Estimating Equivalence Scales for Tax and Benefits Systems*

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1 Introduction

When comparing, say, the welfare derived from income by a family that is comprised of two adults and three children to that of a single adult, it is necessary to take into consideration the relative needs of the respective households. The most common means by which applied studies in economics currently relate the needs of heterogeneous income units is through the use of equivalence scales. Despite a considerable research effort, however, almost every aspect of equivalence scale specification remains controversial. What characteristics should equivalence scales take into account? Should the scales apply an additive or multiplicative adjustment to income? Is the assumption of base independence valid?[1] How should a reference unit be selected? Is it reasonable to assume that there is no inequality within an income unit? What criteria are most sensible for selecting a functional form? And, arguably most important, do the cardinal relations implied by equivalence scales permit income units to be compared in terms of underlying welfare? All of these questions remain largely unresolved.

This paper is concerned with estimating the relativities that are implicit in tax and benefits policy. Using observed tax and benefits payments to estimate equivalence scales may mitigate some of the criticisms to which alternative scale specification criteria have been subject. For example, most econometric estimates of equivalence scales used for distributional analysis are based on consumer demand behaviour. Pollak and Wales (1979, p. 216) have notably criticised this methodology on the basis that "the equivalence scales required for welfare comparisons are logically distinct from those which arise in demand analysis".[2] The central difficulty is that demand analysis fails to provide a

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[1] Base independence (Lewbel, 1989, and Blundell and Lewbel, 1991) requires the equivalence scale to be unaffected by the level of utility (or income). This requirement is referred to as 'equivalence scale exactness' by Blackorby and Donaldson (1993).

[2] Deaton and Muellbauer (1980, chapters 7, 8 and 9) provide a comprehensive discussion of the theoretical underpinnings of the demand based approach for estimating equivalence scales. Muellbauer (1975) pointed out the difficulties of making welfare comparisons, given taste differences of the type used in equivalence scale models.

basis for making cardinal comparisons of welfare between households, and so equivalence scales that are estimated from expenditure data necessarily depend upon exogenously imposed value judgements. In contrast, part of the intuitive appeal of equivalence scales based on a country's transfer system is the perception that such relativities embody a social consensus; that the tax and benefits system, being an observable instrument of government, can be used to infer the value judgements made by government when acting in its role as administrative agent for society.[3]

Comparison of the relativities that underlie taxation with equivalence scales based on the costs borne by heterogeneous income units could also provide a useful means for evaluating the adequacy of associated provisions made by transfer systems. Alternatively, the equivalence scales implicit in transfer systems could be used to compare the provisions made through time and/or between countries. Such equivalence scales could also play a role in the tax design process itself. In view of the fact that the redistributive systems of many countries are comprised of numerous different tax and benefit schemes, it is difficult to ensure that the overall system does in fact achieve the desired redistribution. Information about implicit scales – which may on reflection be found to differ from the values held by policy makers – can be useful in suggesting that certain features of the tax system need adjusting.

The prevailing uncertainty regarding how an equivalence scale should be specified has particular relevance when considering the redistributive effects of taxation. This is because measures of progressivity and horizontal inequity calculated from equivalised income are often observed to depend upon the equivalence scale assumed. Hence, distributional analyses of equivalent income are typically subject to the criticism that horizontal inequity is 'imposed from the outside'.[4] The analysis presented in this paper essentially reverses the prevailing methodology by assuming that *horizontal equity* is, on average, satisfied by a transfer system. This assumption enables the relativities implicit in transfer policy to be inferred from observed tax and benefits payments.

The reversal of methodology suggested here is analogous to the analysis presented in a recent paper by Bourguignon and Spadaro (2002), which considers the social preferences that are consistent with observed marginal tax rates. The traditional approach adopted to evaluate a transfer system using optimal tax theory involves assuming a 'reasonable' social welfare function, and then comparing the implied 'optimal tax schedule' with the schedule that is observed in practice.[5] In contrast, Bourguignon

[3] See, for example, Atkinson & Stiglitz (1980, pp. 404-405), and Foley (1967). This argument in favour of equivalence scales based on tax and benefits systems has been questioned by Coulter *et al.* (1992, p. 100) who, referring to the social security reforms that were made in the UK in April 1988, state that the "controversy that accompanied the social security changes gives little support to the idea that the reforms represented a social consensus opinion." Alternatively, Atkinson & Stiglitz (1980, p. 9) warn that "Tax and expenditure policy may be designed more with a view to electoral success, or the goals of an established bureaucracy, than to social welfare maximisation."

[4] See referee comments cited by Lambert (2003, footnote 2).

[5] This methodology is attributable to Mirrlees (1971, 1986). Recent literature in this field includes Diamond (1998),

and Spadaro (2002) "consider the effective marginal tax rates schedule that corresponds to an actual redistribution system and...look for the social welfare function according to which that schedule would be optimal" (p. 2). The analysis advocated by Bourguignon and Spadaro (2002) thus enables a practitioner to determine whether the social welfare function implied by the observed marginal tax rate schedule is consistent with *a priori* expectations. Similarly, the framework of analysis presented in this paper enables the relativities implicit in transfer policy to be compared against the consistency and monotonicity properties that could be expected of a 'sensible' equivalence scale.[6] Whereas the approach suggested by Bourguignon and Spadaro (2002) interprets observed marginal tax rates in terms of social welfare, the approach presented here interprets the relativities implicit in a transfer system in terms of the equivalence scale methodology.[7]

Very few studies have attempted to estimate the equivalence scales implicit in tax policy, and those that do have focused on a subset of the transfer system.[8] This apparent lack of interest can be attributed to the perception that "income taxes are not typically coherent with equivalence scales" (Lambert, 1993, p. 364). In the absence of a generally agreed optimal solution[9], it is unsurprising that different countries have adopted transfer systems that take a range of different forms. The system that is perhaps the most transparently consistent with the equivalence scale methodology is the *quotient familial* applied in France, as described by Atkinson *et al.* (1988). Others, such as the system of exemptions and credits applied in the UK, bear less resemblance to the equivalence scale framework.[10] The perception that many tax and benefits systems are inconsistent with the equivalence scale methodology is strengthened by the observation that the individual tax and benefit schemes from which transfer systems are comprised often imply different relativities. How should an analyst decide which scheme, and consequently which relativities, to use? Furthermore, given that many benefits provide safety-net incomes, how can an analyst be sure that the implied equivalence scales are relevant for the entire distribution of income, rather than just the bottom tail?

An equivalence scale that embodies only part of a tax and benefits system is evidently of limited

Saez (1998), Salanié (1998), d'Autume (1999), and Bourguignon and Spadaro (2000), cited by Bourguignon and Spadaro (2002).

[6]If the scales implied by transfer policy are found to be inconsistent with *a priori* expectations (are found, for example, to be decreasing in household size), then this may reveal that existing transfer policy is inconsistent with the concept of horizontal equity - which is itself an interesting finding.

[7]The applied analysis presented by Bourguignon and Spadaro (2002) considers two extremes with regard to the relationship assumed between heterogeneous households. In the first instance household differences are ignored. In the second, households are considered exclusively within homogenous demographic subgroupings, from which qualitative comparisons are made.

[8]See, for example, the Royal Commission on the Distribution of Income and Wealth (1978), which calculates equivalence scales based on the short-term scale rates of supplementary benefits.

[9]See, for example, the impossibility result of Moyes and Shorrocks (1998).

[10]See Lambert and Yitzhaki (1997)

interest.[11] Section 2 introduces a formal framework that is consistent with the equivalence scale methodology, and demonstrates how the framework relates to the concept of horizontal equity. It is frequently assumed that the adjustments made by fiscal policy for household heterogeneity describe a range of equivalence scales, rather than a single set of relativities.[12] In Section 3 it is shown that the suggested framework, based on a single equivalence scale, is sufficiently flexible to reflect *any* transfer system. The generality of the model is a product of relaxing all *a priori* restrictions, including the assumption of base independence, upon the equivalence scale. In Section 4, a non-parametric procedure for estimating equivalence scales implicit in tax and benefits systems is introduced. In Section 5 estimates are reported for the relativities implicit in the Australian transfer system.[13] The equivalence scale estimates derived from tax and benefits data are also compared with a range of alternative scales, including those used by government agencies in Australia, the scale proposed by the OECD, and scales estimated from household expenditure data. It is found that the scales implicit in the Australian transfer system compare well with the official and demand based scales, and the relationship with income is both interesting and intuitive. Conclusions from the study are summarised in Section 6.

2 Equivalence Scales and Taxation

Assume that the redistributive objectives of tax policy designers (the government) are framed in terms of income per equivalent adult. Values in adult equivalent terms, as measured by the government, are denoted by a * superscript. Let a_i^* denote the number of equivalent adults in tax unit i. Each a_i^* can be regarded as a function of a set of m observable variables (which may include income) of the tax unit $v_i = (v_{1i}, ..., v_{mi})$, so that:[14]

$$a_i^* = a^* (v_i) \tag{1}$$

Let x_i and y_i denote the pre-tax and post-tax income of tax unit i.[15] Hence, $x_i^* = x_i / a_i^*$ and $y_i^* = y_i / a_i^*$ are respectively pre-tax and post-tax equivalent income.

