Money demand in the Yugoslavian hyperinflation 1991-1994.

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8 December 2004

Summary: Empirical analyses of Cagan's money demand schedule have broadly speaking suffered from the following problems: (*i*) Inability to model the data to the end of the hyperinflation. (*ii*) Difficulties in making congruent models for systems of variables. (*iii*) Discrepancies between "estimated" and "actual" inflation tax. In this paper the extreme Yugoslavian hyperinflation of the 1990's is therefore studied. Two econometric models are presented. First, money, prices and exchange rates are analysed by a vector autoregression allowing for random walk and explosive common trends. This analysis of the sample distribution leads on to the second model of real money and the cost of holding money, rather than the traditional inflation measure. The three outlined problems can then be addressed, giving support to Cagan's model.

Keywords: *Cost of holding money, Co-explosiveness, Cointegration, Explosive processes, Hyper-inflation.*

JEL: C32, E41.

1 Introduction

The money demand equation for hyper-inflation of Cagan (1956) postulates a linear relationship between real money and the expected rate of change in prices. Cagan's own empirical work as well as most later empirical work is essentially single equation regressions of log real money, $m_t - p_t$, regressed on the changes in log prices, Δp_t, measured at a monthly frequency. For extreme hyper-inflations only little progress has

[1]The data used in this paper were collected and previously analysed by Zorica Mladenović and her co-authors. I have benefitted from many discussions with her and with David Hendry, as well as from discussions with Frédérique Bec, Aleš Bulíř, Katarina Juselius, Takamitsu Kurita, and John Muellbauer. Computations were done using PcGive (Doornik and Hendry, 2001) and Ox (Doornik, 1999).

been made to model the joint system of the variables, thereby avoiding the exogeneity assumption underlying a single equation approach, and to describe extreme hyper-inflations to the very end. Typically discrepancies have been found between the "optimal" and the "actual" inflation tax, and, hence only little support for Cagan's model. In this paper, two empirical models for the extreme Yugoslavian hyper-inflation are proposed with a view towards addressing these issues.

In the first model, a vector autoregressive model for nominal money, m_t, nominal prices, p_t, and spot exchange rates, s_t, is constructed for a sample excluding the last few months. Due to the accelerating nature of the data the vector autoregression is found to have an explosive characteristic root. Using econometric methods developed in tandem with this empirical analysis, it is found that real money is like a random walk while changes in log prices are explosive. A regression of real money on changes in log prices is therefore unbalanced and Cagan's model cannot be supported directly. While this first model does achieve the aim of finding a well-specified simultaneous model it is clearly only partially successful.

Since the aim is to find a link between real money and the cost of holding money, but the variables $m_t - p_t$ and Δp_t are unbalanced, the idea of the second model is to consider a transformed set of variables. The cost of holding money is now measured by $c_t = \Delta P_t/P_t = 1-\exp(-\Delta p_t)$. For small values of Δp_t a Taylor expansion shows that $c_t \approx \Delta p_t$ whereas in extreme inflations the two measures are quite different, with c_t being bounded by 1, which opens up for a new interpretation of maximal seigniorage in Cagan's model. A well-specified vector autoregressive model can now be set up for real money, the cost of holding money and a similar measure for the rate of depreciation, d_t, of the Yugoslavian dinar. This transformed time series is integrated of order one and standard cointegration analysis gives one cointegrating relation that is a variant of Cagan's model. The cointegrating relation has an additional component $d_t - c_t$ suggesting that the hyper-inflation is pro-longed by uncertainty in the foreign exchange market. Moreover, the estimated "optimal" and "actual" inflation tax rates are found to be in line.

The outline of the paper is that §2 gives a brief review of the existing literature on money demand in hyper-inflations. §3 introduces the data and some institutional background. The two econometric models are outlined in §4 and §5 while §6 concludes.

2 A brief review of previous work on hyper-inflation

The main theory for hyper-inflation is due to Cagan (1956). In his equations 2 and 5 real cash balances in hyperinflation are modelled through the equations

$$
\begin{aligned}
m_t - p_t &= -\alpha E_t - \gamma, &(2.1)\\
\left(\frac{\partial E_t}{\partial t}\right)_t &= \beta \left(C_t - E_t\right). &(2.2)
\end{aligned}
$$

Here m_t and p_t represent the logarithm of money and prices, $C_t = \partial p_t / \partial t$ is the continuous rate of change in prices, while E_t represents an adaptive expectation of C_t. Other variables, like output, that are usually appearing in quantity theories for money are assumed to have a negligible influence. By solving equation (2.2) backwards from present time, t, to an initial value, $-T$, the expectations term E_t can be expressed as an exponentially weighted average of past values of C, that is

$$E_t = H \exp\left(-\beta t\right) + \beta \int_{-T}^{t} C_x \exp\left\{\beta \left(x - t\right)\right\} dx. \tag{2.3}$$

Inserting this in (2.1), Cagan could then estimate α and β from monthly data as follows. Letting $-T$ represent the beginning of the sample and assuming that prices had been almost constant before time $-T$, then H can be set to zero in (2.3). Assuming, further, C_x is constant within a month, in which case $C_t = \Delta p_t$, the latent expectations process E_t can be approximated by a sum. For a given value of β the parameter α can then be estimated from (2.1) by regression. By varying the value of β a joint estimate for α, β can be found.

In the empirical analysis, Cagan considered data from seven hyperinflations. The infamous German hyperinflation from August 1922 to November 1923 was in this way analysed using data from September 1920 to July 1923 due to difficulties in fitting the data from the last few months. In the German case, α is estimated by $\widehat{\alpha} = 5.76$.

