The Hedge Fund Game:

Incentives, Excess Returns, and Piggy-Backing

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Abstract

We show that it is very difficult to structure incentive schemes that distinguish between unskilled hedge fund managers, who cannot generate excess returns, and highly skilled managers who can consistently deliver such returns. Under any incentive scheme that does not levy penalties for underperformance, managers with no investment skill can "game" the system to earn expected fees that are at least as high, relative to expected gross returns, as they are for the most skilled managers. Various ways of eliminating this "piggy-back problem" are examined, but the nature of the derivatives market means that it cannot be eliminated entirely.

1. Background

Hedge funds are largely unregulated investment pools that have become increasingly important in the marketplace. The typical fee structure is a two-part pricing scheme in which the manager takes a fixed annual percentage of funds under management (the *management fee*), plus another percentage on that portion of returns that exceed some pre-established benchmark (the *incentive fee*). A fairly common arrangement is a management fee between 1% and 2%, an incentive fee of 20%, and a benchmark rate of return in the region of 5-10% [Ackermann, McEnally, and Ravenscraft, 1999].

The purpose of such incentive contracts is to reward exceptional performance and to align the interests of investors and managers as closely as possible. Two-part incentive schemes certainly reward performance, but they are not very satisfactory on the second count. One reason is that the convexity of the fee structure encourages managers to employ strategies with high variance, which is not always in the best interests of the investors, particularly those who are risk averse. A second problem is that the pay-as-you-go feature encourages managers to push high returns forward in time, because poor returns later are not used to offset the amounts earned from high returns early on. These points have been discussed at length in the prior literature; see among others Starks (1987), Carpenter (1998), and Ackermann, McEnally, and Ravenscraft (1999).[1]

This paper considers a different problem that has received less attention, namely, the vulnerability of incentive schemes to manipulation. We show, in particular,

[1] For statistical analyses of recent hedge fund performance see Malkiel and Saha (2005) and Ibbotson and Chen (2006).

that managers can mimic exceptional performance records with high probability (and thereby earn large fees), without delivering exceptional performance *in expectation*. We give a precise formulation of this "piggy-back problem," and show that it is both very large and very difficult to correct. In particular it is very hard to set up an incentive structure that rewards skilled hedge fund managers without at the same time rewarding unskilled managers and outright con artists. Furthermore, any incentive scheme that does not directly penalize underperformance can be gamed by the manager so that his expected fees are at least as high, relative to expected gross returns, as for the most skilled managers.

This rather surprising result, which is established in theorem 1 below, stems from the unusual flexibility of the derivatives market. It also has a disturbing corollary: since the cost of entry is low relative to the potentially enormous fees, the industry risks being inundated by managers who are gaming the system rather than delivering high returns, which could ultimately lead to a collapse in investor confidence. In other words, this is a potential 'lemons' market (Akerlof, 1970) in which lemons can be manufactured at will, and the lemons look good for a long time before their true nature is revealed. This difficulty can be attenuated by insisting on greater transparency in the strategies that funds employ, but designing measures of risk exposure that cannot be circumvented using derivatives turns out to be an extremely challenging problem that will be considered elsewhere.

2. A simple method for generating fake alpha

The nature of the problem can be illustrated by a somewhat whimsical example that we have found useful in communicating these ideas to students and non-specialists. The scheme described below is quite transparent, and is not necessarily the best way to deceive investors or auditors. However, there are more complex schemes with similar characteristics that very likely are in use and would be difficult to detect even in a detailed audit (Lo, 2001).

An enterprising man named Oz sets up a new hedge fund with $100 million. The stated aim is to earn at least 10% in excess of some benchmark rate of return, say the going rate on one-year government bonds. Returns will be uncorrelated with the usual asset classes, that is, the excess returns are supposedly 'pure alpha'. The fund will run for five years and investors can cash out at the end of each year if they wish. The fee is the customary 'two and twenty': 2% annually for funds under management, and a 20% incentive fee for annual investment returns that exceed the benchmark.

Step 1. Oz starts operations on the first business day of the year by writing a certain number of binary calls on an event that will occur with probability 10% by the end of the year. An example would be the event "the S&P 500 ends the year with a gain of at least *x*%," where *x* is chosen so that the market's estimate of its probability is about 10%. Each call pays $1 million if the event occurs; otherwise the call expires worthless. To simplify matters we shall assume that the calls are *fully covered*: there is no borrowing and therefore no margin requirements. This makes the scheme more transparent analytically.