Given the distribution of x_i^* for a population, the government is considered to impose a tax structure that is capable (subject to a budget constraint) of achieving its distributional aims. This involves a

[11] See Ebert and Lambert (1999).

[12] See, for example, Coulter *et al.* (1992).

[13] The Australian transfer system is considered here in response to the prior expertise of the authors. Estimating the equivalence scales implicit in the UK transfer system is planned to be the subject of a subsequent study.

[14] Seneca and Taussig (1971) recognised the importance of allowing the scales implicit in transfer policy to be income dependent. See, also, the empirical results presented by Banks and Brewer (2002).

[15] Throughout this paper, pre-tax and post-tax income refer, respectively, to income gross and net of associated tax and benefit payments.

tax function, $T^*(x_i^*)$, so that the net adult equivalent income of unit i is:

$$y_i^* = x_i^* - T^*(x_i^*) \tag{2}$$

In equation (2), the tax function depends only upon pre-tax equivalent income, which ensures that the requirement of horizontal equity is satisfied.[16] This assumption can be relaxed by adding an individual specific term, ε_i', to the right hand side of equation (2).

Assuming that the same equivalence scale is applied to both pre-tax and post-tax income, equation (2) translates to:

$$y_i = x_i - a_i^* T^* \left(\frac{x_i}{a_i^*} \right) \tag{3}$$

This specification was alluded to by Vickrey (1947, pp. 295-296) who wrote; "A more thoroughgoing and equitable procedure [than exemptions and credits] would be to set up some factor indicative of the needs of the entire family, divide the total income by this factor, compute a per capita tax on this 'per capita income', and multiply the tax so computed by the family size factor to obtain the total tax for the family".[17]

2.1 Family size, horizontal equity, and taxation

Horizontal equity is the command that equals be treated equally by a transfer system. Although there is widespread support for this concept, there is an ongoing debate regarding who should be defined as equals and what constitutes 'equal tax treatment'.[18] Consequently, it is important to provide a clear definition of what horizontal equity means here, and how it may be used to motivate the formal framework considered by this paper.

Horizontal equity is defined here as the requirement that: "*If two individuals would be equally well off (have the same utility) in the absence of taxation, they should also be equally well off if there is a tax*" (Feldstein, 1976, p. 83, emphasis in the original). Consider, for example, a population that is comprised of one and two adult households (singles and couples), where households are further differentiated by their respective ability levels (wage rates). Household size is the only non-income characteristic that is relevant for tax purposes, and is defined exogenously - hence behavioural effects are not considered in this dimension.[19] Define s subscripts for single adult households and c subscripts

[16] The "principle of equity, or *horizontal equity*, is fundamental to the ability-to-pay approach, which requires equal taxation of people with equal ability and unequal taxation of people with unequal ability" (Musgrave, 1959, p. 160). This requirement is discussed further in the following subsection.

[17] Quoted by Lambert and Yitzhaki (1997, p. 346). See also Pyatt (1990), and Ebert (1997).

[18] See Lambert (2003) and Ebert and Lambert (2002) for alternative interpretations of horizontal equity and their relation to equivalence scales.

[19] See, for example, Nerlove *et al.* (1984), and Barro and Becker (1989) for models of endogenous fertility.

for couples. Pre-tax and post-tax income are defined by x and y respectively (as in the previous subsection), and it is assumed that pre-tax income is equal to earnings derived through labour, l, via the (unobserved) wage rate, w, $x = wl$. The variable, l, is defined as the proportion of a household's total time spent on labour, which takes a value between zero and one for both singles and couples. The government is considered to impose a tax schedule, $T_k(x)$, which may differ between couples and singles (hence the subscript k), such that household pre-tax and post-tax income are related by:

$$y = wl - T_k(wl) \tag{4}$$

The household utility function, $u(y, l)$, is assumed to be strictly quasi-concave, (strictly) increasing in y, (strictly) decreasing in l, and continuously differentiable. Furthermore, it is assumed that $u \to -\infty$ as $l \to 1$ from below. An assumption of equal sharing is made, such that every member of a household enjoys the same level of utility, indicated by the function $u(.)$.[20] An equivalence scale, $a_c = a(u_c)$, relates the utility of each member of a couple to a single adult household by:[21]

$$u_c(y_c, l_c) = u_s\left(\frac{y_c}{a_c}, l_c\right) \tag{5}$$

Equation (5) reflects the fact that a couple are likely to convert post-tax income into utility differently to single adults because of the need to share. Labour, in contrast, is not similarly affected. No attempt is made to estimate the utility functions that are considered here - rather the functions $u(.)$ can be interpreted as the utility that the government attributes to households when formulating transfer policy.[22]

Households are assumed to select their labour supply (given perfectly competitive markets, no profits, and a clearing labour market) to maximise their utility, subject to the budget constraint defined by equation (4).[23] For single adult households the solution is obtained by maximising the Lagrangian:

$$\mathcal{L} = u_s(y, l) + \lambda(wl - T_s(wl) - y) \tag{6}$$

[20]This assumption is made in view of the scarcity of data regarding intra-household distributions. See, for example, Kaplow (1996) for an explicit consideration of alternatives to the assumption of equal sharing.

[21]Equation (3) implies that a_c can be stated as a function of (observable) income, whereas it is fundamentally a function of (unobservable) utility. For the government to be able to impose a horizontally equitable tax, it must be able to infer a household's ability by observing its type and income. Hence, it is necessary for there to exist a one-to-one mapping of pre-tax income to ability for any household type. That is, we require $\frac{\partial l}{\partial w} > -\frac{l}{w}$ for all conceivable abilities (wage rates).

[22]Rosen (1978) considers a similar framework to the one defined here, but takes a different view by estimating utility functions from household income and labour data to consider the issue of horizontal equity.

[23]The model considered here assumes that individuals are free to choose how much labour they supply. Involuntary unemployment could be included in the model, by recognising it as an individual characteristic (in much the same way as health or invalidity might be included as a characteristic in the equivalence scale specification).

Defining equivalent post-tax income for couples by $y^* = y/a_c$, and the equivalent wage rate by $w^* = w/a_c$, the decision for couples can be identified by maximising:

$$\mathcal{L} = u_s\left(y^*, l\right) + \lambda\left(w^* l - \frac{T_c\left(a_c w^* l\right)}{a_c} - y^*\right) \quad (7)$$

The first order conditions of equations (6) and (7) indicate that there is a critical wage rate for singles (w_{s0}), and for couples (w_{c0}^*), such that a wage rate in excess of the critical rate is sufficient to induce some labour supply, $l > 0$. To simplify the analysis, the wage rate of any household is assumed to be sufficient to reach an interior solution throughout the remainder of this section.

Substituting $T_s\left(wl\right) = T_c\left(wl\right) = 0$ into equations (6) and (7) reveals that, when single adult and two adult households enjoy the same equivalised ability (w^* in equation (7) $=$ w in equation (6)), they are subject to the same maximising problem in the absence of taxation and hence, given the assumptions made, will enjoy the same utility. We therefore define households as pre-tax equals if they possess the same equivalised ability.[24] The specification of pre-tax equals as households with the same equivalised ability (and the same pre-tax equivalent income[25]) can be contrasted with the framework of Balcer and Sadka (1986), and Banks and Brewer (2002), who define pre-tax equals as households with the same *pre-tax unequivalised* incomes, and to Seneca and Taussig (1971) who define pre-tax equals as households with the same *post-tax unequivalised* incomes.

A horizontally equitable tax in the current context must consequently imply that the same level of post-tax utility will be enjoyed by households with the same equivalised ability. Comparison of equation (6) with (7) reveals that single adult and couple households with the same equivalised ability will enjoy the same utility if their respective tax functions are related by:

$$T_c\left(x\right) = a_c T_s\left(\frac{x}{a_c}\right) \quad (8)$$

which restates equation (3).[26]

Given the strong support that horizontal equity has received from a diverse range of views regarding redistributive justice[27], it seems reasonable to suppose that it does play an important role in the design of transfer policy. This is the premise upon which the remainder of the current paper is based.

[24] This finding implies that, if the equivalence scale a_c embodies economies of scale, then couples will require a lower (average) ability than single adults to enjoy the same utility. Consider, for example, the case when $a_c = 1.6$, and each member of a couple supplies the same number of labour hours and earns the same hourly wage rate. Given these conditions, if each member of a couple are to enjoy the same utility as a single with ability w_s, then $w_c = a_c w_s = 1.6 w_s$. Since w_c is shared equally between the members of the couple, each must consequently earn 80 per cent of the single's wage.

[25] Pre-tax equals will share the same pre-tax equivalent income if there exists a unique solution to the utility maximising problem, which (given the assumptions made regarding $u_s\left(.\right)$) is ensured if $T_s\left(x\right) = T_c\left(x\right) = 0$.

