Short-Run Parameter Changes in a Cointegrated Vector Autoregressive Model

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Abstract

This paper addresses the question of whether a conventional approach to cointegration is applicable to the case where changes are allowed in the parameters for the short term dynamics. We reparametrise a vector autoregressive model such that the short-run parameters exhibiting changes at known points are explicitly given. We then show that the likelihood ratio test statistic for cointegration rank is based on reduced rank regression and has the usual asymptotic distribution. An emperical illustration using US gasoline prices is presented.

KEY WORDS: Cointegration, Parameter Change, Short Term Dynamics, Likelihood Ratio Test.

1 Introduction

The cointegration analysis of Johansen (1988, 1996) is based on a simple vector autoregressive (VAR) model that remains constant throughout the sample period. When there are regime changes within a given sample, it is convenient to allow parameter changes to some extent. However, this will often involve estimation based on numerical optimisation or new asymptotic distributions, either of which making the econometric analysis more complicated. In the following cointegration analysis is presented as a model in which the parameters for short term dynamics can change at known points in time. The statistical analysis of the model is based on reduced rank regression as in the original work of Johansen (1988) and inference can be conducted using the tables presented in his monograph.

The starting point of the analysis is a VAR(k) model of a p-dimensional time series, $X_{-k+1}, ..., X_0, X_1, ..., X_T$, given by

$$\triangle X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \triangle X_{t-i} + \varepsilon_t, \quad \text{for } t > 0,$$

as suggested by Johansen (1988). Cointegration arises when Π has reduced rank r and can be written as $\Pi = \alpha\beta'$ for some $(p \times r)$ matrices α and β. Inspired by the I(2) analysis in Johansen (1996, Ch.9), the autoregressive equation can be reparametrised as

$$\triangle^2 X_t = \alpha\beta' X_{t-1} - \Gamma \triangle X_{t-1} + \sum_{i=1}^{k-2} \Psi_i \triangle^2 X_{t-i} + \varepsilon_t. \tag{1}$$

According to the Granger-Johansen representation theorem, see Johansen (1996, Theorem 4.2), the cointegrating relation $\beta' X_{t-1}$ determines the long run dynamics of the model. We refer to α and Γ as the parameters for medium term dynamics as they describe how the process adjusts to changes in $\beta' X_{t-1}$ and $\triangle X_{t-1}$ respectively. The parameters $\Psi_1, ...\Psi_{k-2}$ indicate the short term dynamics of the model in that they have no impact on the evolution of the common stochastic trends. We consider a situation where the parameters $\Psi_1, ...\Psi_{k-2}$ are allowed to change at given points in time. The main contribution of this paper is to prove that the conventional cointegration analysis is applicable to the case of short-run parameter changes.

This paper is organised as follows. Section 2 gives a model with changes in the parameters for the short-run adjustments and considers its statistical properties such as the Granger-Johansen representation. Section 3 provides the asymptotic analysis of the model, and it is shown that the results are identical to those in Johansen (1996, Ch.10,11). In Section 4 we give a brief survey of related literature and possible extensions of research, and in Section 5 an empirical study is presented based on the results derived in the preceding sections. The summary and conclusion are given in Section 6. Throughout this paper the following notational convention is used: For a matrix a with full column rank, let $\overline{a} = a(a'a)^{-1}$. Further, let $a_\perp$ satisfy $a'_\perp a_\perp = 0$ and have the property that $(a, a_\perp)$ has full rank.

2 Model and Statistical Analysis

For notational convenience, we consider the model (1) in which just one structural change is allowed in the short term dynamics, giving rise to two separate regimes. This can be easily generalised to multiple structural changes. The lengths of the first and second sub-samples are T_1 and T_2 respectively, so the total sample is given by $T = T_1 + T_2$. Thus the series are given by $X_1, ..., X_{T_1}$ and $X_{T_1+1}, ..., X_{T_1+T_2}$. We also extend the model to include a restricted trend and an unrestricted constant, which is the most common setting in applied work. Defining $T_0 = 0$, the model can be written down for each sub-sample as

$$\triangle^2 X_t = (\Pi, \Pi_l) \begin{pmatrix} X_{t-1} \\ t \end{pmatrix} - \Gamma \triangle X_{t-1} + \mu + \sum_{i=1}^{k-2} \Psi_i^{(j)} \triangle^2 X_{t-i} + \varepsilon_t, \qquad (2)$$

$$\text{for } T_{j-1} < t \leq T_{j-1} + T_j \text{ and } j = 1, 2,$$

where the innovations $\varepsilon_1, ..., \varepsilon_T$ are independent, identically normal $N(0, \Omega)$ distributed and the starting values $X_{-k+1}, ..., X_0$ are fixed. The parameters Π, Γ, Ω, $\Psi_i^{(j)} \in \mathbf{R}^{p \times p}$ and Π_l, $\mu \in \mathbf{R}^p$ vary freely so Ω is positive definite. The parameters Π, Π_l, Γ, μ, Ω are common for the two periods while the parameters for the short term dynamics change from $\Psi_1^{(1)}, ...\Psi_{k-2}^{(1)}$ to $\Psi_1^{(2)}, ...\Psi_{k-2}^{(2)}$. The hypothesis of reduced cointegration rank is given by

$$H(r) : rank\,(\Pi, \Pi_l) \leq r \text{ or } (\Pi, \Pi_l) = \alpha(\beta', \gamma'),$$

where α, $\beta \in \mathbf{R}^{p \times r}$ and $\gamma \in \mathbf{R}^r$. For future reference we define $X^*_{t-1} = \left(X'_{t-1}, t\right)'$ and $\beta^* = (\beta', \gamma')'$. In the rest of this section we give the Granger-Johansen representation of (2) and consider the application of reduced rank regression to this model.

2.1 Granger-Johansen Representation

The Granger-Johansen representation theorem of this model is closely related to those of Johansen (1996, Theorem 4.2) and Johansen, Mosconi, and Nielsen (2000). The underlying assumptions for the theorem are given first.

Assumption 2.1

1. The characteristic roots obey the equations det $\left\{A^{(j)}(z)\right\} = 0$ where

$$A^{(j)}(z) = (1-z)^2 I_p - \alpha\beta' z + \Gamma(1-z)z - \sum_{i=1}^{k-2} \Psi_i^{(j)} z^i (1-z)^2$$

   and satisfy $|z| > 1$ or $z = 1$.