Step 2. To cover the calls in case they come due, Oz puts his funds in escrow in one-year US Treasury bills yielding 4%. The funds consist of the initial \$100 million plus the proceeds from selling the calls for their expected value, namely, \$100,000 per call (10% times \$1 million). Thus Oz can cover c calls, where $c = 100 + .10c$, which implies $c \cong 111$.

Step 3. At the end of the year the chances are 90% that the calls expire worthless, that is, Oz owes the call-holders nothing. In this case the fund has earned 4% on an initial investment of \$111.1 million, which comes to about \$115.5 million. The other outcome, which has probability 10%, is that the calls are exercised. In this case Oz pays the call-holders \$111 million, leaving the investors with the interest earned over the year, namely \$4.5 million. In this event Oz might find it prudent to close the fund early; he can always start a new one under another name next year.[2]

Under both outcomes, Oz earns his annual management fee. Furthermore, with probability 90% he earns 20% of the returns in excess of 4%. An important point to notice is that, as long as the fund does not crash it keeps compounding at a substantial rate, namely, at a rate of 11.2% net of fees: 15.5% gross minus a 2% management fee minus a .20(15.5 - 4)% incentive fee. In other words, for every year the fund does not crash Oz increases the stakes by 11.2%. A little arithmetic shows that his expected earnings in the first five years (crash or no crash) are in excess of \$19 million. Moreover, the chances are quite good (59%) that the fund

[2] In fact Oz is being generous by leaving the accrued interest on the table. He could have written another 4 calls (actually about 4.5 calls) using the interest as collateral made a gross return of over 16%. In this case the investors will be cleaned out completely when the fund crashes.

closes after five years without crashing, in which case the gambit will never be exposed.

We shall call Oz's scheme for generating fake alpha a *piggy-back strategy*. While it is doubtful that hedge fund managers use strategies that are this transparent, more sophisticated versions of it probably are in use. Lo (2001) gives the following example: take short positions in S&P 500 put options that mature in 1-3 months and are about 7% out-of-the-money (using the investors' funds as collateral). The chances are high that such options will expire worthless, in which case the manager makes money -- indeed quite a lot of money, as Lo shows by putting the strategy through its paces using historical data. Of course, there is a small probability that the market will decline sharply, the puts will be in-the-money, and the fund will lose a great deal. But this event is rare, and before it happens the manager will earn very large fees for delivering apparently above-average performance, when the *expected* returns are at best average (but this fact is concealed from the investors).[3]

The same logic is at work in the piggy-back strategy and in many other strategies one can devise. Our object is not to concoct the cleverest way to deceive investors using this approach, but to exploit the transparency of the piggy-back strategy to make two general points. First, it is extremely difficult to detect, from a fund's track record, whether a manager is actually able to deliver excess

[3] Agarwal and Naik (2004) show that many equity-oriented funds have payoffs that resemble short positions in out-of-the-money puts on the S&P 500, though of course this does not prove that managers are consciously pursuing such a strategy.

returns, is merely lucky, or is an outright con artist.[4] Second, we show that it is essentially impossible to re-design the incentive structure so that it keeps the con artists out of the market: *any contract that rewards skilled managers will also confer substantial expected rewards on the unskilled (and unscrupulous) managers as well*.

3. The piggy-back theorem

We now show how the scheme described above can be generalized to manufacture, with high probability, a long series of returns that are *consistently higher* than the returns being generated by *any* tradable asset such as a stock or bond index. Before describing the scheme we need to develop some notation.

Consider first the case of a safe asset, such as a government bond with an annual rate of return equal to $r > 0$. Suppose that a fund starts with size 1 at the beginning of year 1 and runs for T years. The stated aim of the fund is to generate returns that exceed *r*. The total return in year *t* will be denoted by $(1+r)X_t$ where $X_t \geq 0$ is a multiplicative random variable generated by the fund manager. There are *excess returns* in year *t* if $X_t > 1$, *deficient returns* if $X_t < 1$, and *ordinary returns* if $X_t = 1$. At the end of T years the fund's *gross return* (before fees) is $R_T = (1+r)^T \prod_{1 \leq t \leq T} X_t$ and the total *excess return* is $E_T = \prod_{1 \leq t \leq T} X_t - 1$.