[26] No attempt is made to generalise this discussion, which remains an issue for subsequent research.

[27] See, for example, Musgrave (1990) for a brief review.

Specifically, we assume that the government makes value judgements regarding the relative needs of different households, and that transfer policy is designed with reference to a population whose preferences are consistent with the value judgements made. It is important to note, however, that the value judgements (implicitly) made by a government may (and arguably *will*) bear little resemblance to the preferences of the actual population. Furthermore, policy objectives that run orthogonal to horizontal equity are likely to complicate the relativities implicit in tax and benefits systems.[28] Hence, although equivalence scales that reflect the value judgements made by government are an important descriptive tool, they have limited appeal as a means of analysing social welfare. Interpretation of the welfare implications of equivalence scales implicit in transfer policy will necessarily depend upon how such scales relate to *a priori* expectations regarding, for example, monotonicity and uniformity.

3 Generality of the equivalence scale framework

Consider a transfer system that is comprised of numerous tax and benefit schemes, where each scheme depends upon household characteristics (such as the number and age of household members, their health status, pre-tax income, and so on), v, and may embody a different set of relativities to those of other schemes. It is clear that there will exist a tax function, $T(x, v)$, which relates pre-tax and benefit and post-tax and benefit income, such that:

$$y_i = x_i - T(x_i, v_i) \tag{9}$$

for each household i.[29] The generality of equation (9) is sufficient to capture a broad range of redistributive systems; from the quotient familial applied in France, to the system of exemptions and credits applied in the UK. How might the system described by equation (9) be related to the equivalence scale framework, using a single set of relativities denoted by a^*? While the equivalence scale framework is, in principle, quite general, a few restrictions, plausible in themselves, can result in significant restrictions on the tax functions. We first illustrate this proposition, and then consider some implications for the properties of the equivalence scale function a^* of assumptions about the tax function $T(x, v)$, such as being progressive and diminishing in household size, given x.

To illustrate restrictions on the tax functions, suppose that the equivalent tax function $T^*(x^*)$

[28] For example, unemployment benefits may be designed to encourage labour market participation, or family benefits specified to affect fertility rates (compare the different fertility objectives of state policy in France and China).

[29] As in the case of equation (2), equation (9) omits any horizontal inequity, consistent with the official specifications commonly implied by tax and benefit policy. Horizontal inequity can be included by adding an error term to the right hand side of equation (9).

takes the quadratic form:

$$T^*(x^*) = \beta_0 + \beta_1 x^* + \beta_2 x^{*2} \tag{10}$$

If a^* is a (potentially base dependent) equivalence scale, then from equation (3) we require:

$$\begin{aligned} a^* T^*(x^*) &= T(x,v) \\ &\therefore \quad \beta_0 a^{*2} + \beta_1 x a^* + \beta_2 x^2 = a^* T(x,v) \\ &\therefore \quad a^* = \frac{T(x,v) - \beta_1 x \pm \sqrt{(\beta_1 x - T(x,v))^2 - 4\beta_0\beta_2 x^2}}{2\beta_0} \end{aligned} \tag{11}$$

Supposing that $T(0,v) < 0$ for all v (consistent with the view that the tax system provides a net benefit to households with zero pre-tax income), then the negative square root of equation (11) must be selected to ensure that $a^* \neq 0$ when $x = 0$. When pre-tax income is equal to zero, equation (11) implies that:

$$a^* = \frac{T(0,v)}{\beta_0} \tag{12}$$

Hence, if the government provides a transfer benefit for all tax units with zero pre-tax income, such that $T(0,v) < 0$ for all v, then β_0 must be less than zero to obtain $a^*(0,v)$ greater than zero for all v. If $\beta_0 < 0$, then we require:

$$T(x,v) - \beta_1 x - \sqrt{(\beta_1 x - T(x,v))^2 - 4\beta_0\beta_2 x^2} < 0 \tag{13}$$

to observe $a^*(x,v) > 0$ for any x and v. Therefore:

$$T(x,v) - \beta_1 x < \sqrt{(T(x,v) - \beta_1 x)^2 - 4\beta_0\beta_2 x^2} \tag{14}$$

It is clear that

$$T(x,v) - \beta_1 x \leq |T(x,v) - \beta_1 x|$$

and that

$$|T(x,v) - \beta_1 x| < \sqrt{(T(x,v) - \beta_1 x)^2 - 4\beta_0\beta_2 x^2}$$

for any $x > 0$ when $\beta_0 < 0$ and $\beta_2 > 0$. Hence, assuming that $T(0,v) < 0$ for all v, we require $\beta_0 < 0$ and $\beta_2 > 0$ for $a^*(x,v)$ to be positive for any tax unit - the equivalent tax function must be strictly progressive in the sense that the average tax rate must be an increasing function of pre-tax income. This restriction on the tax function is the outcome of the three assumptions: a quadratic tax function, $T(0,v) < 0$ for all v,and $a^*(x,v)$ to be positive for any tax unit.

The restriction, $T(0,v) < 0$, considered above is consistent with most practical cases, since transfer systems generally provide a net benefit to households with no pre-tax income. There are, however, exceptions to this rule, particularly for households with large wealth stocks. If $T(0, v_i) < 0$ for some v_i and $T(0, v_j) \not< 0$ for some v_j, then equation (12) indicates that there are no equivalent tax parameters which will ensure that $a^*(0, v) > 0$ for all conceivable specifications of the tax unit. In the case of wealthy households, however, the equivalence scales obtained by the above framework do retain a sensible interpretation. When, for example, the transfer system provides no net benefit to a tax unit, v_j, with zero pre-tax income, $T(0, v_j) = 0$, then it is clear from equation (12) that $a^*(0, v_j) = 0$ for any $\beta_0 \neq 0$. This is consistent with the interpretation that the tax unit v_j has no tax relevant needs when it has no pre-tax income, a value judgement that is possibly justified for households with large stocks of wealth. Furthermore, it is arguably preferable to consider an income concept that includes all pecuniary accruals of wealth measured over a long period of time for households with large wealth stocks, in which case it is less likely that $T(0, v_j) \not< 0$ will be observed for some v_j.

3.1 Properties of the implicit equivalence scale

It is useful to consider how the properties of the equivalence scale a^* defined by equation (3) relate to properties of an observed tax and benefits system. Assume that the observed tax burden of household i depends upon pre-tax income, x_i, and household characteristics including the number, age, and health status of household members, v_i, such that post-tax income is given by:

$$y_i = x_i - T(x_i, v_i) \tag{15}$$

For equations (3) and (15) to be equivalent over the relevant domain of x and v, we need:

$$a^* T^*(x^*) \equiv T(x, v) \tag{16}$$

and:

$$T^*(x) \equiv T(x, v_0) \tag{17}$$

where $a^* = 1$ for the reference household type with characteristics v_0. Equations (16) and (17) can be combined into (18):

$$a^* T\left(\frac{x}{a^*}, v_0\right) \equiv T(x, v) \tag{18}$$

Equation (18) defines an implicit function for a^* in terms of pre-tax income, household characteristics and reference characteristics:

$$a^* = a^*(x, v, v_0) \tag{19}$$

To analyse the properties of the function a^*, consider the following assumptions about $T(.)$. Assume that the tax function $T(.)$ is differentiable in pre-tax income x and in size s, a component of v.[30]

[30] Differentiability with respect to size s, which is an integer, is easily relaxed by considering unit changes $\triangle s = 1$ in what follows.

Furthermore, we assume for the moment that $T(.)$ is progressive in x ($\partial \log T / \partial \log x > 1$) given v[31], and decreasing in household size, given x. Taking logs of equation (18) and differentiating with respect to s yields:

$$\frac{\partial \log a^*}{\partial s} = \frac{t_s}{(1 - t_x^*)} \tag{20}$$

where $t_s = \partial \log T(x, v) / \partial s$ and $t_x^* = \partial \log T(x/a^*, v_0) / \partial \log (x/a^*)$. The assumption of progressivity implies that $t_x^* > 1$. If, as we have assumed, $t_s < 0$ where s is a household size characteristic, then a^* will be increasing in s, as we should expect. If, in contrast, $T(.)$ is locally regressive, then the opposite conclusion may apply to specific pre-tax income domains.

The relationship between the equivalence scale a^* and pre-tax income x is now investigated. Differentiation in logs of equation (18) with respect to x implies:

$$\frac{\partial \log a^*}{\partial \log x} = \frac{t_x - t_x^*}{1 - t_x^*} \tag{21}$$

where $t_x = \partial \log T(x, v) / \partial \log x$. Since $1 - t_x^* < 0$, a^* will be diminishing in x if $t_x > t_x^*$ and increasing in x if $t_x < t_x^*$.[32] Some (progressive tax) examples illustrate the issues. First, consider the case of a linear tax and benefits system with credits:

$$T(x, v) = \alpha x - c(v) \tag{22}$$

Then equation (18) implies:

$$a^* \left[\alpha \frac{x}{a^*} - c(v_0) \right] = \alpha x - c(v) \tag{23}$$

and hence:

$$a^* = c(v) / c(v_0) \tag{24}$$

which implies that the equivalence scale function is base independent. Thus, if actual tax and benefits systems are close to this linear case, at least over some income ranges, we can expect a^* to be approximately independent of x over those ranges.[33]

Now consider a progressive tax and benefits system (in the sense that the marginal tax rate is increasing in pre-tax income) with exemptions:

$$T(x, v) = \alpha x + \beta x^2 - c(v) \tag{25}$$

[31] Empirically, this is often untrue. See, for example, Brewer and Clark (2002) who show that marginal effective tax rates are frequently higher at lower incomes in the UK.

