Optimal Fiscal Stabilisation through Government Spending

Fabian Eser*†

13th October 2009

Abstract

This paper examines under what conditions fiscal policy in the form of government spending should contribute to macroeconomic stabilisation. To this end optimal fiscal targeting rules minimising the microfounded social loss are examined in the following settings. Firstly, for the benchmark New Keynesian model, where monetary policy is unconstrained, a neutrality result for fiscal obtains: fiscal policy should not respond to any shocks. Secondly, if monetary policy is constrained to follow a Taylor rule, a stabilisation role for fiscal policy emerges. Fiscal policy should 'lean against' inflation and be countercyclical relative to output. Crucially, the Taylor principle is shown to remain the key requirement on policy to guarantee equilibrium determinacy. Thirdly, the fiscal targeting rule obtained under a Taylor rule is shown to be optimal, too, when policy is optimal but subject to monetary frictions. Thus, there is a stabilisation role for government spending under monetary frictions, changing the role of monetary and fiscal policy fundamentally.

Keywords: Monetary Policy, Fiscal Policy, Macroeconomic Stabilisation, Discretion, Dynamic General Equilibrium, Sticky Prices, Monetary Frictions, Equilibrium Determinacy

JEL Classification: E5, E6, C62

*Nuffield College, Oxford OX1 1NF, United Kingdom; *fabian.eser@nuffield.ox.ac.uk*.

†This is a substantially revised version of Eser (2006). I am particularly indebted to Florin Bilbiie, Stephen Nickell, Joe Perkins, David Vines as well as two anonymous examiners for helpful comments. At various stages I have also benefited from discussions with Michael Cheng, Stefano Gnocchi, Tatiana Kirsanova, Richard Mash, Roland Meeks. I am grateful to Campbell Leith and Simon Wren-Lewis, co-authors on related work. Financial support from the UK Economic and Social Research Council and the Royal Economic Society is gratefully acknowledged. All errors remain my own.

Contents

1 Introduction

This paper examines in the benchmark New Keynesian model whether and how fiscal policy through adjustments of government spending should contribute to macroeconomic stabilisation. The question appears important for two reasons in particular.

Firstly, among policymakers there appears to be a renewed interest in fiscal stabilisation, as well as the readiness to implement it. This follows a long period during which stabilisation policy of a fiscal nature have been regarded as inappropriate and ineffective, while monetary policy has been considered both better suited and sufficient for stabilisation in normal times. Indeed, the fiscal stabilisation attempts following the financial current credit crunch may be solely motivated by nominal interest rates being close to the lower zero bound. That this can generate a role for stabilisation has, however, been shown in Evans et al. (2008). The present paper, by contrast, investigates whether there is a role for fiscal stabilisation in the absence of special situations like the zero nominal bound.

Secondly, the New Keynesian model which has emerged as the new benchmark framework for the study of macroeconomic policy during this period championing monetary policy has, until very recently, been nearly exclusively developed for and devoted to the study of monetary policy. Woodford (2003) offers the most comprehensive exposition of this benchmark model, while Clarida et al. (1999) provide a concise survey of the workhorse model of monetary policy and its policy conclusions relevant for this paper. In the standard New Keynesian model government spending is usually treated as an exogenous disturbance. This applies even to models with more complex dynamics and rigidities which are designed to match the data, e.g. Smets and Wouters (2003), Christiano et al. (2005) or Smets and Wouters (2007). It thus appears important to study fiscal policy in its own right and to examine the robustness of results on monetary policy to the presence and interaction with fiscal policy.

The structure and most important results are as follows. Section 2 lays out the microfoundations in form of the problems of the optimising households and firms. Price-staggering of the form of Calvo (1983) is the only rigidity in the model. Consumers are Ricardian, financial markets are complete, the economy is cashless, and government spending is financed by lump-sum taxation, while a time-invariant distortional income tax is used to perfectly offset monopolistic mark-ups in equilibrium. There is no investment, all government spending is consumed and financed by lump-sum taxes.

Section 3 derives from the microfoundations structural relations representing the economy as a whole: an IS relation (19), a Phillips curve (18) as well as quadratic social loss (25). Importantly, endogenous government spending enters all of these relations directly, which is not true for the nominal interest rate as instrument of monetary policy.

The model is used in sections 4-6 to examine optimal fiscal policy through adjustment of government spending. Throughout the paper fiscal policy is set optimally, minimising the social loss. The optimality conditions can be interpreted as optimal targeting rules, specifying how to trade off the endogenous variables.[1] In this sense, the entire paper is concerned with optimal targeting rules for fiscal policy in three settings.

As is known from the literature on monetary policy,[2] optimal monetary policy can offset shocks to the natural rate of interest without cost, because the interest rate only enters the IS relation. However, in response to cost-push shocks inflation and output are driven in opposite directions, leading to the standard inflation-output variability problem. The optimal targeting rules then determine this optimal trade-off.

Section 4 identifies the fiscal targeting rule in the benchmark New Keynesian models without monetary frictions when monetary policy, too, is set optimally. In this setting, I identify a 'neutrality' result for fiscal policy. The government spending gap should optimally be set to zero at all times, implying that fiscal policy has no role in stabilising shocks. The optimal monetary targeting rule then has the familiar properties discussed in Clarida et al. (1999), such as restricting output below the natural rate in the face of inflation.

There are two main factors behind the neutrality result. Firstly, in the benchmark model the nominal interest rate does not enter the social loss so that it can be obtained as a function of the optimal levels of inflation, output and the government spending gap. Secondly, while government spending and output enter the model in a nearly parallel fashion, the elasticity of inflation with regard to output is greater than its elasticity with regard to government spending. It thus is optimal to set spending to zero and trade-off inflation and output against each other.

Section 5 assumes that the monetary authority is constrained to set the monetary policy according to a specific Taylor rule. Under these circumstances a stabilisation role for

[1]Cf. i.a. Svensson (1999), Jensen (2002) or Svensson (2003).
[2]Cf. e.g. Clarida et al. (1999), Woodford (2003).

government spending re-emerges. According to the optimal fiscal targeting rule government spending should be restricted in the face of inflation, and conditional on inflation, it should be countercyclical with regard to output. Importantly, I demonstrate that, when monetary policy follows a Taylor rule and fiscal policy the optimal fiscal targeting rule, the Taylor principle remains the central requirement for the monetary authority to achieve equilibrium determinacy.

