 0 & 0 & 0 & 0 & 0 \\
0 & y_{i1} & y_{i2} & 0 & 0 & 0 \\
0 & 0 & 0 & y_{i2} & y_{i3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & y_{i3} & y_{i4}
\end{pmatrix}$$
(9)

Varying l between (T-1) and 3 then creates a set of tests, all of which can be regarded as tests of the null hypothesis that there is no serial correlation in the error term  $(H_0 : E(v_{it}v_{is}) = 0$  $\forall t \neq s$  and  $\forall i$ ). Note that in the case of the AR(1) model considered here, the invalidity of an instrument with a lag length greater than 3 will usually also result in the invalidity of  $y_{i,t-2}$ and  $y_{i,t-3}$  as instruments because of the autoregressive structure of the model. Therefore if the  $S_3$  test were found to have good size and power properties in finite samples, then its failure to reject in some application could be regarded as good evidence for the validity of the full set of DIF moment conditions in that case.

The top half of Table 2 shows the effect on the size and power of the Sargan test of reducing l when N = 100 and T = 15.  $\alpha$  was again set to 0.4 and the estimates of power were obtained under the alternative  $\gamma = 0.3$ . As was noted above, the test has zero rejection frequencies under the null for high values of l. Reducing l improves the size properties of the test, with the  $S_3$  test having a rejection frequency that is reasonably close to the nominal size. (The  $S_2$  test is discussed separately below.) The improvement in the approximation of the finite sample null density by the relevant asymptotic  $\chi^2$  density is depicted in Figure 2. Such an improvement is not surprising given that reducing l results in a significant reduction in the number of moment conditions tested. However, Figure 2 has two important implications. First, a panel with T = 15 is not unusual in applied microeconomics and an unwary researcher might not regard such a value of T as particularly large compared to a sample size of N = 100. Indeed, our experiments showed that the conventional GMM estimator of  $\alpha$  based on l = 14 is well behaved in this context and has a slightly lower root mean squared error than the one based on l = 3. In contrast, the

 $S_{14}$  test has a null distribution which is far from its asymptotic distribution and (as is discussed below) also has no power to detect invalid moment conditions. Second, the simple procedure discussed above for reducing the number of moment conditions tested results in a Sargan test (the  $S_3$  test) with good size properties in a panel of these dimensions.

We now consider the effect on the power of the Sargan test of reducing l (again with N = 100and T = 15). The  $S_{14}$  test based on the nominal critical values has no power to detect the serial correlation in the error term  $(v_{it})$  introduced by setting  $\gamma = 0.3$ , whereas the rejection frequency of the  $S_3$  test is approximately 90%.<sup>7</sup> The size-corrected powers show that the low (uncorrected) powers associated with high values of l are not solely the result of size distortions. Thus, we conjectured that reducing l might also result in increased power to detect serial correlation in settings where Sargan tests based on high values of l are correctly sized. The bottom half of Table 2 shows such a case. All features of the experiment were unchanged except that N was increased to 200 and the powers were estimated under the alternative  $\gamma = 0.2$ . Note that the null rejection frequencies are now very close to the nominal size for all values of l. The power of the  $S_l$  test nevertheless increases as l decreases: the power of the  $S_3$  test is approximately double that of the test based on the full instrument set  $(S_{14})$ . The finding that reducing l can result in a substantial improvement in power when the Sargan test based on the full instrument set is correctly sized was quite widespread. Although the results are not reported here, experiments were also performed for tests of 'SYS-type' moments (that is, tests which also test the validity of (3) for all values of l), for DGP's with  $\alpha = 0.9$  and for DGP's with an AR(1) rather than a MA(1) error term.