Cagan also analysed the seigniorage from printing money, arguing that the revenue from the inflation tax is the product of the rate of tax and the base

$$R = \left(\frac{dP}{dt}\frac{1}{P}\right)\frac{M}{P}, \tag{2.4}$$

where M and P are levels of money and prices, and the timing is left uncertain. He then proceeds to make the counter-factual assumption that the quantity of nominal money rises at a constant rate. This would eventually imply constancy of real money balances, which is contradicted by Cagan's own observation that real money balances tend to fall in hyperinflation. It would also imply that E_t can be replaced by C_t in equation (2.1):

$$\frac{M}{P} = \exp(-\alpha C - \gamma) \tag{2.5}$$

Combining (2.4) and (2.5) gives a revenue of $R = C\exp(-\alpha C - \gamma)$, which has a unique maximum, with respect to C, when

$$C = \frac{1}{\alpha}.$$

The inverse of the semi-elasticity α is therefore interpreted as the rate of inflation that maximises the revenue from seigniorage under the above assumptions.

In the empirical analysis, Cagan estimates for the German hyperinflation that $\hat{\alpha}^{-1} = 0.183$. This is a continuously compounded rate corresponding to a monthly tax of $\exp(\widehat{\alpha}^{-1}) - 1 = 20\%$. In the counter-factual analysis, this is then compared with an average monthly rate of inflation, defined as $\Delta P_t/P_{t-1}$, of 322%. Comparing the two shows a puzzling mismatch between an "optimal" tax rate and the "actual" inflation tax.

Sargent (1977) revisited Cagan's analysis, with a view towards explaining the discrepancy of the "optimal" and the "actual" inflation tax. While keeping the above structure of Cagan's model, the backward looking, adaptive expectations were replaced by forward looking, rational expectations, and a two-equation structural simultaneous equations model was proposed.

In Sargent's econometric analysis, m_t and p_t are implicitly assumed to be integrated of order one, $I(1)$, and a bivariate first order autoregressive - first order moving average model was fitted to the monthly growth rates of these variables. In the case of Germany, the estimate of α is virtually unchanged, $\widehat{\alpha} = 5.97$, but the uncertainty is judged differently with a standard error of 4.6 so the estimated confidence band for the "optimal" inflation tax covers nearly the whole positive real axis. Sargent's analysis therefore lends support, albeit only weakly, to Cagan's model

Taylor (1991) reformulated the real cash balance equation in an $\mathsf{I}(2)$ framework, which pushed the research in hyperinflations a significant step forward. The equation (2.1) was written in discrete time as

$$\begin{aligned} m_t - p_t &= -\alpha \Delta p^e_{t+1} + \zeta_t, & (2.6)\\ \Delta p^e_{t+1} &= \Delta p_{t+1} - \epsilon_{t+1}, & (2.7) \end{aligned}$$

where the variable Δp^e_{t+1} measures the expected inflation in period $t+1$ and ζ_t, ϵ_{t+1} are stationary error terms. Taylor showed that Δp^e_{t+1} can be interpreted as either a rational expectation, an adaptive expectation or an extrapolative expectation. Inserting (2.7) into (2.6), subtracting Δp_t on both sides and then reorganising leads to

$$\Delta^2 p_{t+1} = -\alpha^{-1}\left(m_t - p_t + \alpha \Delta p_t\right) + \left(\epsilon_{t+1} - \alpha^{-1}\zeta_t\right). \quad (2.8)$$

Assuming that m_t and p_t are both $\mathsf{I}(2)$ variables it can be tested whether real money $m_t - p_t$ is $\mathsf{I}(1)$ and in turn whether $m_t - p_t + \alpha\Delta p_t$ cointegrates to $\mathsf{I}(0)$. In this cointegrated framework the coefficient to the expected inflation variable Δp^e_{t+1} therefore shows up as the coefficient to Δp_t in a cointegrating relation.

In the empirical work Taylor considered six of Cagan's cases. As a justification for the $\mathsf{I}(2)$ framework, unit root tests were applied to levels, first and second differences of $m_t - p_t$ and Δp_t with a focus on the left-hand tail of the unit root t-statistics. Considering instead both tails of the t-statistics the results indicate the presence of explosive roots for instance in the German data. For Germany, Taylor estimated α by 5.31 which is in line with previous estimates.

Frenkel (1977) suggested linking real money balances with exchange rates and forward rates to overcome the problem of measuring expected inflation. The rationale is that agents hold real money in foreign currency and adjust holdings of real money to expected exchange rate depreciations. This idea was cast in Taylor's framework by Engsted (1996). Abel, Dornbusch, Huizinga and Marcus (1979) went one step further in suggesting that both inflation and depreciation in exchange rates may influence real money as in

$$m_t - p_t = -\alpha \Delta p^e_{t+1} - \beta \Delta s^e_{t+1} + \gamma + \epsilon_t.$$

Yugoslavia experienced two hyper-inflations in short time. The first had a long build-up during the 1980s and peaked in 1989 reaching high, but not very extreme inflation only briefly. The second and very extreme hyper-inflation which is studied here developed from 1991 to January 1994. Data from these hyperinflations have been studied using the above methods in a number of papers with for instance Petrović and Mladenović (2000) looking at the latter episode following the approaches of Taylor and Engsted. For the first Yugoslavian hyper-inflation, richer data are available such as wages. Recently, Juselius and Mladenović (2002) have re-analysed this period seeking a link between wages and prices. They follow a vector autoregressive approach paying a lot of attention to describing the sample distribution, by which they can free themselves from the exogeneity assumptions underlying univariate approaches. They find an explosive root in the data and they proceed to analyse the data in a way that has inspired the first econometric model that will be presented in §4.