Section 6 returns to optimal monetary policy but allows for the existence of monetary frictions. Monetary frictions turn out to generate a stabilisation role for fiscal policy. More than that, we obtain exactly the same optimal targeting rule for fiscal policy as when monetary policy follows a Taylor rule. With monetary frictions, variations of nominal interest rates enter the social loss function, fundamentally changing the relative costs of using the monetary and fiscal instrument. This underscores that, while the cashless model without monetary frictions may be a useful limiting case when studying monetary policy by itself, excluding monetary frictions, when other instruments like government spending are considered, can have innocuous effects.

Regarding the literature, the closest companion is Eser et al. (2009) who explore the robustness of the neutrality result in several directions. They show that it continues to hold in a small open economy with wage rigidities. The same is true under price and wage-inflation inertia due to rule-of-thumb behaviour among price and wage setters. The introduction of government debt overturns the neutrality result in the sense that following a shock with fiscal consequences the government spending gap will be non-zero. However, government spending is only used to stabilise debt, not to stabilise output and inflation beyond this. In this sense the neutrality result is robust to the introduction of debt. Eser et al. (2009) show, however, that variations in taxes rather than taxes can provide an important stabilisation role. Taxes have the advantage that they can change relative prices directly, rather than affecting demand as is the case for government spending. In a similar vein, Correia et al. (2008) show that with state-contingent labour income and consumption taxes, optimal policy is independent of the extent of price stickiness.

In terms of the New Keynesian model, the paper follows most closely Woodford (2003), while Clarida et al. (1999) provide the most concise comparison with optimal targeting rules for monetary policy, when fiscal policy is exogenous. Regarding the main results for fiscal policy, Galí and Monacelli (2008) study a monetary union and find that, for the union as a whole, government spending should be zero, paralleling the result of section 4 of this paper. As monetary policy set in response to union-aggregate variables cannot react to country-specific shocks, there is a role for fiscal stabilisation at the country-level. This is consistent with the finding of sections 5 and 6.

While the neutrality result is implicit in Galí and Monacelli (2008), their analysis is conducted in deviations of output from government spending and not discussed . In the same framework Gnocchi (2007) makes the result explicit and finds that it does not hold when monetary policy is set under commitment while fiscal policy is set under discretion.[3] While the parallels are consistent with this paper, we explore the neutrality result in much more depth. The present paper also allows for more general constant-relative-risk-aversion-utility and specification of shocks. Sections 5 and 6 are also novel.

Other papers assume that fiscal policy follows a simple rule and study its interaction with monetary policy in various contexts, e.g. Schmitt-Grohé and Uribe (2007), Kirsanova et al. (2007), Hovath (2008). Fiscal and monetary interactions have also been studied in settings where government spending is exogenous and the instrument of fiscal policy are taxes, e.g. Benigno and Woodford (2003), Schmitt-Grohé and Uribe (2005). This literature focuses more on the optimal volatility of various taxes relative to inflation, which can also be a form of taxation, and re-affirms the pursuit of low and stable inflation as central policy objective.[4]

2 Microfoundations

This section introduces the problems faced by households, government and firms. The general form of notation as for instance the definition of variables in logs, or in gap terms, is laid out in appendix A.1 and not re-introduced at every step of the paper.

2.1 Households

The economy consists of one representative agent, or household, who lives infinitely. His preferences $U(C_t, G_t, N_t; \xi_t, \varepsilon_t, A_t)$ are defined over the composite consumption good C_t, leisure $1 - N_t$, where N_t is the supply of labour in terms of hours worked, and composite

[3] Interaction under different degrees of commitment is also studied in a static set-up where fiscal policy operates via a subsidy in Dixit and Lambertini (2003).

[4] See also Kirsanova et al. (2009) for an overview of the literature.

government spending G_t. The agent maximises the expected present discounted value of utility given by

$$U \equiv E_t \sum_{i=0}^{\infty} \beta^i \left[(1-\chi)U^c(C_{t+i};\xi_{t+i}) + \chi W(G_{t+i};\varepsilon_{t+i}) - V(N_{t+i};A_{t+i})\right], \tag{1}$$

where the utility from consumption is given by

$$U^c(C_t;\xi_t) \equiv \frac{(\xi_t C_t)^{1-\sigma}}{1-\sigma}. \tag{2}$$

ξ_t is an exogenous taste shock to consumption for which in steady state $\xi = 1$. σ is the coefficient of relative risk-aversion of consumption which, for CES-utility, equals the inverse of the elasticity of intertemporal substitution. The utility which the agent derives from government spending is characterised by

$$W(G_t;\varepsilon_t) \equiv \frac{(\varepsilon_t G_t)^{1-\gamma}}{1-\gamma}. \tag{3}$$

Government spending, too, can be hit by a taste shock, ε_t, for which in steady state $\varepsilon = 1$. γ is the coefficient of relative risk-aversion of government spending. χ is the relative weight on government spending in the utility function.

Government spending is equal to public consumption, as there is no investment. Goods, firms and types of labour are differentiated along the continuum $j \in [0,1]$. Each differentiated good j is produced by exactly one firm j, which in turn employs exactly one type of labour j. The disutility of labour

$$V(N_t(j);A_t) \equiv \int_{j=0}^{1} \frac{N_t(j)^{1+\eta}}{1+\eta}\, dj \tag{4}$$

aggregates the disutility of supplying labour of different types j. A_t represents an exogenous productivity-factor which augments labour in the production function and for which in steady state $A = 1$. The parameter η represents the inverse of the Frisch-elasticity of labour-supply. Each agent supplies all types of labour and receives his pro rata share of the economy's aggregate wage bill.

$\beta^t \in (0,1)$ is the discount factor at time t. C_t is the index of the agent's consumption of all the individual goods j that are supplied. Following Dixit and Stiglitz (1977), the composite consumption good consists of differentiated products of monopolistically competitive firms. Each firm j produces one good $C_t(j)$. The composite consumption good entering the agent's utility function is $C_t = \left[\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ is the constant elasticity of substitution between any two goods. The corresponding price index, defining the minimum cost of a unit of C_t, is $P_t = \left[\int_0^1 P_t(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}$. Agents earn nominal wage $W_t(j)$ for supplying labour of type j and $W_t N_t = \int_0^1 W_t(j) N_t(j)\, dj$. Agents own shares in the firms producing the differentiated goods. If we assume that agents own equal amounts of shares in all firms, we can write the profits accruing to any agent from the sale of good j as $\Pi_t(j)$, giving him total profits of $\Pi_t = \int_0^1 \Pi_t(j)\, dj$.