[4] We can define a "con artist" as a fund manager who *knows* that he cannot generate excess returns and tries to fool his investors into thinking that he can. An "unskilled manager" is someone who *imagines* that he can generate excess returns even though he cannot. It is rational for both types of managers to use strategies that maximize expected returns, which (given the fee structure) means exposing their investors to large losses with a small but non-negligible probability. In other words, both types of managers produce bad outcomes for the investors irrespective of their intentions.

This set-up can be generalized to situations where the benchmark asset delivers stochastic rather than deterministic returns. Let $\{Y_t\}$ be a stochastic sequence of returns generated by a tradable asset, such as a stock index. These returns are given exogenously and cannot be manipulated by the manager. As before we denote the manager's contribution to the total return in period t by the multiplicative random variable $X_t \geq 0$. In other words the total return in period t is $X_t Y_t$. After T years the gross return is $R_T = \prod_{1 \leq t \leq T} X_t Y_t$ and the excess return is $E_T = \prod_{1 \leq t \leq T} X_t - 1$.

A specific realization of the stochastic processes $\{X_t\}$ and $\{Y_t\}$ over T periods will be denoted by $(\vec{x}, \vec{y})$, where $\vec{x} = (x_1, x_2, ..., x_T)$ and $\vec{y} = (y_1, y_2, ..., y_T)$. Any such realization generates a series of fees for the manager. A *fee contract over T years* maps each realization $(\vec{x}, \vec{y})$ to a series $(\phi_1(\vec{x}, \vec{y}), ..., \phi_T(\vec{x}, \vec{y}))$ where $\phi_t(\vec{x}, \vec{y})$ is the *fee per dollar in the fund at the start of period t* and $0 \leq \phi_t(\vec{x}, \vec{y}) \leq x_t y_t$. The return to investors *net of fees* in period t is $x_t y_t - \phi_t(\vec{x}, \vec{y})$ and the *total net return* over the first t periods is $r_t(\vec{x}, \vec{y}) = \prod_{1 \leq t' \leq t} [x_t y_t - \phi_t(\vec{x}, \vec{y})]$. Let $r_0(\vec{x}, \vec{y}) \equiv 1$. Assuming no funds are withdrawn prematurely, the *total fee per initial dollar in the fund* can be expressed as

$$\phi(\vec{x}, \vec{y}) \doteq \sum_{1 \leq t \leq T} \phi_t(\vec{x}, \vec{y}) r_{t-1}(\vec{x}, \vec{y}). \tag{1}$$

For example, the standard 'two and twenty' contract can be written $\phi_t(\vec{x}, \vec{y}) = .02 x_t y_t + .2[x_t - 1]_+ y_t$ where $[\cdot]_+$ denotes 'nonnegative part of' and it is assumed that the 2% management fee is collected at the end of the period.

It is natural to assume that fees are non-decreasing in the realized returns x_t, y_t, although we shall not actually need this condition for our results. The non-negativity condition will be used, but later (in section 4) we shall examine what happens when penalties are introduced for underperformance.

A manager's strategy generates a series of T stochastic returns $\{X_t\} = (X_1, ..., X_T)$, where each X_t has a conditional distribution that may depend on the prior realizations $(x_1, ...x_{t-1}; y_1, ..., y_{t-1})$. Indeed for present purposes we can *identify a strategy* with such a stochastic sequence, since we will be unconcerned with the particular investments that generated it.

A manager has *no ability* (is *unskilled*) if his strategy generates ordinary returns in every period relative to the benchmark asset, that is, $E[X_t] = 1$ for all t and all prior realizations $(x_1, ...x_{t-1}; y_1, ..., y_{t-1})$. A manager has *ability* $\alpha > 0$ if she consistently generates excess return α in every period relative to the benchmark asset, that is, $E[X_t] = 1 + \alpha$ for all t and all prior realizations $(x_1, ...x_{t-1}; y_1, ..., y_{t-1})$.