3 Data and institutional background

The institutional background for the extreme Yugoslavian hyperinflation of the 1990s is described in Petrović and Vujošević (1996) and Petrović, Bogetić and Vujošević (1999). In short, the former federal republic of Yugoslavia was falling apart in 1991, the civil war started and United Nations embargo was introduced in May 1992. This situation led to decreased output and fiscal revenue, while transfers to the Serbian population in Croatia and Bosnia-Herzegovina as well as military expenditure added to the fiscal problems. The monthly inflation rose above 50% in February 1992 and

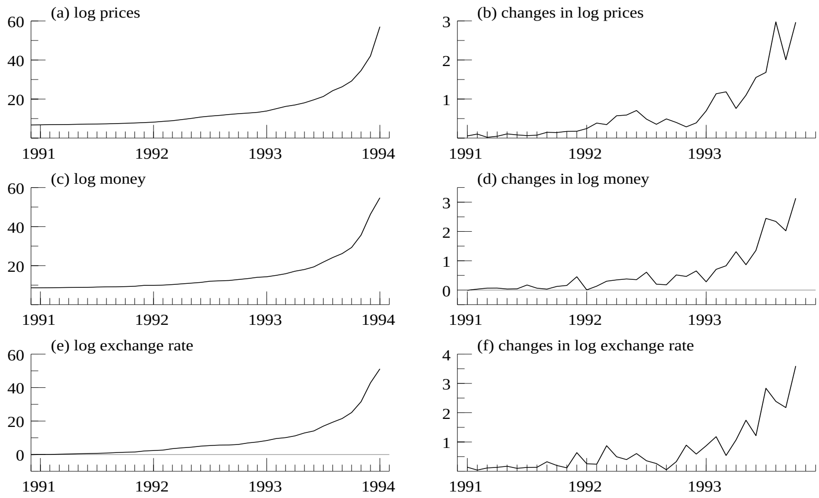

Figure 1: Data in level for full period. Data in differences (using Δ_1-operator) for shorter period

accelerated further, a price freeze was attempted in the end of August 1993 and the inflation finally ended on 24 January 1994 with a currency reform after prices had risen by a factor of 1.6×10^{21} over 24 months.

Figure 1 shows three time series of monthly data relating to the period 1990:12 to 1994:1. The variables are the monthly retail price index, p_t, narrow money measured as M1, m_t, and a black market exchange rate for German mark, s_t, all reported on a logarithmic scale. The sources for the data are documented in Petrović and Mladenović (2000). They consider the prices for 1993:12 and 1994:1 to be unreliable and choose end their analyses end at the latest 1993:11. This is in line with previous studies of hyper-inflation that mostly ignore the last few observations.

Figure 1 also shows first differences of the series. Both in levels and in differences the series show an exponential growth over time and hence an accelerating inflation. Cross-plotting the variables against their lagged values would give approximately straight lines with slopes in the region 1.15-1.35, which would be another indication of explosive behaviour.

The real money series $m_t - p_t$ and $m_t - s_t$ discounted by the price level and the exchange, respectively, are both falling. The former was also observed by Cagan and motivates the negative sign in equation (2.6). Noting that German prices only increase a few percent over the period and that $m_t - s_t$ is falling more than $m_t - p_t$. It follows that $p_t - s_t$ is essentially the real exchange rate and it is mostly falling.

Test	p	m	s	Test	(p,m,s)
$\chi^2_{normality}(2)$	1.3 [0.53]	6.0 [0.05]	4.5 [0.11]	$\chi^2_{normality}(6)$	3.1 [0.79]
$F_{AR(1)}(1,20)$	1.8 [0.19]	1.0 [0.32]	0.1 [0.82]	$F_{AR(1)}(9,39)$	1.5 [0.20]
$F_{AR(3)}(3,18)$	0.6 [0.62]	0.8 [0.53]	0.3 [0.81]	$F_{AR(3)}(27,29)$	1.1 [0.44]
$F_{ARCH(3)}(3,15)$	0.1 [0.94]	0.2 [0.92]	0.1 [0.93]		

Table 1: Misspecification tests for the vector autoregressive model for p, m, s. p-values are given in brackets.

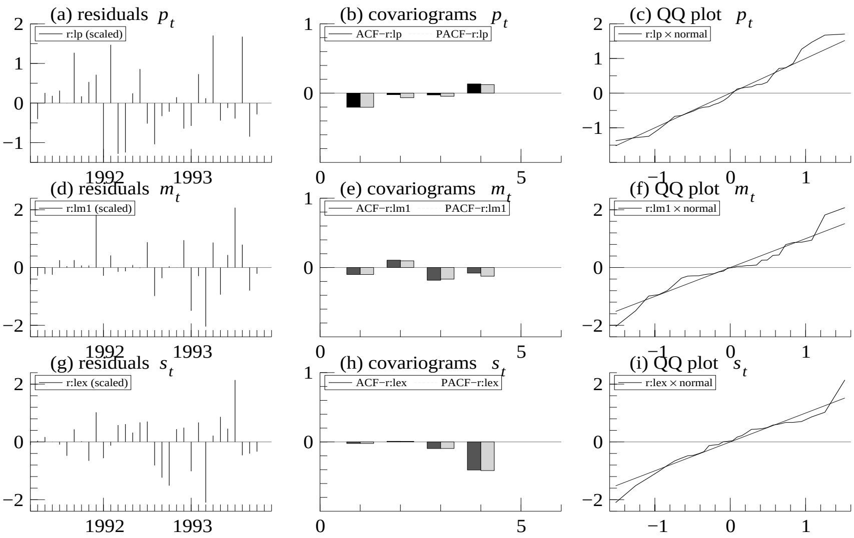

Figure 2: Misspecification graphics for unrestricted model.

This can be seen more clearly from Figure 3 (a),(d).

In the following two econometric models are proposed for the data. The approach is general-to-specific as advocated by for instance Hendry (1995). This gives considerable weight to describing the sample distribution of the data, but is carried out as a dialogue between the economic theory and the data. As seen from the reviewed empirical research Cagan's model is not quite rich enough to describe the sample variation in detail although it of course represents a valuable insight into the basic structure of hyperinflations.

4 Model 1: Variables in levels

With theories linking money both with prices and exchange rates it seems prudent to seek to analyse all three variables in a joint model. It will first be established that

Re(z)	**1.21**	-0.42	-0.42	0.02	0.02	**0.75**	**0.75**	-0.31	0.09
Im(z)	**0**	0.84	-0.84	0.90	-0.90	**0.33**	**-0.33**	0	0
$\lvert z\rvert$	**1.21**	0.94	0.94	0.90	0.90	**0.81**	**0.81**	0.31	0.09

Table 2: Characteristic roots of unrestricted model

Cointegration rank, r	0	1	2	3
Test	79.1 [0.00]	23.1 [0.11]	9.8 [0.14]	
Likelihood	15.30	43.27	49.94	54.84

Table 3: Cointegration rank tests.

a vector autoregression gives a reasonable fit and then an econometric model with random walk and explosive common trends will be applied.