Let $u_c, w_g, v_y, u_{c,\xi}$ etc. denote the non-stochastic steady states of the derivatives of (2), (3) and (4) with regard to the variables in the subscript. Then we have $\frac{v_{yy}Y}{v_y} = \eta$, $\frac{u_{cc}Y}{u_c} = -\frac{\sigma}{1-\psi} \equiv -\tilde{\sigma}$, $\frac{w_{gg}Y}{w_g} = -\frac{\gamma}{\psi} \equiv -\tilde{\gamma}$, where $\psi \equiv \frac{G}{Y}$ is the steady state share of government spending in output, and the consumption share in output is $(1-\psi) = \frac{C}{Y}$.

The flow-budget constraint of the agent can be written as

$$P_t C_t + E_t[Q_{t,t+1} B_{t+1}] \leq (1 + \imath_{t-1}) B_{t-1} + (1-\tau) W_t N_t + \Pi_t - T_t, \tag{5}$$

with the no-Ponzi condition $(1 + \imath_t) B_t = -\sum_{T=t+1}^{\infty} E_{t+1} Q_{t,T} \left[(1-\tau) W_T N_T - T_T\right]$.

Financial markets are assumed to be complete, but the economy is cashless. The monetary authority defines a unit of account in which all assets are denoted. In terms of this unit of account, B_t, denotes the agent's end-of-period portfolio holdings of all assets, both private and public. Wage income $W_t N_t$ represents all non-financial income. The government levies net nominal taxes of T_t and a distortionary income tax τ. Absence of arbitrage requires $B_t = E_t[Q_{t,t+1} B_{t+1}]$, which implies a unique stochastic discount factor $Q_{t,t+1}$. In terms of the stochastic discount factor, the one-period short-term riskless nominal interest rate, $\imath_t$, is given by $\frac{1}{1+\imath_t} = E_t Q_{t,t+1}$.

The representative agent makes three choices. Given any level of aggregate consumption C_t, he chooses the cost-minimising consumption of differentiated goods, $C_t(j)$, given their associated prices $P_t(j)$. The consumption of good j can then be written as $C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} C_t$.

The agent's intertemporal optimisation problem is to choose processes $\{C_t, \frac{B_t}{P_t}\}_{t=0}^{\infty}$ for all $t > 0$ to maximise (1) subject to the budget constraint (5) with equality and the associated no-Ponzi condition. This leads to the standard consumption Euler condition

$$1 + \imath_t = \beta^{-1} \left\{ E_t \left[\frac{\xi_{t+1}^{1-\sigma} C_{t+1}^{-\sigma}}{\xi_t^{1-\sigma} C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right] \right\}^{-1}. \tag{6}$$

Within each period and for each type j, the agent chooses an optimal amount of labour to supply, resulting in the optimal labour supply-relation

$$(1-\tau)\frac{W_t(j)}{P_t} = \frac{N_t(j)^\eta}{(1-\chi)\xi_t^{1-\sigma} C_t^{-\sigma}}\, e^{\mu_w^t}, \tag{7}$$

where $e^{\mu_w^t}$ represents a cost-push shock.

2.2 Government

The government purchases public consumption of G_t. G_t is defined analogously to the private consumption aggregator C_t, with demand $G_t(j) = G_t \left(\frac{P_t(j)}{P_t}\right)^{-\theta}$. The government finances its spending by levying a lump-sum tax T_t. Furthermore, with a distortionary income tax, τ, it can perfectly offset monopolistic mark-ups. Consumers are Ricardian, so the time profile of taxation does not matter.

2.3 Firms

The production of each good j is given by $Y_t(j) = A_t N_t(j)$ so that in aggregate $Y_t = A_t N_t$. With a variable cost of supplying $Y_t(j)$ of $W_t(j)N_t(j) = W_t(j)\frac{Y_t(j)}{A_t}$, the nominal marginal cost of producing good j is $S_t(j) = \frac{W_t(j)}{A_t}$. Using the price-index, the labour-supply relation (7) as well as the production function we obtain the real marginal cost function of supply for every differentiated good as

$$S_{j,t} = \frac{\chi Y_t(j)^\eta}{A_t^{1+\eta}(1-\chi)\xi_t^{1-\sigma} C_t^{-\sigma}} \frac{e^{\mu_w^t}}{(1-\tau)}. \tag{8}$$

When setting $P_t(j)$, producers take Y_t and P_t as given, and apply a mark-up $\frac{\theta}{\theta-1}$ over nominal marginal cost. Thus, they set a relative price of

$$\frac{P_t(j)}{P_t} = \frac{\theta}{\theta-1} S_t(j). \tag{9}$$

3 Equilibrium Dynamics

Goods market clearing requires $Y_t(j) = C_t(j) + G_t(j)$, so that each producer j faces demand of $Y_t(j) = Y_t \left(\frac{P_t(j)}{P_t}\right)^{-\theta}$. For aggregate production we thus have

$$Y_t = C_t + G_t; \qquad \hat{y}_t = (1-\psi)\hat{c}_t + \psi\hat{g}_t. \tag{10}$$

Without loss of generality it is often convenient to work with government spending expressed in terms of log-deviations from output

$$\hat{\mathfrak{g}}_t \simeq \frac{G_t - G}{Y} = \psi\hat{g}_t, \tag{11}$$

so that $\hat{y}_t = (1-\psi)\hat{c}_t + \hat{\mathfrak{g}}_t$.

3.1 Efficient Steady State

The optimal efficient allocation is the solution to the maximisation of (1) subject to the production function $Y_t = A_t N_t$ and the resource constraint $Y_t = C_t + G_t$. This gives two optimality conditions characterising the efficient allocation

$$\frac{N^\eta}{A_t} = (1-\chi)C^{-\sigma} = \chi G^{-\gamma}. \tag{12}$$

Together with the production function and the resource constraint (10), we can characterise the efficient steady state values of Y_t, C_t, G_t, N_t.