[32] Local regressivity implies that the opposite observations may apply over limited pre-tax income domains.

[33] See also, Lambert and Yitzhaki (1997, p. 347).

where $\beta > 0$. For $a^* > 1$, it is possible to show that $t_x > t_x^*$ in this case, so that a^* is decreasing in x. An iso-elastic progressive system with exemptions has the same property. Here:

$$T(x, v) = \alpha x^{1+\beta} - c(v) \tag{26}$$

where $\beta > 0$. Hence the relationship between a^* and x is dependent upon the specification of the tax function. Furthermore, the examples considered here suggest that, where the tax function describes a progressive system with (income independent) exemptions, the equivalence scale will be decreasing in pre-tax income.

4 Estimating the Equivalence Scales Implicit in Transfer Systems

Given that the equivalence scales discussed in this paper are implicit, and hence unobservable, non-parametric estimation methods that impose limited *a priori* restrictions on the specification of the equivalence scale are particularly useful. The remainder of this paper is consequently concerned with non-parametric estimates. Two alternative procedures that use standard econometric regression techniques to estimate assumed tax and equivalence scale functions are described in Appendix A.[34]

4.1 Non-Parametric Estimation

A number of alternative non-parametric procedures can be devised for estimating the equivalence scales implicit in tax and benefits policy. The approach that is described here is one that we found useful when deriving the estimates that are reported in the following section. The approach involves three principal stages:

1. *Population Division.* The sample population used to estimate the equivalence scales is divided into subgroups, within which tax units are considered to be homogenous in all tax relevant respects other than pre-tax and post-tax income.

2. *Tax Function Estimation.* Standard non-parametric methods are used to estimate the tax functions of each individual subgroup identified in (1) above.

3. *Equivalence Scale Inference.* The tax functions estimated in (2) are used to obtain functions of the average tax rate versus pre-tax income, $AVt_i(x)$, for each of the subgroups, i, identified in (1). Let the subscript $i = 0$ denote the reference unit used to define the equivalence scale. The

[34] Regression techniques are complicated by the highly non-linear nature of the model described by equation (3). See Muellbauer and van de Ven (2003) for a detailed discussion of econometric estimation issues.

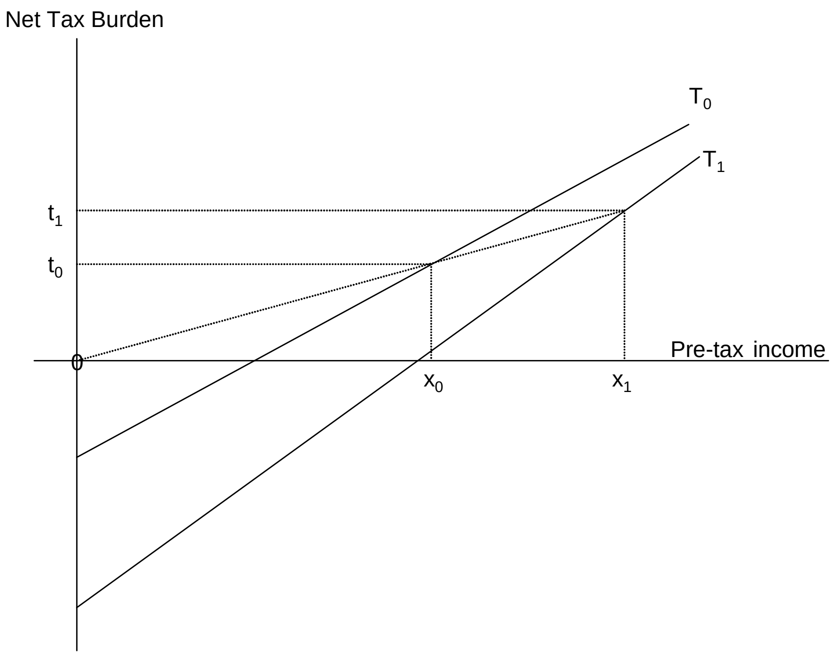


Figure 1: Relating the Average Tax Rates of Alternative Population Subgroups

equivalence scale of subgroup i with pre-tax income x_i is then obtained by:

$$a\left(x_i\right) = \frac{x_i}{x_0}, \text{ where } AVt_i\left(x_i\right) = AVt_0\left(x_0\right) \tag{27}$$

This last stage of the estimation procedure warrants some comment. If it is assumed that the same equivalence scale is applicable for pre-tax and post-tax income then:

$$\frac{y^*}{x^*} = \frac{(y/a)}{(x/a)} = \frac{y}{x} \tag{28}$$

Equation (28) indicates that the equivalence scale does not affect the ratio of pre-tax to post-tax income, or equivalently the average tax rate, $AVt\left(x\right) = T\left(x\right)/x = \left(x - y\right)/x$. The equivalence scale discussed here consequently applies a proportional adjustment to pre-tax and to post-tax income, so that the tax function of any household is mapped onto the tax function of the reference household by way of the average tax rate. This is described graphically in Figure 1.

Figure 1 displays tax functions for two household types, T_0 (singles) and T_1 (couples). Consider an equivalence scale that specifies the tax adjustment made for couples relative to the associated adjustment that is made for singles. When a couple earn pre-tax income x_1, their average tax rate is given by t_1/x_1, and the figure indicates that the equivalence scale is equal to $a\left(x_1\right) = x_1/x_0 = t_1/t_0$, $t_1/x_1 = t_0/x_0$.

It is evident from the above discussion that, if the tax function of the reference unit is not strictly progressive in the sense that the average tax rate is strictly increasing with pre-tax income (or, more accurately, that the average tax rate does not vary strictly monotonically with pre-tax income), then the equivalence scale implicit in tax policy may not be unique.[35] This observation serves to highlight the care that must be exercised when selecting a reference unit, and when calculating the equivalence scales that are implicit in tax and benefits policy. Furthermore, the above discussion does not address the issue of calculating associated standard errors, which remains an issue for further research.[36]

5 Equivalence Scale Estimates for Australia

This section presents equivalence scale estimates calculated using simulated and survey data for Australia. The section begins with a description of the data used, before discussing the non-parametric equivalence scale estimates, and then the parametric estimates obtained. The section concludes by comparing the tax implicit equivalence scales with scales that have received some official recognition, and with scales estimated from household expenditure data.

5.1 Data

Simulated Data

A simulation model is used to generate a synthetic population of households that are differentiated by their number of adults, number of children, and their pre-tax and post-tax incomes. The synthetic population is comprised of single adults with up to two dependant children and couples with up to three dependant children (7 different demographic combinations), for 51 measures of pre-tax annual income ranging between $0 and $100,000. Given the number of adults, the number of children and the pre-tax income of a household, the simulation model generates measures of post-tax income based upon the rates and thresholds of the following 10 tax and benefit schemes, as they were defined in 1997/98:[37]

- *Newstart* (*NS*): An unemployment benefit payable to individuals who are available for, capable of, and actively seeking work between the ages of 18 and 65.

[35] This point is first alluded to by the analysis that is reported in Section 3. Indeed, it is possible to devise examples where the equivalence scale will not be defined at all.

[36] Given that the non-parametric estimates discussed here are sample statistics, it is presumably possible to use resampling methods to obtain bootstrap estimates for associated standard errors.

[37] Post-tax income is generated assuming that; all couples are married; all household income is earned by one individual; no household is eligible for an 'over 60s bonus' for the basic NS rate; there are no part year recipients; there are no wealth tests; if there are children in a household, then at least one child is under 5 years of age (for the Medicare Levy).

- *Family Payment* (FP): A benefit that is structured to support low income families with dependant children.
- *Parenting Payment* (PP): A benefit paid instead of *Newstart* to one member of a married couple with at least one dependant child. It has a more generous income test than *Newstart* and does not require the recipient to be seeking employment.
- *Sole Parent Payment* (SPP): The sole parent equivalent of the *Parenting Payment.*
- *Income Tax*: Income taxation is levied on individual rather than joint incomes, and takes a standard multi-step form with 5 progressive marginal rates.
- *Medicare Levy* (ML): Charged in addition to *Income Tax*, to fund the costs of a universal health care system.
- *Family Tax Initiative* (FTI): A scheme designed to support households with dependant children (in addition to the FP). It is comprised of two parts; Family Tax Assistance for households that earn a sufficiently high taxable income, and Family Tax Payment for low income families.
- *Dependent Spouse Rebate* (DSR): A tax rebate that can be claimed by individuals who have a spouse who earns a sufficiently low income.
- *Sole Parent Rebate* (SPR): A tax rebate that can be claimed by single parents with dependant children.
- *Low Income Earner Rebate* (LIR): A tax rebate that can be claimed by individuals who earn a sufficiently low income.