A model with a constant, a linear trend and three lags is fitted to the data up to 1993:10:

$$X_t = \sum_{j=1}^{3} A_j X_{t-j} + \mu_c + \mu_l t + \varepsilon_t,$$

where the innovations ε_t are assumed independent normal $N_3(0, \Omega)$-distributed. On the one hand this gives a model that has admittedly few degrees of freedom in that each equation has 11 mean parameters which are fitted using $T = 32$ observations. On the other hand a lot of information should be available in these explosively growing time series. Formal mis-specification tests are reported in Table 1 while Figure 2 reports graphical tests for mis-specification. Interpreting these in the usual way indicates that the model is well specified. In doing so it is assumed that the usual asymptotic theory is valid although this has only been proved for the test for autocorrelation in the residuals, see Nielsen (2001a). Some of the test statistics are reported in an F-form as advocated by Doornik and Hendry (2001) in an attempt to deal with finite sample issues for these tests even though it has not yet been argued whether this represents an improvement.

Table 2 reports the characteristic roots of the unrestricted vector regression. It appears as if there is one explosive root and two unit roots as marked with bold face. The explosive root of 1.21 is within the region of 1.15-1.35 discussed above. There is a further set of four complex roots near the unit circle. An interpretation of a seasonal pattern repeating itself every five months seems unlikely. In this analysis these four roots will be ignored, but it is a matter for further research to understand the nature of such roots.

The next step of the analysis is a co-integration analysis using the approach sug-

Re(z)	**1.19**	**1**	**1**	-0.37	-0.37	0.07	0.07	-0.54	0.07
Im(z)	**0**	**0**	**0**	0.88	-0.88	0.83	-0.83	0	0
$\lvert z \rvert$	**1.19**	**1**	**1**	0.95	0.95	0.83	0.83	0.54	0.07

Table 4: Characteristic roots of restricted model with rank one, $r = 1$.

	p	m	s	t
H_1	1	-0.35	-1	0.065
$(\sqrt{LR})$	(6.2)	(-6.5)	(-6.2)	(6.6)
H_1, H_ρ	1	-0.35	-1	0.011

Table 5: Cointegrating vector, $\hat{\beta}^* = \hat{\beta}^*_1$, estimated under H_1 alone and under the joint hypothesis $\mathsf{H}_1, \mathsf{H}_\rho$. Signed likelihood ratio statistic, $\sqrt{LR}$, for insignificance is given in brackets.

gested by Johansen (1996). For this purpose the model is re-parametrised as

$$\Delta_1 X_t = (\Pi, \Pi_l) X^*_{t-1} + \sum_{j=1}^{2} \Gamma_j \Delta_1 X_{t-j} + \mu_c + \varepsilon_t, \tag{4.1}$$

where $\Delta_1 X_t = X_t - X_{t-1}$ is the usual first difference and $X^*_{t-1} = (X'_{t-1}, t')'$. This likelihood can be maximised analytically under the reduced rank hypothesis

$$\text{rank}(\Pi, \Pi_l) \leq r \leq \dim X \qquad \text{so} \qquad (\Pi, \Pi_1) = \alpha \beta^{*\prime}$$

for matrices $\alpha \in \mathbf{R}^{p \times r}, \beta^* \in \mathbf{R}^{(p+1) \times r}$ with full column rank. Although the symbols α, β were used above to describe Cagan's model, they are used here once again to be consistent with Johansen's notation. The interpretation of the cointegrating vectors β is now that $\beta' X_t$ has no random walk component but it could have an explosive component. This statement will be made more precise in connection with the Granger-Johansen representation in (4.2) below. The usual asymptotic critical values are valid in the presence of explosive roots as argued by Nielsen (2001) for the univariate case and Nielsen (2000) for the general case.

The cointegration rank r is determined using the likelihood ratio tests reported in Table 3. It is relatively clear to conclude that $\hat{r} = 1$. The characteristic roots are only little changed by imposing this restriction as seen from comparing Table 4 with Table 2. Once the rank is determined we can impose restrictions on the cointegrating vector β^*. A homogeneity restriction, H_1 say, between prices and exchange rates reduces the likelihood value slightly to 43.0 and such a restriction is therefore easily accepted when comparing the likelihood ratio statistics to a $\chi^2(1)$ distribution. The resulting cointegrating vector is reported in the first line of Table 5. As the cointegrating relation $\beta' X_t$ represents those linear combinations that are explosively growing, but without a random walk component, it can be interpreted as the relation of nominal money, m_t, and real price, $p_t - s_t$, that generates the explosive trend.

To investigate the influence of the explosive trend the model is now reformulated as

$$\Delta_1 \Delta_\rho X_t = \alpha_1 \beta_1^{*\prime} \Delta_\rho X_{t-1}^* + \alpha_\rho \beta_\rho' \Delta_1 X_{t-1} + \psi \Delta_1 \Delta_\rho X_{t-1} + \mu_c + \varepsilon_t$$

where $\beta_1^* = \beta_1$ is the cointegrating vector from before and $\Delta_\rho X_{t-1} = X_t - \rho X_{t-1}$ with ρ being an unknown scale parameter representing the explosive root. The matrix $\alpha_\rho \beta_\rho'$ has rank $\dim X - 1 = 2$ due to the single explosive root. Nielsen (2004) shows that in this model the process X_t has Granger-Johansen representation

$$X_t \approx C_1 \sum_{s=1}^{t} \varepsilon_s + C_\rho \sum_{s=1}^{t} \rho^{t-s} \varepsilon_s + y_t + \tau_c + \tau_l t + \tau_\rho \rho^t \tag{4.2}$$

where y_t can be given a stationary initial distribution. The impact matrices C_1, C_ρ are functions of the parameters and satisfy $\beta_1' C_1 = 0$ and $\beta_\rho' C_\rho = 0$ whereas τ_l satisfies $\beta_1' \tau_l + \delta_1' = 0$ and the coefficients τ_c, τ_ρ are functions of parameters and initial values so $\beta_\rho' \tau_\rho = 0$. The explosive common trend $W_t = \sum_{s=1}^{t} \rho^{-s} \varepsilon_s$ converges almost surely to a random variable W as t increases according to the Marcinkiewicz-Zygmund result, see for instance Lai and Wei (1983).