2. The matrices α and β have full column rank r.

3. The matrix $\alpha'_\perp \Gamma \beta_\perp$ has full rank $p - r$.

The first condition ensures that the process is neither explosive ($|z| < 1$) nor seasonally cointegrated ($|z| = 1$ other than $z = 1$). The second implies that there are at least $p - r$ common stochastic trends and cointegration arises when $r \geq 1$. The final condition prevents the process from being $I(2)$ or of higher order. In combination the second and third conditions ensure that the number of common stochastic trends is exactly $p - r$. Then, the solution of (2) is given by the next theorem.

Theorem 2.2 *Granger-Johansen Representation Theorem.*
Under Assumption 2.1 the equation (2) is solved, for $T_{j-1} < t \leq T_{j-1} + T_j$ and $j = 1, 2$, by

$$X_t = C \sum_{s=1}^{t} \varepsilon_s + Y_t^{(j)} + \tau_c + \tau_l t + A^{(1)} + A^{(2)} 1_{(t>T_1)}. \tag{3}$$

Here $C = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1} \alpha'_\perp$ and the processes $(Y_{T_{j-1}+1}^{(j)}, ..., Y_{T_{j-1}+T_j}^{(j)})$ can be given zero mean, stationary initial distributions. The parameters τ_c and τ_l satisfy

$$\beta'_\perp \tau_c = 0, \quad \beta' \tau_c = \overline{\alpha}'(C\Gamma - I)\mu + \overline{\alpha}'(\Gamma C \Gamma - \Gamma)\overline{\beta}\gamma' - \gamma',$$
$$\tau_l = C\mu + (C\Gamma - I)\overline{\beta}\gamma',$$

whereas $A^{(j)}$ for $j = 1, 2$ depend on the initial values of the jth sub-sample such that $\beta' A^{(j)} = 0$. In particular $\beta' X_t + \gamma' t$ and $\triangle X_t$ can be given stationary initial distributions in each period.

Proof. See Appendix. ■

Thus, in the representation (3), the stationary part $Y_t^{(j)}$ is affected by the change in $\Psi^{(j)}$, whereas the parameters of the random walk, C, and of the linear trend, τ_l, remain unchanged throughout the whole period. This isolation of the parameter change plays a crucial role in asymptotic theories discussed later in this paper.

2.2 Reduced Rank Regression

As explained in Johansen (1996, Ch.6), the test for cointegration rank is based on reduced rank regression through the squared sample canonical correlations, $1 \geq \widehat{\lambda}_1 \geq \cdots \geq \widehat{\lambda}_p \geq 0$, of $\Delta^2 X_t$ and X^*_{t-1} corrected for all the other regressors. In performing such analysis we have to take into account the parameter change in $\Psi^{(j)}$. First we introduce some notation. For any two process V_t and W_t the residuals are defined as

$$(V_t | W_t) = V_t - \sum_{s=1}^{T} V_s W_s' \left(\sum_{s=1}^{T} W_s W_s' \right)^{-1} W_t.$$

Next, write $Z_{0t} = \Delta^2 X_t$, $Z_{1t} = X^*_{t-1}$, and let Z_{2t} be a stacked variable consisting of $\triangle X_{t-1}$ and 1, and Z_{3t} a stacked variable composed of $\triangle^2 X_{t-1}, \ldots \triangle^2 X_{t-k+2}$. Thus, for each period the residuals are derived from the regression of Z_{0t}, Z_{1t}, Z_{2t} on Z_{3t} as follows:

$$\begin{pmatrix} R^{(j)}_{0.3,t} \\ R^{(j)}_{1.3,t} \\ R^{(j)}_{2.3,t} \end{pmatrix} = \left(\begin{array}{c|c} Z_{0t} & \\ Z_{1t} & Z_{3t} \\ Z_{2t} & \end{array} \right). \tag{4}$$

By combining these two periods' residuals, we define the residuals from the second stage regression as

$$\begin{pmatrix} R_{0t} \\ R_{1t} \end{pmatrix} = \left(\begin{array}{c|c} \sum_{j=1}^{2} R^{(j)}_{0.3,t} & \\ \sum_{j=1}^{2} R^{(j)}_{1.3,t} & \sum_{j=1}^{2} R^{(j)}_{2.3,t} \end{array} \right).$$

The sample product moment matrices are given by

$$\begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} = \frac{1}{T} \sum_{t=1}^{T} \begin{pmatrix} R_{0t} \\ R_{1t} \end{pmatrix} \begin{pmatrix} R_{0t} \\ R_{1t} \end{pmatrix}',$$

We are in a position to present the reduced rank regression based on these settings. Since the innovations are normal, the concentrated log likelihood function is

$$\log L(\alpha, \beta, \Omega) = -\frac{T}{2} \log |\Omega| - \frac{1}{2} \sum_{t=1}^{T} \left(R_{0t} - \alpha \beta^{*\prime} R_{1t} \right)' \Omega^{-1} \left(R_{0t} - \alpha \beta^{*\prime} R_{1t} \right).$$

Following Johansen (1996, Ch.6), this is maximised by solving the eigenvalue problem

$$\left| \lambda S_{11} - S_{10} S_{00}^{-1} S_{01} \right| = 0,$$

with eigenvalues $\widehat{\lambda}_i$, $i = 1, \ldots, p+1$. Note that S_{11} is $(p+1) \times (p+1)$, whereas S_{00} is $p \times p$, so the rank of $S_{10} S_{00}^{-1} S_{01}$ is only p and hence $\widehat{\lambda}_{p+1} = 0$. The likelihood ratio (LR) test statistic for the hypothesis of at most r cointegration relations, $H(r)$, against $H(p)$ is given by

$$LR\left[H(r) | H(p) \right] = -T \sum_{i=r+1}^{p} \log(1 - \widehat{\lambda}_i). \tag{5}$$

We took into consideration the effects of the parameter change using the separate regression (4), so that the cointegration rank test (5) appears to be the same as that in Johansen (1996, Theorem 6.1) by formulation. However, it is not necessarily clear whether the limiting distribution of (5) is also unchanged. This issue is discussed in the next section.

3 Asymptotic Analysis of Cointegration

Asymptotic analysis presented in this section basically follows Johansen (1996, Ch 10, 11). However, care is needed since the parameter change contaminates the Granger-Johansen representation. In this section we first present the asymptotic distribution of the cointegration test as a theorem. Then, we investigate the asymptotic properties of stationary and non-stationary product moment matrices, which are given as lemmas needed to prove the theorem.