Given a contract ϕ, a strategy $\{X_t\}_{1 \le t \le T}$, and a benchmark asset with returns $\{Y_t\}_{1 \le t \le T}$, define the manager's *take* to be the expected total fee divided by the expected gross return:[5]

$$\tau(\phi, \{X_t\}, \{Y_t\}) = \frac{E[\phi(\vec{X}, \vec{Y})]}{E[\prod_{1 \le t \le T} X_t Y_t]}. \qquad (2)$$

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[5] An alternative definition would be $E[\phi(\vec{X}, \vec{Y}) / \prod_{1 \le t \le T} X_t Y_t]$, that is, the expected ratio rather than the ratio of the expected values. A problem with this alternative is that the manager's strategy may sometimes produce zero gross returns (the fund is cleaned out), in which case the expected ratio is undefined. Under definition (2) this problem does not arise as long as the fund's gross return is positive in expectation, which we shall assume in what follows.

We shall assume that the returns are such that the expected gross return (the denominator of (2)) is positive. We shall further assume that the tradable asset $\{Y_t\}_{1\le t\le T}$ satisfies one of the following conditions:

Y_t *is safe – it takes a constant value* y_t *for each t, or* (3)

Y_t *is stochastic with continuous conditional density* $f_t(y_t : y_1,..., y_{t-1})$ *and*

$$0 < E[\prod_{1\le t\le T} Y_t] < \infty . \tag{4}$$

Theorem 1. Let $\{Y_t\}_{1\le t\le T}$ *be a benchmark tradable asset with returns that satisfy (3) or (4), and let* ϕ *be a non-negative contract.*

i) A manager with no ability can generate any desired sequence of excess returns $(x_1,..., x_T) \ge (1,...,1)$ *with probability* $1/\prod_{1\le t\le T} x_t$ *for all realizations of* $\{Y_t\}_{1\le t\le T}$.

ii) The resulting take of the no-ability manager is at least as large as the take of a manager who delivers the same series $\vec{x} = (x_1,..., x_T)$ *with certainty. In particular, the take of the no-ability manager is at least*

$$\frac{E_{\vec{Y}}[\phi(\vec{x},\vec{Y})]}{[\prod_{1\le t\le T} x_t]E_{\vec{Y}}[\prod_{1\le t\le T} Y_t]}. \tag{5}$$

The idea of the proof is to show that an unskilled manager can piggy-back on any series of excess returns that a more skilled manager could generate by creating a series of covered options that *mimic* these returns with positive probability, namely, with probability $1/\prod_{1\le t\le T} x_t$. Hence the unskilled manager gets the fees associated with these returns with probability $1/\prod_{1\le t\le T} x_t$, and furthermore this holds for all realizations of the underlying asset.

Proof of theorem 1. Without loss of generality assume that the initial size of the fund is $x_0 = 1$. Choose a *target sequence of excess returns* $(x_1,...,x_T) \ge (1,...,1)$. Consider first the case where the benchmark asset $\{Y_t\}_{1\le t\le T}$ is stochastic and satisfies condition (4). At the start of each period $t \le T$ write a binary call Z_t on the benchmark asset with strike price *s*, where *s* is chosen such that $P[Y_t \le s : y_1,..., y_{t-1}] = 1/x_t$. Such a strike price exists because, by assumption, Y_t has a continuous density conditional on the realized prior returns $y_1,..., y_{t-1}$. Thus Z_t is a binary random variable satisfying

$$Z_t = 0 \text{ if } Y_t \le s\,,\ Z_t = 1 \text{ if } Y_t > s\ . \tag{6}$$

$$E[Z_t] = 1 - 1/x_t\,. \tag{7}$$

We assume the manager can sell such an option for its expected value, namely, $E[Z_t] = 1 - 1/x_t$. Suppose the manager sells *c* calls. To cover them he has the current amount in the fund, say $w_{t-1} > 0$ plus the proceeds from the sale of the calls, $c(1-1/x_t)$. Therefore he can sell *c* calls, where $c(1-1/x_t) + w_{t-1} = c$, which implies $c = x_t w_{t-1}$.

It follows that, conditional on having $w_{t-1} > 0$ at the end of period $t-1$, by the end of period t he will have $x_t y_t w_{t-1}$ with probability $1/x_t$ and 0 with probability $1-1/x_t$ *for all realized values* y_t.

Therefore, after T periods, he will have generated the sequence of excess returns $(x_1, x_2, ..., x_T)$ with probability $1/\prod_{1\le t\le T} x_t$ irrespective of the realized returns $(y_1, ..., y_t)$ of the benchmark asset. (By assumption (4) these are all positive with probability one.) With probability $1-1/\prod_{1\le t\le T} x_t$ the fund crashes at or before period T and its final value is zero.