The procedures that are used to simulate these transfer schemes are based upon a study by Creedy and van de Ven (1999). In 1997/98, the transfer schemes listed above accounted for 73.0 per cent of all social security expenditure excluding benefits for the elderly, and 82.0 per cent of individual taxation liability.

Survey Data

The survey data are derived from the Confidentialised Unit Record File (CURF) of the 1997-1998 Survey of Income and Housing Costs (SIHC) for Australia. This survey provides income and demographic data for individuals who are aggregated into households.[38] The SIHC records annual household income measured in 1997 Australian dollars, and attempts to account for all direct pecuniary flows.

[38] Refer to the Australian Bureau of Statistics for detailed information regarding the SIHC.

Importantly, for the analysis that is undertaken here, no attempt is made to impute indirect taxes or transfer benefits. The results presented here must consequently be interpreted bearing this limitation in mind.

5.2 Non-Parametric Estimates

Figures 2 to 4 display non-parametric estimates of the equivalence scales implicit in the Australian tax and benefits system. Also included in Figures 2 to 4 are equivalence scale estimates based on household expenditure data, the scales recommended by the OECD, and Henderson equivalence scales.[39] The alternatives to the tax implicit equivalence scales considered, are the scales that are most commonly applied in the existing literature, and consequently allow a comparison with the status quo. The comparative analysis undertaken here is preliminary and incomplete insofar as it omits reference to associated standard errors. A more thorough interpretative analysis remains an issue for further research.

Four data series are displayed in each of the panels of Figures 2 to 4, three of which are derived from simulated data, and one from survey data. The series denoted 'smoothed - bandwidth = 0.2' describes the non-parametric estimates of equivalence scales derived from simulated data, using tax functions estimated by the 'lowess' procedure in STATA (with a bandwidth equal to 0.2). Similarly, 'smoothed - bandwidth = 0.8' refers to non-parametric estimates obtained from simulated data using lowess with a bandwidth of 0.8.[40] Equivalence scale estimates obtained without smoothing the simulated tax functions are denoted as 'no smoothing' in each of the figures. Finally, 'survey data (bandwidth = 0.8)' refer to non-parametric estimates obtained from survey data using a bandwidth of 0.8.[41] The alternatives to the tax implicit equivalence scales all take base independent specifications, and are consequently displayed as points on the vertical axis to avoid excessive clutter.

The tax implicit equivalence scale estimates obtained from simulated data are easier to analyse than those derived from survey data because of the limited sources of household heterogeneity involved. These are consequently discussed, before making comparisons with the estimates obtained from survey data, and the alternative equivalence scales displayed in each graph. In all cases, the equivalence scale estimates are specified with reference to single adults without dependant children (for whom the

[39] See Appendix B for a detailed description of the demand based, OECD and Henderson scales referred to here.

[40] In STATA, lowess calls a non-parametric regression procedure that obtains a separate estimate, $\hat{y}_i$, for each observation in a data set, (x_i, y_i), by regressing y on x for a limited proportion of the data set (defined by the bandwidth). The regression weights the data such that points further from the central point (x_i, y_i) receive less weight. See Cleveland (1993) for further details.

[41] A bandwidth of 0.8 was selected for the survey based estimates displayed here, after testing a number of alternatives. The authors may be contacted for associated results.

Figure 2: Australian Non-Parametric Equivalence Scale Estimates - Single Adults with Children

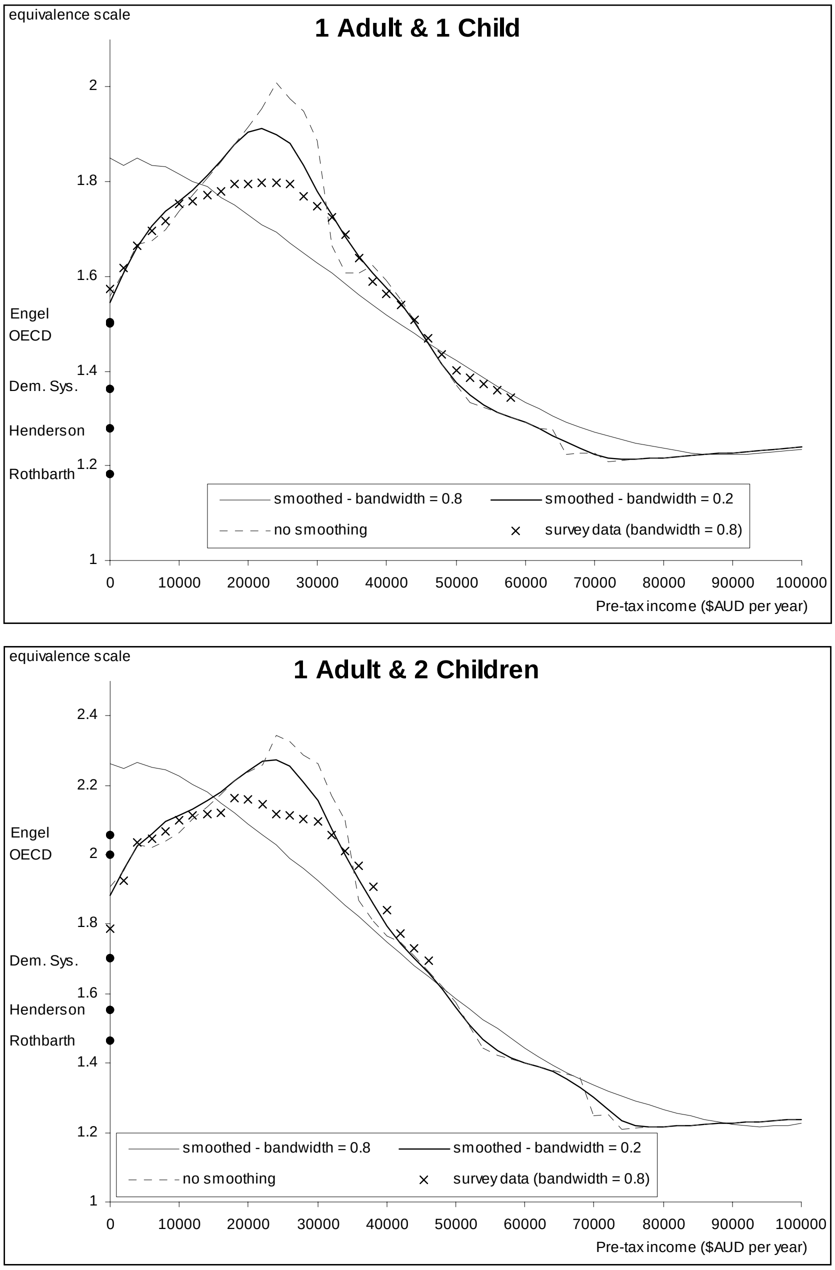
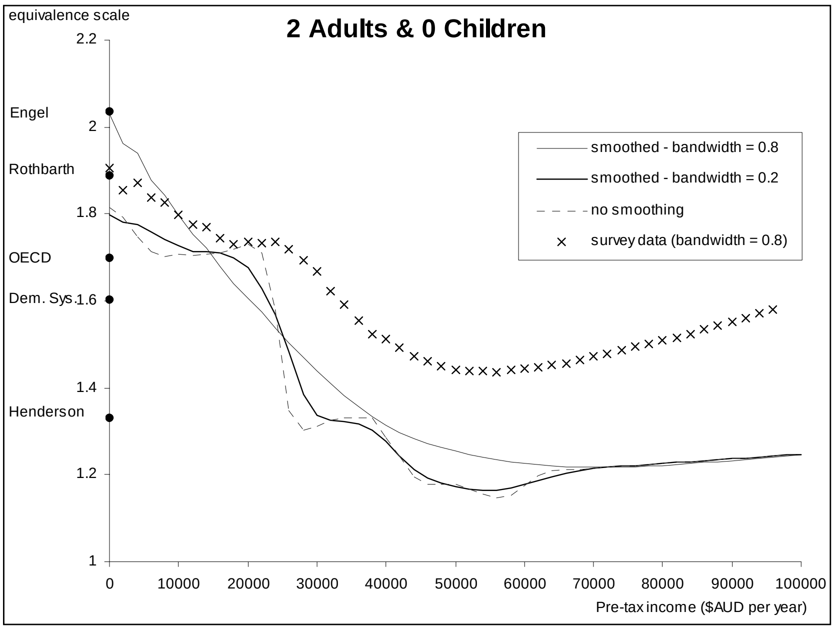


Figure 3: Australian Non-Parametric Equivalence Scale Estimates - Couples without Children