Theorem 3.1 *Suppose that (i) $T_j/T = a^{(j)}$ is fixed while $T \to \infty$ and (ii) Assumption 2.1 is satisfied. Then, as $T \to \infty$,*

$$LR\left[H(r)|H(p)\right] \xrightarrow{w} tr\left[\int_0^1 (dB)F'\left(\int_0^1 FF'du\right)^{-1}\int_0^1 F(dB)'\right],$$

where B is a $(p-r)$ dimensional standard Brownian motion and F is a $(p-r+1)$ dimensional process consisting of

$$F_i(u) = \begin{cases} B_i(u) - \int_0^1 B_i(u)du, & i = 1, ..., p-r, \\ u - \frac{1}{2}, & i = p-r+1. \end{cases}$$

Proof. Follow the proof of Theorem 11.1 in Johansen (1996) by using Lemmas 3.2 and 3.3 given below instead of his Lemmas 10.1 and 10.3. ■

This theorem ensures that the parameter change has no effect on the asymptotic distribution of the cointegration test. Therefore, the tables in Johansen (1996, Ch.15) are applicable to the case where changes occur in the parameters for the short term dynamics, and so is conventional hypothesis testing on cointegration and adjustment spaces. This is a quite useful finding in that some economies appear to be subject to regime shifts which can be captured by changes in the short-run parameters. An empirical example is given in Section 5.

In the following we investigate the asymptotic properties of the product moment matrices, S_{00}, $S_{01}\beta^*$, $\beta^{*\prime}S_{11}\beta^*$, in order to adapt Lemmas 10.1 and 10.3 of Johansen (1996) to the present model. As a prerequisite setting we need to define the variance-covariance matrices of the stationary processes for each sub-sample

$$\begin{pmatrix} \Sigma_{00.3}^{(j)} & \Sigma_{0\beta.3}^{(j)} & \Sigma_{02.3}^{(j)} \\ \Sigma_{\beta 0.3}^{(j)} & \Sigma_{\beta\beta.3}^{(j)} & \Sigma_{\beta 2.3}^{(j)} \\ \Sigma_{20.3}^{(j)} & \Sigma_{2\beta.3}^{(j)} & \Sigma_{22.3}^{(j)} \end{pmatrix} = Var\left(\begin{array}{c} \triangle X_t \\ \beta^{*\prime}X_{t-1}^* \\ \triangle X_{t-1} \end{array}\middle| \triangle^2 X_{t-1}, ... \triangle^2 X_{t-k+2}\right).$$

Furthermore, we define

$$\Sigma_{lm.3} = \sum_{j=1}^{2} a^{(j)}\Sigma_{lm.3}^{(j)}, \quad \text{for } l, m = 0, 2, \beta, \tag{6}$$

and

$$\begin{aligned} \Sigma_{00} &= \Sigma_{00.3} - \Sigma_{02.3}\Sigma_{22.3}^{-1}\Sigma_{20.3}, \\ \Sigma_{0\beta} &= \Sigma_{0\beta.3} - \Sigma_{02.3}\Sigma_{22.3}^{-1}\Sigma_{2\beta.3}, \\ \Sigma_{\beta\beta} &= \Sigma_{\beta\beta.3} - \Sigma_{\beta 2.3}\Sigma_{22.3}^{-1}\Sigma_{2\beta.3}. \end{aligned} \tag{7}$$

Based on these definitions, the asymptotic properties of the sample product moment matrices are given by the next lemma.

Lemma 3.2 *Suppose that (i) $T_j/T = a^{(j)}$ is fixed while $T \to \infty$ and (ii) Assumption 2.1 is satisfied. Then,*

$$\begin{pmatrix} S_{00} & S_{01}\beta^* \\ \beta^{*\prime}S_{10} & \beta^{*\prime}S_{11}\beta^* \end{pmatrix} \xrightarrow{p} \begin{pmatrix} \Sigma_{00} & \Sigma_{0\beta} \\ \Sigma_{\beta 0} & \Sigma_{\beta\beta} \end{pmatrix}, \tag{8}$$

where

$$\Sigma_{00} = \alpha\Sigma_{\beta 0} + \Omega, \quad \Sigma_{0\beta} = \alpha\Sigma_{\beta\beta}, \quad \Sigma_{00} = \alpha\Sigma_{\beta\beta}\alpha' + \Omega. \tag{9}$$

Proof. See Appendix. ∎

The derived results (8) and (9) match those in Johansen (1996, Lemmas 10.1 and 10.3). As shown in Appendix, this is because the sample product moments in each subsample converge to their population values, thereby their linear combinations using $a^{(j)}$ can also be defined accordingly. As shown above, Lemma 3.2 is required in the proof of Theorem 3.1.

Next, we investigate the asymptotic properties of non-stationary components with a view to adjusting Lemma 10.3 of Johansen (1996) to the present model. We first consider the next transformation as a prerequisite setting

$$\begin{aligned} B_T' X_{t-1}^* &= \begin{pmatrix} \alpha_\perp'\Gamma & -\alpha_\perp'\Gamma\tau_l \\ 0 & T^{-1/2} \end{pmatrix} \begin{pmatrix} X_{t-1} \\ t \end{pmatrix} \\ &= \begin{bmatrix} \alpha_\perp'\Gamma\left(C\sum_{s=1}^{t-1}\varepsilon_s + Y_{t-1}^{(j)} + \tau_c - \tau_l + A^{(1)} + A^{(2)}1_{(t-1>T_1)}\right) \\ T^{-1/2}t \end{bmatrix}, \end{aligned} \tag{10}$$

which isolates the deterministic trend in the final row of the vector. Then, we prove the following lemma.