Simple hypotheses on the co-explosive vectors β_ρ can be tested using χ^2-inference. The underlying asymptotic result, due to Lai and Wei (1985) and Nielsen (2003) is that the stationery component, the random walk and the explosive trend are asymptotically uncorrelated. Nielsen (2004) then uses this to show that simple hypotheses on the co-explosive vectors β_ρ can be tested using χ^2-inference under the normality assumption to the innovations, which was checked above.

The hypothesis that β_ρ is known and given by

$$\mathsf{H}_\rho: \qquad \beta_\rho' = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix},$$

implies that each of $m_t - p_t$, $m_t - s_t$ and $s_t - p_t$ are co-explosive relations and thus have random walk behaviour. Since β_ρ is completely specified, the model can be estimated by reduced rank regression for each value of ρ. This in turn results in a profile likelihood in ρ which can then be maximised by a grid search. Searching in the region $\rho > 1$ there appears to be a unique maximum to the likelihood function of 41.3 with $\hat{\rho} = 1.175$ and a slightly changed cointegrating vector β_1 as given in Table 5. Once $\hat{\rho}, \hat{\beta}_\rho, \hat{\beta}_1^*$ are known the remaining parameters can be estimated by regression. The test statistic for H_ρ against H_1 is 3.4 which is small compared to the $\chi^2(2)$ distribution.

In summary, the above analysis shows that the three variables p, m, s each has a common explosive trend and two common random walk trends. The series co-explode so $m-p$, $m-s$ and $p-s$ are all non-exploding random walks, while the differenced series $\Delta p, \Delta m, \Delta s$ are explosive with no random walk. This indicates that linking for instance $m-p$ with Δp as in the model of Taylor (1991) may not give a balanced regression in this situation. A suggestion for getting around this issue and for recovering Cagan's money demand schedule is given in the following.

5 Model 2: transformed variables

In order to overcome the difficulty that $m-p$ is $\mathsf{I}(1)$ while Δp is explosive another inflation measure is constructed. A three dimensional vector autoregression is then set up and analysed as a cointegration model using the parametrisation in (4.1).

One central question is how to measure the cost of holding money. While Cagan approximates $C_t = \partial p_t/\partial t = (\partial P_t/\partial t)/P_t$ essentially by Δp_t another measure is

$$c_t = 1 - \frac{P_{t-1}}{P_t} = 1 - \exp\left(-\Delta p_t\right),$$

showing the relative loss in purchasing power over one period. This can be motivated by the following argument inspired by Hendry and von Ungern-Sternberg (1981). The nominal money stock growths according to

$$M_t = M_{t-1} + \delta_t,$$

where δ_t represents net money issues. Dividing through by P_t then shows that real money satisfies

$$\frac{M_t}{P_t} = \frac{M_{t-1}}{P_{t-1}}\left(\frac{P_{t-1}}{P_t}\right) + \frac{\delta_t}{P_t}$$

where the coefficient $c_t = 1 - P_{t-1}/P_t$ is the proportion of the real money stock that is lost from period to period. The variable c_t has the property of being bounded by 1 indicating that in each period one can at most loose all money. This fits nicely with interpreting inflation as seigniorage, giving a maximal tax rate of 100%. When the quantity $\Delta p_t = p_t - p_{t-1} = \log(P_t/P_{t-1})$ is small, a Taylor expansion shows

$$c_t = 1 - \exp\left(-\Delta p_t\right) \approx \Delta p_t.$$

The measure c_t is closely related to the inflation measure $\Delta p_t/(1+\Delta p_t)$, which, however, has an asymptote for $\Delta p_t = -1$.

While the quantity Δp_t is indeed the preferred inflation measure when analysing economies without severe inflation the choice of measure becomes increasingly important as the inflation accelerates. As the price series p_t accelerates, c_t approaches

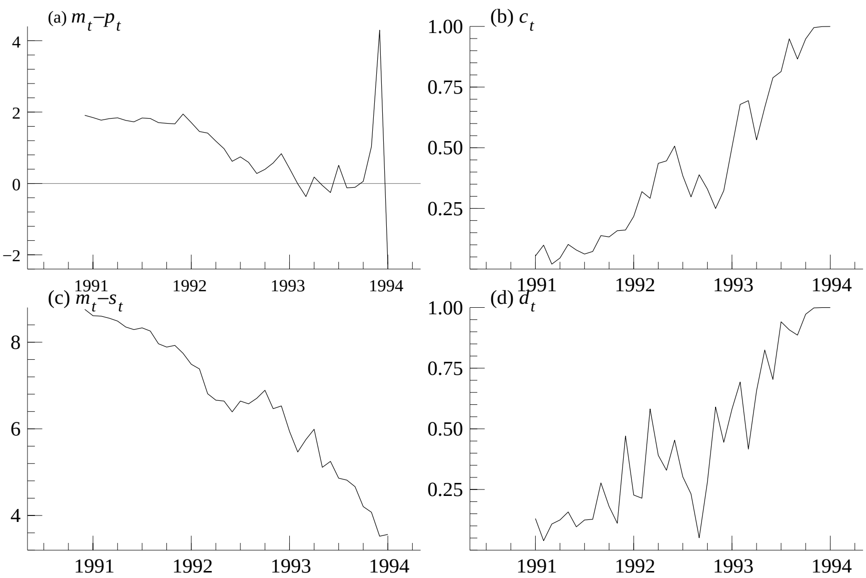

Figure 3: Transformed variables

1 indicating a nearly complete loss in value of money. This type of transformation is related to the non-linear models suggested by Frenkel (1977) linking real money, $m-p$, with either $\log(\Delta p_t^e)$ or $(\Delta p_t^e)^\gamma$ although such an approach would maintain Δp_t as the central measure of the cost of holding money. A measure like c_t appears to give a more direct measure of the cost of holding money and can more easily be used in a linear model. It has the added benefit of reducing the impact of measurement error as prices accelerate.