Now we see under what conditions the efficient allocation can be achieved as a decentralised equilibrium rather than the solution to the social planner problem. From (8) and (9) we know that under flexible prices $\frac{1-\frac{1}{\theta}}{(1-\tau)} = \frac{\frac{N^\eta}{A_t}}{(1-\chi)C^{-\sigma}}$. For a labour tax of

$$\tau = \frac{1}{\theta} \tag{13}$$

this equals one, so that the efficient allocation is achieved. The second efficiency condition is then fulfilled if and only if steady state government spending is chosen so that

$$(1-\chi)C^{-=\sigma} = \chi G^{-\gamma}. \tag{14}$$

If (13) and (14) are met, the flexible price equilibrium is efficient in the decentralised equilibrium.

3.2 Phillips Curve

Price-setting follows a discrete-time variant of Calvo (1983). The opportunity of firms to adjust prices follows an exogenous Poisson process. There is a constant $(1-\omega)$ probability that a firm can adjust its price, so that each period a fraction ω of firms leaves the prices of their product unchanged. Given this assumption a New Keynesian relationship between inflation and real marginal cost aggregated over all goods can be obtained:[5]

$$\pi_t = \beta E_t \pi_{t+1} + \delta \hat{s}_t, \tag{15}$$

where $\hat{s}_t$ is the real marginal cost of supply aggregated over all goods j and $\delta = \frac{(1-\omega)(1-\beta\omega)}{\omega}$ is the elasticity of inflation with regard to real marginal cost.

Log-linearising and combining (8), (9) as well as the demand for good j, we obtain the real-marginal cost of supply for good j. Integrating that expression over all goods, we obtain the aggregate real marginal cost of output as

$$\zeta \hat{s}_t = \eta \hat{y}_t - (1+\eta)\hat{a}_t - (1-\sigma)\hat{\xi}_t + \sigma \hat{c}_t + \mu_w^t, \tag{16}$$

where $\zeta \equiv (1+\eta\theta)$.[6] Under flexible prices the inflation shock and the log-deviation of real marginal cost must equal zero. We then find a relation between the natural rate of output and the natural rate of government spending by imposing these conditions on (16) and using the economy resource constraint so that

$$(\tilde{\sigma} + \eta)\, \hat{y}_t^n = \tilde{\sigma} \hat{\mathfrak{g}}_t^n + (1+\eta)\hat{a}_t + (1-\sigma)\hat{\xi}_t. \tag{17}$$

Combining this with (16) and (15), we obtain the Phillips curve in terms gap variables, i.e. deviations of output and government spending from flexible price values, as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_y \tilde{y}_t - \kappa_g \tilde{\mathfrak{g}}_t + \mu_t, \tag{18}$$

[5] The derivation is standard and not repeated here, cf. Woodford (2003), Galí (2008).

[6] As discussed in Woodford (2003) ch.3, if $\eta/\zeta > (<)\ 1$ pricing decisions by different firms are strategic substitutes (complements).

where the elasticities of inflation with regard to the output gap and government spending are

$$\kappa_y \equiv \delta\left(\tilde{\sigma}+\eta\right)=\delta\left(\frac{\sigma}{1-\psi}+\eta\right); \qquad \kappa_g \equiv \delta\tilde{\sigma}=\delta\frac{\sigma}{1-\psi}.$$

The government spending gap does enter the Phillips curve. From (15) we know that inflation is a function of real marginal cost. Real marginal cost, as seen in (16), increases in consumption. Since, due to the economy resource constraint (10), consumption is a function of output, government spending has an effect on marginal cost through its effect on consumption. This way we can also explain the fact that in the Phillips curve inflation increases in the output gap, and decreases in the government spending gap, conditional on the output gap remaining constant.[7] For a given level of output, an increase in government spending crowds out domestic consumption, reducing marginal cost and thus inflation. By contrast, the nominal interest rate does not affect the real marginal cost of supply. Hence, it does not have any direct effect on inflation.

3.3 IS Relation

Combining the consumption Euler equation (6) with the economy resource constraint we obtain an IS relation $\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma}(\hat{\imath} - E_t\pi_{t+1}) + \frac{\sigma-1}{\sigma}(E_t\hat{\xi}_{t+1} - \hat{\xi}_t)$. This can be expressed in terms of gaps deviations from flexible price equilibria as

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - (E_t\tilde{\mathfrak{g}}_{t+1} - \tilde{\mathfrak{g}}_t) - \tilde{\sigma}^{-1}(\hat{\imath}_t - E_t\pi_{t+1} - r_t^n). \tag{19}$$

where $\hat{\imath}_t \equiv \frac{1+\imath_t}{1+\imath}$ and the real natural rate of interest is given by

$$r_t^n \equiv \tilde{\sigma}\left(\Delta E_t\hat{y}_{t+1}^n - \Delta E_t\hat{\mathfrak{g}}_{t+1}^n + (\sigma-1)\tilde{\sigma}^{-1}\Delta E_t\hat{\xi}_{t+1}\right). \tag{20}$$

We see that the output gap is a positive function of the expected output gap and of the current government spending gap. However, the output gap is a negative function of the expected growth of the government spending gap, as well as the real rate of interest. Since $\tilde{\sigma} = \frac{\sigma}{1-\psi}$, we see that the elasticity of output with regard to the interest rate increases in the steady state share of government spending ψ. Note that the natural rate of interest increases in the expected growth rate of the natural rate of output and decreases in the

[7] See also Galí and Monacelli (2008).

increase of the natural rate of government spending.