Since the contract carries no penalties for underperformance (ϕ is non-negative), the manager's *expected fees are at least*

$$E_{\vec{Y}}[\phi(\vec{x}, \vec{Y})]/[\prod_{1\le t\le T} x_t]. \tag{8}$$

By the end of period T the manager will have generated a gross return of $[\prod_{1\le t\le T} x_t] E_{\vec{Y}}[\prod_{1\le t\le T} Y_t]$ with probability $1/\prod_{1\le t\le T} x_t$, and a gross return of zero with probability $1-1/\prod_{1\le t\le T} x_t$. Hence his expected gross return is

$$E_{\vec{Y}}[\prod_{1\le t\le T} Y_t] > 0. \tag{9}$$

It follows that the no-ability manager's *take* is *at least* the ratio of (8) to (9), that is, at least $\frac{E_{\vec{Y}}[\phi(\vec{x},\vec{Y})]}{[\prod_{1\le t\le T} x_t]E_{\vec{Y}}[\prod_{1\le t\le T} Y_t]}$, as claimed in part ii) of the theorem.

It remains to show that these conclusions hold when the benchmark asset is safe instead of stochastic. This follows by writing the binary call on *any* stochastic asset, because all that matters in this case is that the calls have the right probability distribution. They will generate the target sequence of excess returns $(x_1, x_2, ..., x_T)$ with probability $1/\prod_{1\le t\le T} x_t$, and the returns on the benchmark asset are guaranteed. This concludes the proof of theorem 1.

Remark 1. The type of strategy used in the proof of the theorem will be called a *piggy-back strategy*. It allows an unskilled manager to mimic a target series of excess returns *without having the slightest idea about how a skilled manager would actually generate them.* All that is required is the ability to write binary calls on the target series.

Remark 2. The equivalent of a binary call can be constructed from more commonly traded options. Suppose, for example, that the manager wishes to sell a binary call that is triggered if a given index exceeds some value *s* by the end of period *t*. Instead he can go short a European call with strike price *s* and date *t* and buy another European call with the same date and a slightly higher strike price $s + \delta$. If the index closes below *s* neither call is exercised and he makes money on the spread (known as a *bull spread*). If the index closes above $s + \delta$ he loses δ, however, *s* and δ are chosen so that the probability of this event is small.

(Moreover, since δ is small the probability that the index closes between s and $s + \delta$ is extremely small.)

Remark 3. Piggy-backing is not the only method for generating 'fake alpha'. For example, one could take short positions in puts that are far out-of-the-money, or dynamic positions that roll through a series of short-term options. The piggy-back strategy has the advantage that it is particularly easy to analyze and full coverage of the option positions is never an issue.

Remark 4. We focused on the situation in which an unskilled manger generates fake alpha, but of course a skilled manager can use the same strategy to generate fake alpha on top of his true alpha (to pad his returns). The theorem does not address the question of how much fake alpha different types of managers (skilled and unskilled) will generate in equilibrium. To answer this question we would need to make assumptions about managers' levels of risk aversion, the wealth they hold in other assets, and (quite importantly) the effect that different levels of alpha will have on the competition for customers. An analysis of these matters would take us too far afield here. The key point established by the theorem is to show that a manager can look very good for a long period of time without having any skill. Skilled managers can do even better.

Remark 5. The theorem also shows that a piggy-back strategy can result in high expected earnings. To appreciate the magnitude of the effect, consider a skilled manager who can generate excess returns $\alpha = .03$ every year for ten years in a row. Let M be the total expected monetary fee to such a manager under the contract. Then the skilled manager's *take* is $M/[(1.03)^{10} R] = .74M / R$, where

$R = E_{\vec{Y}}[\prod_{1 \le t \le T} Y_t]$ is the expected gross return of the benchmark asset over the period. Theorem 1 shows that an unskilled manager with the same size fund initially can get *at least* .74*M* in expectation, and he gets even more if the fees are positive when the fund crashes. Since in reality he is unable to generate excess returns, in expectation his gross return is exactly *R*. Hence his take is bounded below by *.74M/R*, which is the same as for the skilled manager.