The transformed variables $m_t - p_t$, $m_t - s_t$ and c_t as well as a depreciation rate $d_t = 1 - \exp(-\Delta s_t)$ are plotted in Figure 3. While the measurement problem in prices show up in real money, $m_t - p_t$, money deflated by exchange rate, $m_t - s_t$, is more benignly behaved. Concentrating on the variables $m_t - s_t, c_t, d_t$ at first it is possible to set up a model for the entire period up to 1994:1.

A third order vector autoregression with a restricted constant is fitted to the data 1991:1 to 1994:1 giving a sample size of $T = 37 - 3 = 34$. Mis-specification graphics and tests are reported in Figure 4 and Table 6. Neither the formal tests nor the graphical tests indicate any serious mis-specification. In Figure 4, the first three rows relate directly to the three variables, $m_t - s_t, c_t$ and d_t, while the last row relates to a linear transformation thereof, $d_t - c_t$, exploiting that the likelihood for an unrestricted vector autoregression is invariant to linear transformations of the variables involved. The last column of Figure 4 consists of recursive plots testing the temporal invariance

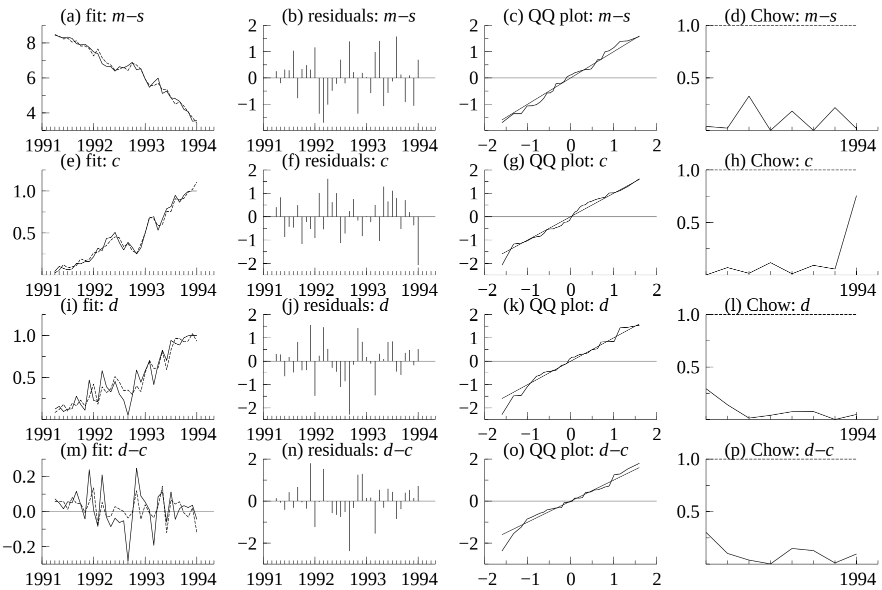

Figure 4: Misspecification graphics for model for transformed data. First column show quality of the fit, so solid line is the observed series and dashed line is fit. Second column: standardised residuals. Third column: QQ plots. Fourth column: One-step-ahead recursive Chow tests. First row: $m_t - s_t$. Second row: c_t. Third row: d_t. Fourth row: $c_t - d_t$.

of the model, which is something that could be questioned for hyperinflation data. The software PCGive has three different recursive plots which are all fine, with the one-step-ahead Chow test reported here.

There is now one characteristic root at 1.035 while the remaining roots are well inside the unit circle. The cointegration rank tests reported in Table 7 point to a rank of 1. Under that hypothesis the slightly explosive root is restricted to 1 and all characteristic roots, but two unit roots, are well inside the unit circle.

The estimated cointegrating relation is given by

$$\underset{(\sqrt{LR})}{ecm_t} = \underset{(2.8)}{1}(m_t - s_t) + \underset{(2.0)}{3.26}c_t + \underset{(5.7)}{10.27}(c_t - d_t) - \underset{(-2.7)}{8.48}.$$

The signed log-likelihood ratio test statistics for individual exclusion restrictions are reported in brackets and are asymptotically standard normal distributed, so one-sided tests 5% level tests would have a critical value of about plus or minus 1.65. This cointegrating vector shows that real money, deflated by exchange rates, moves both with c_t and d_t and not c_t alone. Indeed, excluding d_t, by eliminating $c_t - d_t$, but

Test	$m_t - s_t$	c_t	d_t	Test	$(m_t - s_t, c_t, d_t)$
$\chi^2_{normality}(2)$	0.1 [0.95]	1.2 [0.54]	1.9 [0.38]	$\chi^2_{normality}(6)$	2.8 [0.83]
$F_{AR(1)}(1, 23)$	0.1 [0.71]	0.1 [0.70]	1.4 [0.25]	$F_{AR(1)}(9, 46)$	0.5 [0.87]
$F_{AR(3)}(3, 21)$	0.8 [0.49]	1.3 [0.31]	2.1 [0.13]	$F_{AR(3)}(27, 38)$	0.9 [0.59]
$F_{ARCH(3)}(3, 18)$	1.4 [0.28]	0.2 [0.91]	0.2 [0.88]		

Table 6: Misspecification tests for the vector autoregressive model for $m_t - s_t, c_t, d_t$. Asymptotic p-values are given in brackets.

Hypothesis	$H(0)$	$H(1)$	$H(2)$	$H(3)$
Test	60.1 [0.00]	15.5 [0.20]	4.2 [0.40]	
Likelihood	80.03	102.31	107.97	110.06

Table 7: Cointegration rank tests for transformed model. Asymptotic p-values are given in brackets.

keeping c_t, is strongly rejected, whereas the decision to keep c_t is slightly marginal. The cointegrating equation is approximately of the same form as Cagan's with real money stock measured in foreign currency falling with c_t. In addition, the term, $d_t - c_t$, which can be interpreted as the real appreciation rate of the German mark, enters so that if the German mark appreciates faster than prices rise, goods become relative cheaper, and the money circulation rises. Comparing the Figures $5(a, b)$ shows how the sign of $c_t - d_t$ varies over time with $m_t - s_t$ tending to increase when $c_t - d_t$ is negative. The cointegrating relation itself, normalised on real money is plotted in Figure $5(c)$. Due to the cointegration framework the coefficient to c_t can be thought of as the semi-elasticity for the expected future cost of holding money as in the setup of Taylor (1991).