The natural rates of output, public spending and interest can be expressed in terms of exogenous shocks only. Approximating to first order the steady state relationship between the marginal utility of consumption and government spending, we obtain

$$\mathfrak{g}_t^n = \frac{\tilde{\sigma}}{\tilde{\sigma}+\tilde{\gamma}}\hat{y}_t^n - \frac{(1-\sigma)}{\tilde{\sigma}+\tilde{\gamma}}\hat{\xi}_t + \frac{(1-\gamma)}{\tilde{\sigma}+\tilde{\gamma}}\hat{\varepsilon}_t. \tag{21}$$

Combining this expression with (17), we can solve for

$$\hat{y}_t^n = \eta_{ya}\hat{a}_t + \eta_{y\xi}\hat{\xi}_t + \eta_{y\varepsilon}\hat{\varepsilon}_t, \tag{22}$$

with the elasticities $\eta_{ya} = \frac{1+\eta}{\tilde{\sigma}}\bar{\eta} > 0$, where $\bar{\eta} \equiv \left(\frac{\eta}{\tilde{\sigma}} + \frac{\tilde{\gamma}}{\tilde{\sigma}+\tilde{\gamma}}\right) > 0$. Furthermore, $\eta_{y\xi} = \frac{(1-\sigma)}{(\tilde{\sigma}+\tilde{\gamma})}\frac{\tilde{\gamma}}{\tilde{\sigma}}\bar{\eta}$ and $\eta_{y\epsilon}\frac{(1-\gamma)}{(\tilde{\sigma}+\tilde{\gamma})}\bar{\eta}$. Substituting (22) into (21), we obtain the natural rate of government spending

$$\hat{g}_t^n = \eta_{ga}\hat{a}_t + \eta_{g\xi}\hat{\xi}_t + \eta_{g\varepsilon}\hat{\varepsilon}_t, \tag{23}$$

with the elasticities $\eta_{ga} = \frac{1+\eta}{\tilde{\sigma}+\tilde{\gamma}}\bar{\eta} > 0$, $\eta_{g\xi} = \frac{(1-\sigma)}{(\tilde{\sigma}+\tilde{\gamma})}\left(\bar{\eta}\frac{\tilde{\gamma}}{\tilde{\sigma}+\tilde{\gamma}} - 1\right)$ and $\eta_{g\varepsilon} = \frac{(1-\gamma)}{(\tilde{\sigma}+\tilde{\gamma})}\left(\bar{\eta}\frac{\tilde{\sigma}}{\tilde{\sigma}+\tilde{\gamma}} + 1\right)$.

Assume the exogenous shocks follow the following processes $E_t\hat{a}_{t+1} \equiv \rho_a\hat{a}_t$, $E_t\hat{\varepsilon}_{t+1} \equiv \rho_\varepsilon\hat{\varepsilon}_t$, $E_t\hat{\xi}_{t+1} \equiv \rho_\xi\hat{\xi}_t$, all with persistence parameters $0 \leq \rho < 1$. Using these as well as (22) and (23), the natural real rate of interest can be expressed as

$$r_t^n = -\eta_{ra}\hat{a}_t + \eta_{r\xi}\hat{\xi}_t + \eta_{r\varepsilon}\hat{\varepsilon}_t, \tag{24}$$

with the elasticities $\eta_{ra} = \tilde{\sigma}(1-\rho_a)(\eta_{ga}-\eta_{ya}) > 0$, $\eta_{r\xi} = \tilde{\sigma}(1-\rho_\varepsilon)\left[(1-\sigma)\tilde{\sigma}^{-1} + (\eta_{g\xi} - \eta_{y\xi})\right]$, $\eta_{r\varepsilon} = \tilde{\sigma}(1-\rho_\varepsilon)(\eta_{g\varepsilon} - \eta_{y\varepsilon})$.

What emerges from the analysis of the elasticities is that, while the natural rates of output and government spending increase in response to temporary productivity shocks, the reverse is true for the natural rate of interest. We also see that if utility in consumption and government spending are assumed to be of the log-utility type with $\gamma = \sigma = 1$, then the natural rates of output, government spending and interest are only a function of the technology shock. Furthermore, the sign of the taste shocks to private consumption and public spending in turn depends on whether the coefficient of risk-aversion is greater or smaller than one.

4 Optimal Fiscal Policy when Monetary Policy is Optimal

Throughout this paper fiscal policy is set optimally, minimising the social loss function. The latter is obtained through a second order approximation of the representative agent's utility around the efficient non-stochastic steady state. The conditions necessary for efficiency are discussed in section 3.1. As shown in appendix A.2, the loss-function is

$$\sum_{t=0}^{\infty} U(C_t, G_t, N_t; \xi_t, \varepsilon_t, A_t) = -\frac{1}{2} v_y Y \delta \frac{\theta}{1+\eta\theta} \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + O\|\theta, \xi, \varepsilon, A\|^3,$$

where the loss term is given by

$$L_t = \pi_t^2 + \lambda_y \tilde{y}_t^2 + \lambda_g \tilde{\mathfrak{g}}_t^2 + \lambda_f \left(\tilde{\mathfrak{g}}_t - \tilde{y}_t\right)^2, \tag{25}$$

with

$$\lambda_y \equiv \eta\delta \frac{1+\eta\theta}{\theta}; \qquad \lambda_g \equiv \frac{\gamma}{\psi}\delta \frac{1+\eta\theta}{\theta}; \qquad \lambda_f \equiv \frac{\sigma}{1-\psi}\delta \frac{1+\eta\theta}{\theta}.$$

Recalling that $\tilde{\mathfrak{g}}_t = \psi \tilde{g}_t$, it is easy to see that if we set the share of government spending to zero, i.e. $\psi = 0$, then (25) as well as the Phillips curve (18) and the IS relation (19) reduce to the benchmark model without government spending.

The monetary and fiscal authority both minimise the social loss (25). Both authorities thus essentially cooperate. Inflation and output are functions of the government spending gap and the nominal interest rate. Thus, four processes are chosen optimally in the problem

$$\max_{\tilde{\mathfrak{g}}_t, \hat{\imath}_t, \pi_t, \tilde{y}_t} -\frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \pi_t^2 + \lambda_y \tilde{y}_t^2 + \lambda_g \tilde{\mathfrak{g}}_t^2 + \lambda_f (\tilde{\mathfrak{g}}_t - \tilde{y}_t)^2 \\ -2\Lambda_t^{pc}(\beta E_t \pi_{t+1} + \kappa_y \tilde{y}_t - \kappa_g \tilde{\mathfrak{g}}_t - \mu_t - \pi_t) \\ -2\Lambda_t^{is}(E_t \tilde{y}_{t+1} - (E_t \tilde{\mathfrak{g}}_{t+1} - \tilde{\mathfrak{g}}_t) - \tilde{\sigma}^{-1}(\hat{\imath}_t - E_t \pi_{t+1} - r_t^n) - \tilde{y}_t) \end{array} \right\}, \tag{26}$$

where Λ_t^{pc} and Λ_t^{is} are the Lagrange-multipliers on the Phillips curve and the IS relation.