Lemma 3.3 *Suppose the assumptions of Lemma 3.2 are satisfied. Then as $T \to \infty$ and $u \in [0,1]$*

$$T^{-1/2}B_T'X_{[Tu]}^* \xrightarrow{w} \begin{pmatrix} \alpha_\perp' W(u) \\ u \end{pmatrix},$$

where

$$T^{-1/2}\sum_{s=1}^{[Tu]}\varepsilon_s \xrightarrow{w} W(u),$$

and $W(u)$ is a Brownian motion in $p-r$ dimensions with variance matrix Ω, and $\xrightarrow{w}$ denotes weak convergence. The asymptotic distributions of the non-stationary product moments are

$$\begin{gathered} T^{-1}B_T'S_{11}B_T \xrightarrow{w} \int_0^1 GG'du, \\ B_T'(S_{10} - S_{11}\beta^*\alpha') \xrightarrow{w} \int_0^1 G(dW)', \\ B_T'S_{11}\beta^* \in O_p(1). \end{gathered}$$

where

$$G(u) = \begin{bmatrix} \alpha_\perp'\left(W(u) - \int_0^1 W(u)du\right) \\ u - \frac{1}{2} \end{bmatrix}.$$

Proof. The stationary processes $Y_{t-1}^{(j)}$ and the terms τ_c, τ_l, $A^{(1)}$, $A^{(2)}1_{(t-1>T_1)}$ appearing in (10) are all of order $O_p(1)$ uniformly in t. Therefore,

$$B_T' X_{t-1}^* = \begin{pmatrix} \alpha_\perp' \Gamma C \sum_{s=1}^{t-1} \varepsilon_s + O_p(1) \\ T^{-1/2} t \end{pmatrix},$$

The desired results follow as in the proofs of Lemmas 10.2 and 10.3 in Johansen (1996) since the random walk term is of order $O_p(T^{1/2})$ and thus dominates other terms of order $O_p(1)$. ■

The limiting distributions in Lemma 3.3 are identical to those in Johansen (1996, Lemma 10.3). This is because all the stationary processes are irrelevant asymptotically and the parameter change has no impact on C as shown in Theorem 2.2. This lemma is also required in the proof of Theorem 3.1.

4 Related Work and Further Extensions

The issue of structural changes in cointegrated processes has been addressed by a number of authors. Hansen (1992), Quintos and Phillips (1993), and Campos, Ericsson, and Hendry (1996) would be the earliest pieces of work, though not within the VAR framework. Parameter changes in cointegrated VAR models were studied in several papers such as Seo (1998), Kleibergen (1998), Hansen and Johansen (1999), and Hansen (2003). The final paper, in particular, considered a number of possible patterns of parameter changes using generalised reduced rank regression. However, the existing literature is limited in terms of testing cointegration rank in the presence of parameter changes i.e. the number of cointegration rank is often assumed to be given. Exceptions are Inoue (1999) and Johansen *et al.* (2000), which addressed rank tests with breaks in deterministic terms such as a linear trend. The latter paper provided a general framework for cointegration analysis in such cases.

We should note that the model (2) is transformed into a constant-restricted model without a linear trend, to which the asymptotic results developed above are also applicable. Furthermore, the combination of the present paper and Johansen *et al.* (2000) enables us to extend the model (2) to

$$\triangle^2 X_t = \alpha \begin{pmatrix} \beta \\ \gamma^{(j)} \end{pmatrix}' \begin{pmatrix} X_{t-1} \\ t \end{pmatrix} - \Gamma \triangle X_{t-1} + \mu^{(j)} + \sum_{i=1}^{k-2} \Psi_i^{(j)} \triangle^2 X_{t-i} + \varepsilon_t, \tag{11}$$

where structural breaks occur in the deterministic trend and constant as well as in the short term dynamics. We can also set up the following model

$$\triangle^2 X_t = \alpha \begin{pmatrix} \beta \\ \mu^{(j)} \end{pmatrix}' \begin{pmatrix} X_{t-1} \\ 1 \end{pmatrix} - \Gamma \triangle X_{t-1} + \sum_{i=1}^{k-2} \Psi_i^{(j)} \triangle^2 X_{t-i} + \varepsilon_t, \tag{12}$$

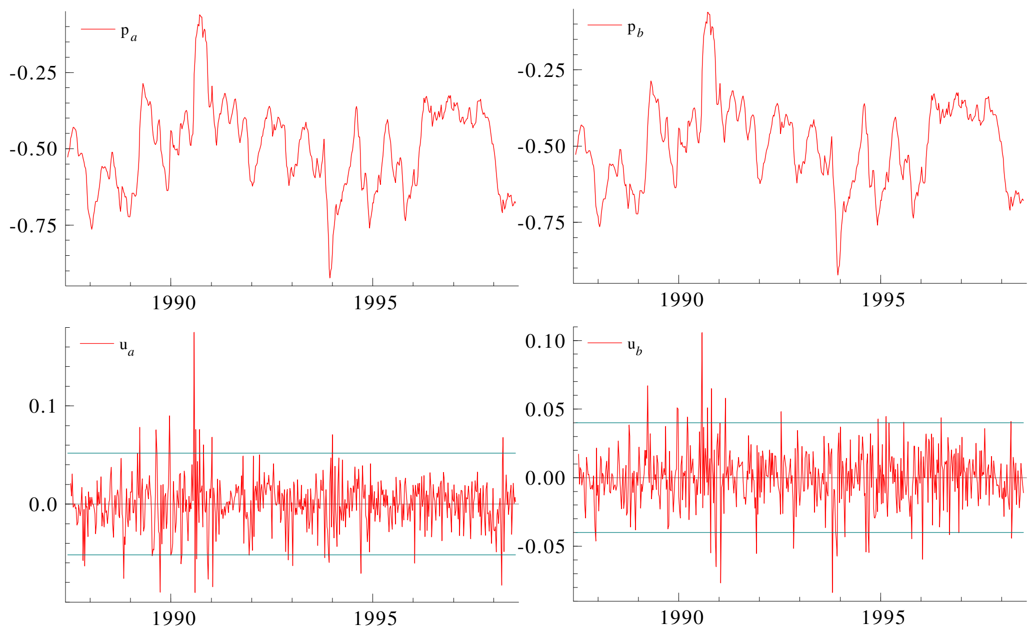
in which broken constant levels are allowed in addition to the changes in the short-run parameters. Some economies which have experienced relatively large regime changes could be described well by these models. In order to perform the rank test in (11) and (12) we have to rely on the results of the response surface analysis in Johansen *et al.* (2000), which was conducted for the cases of deterministic shifts embodied in $\gamma^{(j)}$ and $\mu^{(j)}$. We are justified in using the same results for the generalised models (11) and (12), since $\Psi_i^{(j)}$ dose not affect the asymptotic properties of the LR test statistics as we proved above. In practice, we can introduce additional indicator variables to let the likelihood function be conditional on the initial values of each sub-sample. See Johansen *et al.* (2000) for details.

The main contribution of this paper is proving that the conventional cointegration rank test and hypothesis testing can be applied to the case of changes in the short term parameters. Natural extension is to allow changes in the medium and long term parameters, α, Γ, and β. These types of changes, however, affect the limiting distributions of the common stochastic trends through the corresponding changes in the C matrix (see the definition of C in Theorem 2.2). Therefore, the asymptotic arguments in such cases can be much more involved than those of changes in Ψ_i. Hansen (2000) discussed these issues but further research would be required for an implementation.