More generally, suppose that $M(\alpha, T)$ is the expected monetary reward to a manager of ability α over T periods under a given contract ϕ. An unskilled manager who mimics this "alpha" will not be exposed in T periods with probability $(1+\alpha)^{-T}$. Hence such a manager earns, in expectation, at least $M(\alpha, T)/(1+\alpha)^T$. Table 1 shows how the probability of not being exposed varies with α and T.

α	$T = 5$	$T = 10$	$T = 20$
.01	95%	91%	82%
.03	86	74	55
.05	78	61	38
.10	62	39	15
.20	40	16	3

Table 1. Probability that a piggy-back strategy is not exposed for various values of α and T. (All probabilities rounded to the nearest whole percent.)

Remark 6. The *length* of the reporting period is immaterial to the results in theorem 1 or table 1. Suppose, for example, that managers were required to report their returns at the end of each day instead of at the end of each year. A manager who can generate excess returns of α per year can, on average, generate excess returns of *per day* equal to $(1+\alpha)^{1/365}-1 \approx \alpha/365$. An unskilled manager can mimic each of these daily returns with probability $(1+\alpha)^{-1/365}$. Hence, over T years, the unskilled manager can mimic this series with probability $(1+\alpha)^{-T}$ just as before.

4. Restructuring the incentives

We now consider whether the difficulty can be corrected by restructuring the manager's incentives. There are two separate problems that a properly designed incentive scheme needs to address. The first is how to align the interests of the manager and the investors more closely. The second is how to distinguish between skilled and unskilled managers. The former is the *alignment problem* whereas the second is the *separation problem*.

The alignment problem is a standard one in the theory of contracts, and can be addressed in several ways: a) by rewarding the manager only on the basis of final total returns; b) by forcing the manager to hold an equity stake; c) by levying penalties for underperformance. We shall consider each of these remedies in turn. A basic conclusion is that, while they partially alleviate the alignment problem, they do not solve the separation problem: under almost any arrangement, unskilled managers will be able to piggy-back on the rewards of skilled ones to some extent.

a) Payments based on final returns

One possibility is to make the manager's payments depend only on the gross return achieved by the final period $R_T = \prod_{1 \le t \le T} x_t y_t$. Let us therefore assume that the contract takes the form $\phi(\vec{x}, \vec{y}) = g(\prod_{1 \le t \le T} x_t y_t)$ for some nonnegative, monotone increasing function $g(\cdot)$. To separate the skilled from the unskilled, we need the payments to be zero whenever $\prod_{1 \le t \le T} x_t \le 1$. However, this means that an unskilled manager can make at least

$$\max_{\vec{x}} E_{\vec{Y}}[g(\prod_{1 \le t \le T} x_t Y_t)] / \prod_{1 \le t \le T} x_t . \tag{10}$$

This may still represent an enormous potential payoff with relatively little risk. For example, suppose that the fee is 20% of the gross excess return at the end of ten years. Suppose also that the expected value of the benchmark asset doubles in that time. If F is the initial fund size, a manager who fakes a 3% annual excess return will get .138F with probability 74% and 0 with probability 26%; moreover the costs of setting up the scheme are negligible. The optimum level of fake alpha will obviously depend on the manager's risk aversion, discount rate, and so forth. However, as is standard in these settings, a manager will typically fake some level of alpha no matter how risk averse he is. Furthermore, risk aversion is not the only relevant consideration: by faking still higher levels of alpha he may be able to pull in more investors. We shall not attempt to model this effect explicitly; it suffices to note that it is an additional motivation that can seriously undercut any deferred payment scheme.

b) *Require the manager to hold an equity stake in the fund*

Suppose that the fund manager is required to hold an equity stake in the fund. Let $\theta \in (0,1)$ be the proportion of the fund's value that he is required to hold during the fund's lifetime T. We begin by noting that this requirement is easy to undermine, because the manager can always take positions in the derivatives market that effectively offset the gains and losses generated by his share of the fund. However, even if such offsetting positions can be prohibited, the requirement does not do much to solve the piggyback problem.