Arellano, Hansen and Sentana (1999) have recently proposed testing for a lack of identification in the AR(2) DPD model

$$y_{it} - \eta_i = \alpha_1 (y_{i,t-1} - \eta_i) + \alpha_2 (y_{i,t-2} - \eta_i) + v_{it}$$
(10)

by calculating a Sargan test of the moment conditions given in (11) below.

$$E[y_{i,t-s}(\Delta y_{it} - \delta \Delta y_{i,t-1})] = 0 \text{ for } t = 3, .., T \text{ and } s = 1, .., (t-1)$$
(11)

Although detailed results are not presented here, we found that similar concerns about the size and power properties of Sargan tests based on a 'full instrument set' also apply in this context. In an experiment with N = 100, T = 14 the Sargan test of (11) was found to have a zero rejection frequency both under the null of no identification ( $\alpha_1 = 1.4, \alpha_2 = -0.4$ ) and under the alternative of overidentification ( $\alpha_1 = 0.2, \alpha_2 = 0.3$ ). That is, the test has no power when Tbecomes too large relative to N. Again we found that this problem could be avoided by testing a subset of the moment conditions in (11).

### 4 Conclusions

The finite sample performance of alternative tests of overidentifying restrictions has been investigated in the context of the AR(1) DPD model. Our results highlight the importance of the dimensionality of the moment conditions relative to the cross-sectional sample size.

In contrast to the moment condition models studied by Imbens, Spady and Johnson (1998), the Tilting Parameter test is found to have worse size properties in this context than the conventional Sargan test. Indeed, the Tilting Parameter test is oversized except in cases where very few moment conditions are tested. This limits its potential usefulness as a test of DPD model specification to panels where T is 'small' and there are few (or no) predetermined or exogenous regressors.

Despite being more robust than the Tilting Parameter test to the number of moment conditions becoming larger (with N fixed), the Sargan test based on the full instrument set was nevertheless found to have a zero null rejection frequency and 'no power' in panels where (N, T)would not be judged unusual by the standards of data sets currently available. We thus advise caution in the use of Sargan tests based on the full instrument set. A simple procedure ('reducing l') is considered which often results in a Sargan test with good size and power properties in cases where T is large relative to N. Furthermore, Sargan tests based on a restricted instrument set can offer substantial gains in terms of power to detect serial correlation in the error term even when the test based on the full instrument set is correctly sized.

The main implications of our findings for applied work are as follows. In the case of the AR(1) model, it is advisable always to calculate the  $S_3$  test statistic. For models with one or several  $x_{it}$  regressors, the tendency for the number of moments to become very large as T increases is even more pronounced. A suitable means of reducing that number when T is moderately large (perhaps involving the calculation of several test statistics, each of which tests the validity of some subset of the moment conditions) will need to be devised.

A diagnostic procedure that is able to detect cases where the Sargan test has very low power – due to the number of moment conditions tested being too large relative to N – would have obvious appeal. In the DPD context, we propose the calculation of the Sargan statistic for an 'expanded instrument set' that consists of the instrument set of interest together with  $y_{i,t-1}$ as an instrument for each differenced equation.<sup>8</sup> Since  $y_{i,t-1}$  is an invalid instrument under the assumptions of the maintained model, the failure of this 'diagnostic statistic' to reject in a particular setting can be interpreted as evidence that the usual Sargan test of moment condition validity is likely to be unreliable. In experiments using the AR(1) model, we found that this diagnostic procedure has good finite sample properties. Testing overidentifying restrictions in DPD models with several  $x_{it}$  regressors (and thus a much larger number of moment conditions) and the use of the diagnostic statistic in this context will be the subject of future research.

#### Notes

<sup>1</sup> Hereafter this test is referred to simply as a Sargan test.

<sup>2</sup> The TP test is denoted by  $T_{gmm(r)}^{LM}$  in Imbens, Spady and Johnson (1998).

<sup>3</sup> This description of the initial conditions process holds exactly for the experiments in section 3.1. For those in section 3.2, the  $y_{i1}$ 's were generated by the same dynamic process as (4) initialised at zero and after a 'burn in period' of 20 observations. When  $\gamma = 0$  this initial conditions process can be made arbitrarily close to the one in equation (6) by making the 'burn in period' sufficiently large.

<sup>4</sup> When  $\gamma = 0$  our specification of  $u_i$  implies that  $y_{it}$  is a covariance stationary process. As discussed in Section 1, BB establish that this is sufficient but not necessary for the validity of the additional moment conditions in (3).

<sup>5</sup> The QQ plots graph the quantiles of the Monte Carlo distribution against the corresponding quantiles of the relevant asymptotic  $\chi^2$  distribution on the abscissa.