Ignoring the differential of the cost of holding money and the depreciation, Cagan's semi-elasticity α can be estimated by $\widehat{\alpha} = 3.26$. This value is in line with both Cagan's and Sargent's estimates for the German hyperinflation. According to Cagan the maximal revenue from seigniorage, assuming money rises at a constant rate, is then estimated by $\exp(\widehat{\alpha}^{-1}) - 1 = 36\%$. It seems natural to compare this with the average cost of holding money for a month, $c_t = \Delta P_t / P_t$, rather than the average of inflation measure through $\Delta P_t / P_{t-1}$ since the former is precisely a measure for how much value is lost over a month. For the full sample this average is 42.6% increasing to 44.6% when the three initial values are discarded.

Having the cointegrating relation in place, the short term dynamics of the system can be analysed in order to understand how the variables influence each other. The notion of weak exogeneity introduced by Engle, Hendry and Richard (1983) is helpful and can be implemented in the cointegration analysis as zero row restrictions of the α

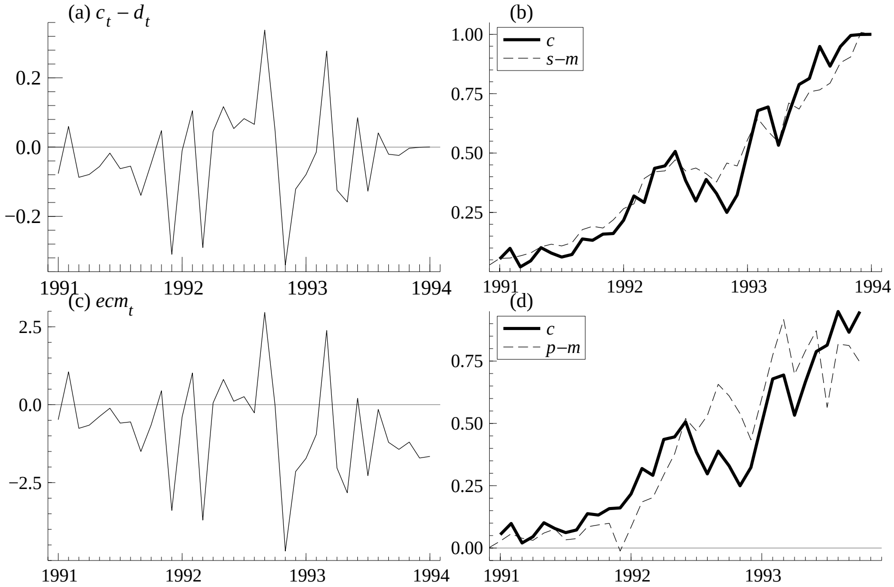

Figure 5: (a) (Minus) real appreciation rate for German mark. (c) Cointegrating relation from Table 8. (b, d) Cost of holding money compared with (minus) real money measured by deflating with exchange rate and price level, respectively. The scale and mean of real money have been adjusted to match the (0,1) range.

vector, see Johansen (1996, §8). After exploration of weak exogeneity properties the approach of Hendry (1995, §16.8) is followed in obtaining a parsimonious vector autoregression by single equation regressions using the estimated cointegrating relation as regressor.

In a first and partly unsuccessful attempt to reducing the model, the depreciation rate d_t is investigated as a candidate for weak exogeneity. The log likelihood ratio test statistic is 3.02 [$p = 0.08$] which gives a rather marginal decision given the small sample. Imposing weak exogeneity, however, only results in minor changes to the cointegrating vector. This analysis has two interesting features, in that real money growth, $\Delta(m_t - s_t)$, only enters marginally in the equation for Δc_t and that neither $\Delta(m_t - s_t)$ nor Δc_t are significant in the equation for Δd_t indicating that these variables are non-Granger causing for Δd_t. This analysis only gives a poor understanding of the dynamics of the system in that it appears as if the exchange rate is driving the inflation. In other words there is only marginal statistical support and a weak economic interpretation for imposing weak exogeneity of d_t.

An advantage of Johansen's method for cointegration analysis is its invariance to linear transformations of the variables, hence it is equivalent to consider the variable

	$m_t - s_t$	c_t	$c_t - d_t$	1
$\hat{\beta}'$	$\underset{(2.7)}{1}$	$\underset{(1.9)}{3.22}$	$\underset{(6.0)}{10.3}$	$\underset{(-2.7)}{-8.50}$
$\hat{\alpha}'$	$\underset{(5.5)}{0.33}$	$\underset{(-5.5)}{-0.088}$	0	

Table 8: Cointegrating vectors $\hat{\beta}$ and adjustment vector $\hat{\alpha}$ for transformed model. Signed likelihood ratio statistic, $\sqrt{LR}$, for insignificance is given in brackets.

vectors $(m_t - s_t, c_t, d_t)$ and $(m_t - s_t, c_t, c_t - d_t)$. The fourth row in Figure 4 indicates the fit for the variable $c_t - d_t$. The test for weak exogeneity of the real depreciation rate $c_t - d_t$ is given by a test statistic of 0.51 $[p = 0.47]$. The estimated α, β under that restriction are reported in Table 8. Using the cointegrating relation normalised on $m_t - s_t$ as a regressor, ecm_t say, a parsimonious vector autoregression was found as