For notational simplicity we shall consider the case of a safe benchmark asset that generates a fixed stream of returns $\vec{y}$ and drop them from the notation. Thus we shall write the fees in the form $\phi(\vec{x})$, where $\vec{x} = (x_1, x_2, \ldots x_T)$ is a series of excess returns over T years. A skilled manager who generates these returns with certainty will have final wealth (per dollar initially in the fund) equal to the

$$\theta \prod_{1 \le t \le T} x_t + (1-\theta)\phi(\vec{x}) . \tag{11}$$

The piggy-back theorem shows that an unskilled manager can generate this same series with probability $1/\prod_{1 \le t \le T} x_t$. His expected wealth at the end of the period is composed of two parts: the expected value of his own stake before fees, which is exactly θ (because in expectation he cannot generate excess returns); and the expected fees from the investors, which amount to $(1-\theta)\phi(\vec{x})/\prod_{1 \le t \le T} x_t$. Hence the unskilled manager's expected end-wealth, per dollar of initial fund value, is

$$\theta + (1-\theta)\phi(\vec{x}) / \prod_{1 \le t \le T} x_t \ . \qquad (12)$$

It follows from (11) and (12) that the *ratio* of the unskilled to the skilled manager's end-wealth is $1/\prod_{1 \le t \le T} x_t$, which is the same as the ratio of their earnings when they are not required to hold an equity stake. As in case a), risk aversion and competition for customers may modify the excess returns that an unskilled manager wishes to target, but it does not alter the conclusion that such a scheme is not much of a deterrent.

c) Assess penalties for underperformance

Theoretically this is the most satisfactory approach, but it still does not solve the problem. As before we shall assume a safe benchmark asset and drop the y_t's from the notation. Consider a contract $\phi(\vec{x})$ that calls for negative payments for some sequences $\vec{x}$ that deliver sub-normal returns. We do not need to specify which returns trigger negative payments, but we will assume that the *largest penalty* that could ever arise is δ per dollar of the fund's initial value. The payments must be enforceable, so the manager must put δ in escrow until the end of period T.

Consider some series of returns $\vec{x} = (x_1, x_2, ...x_T)$ that can be generated with certainty by a skilled manager. Conditional on this realization of returns, the skilled manager's end-wealth, per dollar of initial fund size, is $\delta + \phi(\vec{x})$. He realizes, however, that if he had not opened the fund to investors, but simply applied his skills to the amount held in escrow (δ), he would have had $\delta \prod_{1 \le t \le T} x_t$.

Therefore his participation constraint is

$$\phi(\vec{x}) > \delta(\prod_{1 \le t \le T} x_t - 1) \quad . \tag{13}$$

Now consider an unskilled manager who piggy-backs on the sequence $\vec{x}$. His expected end-wealth, per dollar of initial fund size, is

$$\phi(\vec{x}) / \prod_{1 \le t \le T} x_t - \delta(1 - 1/\prod_{1 \le t \le T} x_t) \, . \tag{14}$$

Since he does not know how to generate excess returns in reality, and the piggy-back strategy is essentially costless, his participation constraint (assuming risk neutrality) is

$$\phi(\vec{x}) / \prod_{1 \le t \le T} x_t > \delta(1 - 1/\prod_{1 \le t \le T} x_t) \, . \tag{15}$$

It follows that *any contract with penalties that keeps out the unskilled risk neutral managers keeps out all the skilled managers as well.*

5. Discussion

In this paper we have shown how easy it is to mimic a series of excess returns without being able to generate such returns in expectation. It suffices to place a series of bets, each of which generates a modest excess return with high probability and a large loss with low probability. As long as the excess returns are not too excessive, and the series not too long, the probability of being exposed is low. Furthermore, it is essentially impossible to design an incentive scheme that keeps out people who are pursuing such strategies (either unwittingly or by design) without keeping out everybody.

We draw two conclusions. First, it is extremely difficult for investors to tell whether a given series of excess returns was generated by superior skill, by mere luck, or by duplicity. Second, because it is easy to fake excess returns and earn a lot of money in the process, mediocre managers and con artists could be attracted to the market. The situation is analogous to an automobile 'lemons' market with the added feature that 'lemons' can be manufactured at will (Akerlof, 1970). Indeed, it is analogous to a car market with the following characteristics: i) every car is one of a kind; ii) the car's engine is locked in a black box and no one can see how it works (it's not protected under patent law); iii) anyone can cobble together a car that delivers apparently superior performance for a period of time and then breaks down completely. In such a case one would expect the price of cars -- both good and bad – to collapse, because buyers cannot tell the difference between them. A similar fate may await the hedge fund industry unless ways are found to make their functioning more transparent.

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