Inspecting the problem we see that in the absence of both cost-push and natural rate of interest shocks, inflation, the output and government spending gaps should be set to zero. As long as cost-push shocks are absent, inflation, the government spending and output gaps can all be set to zero as long as the nominal interest rate is set equal to the natural rate of interest. In that case, the social loss is zero, too. In this sense monetary policy can offset shocks to the natural rate of interest at no cost.

In the presence of cost-push shocks and sticky prices, social loss is inevitable and we have to solve explicitly for the first order conditions. The first order conditions for π_t, $\tilde{y}_t$, $\tilde{\mathfrak{g}}_t$ and $\hat{\imath}_t$ are, in that order:

$$-\pi_t - \Lambda_t^{pc} = 0, \tag{27}$$

$$-\lambda_y \tilde{y}_t + \lambda_f(\tilde{\mathfrak{g}}_t - \tilde{y}_t) + \kappa_y \Lambda_t^{pc} - \Lambda_t^{is} = 0, \tag{28}$$

$$-\lambda_g \tilde{\mathfrak{g}}_t - \lambda_f(\tilde{\mathfrak{g}}_t - \tilde{y}_t) - \kappa_g \Lambda_t^{pc} + \Lambda_t^{is} = 0, \tag{29}$$

$$-\tilde{\sigma}^{-1}\Lambda_t^{is} = 0. \tag{30}$$

The last condition shows that monetary policy makes the IS relation non-binding as a constraint. Imposing this condition on (28) and (29), we can combine these equations. Substituting for output we obtain

$$\left(\underbrace{\frac{\lambda_y + \lambda_f}{\kappa_y} - \frac{\lambda_f}{\kappa_g}}_{=0}\right)\Lambda_t^{pc} = \left(\frac{\lambda_f}{\kappa_y} - \frac{\lambda_g + \lambda_f}{\kappa_g}\right)\tilde{\mathfrak{g}}_t, \tag{31}$$

so that

$$\tilde{\mathfrak{g}}_t = 0. \tag{32}$$

Naturally, the same result for government spending obtains if we instead combine (28) and (29) substituting for Λ_t^{pc}. This result is not contingent on $\tilde{\mathfrak{g}}_t$ being defined as deviation from steady state output rater than government spending. (32) holds if and only if $\tilde{g}_t = 0$. This proves the central proposition of this section.

PROPOSITION 1 [NEUTRALITY OF FISCAL POLICY] *If, in the New Keynesian model with microfounded loss, both the nominal interest rate and the government spending gap are set optimally, then fiscal policy should be neutral. The government spending gap is set to zero at all times, playing no role in macroeconomic stabilisation.*

As shown in appendix A.4, Proposition 1 applies equally if policy is set under commitment rather than discretion. Given (32), the optimal targeting rule for inflation and output is obtained from (27) and (28) as

$$\pi_t = -\frac{\lambda_f + \lambda_y}{\kappa_y}\tilde{y}_t. \tag{33}$$

Since government spending is set to zero, the optimal targeting rule (33) exactly matches that of optimal monetary policy when government spending is exogenous, as described in Clarida et al. (1999). Government spending cannot improve the optimal trade-off implied by (33) in any way. Why is this the case?

Firstly, variations in the nominal interest rate do not enter the loss function (25). The optimal nominal interest rate can be obtained residually from the IS relation (19) as a function of the optimal levels of inflation, the output and government spending gaps. This is possible as the nominal interest only enters the IS relation.

By contrast, government spending enters the loss function $L_t = \pi_t^2 + \lambda_y \tilde{y}_t^2 + \lambda_g \tilde{\mathfrak{g}}_t^2 + \lambda_f (\tilde{\mathfrak{g}}_t - \tilde{y}_t)^2$ twice. The term weighted by λ_g is directly due to variations of government spending. By contrast, the term weighted by λ_f is due to variations in government spending entering through their effect on consumption. Output, however, enters this loss function as well as the IS relation (19) and the Phillips curve (18) in a symmetric way. So that only output should be traded off with inflation as in (33) while government spending is set to zero, is not obvious prima facie.

As is evident from (31), Proposition 1 crucially depends on the exact ratios of the microfounded weights in the loss function λ_y, λ_f to the elasticities of inflation with regard to output and government spending κ_y, κ_g. Notably, the neutrality result does not depend at all λ_g. λ_f determines the loss due to variations in government spending due to its effect on consumption, while κ_g is the elasticity of inflation with regard to the government spending, where the latter again is effective through its effect on consumption. What matters is their relative ratio $\frac{\lambda_f}{\kappa_g}$. The second ratio which matters is $\frac{\lambda_y + \lambda_f}{\kappa_y}$.

The denominator is the elasticity of inflation with regard to output, which operates through the effect of output on consumption and the labour supply. The numerator is the loss from variations in output, again due to the effect of the latter on consumption and output. Fiscal neutrality is optimal when these two ratios of losses relative to gain in reduced inflation per unit of output or government spending change is the same. When this ratio is the same, government spending has nothing to add to stabilisation policy.

The key reason why it is government spending rather than output which should optimally be set to zero is the following. Due to the greater elasticity of inflation with regard to output than with regard to government spending $\kappa_y > \kappa_g$, lower movements in output than government spending are required to temper inflation. As output enters the resource constraint and the production function, lower output affects inflation both through its effect on consumption and labour supply. Thus, the output-elasticity of inflation is a function of elasticity of labour-supply and the intertemporal elasticity of consumption. By contrast, government spending only enters the resource constraint and thus affects only consumption directly and the labour supply only indirectly. Therefore government-spending-elasticity of inflation only function of intertemporal elasticity of consumption. Both the interest rate and government spending attempt to influence demand through their effect on consumption, the interest rate, however, is the more effective tool to do so.