5 Empirical Illustration

We give an empirical illustration using data analysed in Hendry and Juselius (2001), denoted HJ hereafter. All the empirical analyses and graphics in this section use *GiveWin / PcGive* (see Doornik and Hendry, 2001). The data set is composed of two weekly gasoline prices at different locations in the United States over the period 1987 to 1998. The logs of the prices ($p_{a,t}$ and $p_{b,t}$) are displayed in Figure 1, together with residuals (u_a and u_b) from a VAR(2) and with straight lines indicating $\pm 2 \times$ their standard errors.

Figure 1: Gasoline Prices and Residuals from the Constant Parameter VAR(2)



The two log prices start at non-zero level and appear to exhibit random-walk behaviour with no trend. Their comovements are fairly evident due to price arbitrage mechanisms. Thus the first reasoning is that a cointegrated VAR with a restricted intercept could give a reasonable description of the data. Next, we notice an upsurge in both of the series at 31st week in 1990 (denoted as 1990.31 henceforth), which corresponds to the outbreak of the Gulf war. This is quite conspicuous and may have had some impacts on the level coefficient of the underlying cointegrating relation. HJ introduced a step dummy being zero up to the outbreak of the war and one after that, restricted to lie in the cointegration space. This specification led to a reasonable data representation in their analysis. However, this kind of step dummy can have significant effects on the limiting distribution of the cointegration test, in which case inferences based on the ordinary quantiles may not be reliable. Thus, in order to avoid such a possible problem, it would be reasonable to allow a broken constant in the cointegration space by using the results of Johansen *et al.* (2000) instead of adopting a step dummy. Moreover, the phenomenon of autoregressive conditional heteroscedasticity (ARCH) is relatively clear in both of the residuals, and this is particularly obvious for u_a in the period prior to the Gulf war period. This may be captured by allowing a change in the short term dynamics in the cointegrated VAR. Our summarised conjecture is that these price series are cointegrated over the whole sample by price arbitrage mechanisms in the oil markets, but the Gulf war may have had some effects on the underlying mechanisms. Such effects may be reasonably described by the parameter-changing cointegrated VAR model formulated by (12) in the previous section.

The sample period runs from 1987.29 to 1998.29 and the VAR(2) was extended to a VAR(5) with a change in the short-run parameters as well as in the restricted intercept. The change point was set in the outbreak of the war (1990.31) and this determines the relative length of each sub-sample, which is required in using the results of the response surface analysis in Johansen *et al.* (2000). Furthermore, the likelihood function was conditional on the initial values of both sub-samples by introducing indicator variables. The price series exhibit large spikes at some points over the sample period, as is often the case with market data at high-frequency. Thus we included the same impulse and blip dummy variables as HJ so as to manage these outliers (see Appendix for details). Starting with a VAR(5) with no parameter shift, we were able to reduce the model to a VAR(2) model with $F(12, 1104) = 1.19[0.29]$. However, if starting with the parameter-changing VAR(5) model such reduction was rejected with $F(24, 1084) = 2.25[0.00]^{**}$. This could reflect the importance of allowing for a parameter change in model setting.

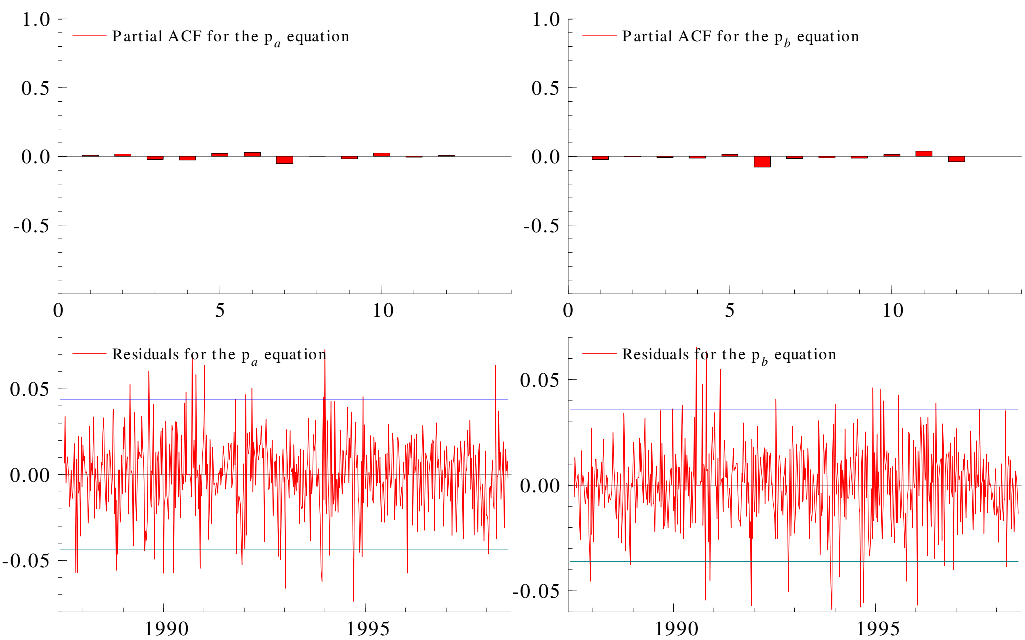
Table 1: Diagnostic and Cointegration Tests for the Models with Constant Parameters and Change in Parameters

	$AR_{F(7,552)}$	$ARCH_{F(7,545)}$	$Skew.$	$Kurt.$	λ	$r \leq$	$Q(r)$	Q_{99}	Q_{95}
$p_{a,t}$	1.53[0.15]	3.63[0.00]**	−0.18	4.50	0.12	0	88.48	24.74	19.99
$p_{b,t}$	2.44[0.02]*	2.32[0.02]*	0.12	3.94	0.02	1	14.04	12.73	9.13
$p_{a,t}$	0.82[0.57]	1.44[0.19]	−0.20	4.31	0.08	0	58.03	31.17	25.87
$p_{b,t}$	1.08[0.37]	2.76[0.01]**	0.13	4.02	0.02	1	13.28	16.66	12.59

Note. Figure in square brackets are p-values.