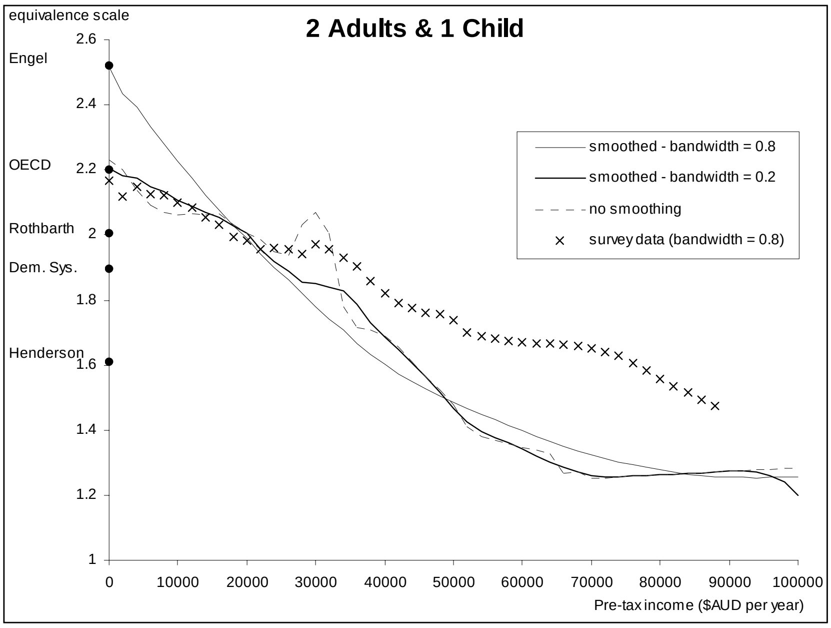


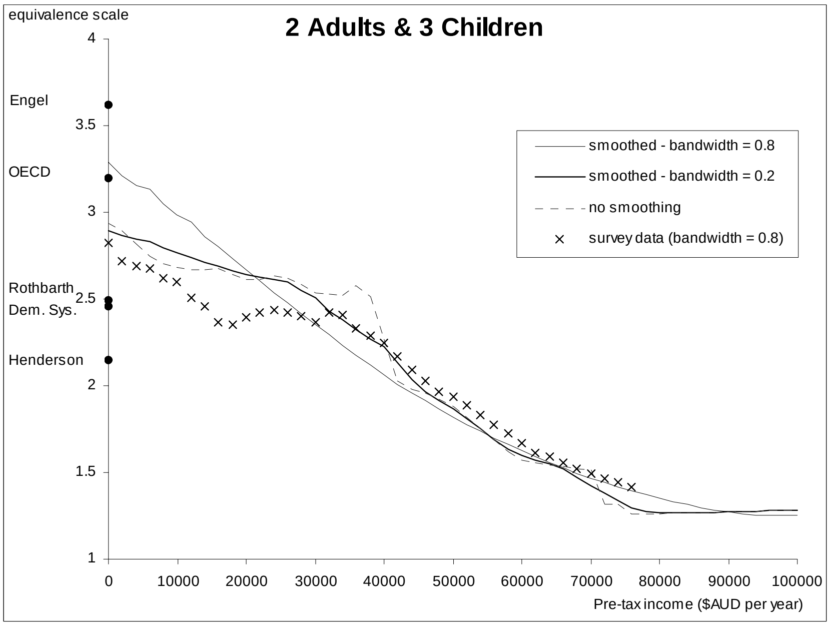
equivalence scale takes a value of one). Muellbauer and van de Ven (2003) note that tax implicit equivalence scales may be sensitive to the reference unit adopted. Single adults are adopted for the analysis presented in this paper because of their prevalence in the survey data used, and the relatively wide range of their average tax rates. A relatively high prevalence in the survey data improves the accuracy of the estimate obtained for the tax function of reference households, and a wide range of average tax rates is useful because of the comparisons that are made to infer tax implicit scales as discussed in Section 4.

Figure 2 indicates that the Australian tax and benefit schemes included in the simulation analysis tend to treat single parents more generously than they do single adults without dependant children, where the equivalence scales for single parents are greater than one for all measures of pre-tax income considered. Focussing upon the estimates obtained from simulated data that were not smoothed (no smoothing), the scales for single parents with one and two children are highly base dependant, increasing at approximately the same rate from $0 to peak at $24,000, before falling away at higher pre-tax incomes. Furthermore, the scale estimates obtained for single parents with one child are strictly less than the estimates for single parents with two children when pre-tax income is less than $74,000, where the two are equal for higher incomes.

The trends observed for the equivalence scales relate to elements of the simulated tax and benefit schemes. *SPP* provides a higher benefit rate and less punitive means testing than *NS*. This drives

Figure 4: Australian Non-Parametric Equivalence Scale Estimates - Couples with Children





a wedge between the post-tax incomes of single people with and without dependant children, which increases in size until pre-tax income reaches \$14,000, before falling away at higher incomes. The effects of FP, FTI, and SPR also increase the post-tax incomes of single parents relative to otherwise similar single adults without children. It is not until the means testing of FP takes effect that the equivalence scales of single adults tend to fall with higher pre-tax income. The fact that SPR is not means tested implies that the wedge between single parents and otherwise similar single adults without children will not go to zero as pre-tax income is increased, and hence the equivalence scales of single parents remain greater than one for higher incomes.

The scale estimates derived from unsmoothed simulated data displayed for single adults in Figure 2 suggest that the government takes greater consideration of the needs of single parents (relative to single adults without dependant children) as incomes rise up to a threshold, but that this consideration decreases thereafter. This is consistent with the view that single parents face a greater burden associated with working than those without children due, for example, to the financial costs associated with childcare and the emotional strain of parental responsibility. The fact that the equivalence scales for single parents fall away after a threshold income level is consistent with the view that, at higher incomes, parenthood becomes a consumption decision that should be borne by the individual rather than by society.

The amount of smoothing increases with the bandwidth used for the lowess estimation procedure - this is reflected by the three series derived from simulated data displayed in Figure 2 (for reference, the 'no smoothing' condition is equivalent to assuming a bandwidth of 0). It is interesting to note that the equivalence scale estimates obtained from survey data, which were derived using a bandwidth of 0.8, bear a closer relationship to the estimates obtained from simulated data using a bandwidth of 0.2 than to those obtained using a bandwidth of 0.8. This result is observed because the distribution of the survey population is concentrated between \$15,000 and \$30,000 (AUD), as opposed to the uniform distribution that is generated by the simulation model. Furthermore, and perhaps more importantly, the survey data include more noise than the simulated data do.

With regard to the alternatives to the tax implicit equivalence scales displayed in Figure 2, it can be seen that the demand system, Henderson, and OECD scales take values that are less than the Engel scales and greater than the Rothbarth scales. The values of the demand system, Henderson, and OECD scales are consequently consistent with biasses that have been associated with the Engel and Rothbarth methods, which tend respectively to overstate and understate the actual child costs borne by households.[42] The tax implicit equivalence scale estimates of single parents with no pre-tax

[42] On biasses associated with the Engel and Rothbarth scales see, for example, Deaton and Muellbauer (1986, p. 732)

income and one dependant child are higher than the Engel scales, and rise (except for the scales with a band-width of 0.8) to a maximum at approximately \$20,000 per annum, or 65 per cent of mean pre-tax income (MPTI, \$31,450), before falling to approach the Rothbarth scales at incomes in excess of 200 per cent of MPTI.

To assign some normative interpretation to these observations, consider the implications of assuming that the demand system estimates provide an accurate reflection of the relative costs borne by single parents, and that the non-parametric estimates obtained from unsmoothed simulated data provide an accurate reflection of the relativities implicit in transfer policy. The top panel of Figure 2 consequently suggests that the Australian transfer system makes a greater proportional adjustment for household need than the proportional increase in the costs incurred due to the addition of a dependant child for single parents on very low pre-tax incomes, and that this disparity increases up to 60 per cent of MPTI. This variation is consistent with the view that the children of single parents should be adequately provided for, either because they are not responsible for the decisions of their parents or because there are social benefits to be enjoyed from such provision. In contrast policy makers may take the view that the onus of adequate provision for single adults without dependant children should rest with the individual (and hence under-provide for their associated costs). Above 60 per cent of MPTI the excess proportional adjustment for household need made by the transfer system falls, and drops below the proportional increase in costs incurred due to the addition of a dependant child when income exceeds 150 per cent of MPTI.

A similar profile is observed for the equivalence scales of single parents with two children, displayed in the lower panel of Figure 2. The tax implicit equivalence scales for single parents with two children are, however, lower relative to the associated demand based and official scales than for single parents with one child. This suggests that Australian policy makers may take into consideration the incentive effects associated with making larger provisions for the needs of single parents relative to the costs that parents actually incur due to their dependant children.

Figures 3 and 4 suggest that the Australian tax and benefits system is more generous to couples than to single adults without dependant children, and tends to make a larger adjustment for household need as the number of children increases. Focussing upon the series associated with the unsmoothed simulated data, the equivalence scale estimates obtained for couples without children take a value of approximately 1.7 until pre-tax income reaches \$20,000, after which the estimates fall away sharply to restabilise at approximately 1.2. These observations are principally driven by two schemes; at low pre-tax incomes, the NS benefit is more generous and imposes a less severe means test for couples

and van de Ven (2003).

than for singles; and at high incomes the DSR (which is not means tested) drives a wedge between the post-tax incomes of couples and singles. Comparison of Figure 3 with 4 indicates that the base dependence of the equivalence scales derived for couples with children is more smooth than for couples without dependant children. This is attributable to the gradual way that means testing is applied for NS, PA, FP, and FTI.