A different way to see this is the following argument. (33) shows that variations of inflation and at least one other variable have to be traded off. In principle this could be either (a) only the output gap, (b) only the government spending gap, or (c) both the output and government spending gaps. To gain some more intuition, recall the losses for the three cases as

$$(a): \quad (\lambda_y + \lambda_f)\,\tilde{y}_t^2$$
$$(b): \quad (\lambda_g + \lambda_f)\,\tilde{\mathfrak{g}}_t^2$$
$$(c): \quad \lambda_y \tilde{y}_t^2 + \lambda_g \tilde{\mathfrak{g}}_t^2 + \lambda_f \left(\tilde{\mathfrak{g}}_t - \tilde{y}_t\right)^2 \quad = \quad (\lambda_y + \lambda_f)\,\tilde{y}_t^2 + (\lambda_g + \lambda_f)\,\tilde{\mathfrak{g}}_t^2 - \lambda_f \tilde{y}_t \tilde{\mathfrak{g}}_t.$$

Proposition 1 shows that (a) is optimal. It is easy to see that (a) is less costly than (b). Due to the greater elasticity of inflation with regard to output than with regard to government spending $\kappa_y > \kappa_g$, lower movements in output than government spending are required to temper inflation. (a) also leads to a lower loss than (c). Consider the right equation of (c), assuming that output is at the level which is optimal when government spending is zero. Conditional on this level of output, we see that increasing government spending leads to a loss of second order while only an improvement of first order. Hence, government spending is optimally zero.

To summarise, the two key features behind Proposition 1 are, firstly, that the optimal nominal interest rate does not enter the loss function and thus can be obtained residually as a function of the optimal levels of inflation, output and government spending. That government spending in the current benchmark setting is optimally zero, in turn, is due to the fact that government spending and output enter the model virtually symmetrically. However, while government spending is only effective through its effect on consumption, output has an effect both on consumption and the labour supply. Thus, the elasticity of inflation with regard to output is lower than that of government spending. As a result variations in output rather than government spending can be traded-off against inflation at lower cost.

When government spending is exogenous, the loss function only contains quadratic terms in inflation and output. As noted by Tinbergen (1952), if we have as many instruments as targets, all targets can be achieved. With two instruments one thus should be able to hit two targets. One may have thought that using government spending as an additional instrument not only shocks to the natural rate of interest but also cost-push shock can be perfectly offset so as to always achieve the inflation and output targets. In appendix A.3 I show that this is indeed the case if government spending did not enter the loss function (25). However, when government spending is endogenous the microfounded loss function contains four quadratic terms, and thus four targets, so that the inflation variability problem does not disappear with government spending as additional instrument.

5 Optimal Fiscal Policy when Monetary Policy follows a Taylor Rule

This section returns to the benchmark New Keynesian model without monetary frictions. However, unlike in section 4 we assume now that monetary policy is not set optimally. Rather it is constrained to follow a specific Taylor rule.

This case is interesting to study for several reasons. Firstly, going back to Taylor (1993) Taylor rules are generally considered a reasonably close approximation to the policy which central bankers implement.[8] Given that a Taylor rule is a reasonable assumption for monetary policy, it is important to investigate what the optimal fiscal rule is in that case.

5.1 Optimal Fiscal Targeting Rule

Let us assume that the monetary authority sets the interest rate endogenously in reaction to expected future inflation

$$\hat{\imath}_t = \varphi_\pi E_t \pi_{t+1}. \tag{34}$$

[8] C.f. Orphanides (2007) for a concise overview on Taylor rules.

Fiscal policy, with the government spending gap $\tilde{\mathfrak{g}}_t$ as its instrument, continues to be set optimally. The Taylor rule can simply be substituted into the IS relation so that the problem is

$$\max_{\tilde{\mathfrak{g}}_t,\tilde{y}_t,\pi_t} -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t \left\{ \begin{array}{c} \pi_t^2+\lambda_y\tilde{y}_t^2+\lambda_g\tilde{\mathfrak{g}}_t^2+\lambda_f(\tilde{\mathfrak{g}}_t-\tilde{y}_t)^2 \\ -2\Lambda_t^{pc}(\beta E_t\pi_{t+1}+\kappa_y\tilde{y}_t-\kappa_g\tilde{\mathfrak{g}}_t-\pi_t) \\ -2\Lambda_t^{is}(E_t\tilde{y}_{t+1}-(E_t\tilde{\mathfrak{g}}_{t+1}-\tilde{\mathfrak{g}}_t)-\tilde{\sigma}^{-1}(\varphi_\pi-1)E_t\pi_{t+1}+\tilde{\sigma}^{-1}r_t^n-\tilde{y}_t) \end{array} \right\}.$$

The first order conditions for inflation, the output gap and the government spending gap remain (27)-(29). There is now, however, no optimality condition for the nominal interest rate as it follows the rule (34) instead.

The optimal rule for trading off inflation, the output gap and the government spending gap is found by eliminating the Lagrange-multipliers and combining the three optimality conditions (27)-(29) into one optimality condition for the conduct of $\tilde{g}_t$:

$$\pi_t = -\frac{\lambda_y}{\kappa_y-\kappa_g}\tilde{y}_t-\frac{\lambda_g}{\kappa_y-\kappa_g}\tilde{\mathfrak{g}}_t \tag{35}$$

Note that the terms for inflation and output are identical to those obtained for optimal monetary policy in (33). Rewriting (35) we obtain:

Proposition 2 Optimal Fiscal Targeting Rule for Forward-Looking Taylor rule *If, in the microfounded New Keynesian model, the monetary authority follows the Taylor rule* $\hat{\imath}_t=\varphi_\pi E_t\pi_{t+1}$*, the optimal fiscal targeting rule is*

$$\tilde{\mathfrak{g}}_t = -\frac{\kappa_y-\kappa_g}{\lambda_g}\pi_t-\frac{\lambda_y}{\lambda_g}\tilde{y}_t. \tag{36}$$

Proceeding analogously it is straightforward to identify the optimal targeting rule for the current-looking Taylor rule $\hat{\imath}_t=\varphi_\pi\pi_t$. For the current-looking rule we obtain the following result.