The second and third rows of Table 1 report the results of diagnostic and cointegration rank tests from a VAR(2) with no parameter shift. Note that this model is slightly different from that in HJ in the following point: A constant was included restrictedly in this model, whereas both a step dummy and constant were introduced restrictedly in the model in HJ. In other words, the latter model took account of a possible level shift effect of the Gulf war. The fourth and fifth rows give the results for a VAR(5) with a parameter change in both of the intercept, $\mu^{(j)}$, and short term dynamics, $\Psi_i^{(j)}$. We first focus on the diagnostics. In the left part of the table the following test results are given: kth-order serial autocorrelation ($AR_{F(k,\cdot)}$: see Godfrey, 1978); kth-order ARCH ($ARCH_{F(k,\cdot)}$: see Engle, 1982); Skewness and kurtosis of the residual distribution ($Skew.$ and $Kurt.$). The results of the parameter-constant model suggest that, although the impulse and blip dummies are included to manage outliers, there still exist autocorrelation for the $p_{b,t}$ equation and ARCH effects for both equations when choosing lag length 7 in these tests. In contrast, the parameter-changing model is free from autocorrelation and no evidence of ARCH is found in the equation for $p_{a,t}$. These findings are quite encouraging and can be confirmed by an overview of Figure 2, in which its partial autocorrelation functions (partial ACF) as well as residuals are shown. Even in allowing a shift in the parameters some evidence of excess kurtosis remains in addition to ARCH effects for the $p_{b,t}$ equation. However, as HJ suggested, it is known that cointegration analysis is not sensitive to these problems (see Gonzalo, 1994; Rahbek, Hansen, and Dennis, 2002), in contrast to possible serious effects of residual autocorrelation. Thus, we are justified in proceeding to tests for cointegration rank.

Figure 2: Partial ACF and Residuals for the Parameter-Changing VAR(5)



The right part of Table 1 reports the eigenvalues (λ), the LR test statistics ($Q(r)$), and the 99% and 95 % quantiles (Q_{9i} or Q_{9i}^R for $i = 9, 5$). Note that the values in Q_{9i} were taken from the conventional table in Johansen (1996), whereas those in Q_{9i}^R were derived from the response surface analysis in Johansen *et al.* (2000). In the case of no parameter change the LR test accepts two cointegration rank ($r = 2$) irrespective of the quantiles, which suggests that both series could be stationary rather than non-stationary. HJ also reached the same finding and, using other information such as the characteristic roots of the model, they explained that the first cointegrating relation is stationary and the second is near integrated but with significant mean-reversion. Thus, they concluded that the second relation should be approximated as integrated series for the purpose of robust inference. This is convincing reasoning, but ideally the second relation should be judged to be non-stationary by the LR test statistic to justify cointegration analysis. As shown in the final row of the table, the LR test supports $r = 1$ based on Q^R_{99} ($Q(1) = 13.28 < 16.66 = Q^R_{99}$) if we allow a parameter shift. Considering that the number of observations is quite large and over 570, we could be justified in using the 99% quantiles instead of those for 95%. Thus we could argue that the parameter-changing formulation led to a reasonable statistical finding in the choice of cointegration rank.

Table 2: Restrictions on the Cointegration and Adjustment Space of the Parameter-Changing Model

	$p_{a,t}$	$p_{b,t}$	$\mu^{(1)}$	$\mu^{(2)}$		$p_{a,t}$	$p_{b,t}$	$\chi^2_{(q)}$
β	1 (-)	-1 (-)	0.003 (0.012)	-0.016 (0.007)	α	-0.11 (0.02)	0 (-)	$0.43[0.81]_{(q=2)}$
β	1 (-)	-1 (-)	0 (-)	-0.016 (0.007)	α	-0.11 (0.02)	0 (-)	$0.50[0.92]_{(q=3)}$

Note. Standard errors are given under coefficients.

Setting $r = 1$ based on the above arguments, we proceed to placing restrictions on the cointegration as well as adjustment space. We imposed on β the restriction that the coefficients of both price series take an equal value with opposite sign. HJ found that $p_{b,t}$ is weakly exogenous for the parameters of interest (see Engle, Hendry, and Richard, 1983, for weak exogeneity), so we imposed an additional zero restriction on α with respect to $p_{b,t}$ (see Johansen, 1992). The second row of Table 2 reports the test result, in which the joint restrictions are accepted with a relatively large p-value. The intercept of the first sub-sample, $\mu^{(1)}$, is very small, having a positive sign, and its standard error is larger than that, which is in marked contrast to $\mu^{(2)}$. Thus we placed an additional zero restriction on $\mu^{(1)}$, and the overall restrictions are accepted with a p-value of 0.92, as shown in the final row of Table 2. The standard error of $\mu^{(2)}$ is so small that $\mu^{(2)}$ should remain in the cointegrating space. We are therefore justified in having found evidence of a broken constant in the cointegrating relation, representing a possible effect of the Gulf war in line with HJ.

Table 3: Equilibrium Correction Model with Change in Parameters

	$\triangle^2 p_{b,t}$	$\triangle p_{a,t-1}$	$\triangle p_{b,t-1}$	$\triangle^2 p_{a,t-1} 1_{(t<90.31)}$	$\triangle^2 p_{a,t-2} 1_{(t<90.31)}$	ecm_{t-1}
$\triangle^2 p_{a,t}$	0.788 (0.038)	-0.561 (0.039)	0.535 (0.05)	0.161 (0.046)	0.099 (0.045)	-0.12 (0.015)

Note. $ecm_t = p_{a,t} - p_{b,t} - 0.016 \times 1_{(t \geq 90.31)}$

Table 3 gives the summarised results of an equilibrium correction model (ECM), in which the equilibrium correction term, ecm_{t-1}, was derived from the restricted β. Since $\triangle^2 p_{b,t}$ can be treated as a conditioning variable due to weak exogeneity, the model was reduced to a single equation ECM describing the behaviour of $\triangle^2 p_{a,t}$. Some of the short term dynamics in the pre-war period are significant as shown in the table, whereas those for the post-1995 period were far from significant and eliminated from the model together with other insignificant dynamics. This reduction was accepted with $F(10, 546) = 1.14[0.33]$.

The overall findings allow us to conclude that a change occurred in the short-run dynamics of the data generation process, in which the cointegrated relation itself remains stable except for a break in the restricted constant. While this empirical illustration is based on a very simple framework with only two variables, it provides encouraging results for the applicability of the proposed parameter-changing model.