The equivalence scale estimates displayed in Figures 2 to 4 suggest that, in general, the government implicitly assumes some economies of scale when adjusting for household need. This is indicated by the fact that smaller vertical shifts are observed as household size increases. Furthermore, the equivalence scales for couples without dependant children are consistent with the value judgement that adults should be supported up to a pre-tax income threshold (of approximately $22,000), after which they are largely responsible for taking care of their own needs. The 20 per cent adjustment made for adults with high incomes and a dependant spouse relative to otherwise similar single individuals is likely to be less than the costs that are actually incurred by the addition of a spouse (a proposition that is supported by the demand based equivalence scale estimates discussed below). This is consistent with the view that having one member of a childless couple take on a 'home-maker' role becomes a consumption decision at higher household incomes. It also suggests that the adjustment for household need takes into consideration the value of home production for couples.

Turning attention to the alternative equivalence scales displayed in Figures 3 and 4 for two adult Australian households, it can be seen that both the Engel and Rothbarth scales for couples without dependant children exceed the OECD, demand system, and Henderson scales. van de Ven (2003) suggests that this observation is attributable to biasses that are associated with the Engel and Rothbarth methods, which cause both methods to over-estimate the costs to households of adult members. Given that biasses associated with the Rothbarth method tend to underestimate the costs to households of children, the observation that the OECD, demand system and Henderson scales tend to increase relative to the Rothbarth scales as the number of children in a household increases is expected.

In general, the tax implicit equivalence scale estimates for two adult households exhibit a negative relationship with income. In all three panels displayed in Figures 3 and 4, the non-parametrically estimated equivalence scales for tax and benefits policy cross through the demand system estimates at approximately average pre-tax income. Hence, if the demand system equivalence scale estimates accurately reflect the costs of heterogeneous households relative to a single adult without dependents, and if the tax implicit estimates obtained by the non-parametric procedure embody the relativities implicit in the Australian transfer system, then couples appear to be treated preferentially at low incomes, and relatively harshly at higher incomes. The equivalence scales at low incomes consequently suggest that the government may perceive a larger social benefit to be gained by supporting couples than singles (potentially for social reasons). In contrast, the equivalence scale estimates observed at high incomes are consistent with the view that couples enjoy benefits beyond economies of consumption due, for example, to their ability to share household responsibilities.

Comparing the estimates obtained from survey data with those derived using simulated data indicates the extent to which the relativities described by the survey data are captured by the simulation model. The estimates displayed in Figure 3, for example, indicate that couples are treated more generously on average relative to single adults in the survey data than by the simulation model. This can be attributed to some form of income splitting that is undertaken by couples when calculating their tax burden, which is not taken into consideration by the simulation model.[43] Nevertheless, despite the simplicity of the simulated tax and benefits system, most of the estimates derived from simulated data displayed in the five panels of Figures 2 to 4 bear a close relationship to the associated estimates derived from survey data, suggesting that the simulation model does a (surprisingly) good job at mimicking the relativities implicit in the actual Australian tax and benefits system.

6 Conclusions

Redistributive policy that makes explicit reference to non-income characteristics of households - such as their size and demographic composition - embodies an implicit set of relativities. Tax and benefits policy consequently provides an important, and as yet under-utilised source of information for identifying the value judgements (implicitly) made by policy makers regarding the needs of heterogeneous households. In this paper it is suggested that the equivalence scale framework can be used to make the relativities embedded in transfer policy explicit.

A formal model for considering observed tax and benefits systems within the equivalence scale framework is introduced, and its relationship with the concept of horizontal equity is described. Furthermore, it is shown that the ability of the model to reflect transfer systems that are commonly observed depends crucially upon the assumptions made regarding the equivalence scale specification. Our equivalence scale specification is general enough to capture transfer systems observed in practice.

A number of methods can be used to estimate the equivalence scale implicit in tax and benefits policy. We consider a non-parametric approach here that infers the relativities implicit in transfer

[43] The simulation model assumes that all household income is earned by a single individual, which is not the case for survey data.

policy using data that are described by common household surveys, such as the Survey of Income and Housing Costs in Australia. The fact that the estimation procedure considered here is based upon survey data, rather than relying upon microsimulation models, is of particular importance because simulation models can provide only a limited reflection of transfer systems; the effects of imperfect take-up rates, tax avoidance and various tax minimisation strategies imply that the real world impact of a transfer system may be quite different from the impact that is implied by official rates and thresholds.

The estimation method is used to explore the relativities implicit in the Australian transfer system, based upon simulated data for a subset of the system, and survey data from the SIHC. The estimates derived using tax and benefits data are compared with a range of alternative scales, including estimates obtained from expenditure data, scales applied by government statistical agencies in the two countries, and the scales suggested by the OECD. The implicit equivalence scale estimates obtained suggest that the adjustments made by the Australian transfer system for differences in household need describe an interesting and intuitive set of value judgements.

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A Regression Estimation Methods

The regression methods that are discussed here are of interest because they enable functional forms to be estimated directly from survey data, and because they allow for an explicit treatment of associated error terms. In practice, however, practical application of the regression methods can prove difficult because of the highly non-linear nature of the regression models. A study that is principally concerned with practical application of these models is currently a work in progress - see Muellbauer and van de Ven (2003) for some preliminary analysis.

A.1 Joint Econometric Estimation

The term 'joint estimation' is used to refer to the following procedure because it involves estimating the equivalence scale and equivalent tax functions jointly.[44] Restating equation (3):

$$y_i = x_i - a_i^* T^* \left(\frac{x_i}{a_i^*} \right) + a_i^* \varepsilon_i' \tag{29}$$

where ε_i' allows for the possibility of horizontal inequity and measurement error. Assuming:

$$\begin{aligned} T^* \left(x_i^* \right) &= T \left(x_i^* \right) + \omega_i \\ a_i^* &= a \left(v_i \right) + \lambda_i \end{aligned}$$

and substituting into equation (29) obtains:

$$y_i = x_i - \left(a \left(v_i \right) + \lambda_i \right) T \left(\frac{x_i}{a \left(v_i \right) + \lambda_i} \right) + \left(a \left(v_i \right) + \lambda_i \right) \left(\varepsilon_i' + \omega_i \right) \tag{30}$$

which can be estimated using standard non-linear regression techniques. It is evident from the specification of equation (30), however, that the error term will be heteroscedastic. Consider, for example, the case in which the function $T(.)$ takes the form of an Nth order polynomial. Then:

$$y_i = x_i - \left(a_i + \lambda_i \right) \left\{ \sum_{j=0}^{N} \beta_j \left(\frac{x_i}{a_i + \lambda_i} \right)^j \right\} + \left(a_i + \lambda_i \right) \varepsilon_i \tag{31}$$

[44] This is distinct from the common use of the term 'joint estimation' in the econometric literature, where it refers to a procedure that takes into consideration hypothesised correlations between the error terms of two regression models. In the analysis undertaken here, the correlation between the error terms of the equivalent tax function and the equivalence scale is unidentifiable, and is consequently assumed to be zero. See below for further discussion.

where the β_j terms are tax function coefficients, $a_i = a(v_i)$, and $\varepsilon_i = \varepsilon_i' + \omega_i$. For the jth term of the tax function:

$$\begin{aligned} (a_i + \lambda_i)\beta_j \left(\frac{x_i}{a_i + \lambda_i}\right)^j &= \beta_j \frac{x_i^j}{(a_i + \lambda_i)^{j-1}} \\ &\simeq \beta_j \frac{x_i^j}{a_i{}^{j-1}\left(1 + (j-1)\frac{\lambda_i}{a_i}\right)} \\ &\simeq \beta_j \frac{x_i^j}{a_i^{j-1}}\left(1 - (j-1)\frac{\lambda_i}{a_i}\right) \\ &= \beta_j \frac{x_i^j}{a_i^{j-1}} - \beta_j (j-1)\left(\frac{x_i}{a_i}\right)^j \lambda_i \end{aligned} \tag{32}$$

where the approximations assume small λ_i. Substituting for the tax function terms in equation (31), we obtain the following:

$$\begin{aligned} y_i = {} & x_i - \sum_{j=0}^{N} \beta_j \frac{x_i^j}{a_i^{j-1}} \\ & + a_i\varepsilon_i + \lambda_i\varepsilon_i \\ & + \sum_{k=0}^{N} \beta_k (k-1)\left(\frac{x_i}{a_i}\right)^k \lambda_i \\ & + \psi_i \end{aligned} \tag{33}$$

where ψ_i accounts for the approximations made to obtain equation (32). The error term of this equation is defined by:

$$\epsilon_i = a_i\varepsilon_i + \lambda_i\varepsilon_i + \sum_{k=0}^{N} \beta_k (k-1)\left(\frac{x_i}{a_i}\right)^k \lambda_i + \psi_i \tag{34}$$

We assume that the error terms, ε_i, λ_i, and ψ_i have an expectation of zero, and a constant variance for all i. Furthermore, we assume that the error terms are all independent of the exogenous variables of the model (that the model is correctly specified), and that the error terms are independent of one another. This last assumption warrants some discussion. Specifically, it is clear from equation (30) that the tax function and the equivalence scale are related, which implies that the associated error terms, ε and λ, might also be correlated. Equation (34), however, indicates that it is not possible to derive explicit estimates for the individual error terms; following regression of equation (33) we have only one equation - (34) - and three unknowns - ε, λ, and ψ. Hence the assumption made here regarding the independence of ε and λ cannot be tested using the model. It is possible, for example, to assume that no error is associated with the equivalence scale ($\lambda = 0$), in which case (assuming $\psi_i = 0$ for all i) all of the error associated with an estimate of equation (33) will be attributed to the tax function (ε).