$$\begin{aligned}
\Delta(m-s)_t &= \underset{(0.05)}{0.33} ecm_{t-1} - \underset{(0.18)}{0.86}\Delta(m-s)_{t-1} + \underset{(0.5)}{1.1}\Delta c_{t-2} \\
&\qquad - \underset{(0.4)}{1.9}\Delta(d-c)_{t-1} + \underset{(0.3)}{1.6}\Delta(d-c)_{t-2} + 0.20\hat{\varepsilon}_t. &(5.1)\\
\Delta c_t &= -\underset{(0.013)}{0.090} ecm_{t-1} + \underset{(0.04)}{0.10}\Delta(m-s)_{t-1} + \underset{(0.04)}{0.20}\Delta(m-s)_{t-2} \\
&\qquad + \underset{(0.11)}{0.60}\Delta\left(d-c\right)_{t-1} - \underset{(0.06)}{0.23}\Delta\left(d-c\right)_{t-2} + 0.046\hat{\varepsilon}_t, &(5.2)\\
\Delta\left(d-c\right)_t &= \qquad -\underset{(0.08)}{0.25}\Delta(m-s)_{t-1} \\
&\qquad -\underset{(0.14)}{0.47}\Delta\left(d-c\right)_{t-1} - \underset{(0.14)}{0.42}\Delta\left(d-c\right)_{t-2} + 0.14\hat{\varepsilon}_t &(5.3)
\end{aligned}$$

These equations have 2,2 and 3 restrictions imposed, respectively, and are valid even at a 20% level since the respective marginal log likelihood ratio statistics of 1.80, 1.12 and 1.26 are small compared to χ^2-distributions. Apart from the cointegrating relation that is driving real money and inflation directly, it appears as if most of the dynamics is generated by the growth of real money, $\Delta(m_t - s_t)$, and the real depreciation rate, $d_t - c_t$, while inflation growth, Δc_t, only enters in the equation for real money. The cointegrating relation ecm_t enters with positive sign in the $m_t - s_t$ equation and negative sign in the c_t equation reflecting the much larger coefficient to c_t in the cointegrating vector.

The only outstanding issue is whether a model with real money measured by $m_t - p_t$ rather than $m_t - s_t$ can be constructed. This turns out to be difficult. As a start, it is actually easy to fit well specified vector autoregressions to the bivariate system $(m_t - p_t, c_t)$ as well as $(m_t - p_t, c_t, d_t - c_t)$. This is under the proviso that the last three observations are discarded and a dummy is introduced for July 1993 which is around the time of the attempted prize freeze. However, in the bivariate system there is no evidence of cointegration whereas c_t is insignificant in the single cointegrating vector of the tri-variate system. This point can be illustrated graphically. In Figure $5(b, d)$, the negative of the the real money variables, $s_t - m_t$ and $p_t - m_t$, respectively, are plotted with c_t with ranges and means adjusted to the latter. It is clear that $s_t - m_t$ follows c_t nicely with discrepancies matched by $d_t - c_t$ of Figure 5(a) while $p_t - m_t$ does not track c_t well. Further research would be needed to see whether this is a problem particular to the Yugoslavian case, or whether the relative ease of measuring exchange rates rather than prices makes $m_t - s_t$ a better measure for real money in hyperinflations.

6 Discussion

Since the work of Taylor (1991) hyperinflationary money demand schedules have typically been analysed using an $\mathsf{I}(2)$ approach, where real money, $m_t - p_t$, and price growth, Δp_t, have been modelled as $\mathsf{I}(1)$ variables. The first of the presented econometric models for the extreme Yugoslavian hyperinflation therefore looks at an unrestricted vector autoregression for nominal money, m_t, prices, p_t, and exchange rates, s_t. Using a recently developed techniques for analysing explosive variables it is found that real money appears to be $\mathsf{I}(1)$ whereas price growth is explosive. This suggests that at least for the Yugoslavian case Taylor's approach should be modified to some extent.

In the second econometric model, the cost of holding money, c_t, is therefore considered instead of Δp_t. This measure has the advantage of being bounded by one, and it is thus easier to model empirically, and it also seems to be a more reasonable ingredient in a discrete time version of the seigniorage interpretation of hyperinflation as presented by Cagan (1956). A further advantage is that the full sample can now be analysed in contrast to earlier work on the Cagan data.

When analysing money deflated by the exchange rate, $m_t - s_t$, or alternatively $m_t - p_t$, together with c_t and the exchange rate depreciation, d_t, it appears as if all three variables enter a cointegrating relation. With the new measure for inflation there is only a small discrepancy between "actual" and "optimal" inflation tax as introduced by Cagan. The depreciation cannot be excluded from the cointegrating relation which supports the model of Abel, Dornbusch, Huizinga and Marcus (1979) as opposed to the simpler models of Cagan (1956) and Frenkel (1977). This suggests that "dollarisation", in terms of the German mark, played an important role in the Yugoslavian hyperinflation as found by Petrović and Mladenović (2000). The real depreciation rate $d_t - c_t$ that is now entering the money demand schedule, is found to be weakly exogenous showing that only real money and inflation are directly disequilibrium correcting. A parsimonious vector autoregression can now be constructed indicating that the dynamics is driven mainly by real money growth and real depreciation rate growth with the lagged inflation growth only entering in the equation for real money.

The results open up for various lines of future research. A first issue is how much information is available in a dataset like this. The explosive growth explored in the first model seems to generate a lot of sample variation, which is largely eliminated when moving on to the second model, so the second model could be more prone to finite sample issues. For the same reason it is probably right to follow Cagan in ignoring output in the first model, whereas in the second model the variables are considered on a scale where a variable like output may matter. Petrović, Bogetić and Vujošević (1999) suggest that the output was reduced by about 50% in US dollar terms in this period, whereas real money measured as $m_t - s_t$ is reduced by $\exp(-4.39) \approx 99\%$ over the period, indicating that the velocity of money changes considerably in hyperinflations. If output data were available to be included in the money demand schedule the semi-elasticity for the cost of holding money could very well be found to be more significant. Expectations have not played a large role in this analysis. On the one hand Taylor (1991) pointed out that cointegrating relations can be interpreted in terms of different expectation hypotheses, so the results are nonetheless of interest also in a expectations setup, and on the other hand, it is questionable how agents form their expectations in an extreme hyperinflation when a country is under embargo and on war-footing. A final step forward is that the second model actually follows the hyperinflation to the end, which makes it easier to consider how the hyperinflation actually ends.

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