6 Summary and Conclusion

In this paper we proved that the conventional cointegration analysis is applicable to the case where changes occur in the parameters for the short term dynamics at known points in time. The VAR model was reparametrised in the same manner as I(2) cointegration analysis such that the short-run parameters exhibiting changes were explicitly given. Then, the Granger-Johansen representation was presented, and we found that the evolution of the common stochastic trends is independent of such changes. The asymptotic properties of the product moment matrices were investigated, and it was proved that the parameter changes have no significant impact on their asymptotic properties. Based on this finding we considered the limiting distribution of the LR test statistic for cointegration rank, reaching the conclusion that the ordinary argument can hold asymptotically.

This type of model setting is useful for the description of some economies, as shown in the empirical illustration using the US gasoline prices. The results here do not apply when the parameters for the medium and long term dynamics change and further research would be needed for empirical analyses of such cases.

Appendix

A Proof of Theorem 2.2

Consider first the homogenous case where μ and γ are both zero. For each period the usual Granger-Johansen representation theorem can be derived. Thus we define here

$$Z_t = (X_t'\beta, \triangle X_t', ..., \triangle X_{t-k+1}')',$$

which implies that the processes $\left(Z_{T_{j-1}+1}, ..., Z_{T_{j-1}+T_j}\right)$ for $j = 1, 2$ can be given zero-mean stationary distributions under the stated assumptions. It is left to check that the individual representation for each period can be combined as stated. In the model (2) with the homogenous assumption, we replace $\triangle X_{t-1}$ by $\triangle X_t - \triangle^2 X_t$, rearrange it, and multiply both sides by $\alpha_\perp'$, and then arrive at the following equation

$$\alpha_\perp' \Gamma \triangle X_t = \alpha_\perp' \left[\varepsilon_t + (\Gamma - I) \triangle^2 X_t + \sum_{i=1}^{k-2} \Psi_i^{(j)} \triangle^2 X_{t-i} \right].$$

Summing up $\alpha_\perp' \Gamma \triangle X_s$ over $s = 1, ...t$ yields

$$\alpha_\perp' \Gamma (X_t - X_0) = \alpha_\perp' \left[\sum_{s=1}^{t} \varepsilon_s + (\Gamma - I)(\triangle X_t - \triangle X_0) + \sum_{i=1}^{k-2} \Psi_i^{(1)} (\triangle X_{t-i} - \triangle X_{-i}) \right],$$
$$\text{for } 0 < t \leq T_1,$$

$$\begin{aligned} \alpha_\perp' \Gamma (X_t - X_0) = \alpha_\perp' &\left[\sum_{s=1}^{t} \varepsilon_s + (\Gamma - I)(\triangle X_t - \triangle X_0) + \sum_{i=1}^{k-2} \Psi_i^{(1)} (\triangle X_{T_1-i} - \triangle X_{-i}) \right. \\ &\left. \sum_{i=1}^{k-2} \Psi_i^{(2)} (\triangle X_{t-i} - \triangle X_{T_1-i}) \right], \quad \text{for } T_1 < t \leq T. \end{aligned} \tag{13}$$

Post-multiplying $\alpha_\perp'\Gamma$ in (13) with the identity $\beta_\perp\overline{\beta}_\perp' + \overline{\beta}\beta' = I$, pre-multiplying with $\beta_\perp(\alpha_\perp'\Gamma\beta_\perp)^{-1}$ noting that $\alpha_\perp'\Gamma\beta_\perp$ is invertible by Assumption 2.1, and then rearranging, we reach the following general formulation covering both of the above two cases

$$\begin{aligned}\beta_\perp\overline{\beta}_\perp' X_t &= C\Bigg\{\sum_{s=1}^{t}\varepsilon_s - \Gamma\overline{\beta}\beta' X_t + (\Gamma - I)\triangle X_t + \sum_{i=1}^{k-2}\Psi_i^{(j)}\triangle X_{t-i} + \Big[\Gamma X_0 \\ &\quad - (\Gamma - I)\triangle X_0 - \sum_{i=1}^{k-2}\Psi_i^{(1)}\triangle X_{-i} + \sum_{i=1}^{k-2}\left(\Psi_i^{(1)} - \Psi_i^{(2)}\right)\triangle X_{T_1-i}1_{(t>T_1)}\Big]\Bigg\}.\end{aligned}$$

Adding $\overline{\beta}\beta' X_t$ on both sides gives the desired representation with $\tau_c = \tau_l = 0$ and

$$\begin{aligned}Y_t^{(j)} &= (I - C\Gamma)\overline{\beta}\beta' X_t + C(\Gamma - I)\triangle X_t + \sum_{i=1}^{k-2} C\Psi_i^{(j)}\triangle X_{t-i}, \\ A^{(1)} &= C\left[\Gamma X_0 - (\Gamma - I)\triangle X_0 - \sum_{i=1}^{k-2}\Psi_i^{(1)}\triangle X_{-i}\right], \\ A^{(2)} &= C\sum_{i=1}^{k-2}\left(\Psi_i^{(1)} - \Psi_i^{(2)}\right)\triangle X_{T_1-i}.\end{aligned}$$

Note that $Y_t^{(j)}$ is a function of Z_t, showing that for each j the process $(Y_{T_{j-1}+1}^{(j)}, \ldots, Y_{T_{j-1}+T_j}^{(j)})$ can be given a stationary initial distribution.

Consider next the inhomogenous case where μ and γ can be different from zero, and replace X_t by $\widetilde{X}_t + \tau_c + \tau_l t$ in the model equation (2). It is seen that if

$$\beta'\tau_l + \gamma' = 0, \quad \alpha\beta'(\tau_c - \tau_l) - \Gamma\tau_l + \mu = 0, \tag{14}$$

then a homogenous equation for $\widetilde{X}_t$ arises and the result derived above can be used for $\widetilde{X}_t$. The equations in (14) do not depend on the period j and therefore have the stated solutions as found in Johansen *et al.* (2000).