<sup>6</sup> The results of experiments performed in connection with the content of this paper but not reported here are available from the author on request.

<sup>7</sup> Examination of Table 2 reveals that the  $S_2$  test has very similar rejection frequencies under the null and the alternative. This is because the source of overidentification when l = 2is parameter constancy ( $\alpha_t = \alpha$  for t = 3, ..., T) and as a result the  $S_2$  test only has power to detect parameter non-constancy.

<sup>8</sup> The diagnostic statistic we suggest is thus similar to the Arellano, Hansen and Sentana (1999) test for underidentification. Their concern is to test for lack of identification whilst maintaining that the Sargan test of overidentifying restrictions has some power. Our concern here is rather to investigate whether the Sargan test has any power, whilst maintaining that the model is identified.

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	DIF moments		SYS moments	
Nominal size	Sargan	ТР	Sargan	TP
T=5				
0.10	0.104	0.112	0.102	0.110
0.05	0.051	0.051	0.048	0.056
T=7				
0.10	0.106	0.153	0.111	0.207
0.05	0.048	0.081	0.051	0.118
T=9				
0.10	0.112	0.351	0.116	0.536
0.05	0.042	0.226	0.046	0.408

TABLE 1. – THE EFFECT OF INCREASING T on the Null Rejection Frequencies of the Sargan and TP Tests

Notes: <sup>a</sup> Each reported result is based on a DGP with  $N = 100, \alpha = 0.4$ , and  $\gamma = 0$ ; the test statistics were calculated using the full instrument set for the first differenced equations; 5000 replications were performed. <sup>b</sup> Rejection frequencies using both the nominal 10% and 5% critical values are shown.

Figure 1. – QQ Plots Showing the Effect of Increasing T on the Size Properties of the Sargan ( —— ) and TP ( - - - - ) Tests



Notes: <sup>a</sup> N is held fixed at 100 in all cases. Each row contains a pair of QQ plots for the indicated value of T, one for each set of moment conditions. <sup>b</sup> Solid lines indicate Sargan tests and broken lines TP tests; the vertical lines are drawn at the 10% and 5% nominal critical values. <sup>c</sup> The DGP used in each case set  $\alpha = 0.4$  and  $\gamma = 0$ ; the test statistics were calculated using the full instrument set for the first differenced equations; 5000 replications were performed.

			l			
	14	11	7	5	3	2
N = 100						
Null rejection frequency	0.000	0.000	0.024	0.051	0.085	0.073
Uncorrected power	0.000	0.000	0.234	0.596	0.905	0.076
Size-corrected power	0.217	0.258	0.487	0.732	0.921	0.097
N = 200						
Null rejection frequency	0.099	0.104	0.096	0.096	0.103	0.100
Uncorrected power	0.388	0.421	0.555	0.651	0.792	0.097
Size-corrected power	0.391	0.412	0.566	0.659	0.789	0.102

TABLE 2. – The Effect of Decreasing l on the Size and Power of the Sargan Test

Notes: <sup>a</sup> Results are shown for  $(N = 100, T = 15, \alpha = 0.4)$  and  $(N = 200, T = 15, \alpha = 0.4)$ ; null rejection frequencies and uncorrected powers were obtained using the 10% nominal critical values; the estimates of power were obtained under the alternative  $\gamma = 0.3$  when N = 100 and  $\gamma = 0.2$  when N = 200. <sup>b</sup> The results are based on 5000 replications.

Figure 2. – The Effect of Decreasing l on the Finite Sample Density of the Sargan Test under the Null



Notes: <sup>a</sup> Nonparametric estimates of the null densities of the  $S_l$  tests (solid lines) are compared with their asymptotic counterparts (broken lines); the vertical lines indicate the 10% nominal critical values. <sup>b</sup> N = 100, T = 15,  $\alpha = 0.4$ ,  $\gamma = 0$  in all four cases; panels (a),(b),(c) and (d) are for l = 14, 7, 5, and 3 respectively. <sup>c</sup> the results are based on 5000 replications; the density estimates were performed using the standard normal kernel and a window width proportional to 5000.<sup>-0.2</sup>