The assumption of zero correlation between ε and λ implies that there is some separability between the process by which the transfer system identifies a household's 'type' (and hence its needs), and the process by which it determines the household's net transfer payment given it's allocated type. In our view, this assumption appears reasonable.

Given the following assumptions:

$$\begin{array}{l} E(\varepsilon_i) = E(\lambda_i) = E(\psi_i) = 0 \\ var(\varepsilon_i) = \sigma_\varepsilon^2 \\ var(\lambda_i) = \sigma_\lambda^2 \\ var(\psi_i) = \sigma_\psi^2 \\ cov(\varepsilon_i, \lambda_i) = cov(\varepsilon_i, \psi_i) = cov(\lambda_i, \psi_i) = 0 \\ cov(\varepsilon_i, z_i) = cov(\lambda_i, z_i) = cov(\psi_i, z_i) = 0 \end{array}$$

where z_i defines all exogenous variables of equation (33), $E(.)$ defines the expectation, $var(.)$ the variance, and $cov(.)$ the covariance (which is also assumed to apply to all higher moments), the properties of ϵ_i are defined by:

$$E(\epsilon_i) = 0 \tag{35}$$

$$var(\epsilon_i) = a_i^2\sigma_\varepsilon^2 + \sigma_\lambda^2\sigma_\varepsilon^2 + \left[\sum_{k=0}^{N} \beta_k (k-1) \left(\frac{x_i}{a_i}\right)^k\right]^2 \sigma_\lambda^2 + \sigma_\psi^2 \tag{36}$$

$$cov(\epsilon_i, z_i) = 0 \tag{37}$$

Since the form of the heteroscedasticity is known, but the associated parameter estimates are not, this analysis suggests that it is appropriate to estimate equation (33) by either Weighted Least Squares or Generalised Methods of Moments.

A.2 Two-stage or mixed estimation

In practice, using the joint estimation procedure described above to estimate the equivalence scale and equivalent tax functions is complicated by a number of factors. Most important of these is the income dependence of the equivalence scale, which can take a highly non-linear form. This, and the fact that such a relationship is unlikely to be known *ex ante*, implies that the function adopted for the equivalence scale may not be sufficiently flexible to accurately reflect the relativities implicit in the transfer system, which will result in omitted variable bias.

When an equivalence scale is desired for distributional, rather than interpretive purposes, some forms of omitted variable bias may actually help to improve the estimates obtained. Omitting relevant health related characteristics may, for example, lead to an upward bias of coefficients on old age identifiers, which could help to correct the estimated equivalence scale for distributional analysis. However, omitting pertinent variables from the equivalence scale specification, and income related variables in particular, is likely to bias the coefficients of the tax function as well as those of the equivalence scale when derived from a joint estimation. Estimating the equivalence scale and equivalent tax functions jointly means that the biasses of one function can produce biases in the other function, which complicates interpretation of the parameter estimates obtained. Specifically, interpretation of the equivalence scale as a proportional adjustment to income which ensures that the same tax function, primarily applicable for reference households, is applicable for the entire population, does not hold when biassed estimates are obtained for the equivalent tax function of reference households.

One useful test for determining the adequacy of the equivalence scale and tax functions used is to check whether the tax function coefficient estimates derived from a non-linear regression of the entire population are significantly different from the tax function coefficient estimates obtained for the reference population when taken in isolation. In practice, it may be difficult to find a specification for which the tax function parameter estimates are stable when calculated using the restricted and unrestricted populations. When this is the case an alternative method of estimation is required. The test described above suggests one possibility; a two-stage estimation procedure in which the tax function and equivalence scale are estimated separately.[45]

Two-stage estimation is available in two alternative forms. The iterative method set out in Section 4.1 can be used to generate values of a_i^* for observed households with an income x_i and characteristics v_i. A parametric function for $a_i^*(x, v, v_0)$ can then be estimated. Note that this method is consistent with parametric or non-parametric estimates of net tax functions.

The alternative two-stage method takes the parametric form (30) and estimates the β's for the reference household type. Given these estimated values, now substitute the equivalence scale function (19) into (30) and estimate the parameters of (19) through weighted non-linear least squares.

Two-stage estimation prevents the biasses of the equivalence scale feeding into the tax function (and vice versa). As such, where the tax function used provides a close approximation to the observed tax function of reference households, the biasses that remain with regard to the equivalence scale will be largely confined to the type that tend to improve the estimates for distributional purposes, as discussed above.

[45] Again, the use of the term 'two-stage estimation' that is adopted here should not be confused with its common use in the econometric literature. In the current context, it refers to separate estimation of the equivalent tax function and equivalence scale in two distinct stages, as opposed to a procedure that is designed explicitly to adjust for heteroscedasticity of the associated error terms.

The indeterminacy that is associated with the error structure of equation (34), will continue to apply to the second stage of the two-stage regression procedure. Specifically, we might assume that reference households are only those for which the tax function estimate derived in the first stage of the procedure provides an accurate description of the relationship between pre-tax and post-tax income, in which case $\varepsilon_i = 0$ for all i, and hence all of the error observed for the second stage of the estimation is associated with the equivalence scale where:[46]

$$\epsilon_i = \sum_{k=0}^{N} \beta_j (j-1) \left(\frac{x_i}{a_i} \right)^j \lambda_i \tag{38}$$

Alternatively, it is possible to assume that both the tax function and the equivalence scale are subject to error, in which case the error structure of the second stage of the procedure will be described by equation (34). These observations regarding the error structure highlight the importance of the precise definition that is adopted for the reference household when estimating an equivalence scale that is implicit in transfer policy.

B Alternative Equivalence Scales

B.0.1 Demand based equivalence scales

Equivalence scale estimates based on household expenditure data are considered by van de Ven (2003), using the base independent specification defined by:

$$a_i = \exp\left\{ \phi \frac{c_i}{n_i} + \gamma \ln(n_i) \right\} \tag{39}$$

where c_i defines the number of children (under the age of 18) in a household, and n_i denotes the total number of household members. Three demand based estimation methods are considered by van de Ven (2003); the Engel method, the Rothbarth method, and a method based on a demand system. All of the scales considered by van de Ven (2003) are assumed to be base independent (as indicated by the specification of equation (39)), which is consistent with the demand based literature. Equivalence scale estimates derived using each of the three approaches are displayed in Table 1, which are used to calculate the associated equivalence scales displayed in Figures 3 to 4.

B.0.2 OECD scales

The OECD scale is recommended by the OECD for use when there exists no other preferred scale, or where international comparisons are to be made. The scale considered here assigns a value of one to

[46] Assuming $\psi_i = 0$ for all i.

Table 1: Demand Based Equivalence Scale Estimates

parameter	Engel	Rothbarth	Dem. Sys.
ϕ	-0.60810	-0.93955	-0.32349
	(0.0563)	(0.0566)	(0.0253)
γ	1.02571	0.91798	0.67985
	(0.0275)	(0.0291)	(0.0130)

Coefficient standard errors provided in parentheses

the reference person of a household, 0.7 to their partner, and 0.5 to any dependant children regardless of household income. Hence a household that is comprised of a couple with two dependant children is allocated a scale of 2.7.[47]

B.0.3 Henderson scales

The Henderson equivalence scale is commonly used for analysis of Australian income data - see, for example, the Australian Bureau of Statistics publication, *Income Distribution, Australia 1997-1998*.[48] This scale allocates points depending upon household member characteristics, and the number of people in a household. The relevant points, which are independent of household income, are defined in Table 2.

Table 2: Henderson Scale

	Points
Each Individual	
First Adult	20.0
Spouse	9.5
Dependent Child	7.5
Household Size (persons)	
1	17.0
2	19.8
3	22.5
4	25.0
5	27.5
6	30.0
7	32.0

[47] See *The OECD List of Social Indicators*, OECD, 1982.

[48] ABS Catalog Number 6523.0. See, also, Henderson, *et. al* (1970), and Appendix F of the Australian Government Commission of Inquiry into Poverty (1975).