B Proof of Lemma 3.2

For each sub-sample, the residuals from the regression of Z_{0t}, Z_{1t}, Z_{2t} on Z_{3t} satisfy the equation

$$R_{0.3,t}^{(j)} = \alpha\beta^{*\prime}R_{1.3,t}^{(j)} + \Psi R_{2.3,t}^{(j)} + \widehat{\varepsilon}_t. \tag{15}$$

Under the satisfaction of Theorem 2.2, the initial values can be given stationary distributions. Thus, each term in this equation is a stationary and ergodic process, leading to the next result by the law of large numbers

$$\frac{1}{T_j}\sum_{t=T_{j-1}+1}^{T_{j-1}+T_j}\begin{pmatrix} R_{0.3,t}^{(j)} \\ \beta^{*\prime}R_{1.3,t}^{(j)} \\ R_{2.3,t}^{(j)}\end{pmatrix}\begin{pmatrix} R_{0.3,t}^{(j)} \\ \beta^{*\prime}R_{1.3,t}^{(j)} \\ R_{2.3,t}^{(j)}\end{pmatrix}' = \begin{pmatrix} S_{00.3}^{(j)} & S_{0\beta.3}^{(j)} & S_{02.3}^{(j)} \\ S_{\beta0.3}^{(j)} & S_{\beta\beta.3}^{(j)} & S_{\beta2.3}^{(j)} \\ S_{20.3}^{(j)} & S_{2\beta.3}^{(j)} & S_{22.3}^{(j)}\end{pmatrix} \xrightarrow{p} \begin{pmatrix} \Sigma_{00.3}^{(j)} & \Sigma_{0\beta.3}^{(j)} & \Sigma_{02.3}^{(j)} \\ \Sigma_{\beta0.3}^{(j)} & \Sigma_{\beta\beta.3}^{(j)} & \Sigma_{\beta2.3}^{(j)} \\ \Sigma_{20.3}^{(j)} & \Sigma_{2\beta.3}^{(j)} & \Sigma_{22.3}^{(j)}\end{pmatrix}.$$

Next, we consider the asymptotic properties of the product moment matrices over the whole sample. Define the moment matrix $S_{00.3}$ as

$$S_{00.3} = \frac{1}{T}\sum_{j=1}^{2}\sum_{t=T_{j-1}+1}^{T_{j-1}+T_j}\left(R_{0.3,t}^{(j)}\right)\left(R_{0.3,t}^{(j)}\right)'.$$

Slutsky's theorem yields

$$S_{00.3} = \sum_{j=1}^{2} \frac{T_j}{T} \frac{1}{T_j} \sum_{t=T_{j-1}+1}^{T_{j-1}+T_j} \left(R_{0.3,t}^{(j)}\right) \left(R_{0.3,t}^{(j)}\right)' \xrightarrow{p} \sum_{j=1}^{2} a^{(j)} \Sigma_{00.3}^{(j)} \stackrel{def}{=} \Sigma_{00.3},$$

where the final equality follows (6). The same argument is applied to the remaining moment matrices, deriving the following asymptotic result

$$\begin{pmatrix} S_{00.3} & S_{01.3}\beta^* & S_{02.3} \\ \beta^{*\prime} S_{10.3} & \beta^{*\prime} S_{11.3}\beta & \beta^{*\prime} S_{12.3} \\ S_{20.3} & S_{21.3}\beta^* & S_{22.3} \end{pmatrix} \xrightarrow{p} \begin{pmatrix} \Sigma_{00.3} & \Sigma_{0\beta.3} & \Sigma_{02.3} \\ \Sigma_{\beta 0.3} & \Sigma_{\beta\beta.3} & \Sigma_{\beta 2.3} \\ \Sigma_{20.3} & \Sigma_{2\beta.3} & \Sigma_{22.3} \end{pmatrix}. \tag{16}$$

The final formulation of the sample product moment matrices is given by

$$\begin{pmatrix} S_{00} & S_{01}\beta^* \\ \beta^{*\prime} S_{10} & \beta^{*\prime} S_{11}\beta^* \end{pmatrix} = \begin{pmatrix} S_{00.3} & S_{01.3}\beta^* \\ \beta^{*\prime} S_{10.3} & \beta^{*\prime} S_{11.3}\beta^* \end{pmatrix} - \begin{pmatrix} S_{02.3} \\ \beta^{*\prime} S_{12.3} \end{pmatrix} S_{22.3}^{-1} \begin{pmatrix} S_{20.3} & S_{21.3}\beta^* \end{pmatrix}. \tag{17}$$

Applying (16) to (17) and using the definition given by (7) proves (8). In order to prove (9), we introduce the Yule-Walker equations for each period corresponding to the residual equation (15)

$$\Sigma_{00.3}^{(j)} = \alpha \Sigma_{\beta 0.3}^{(j)} + \Psi \Sigma_{20.3}^{(j)} + \Omega, \tag{18}$$

$$\Sigma_{0\beta.3}^{(j)} = \alpha \Sigma_{\beta\beta.3}^{(j)} + \Psi \Sigma_{2\beta.3}^{(j)}, \tag{19}$$

$$\Sigma_{02.3}^{(j)} = \alpha \Sigma_{\beta 2.3}^{(j)} + \Psi \Sigma_{22.3}^{(j)}. \tag{20}$$

Inserting (19) and (20) into (18) gives us

$$\Sigma_{00.3}^{(j)} = \alpha \Sigma_{\beta\beta.3}^{(j)} \alpha' + \alpha \Sigma_{\beta 2.3}^{(j)} \Psi' + \Psi \Sigma_{2\beta.3}^{(j)} \alpha' + \Psi \Sigma_{22.3}^{(j)} \Psi' + \Omega. \tag{21}$$

Then substituting (18)-(21) into (6) yields

$$\begin{aligned} \Sigma_{0\beta.3} &= \alpha \Sigma_{\beta\beta.3} + \Psi \Sigma_{2\beta.3}, \\ \Sigma_{02.3} &= \alpha \Sigma_{\beta 2.3} + \Psi \Sigma_{22.3}, \\ \Sigma_{00.3} &= \alpha \Sigma_{\beta\beta.3} \alpha' + \alpha \Sigma_{\beta 2.3} \Psi' + \Psi \Sigma_{2\beta.3} \alpha' + \Psi \Sigma_{22.3} \Psi' + \Omega. \end{aligned}$$

Combining these with (7) proves (9).

C Definitions of Dummy Variables in Section 5

HJ defined impulse and blip dummies as follows: $Dixx.yy_t$ is unity for $t = 19xx.yy$ and zero otherwise; $Dtixx.yy_t$ is unity for $t = 19xx.yy$, -1 for $t = 19xx.yy + 1$, and zero otherwise. Following HJ, we also included the following dummy variables unrestrictedly in our models: $Di89.13_t$, $Di89.39_t$, $Di89.51_t$, $Di90.31_t$, $Di90.31_{t-1}$, $Di90.49_t$, $Di91.03_t$, $Di93.43_t$, and $Dti98.11_t$.

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