Proposition 3 Optimal Fiscal Targeting Rule for Current-Looking Taylor rule *If, in the microfounded New Keynesian model, the monetary authority follows the Taylor rule* $\hat{\imath}_t=\varphi_\pi\pi_t$*, the optimal fiscal targeting rule is*

$$\tilde{\mathfrak{g}}_t = -\frac{\tilde{\sigma}(\kappa_y-\kappa_g)}{\tilde{\sigma}\lambda_g+\eta_{\pi g}\varphi_\pi}\pi_t-\frac{\tilde{\sigma}\lambda_y}{\tilde{\sigma}\lambda_g+\eta_{\pi g}\varphi_\pi}\tilde{y}_t, \tag{37}$$

where $\eta_{\pi g} = \lambda_f(\kappa_y - \kappa_g) + \lambda_g \kappa_y > 0$.

Note that for $\eta_{\pi g} = 0$, (37) reduces to (36). Rewriting (37) as $\pi_t = -\frac{\lambda_y}{\kappa_y - \kappa_g}\tilde{y}_t - \frac{\tilde{\sigma}\lambda_g + \eta_{\pi g}\varphi_\pi}{\tilde{\sigma}(\kappa_y - \kappa_g)}\tilde{\mathfrak{g}}_t$, we see that output and inflation have the same coefficients as in (36). However, the response from government spending required for given values of inflation and the output gap is smaller than in (36). Hence, less activism on the part of fiscal policy is required in a purely forward-looking system when the monetary authority itself follows a Taylor rule responding to current rather than expected inflation. On the basis of (36) and (37) we can establish the main properties of the optimal fiscal targeting rule as follows.

Proposition 4 [Properties of the Optimal Fiscal Targeting Rule] *For a given level of the output gap, the fiscal instrument follows a lean-against-the-wind policy in the following sense: the government spending gap should be negative when inflation is above equilibrium. Conditional on the level of inflation, fiscal policy should be countercyclical in the following sense: the government spending gap should be negative if the output gap is positive.*

The conditionality is due to the fact that a stag-flation period with rising inflation and negative output gap can arise where the optimal fiscal policy should be pro-cyclical.

5.2 Determinacy

Equilibrium determinacy has become a major yardstick for the evaluation of monetary policy. If equilibrium determinacy obtains variations in the monetary instrument achieve a unique equilibrium path for the variables, ruling out self-fulfilling rational expectations equilibria. Here we examine the condition on the monetary authority to ensure determinacy in the presence of optimal fiscal stabilisation policy.

First, we look at the forward-looking Taylor rule (34) and the first order condition (35). Together with the Phillips curve (18) and the IS relation (19), we have four equations with which we can solve for the four processes of inflation: the output gap, the government spending gap and the interest rate. However, using the Taylor rule and the first order condition we can reduce the four-variable system to the two-dimensional one

$$E_t z_{t+1} = \Gamma_1 z_t + \Psi_1 v_t, \tag{38}$$

with the vector of endogenous variables $z_t \equiv \begin{bmatrix} \tilde{y}_t & \tilde{\mathfrak{g}}_t \end{bmatrix}'$ and the vector of exogenous disturbances $v_t \equiv \begin{bmatrix} \mu_t & r_t^n \end{bmatrix}'$, where the coefficient matrix Γ_1 is given by

$$\Gamma_1 = \begin{bmatrix} \frac{\lambda_g}{\lambda_y+\lambda_g} + \frac{\left(\frac{\lambda_y}{\kappa_y-\kappa_g}+\kappa_y\right)[(\lambda_g(1-\varphi_\pi)+\tilde{\sigma}(\kappa_y-\kappa_g)]}{\beta\tilde{\sigma}(\lambda_y+\lambda_g)} & -\frac{\lambda_g}{\lambda_y+\lambda_g} + \frac{\left(\frac{\lambda_g}{\kappa_y-\kappa_g}-\kappa_g\right)[(\lambda_g(1-\varphi_\pi)+\tilde{\sigma}(\kappa_y-\kappa_g)]}{\beta\tilde{\sigma}(\lambda_y+\lambda_g)} \\ -\frac{\lambda_y}{\lambda_y+\lambda_g} - \frac{\left(\frac{\lambda_y}{\kappa_y-\kappa_g}+\kappa_y\right)[(\lambda_y(1-\varphi_\pi)-\tilde{\sigma}(\kappa_y-\kappa_g)]}{\beta\tilde{\sigma}(\lambda_y+\lambda_g)} & \frac{\lambda_g}{\lambda_y+\lambda_g} - \frac{\left(\frac{\lambda_g}{\kappa_y-\kappa_g}-\kappa_g\right)[(\lambda_y(1-\varphi_\pi)-\tilde{\sigma}(\kappa_y-\kappa_g)]}{\beta\tilde{\sigma}(\lambda_y+\lambda_g)} \end{bmatrix}.$$

Both endogenous variables in z_t are non-predetermined. By Blanchard and Kahn (1980) determinacy, i.e. the existence of a unique rational expectations equilibrium of (38), requires the two roots of Γ_1 have to lie outside the unit circle. With this condition I show in appendix A.5 the following.

PROPOSITION 5 [DETERMINACY UNDER FORWARD-LOOKING TAYLOR RULE] *If, in the New Keynesian model, monetary policy follows the Taylor rule* $\hat{\imath}_t = \varphi_\pi E_t \pi_{t+1}$ *while fiscal policy implements the optimal targeting rule* $\tilde{\mathfrak{g}}_t = -\frac{\kappa_y-\kappa_g}{\lambda_g}\pi_t - \frac{\lambda_y}{\lambda_g}\tilde{y}_t$*, then equilibrium determinacy obtains if and only if*

$$1 < \varphi_\pi < 1 + 2\frac{\tilde{\sigma}\left[(\kappa_y-\kappa_g)^2 + (1+\beta)(\lambda_y+\lambda_g)\right]}{(\kappa_y\lambda_g+\kappa_g\lambda_y)}. \tag{39}$$

As shown in appendix A.6, the analogous condition for the current-looking Taylor rule $\hat{\imath}_t = \varphi_\pi \pi_t$ is the following.

PROPOSITION 6 [DETERMINACY UNDER CURRENT-LOOKING TAYLOR RULE] *If, in the New Keynesian model, monetary policy follows the Taylor rule* $\imath_t = \varphi_\pi \pi_t$ *while fiscal policy implements the optimal targeting rule* $\tilde{\mathfrak{g}}_t = -\frac{\tilde{\sigma}(\kappa_y-\kappa_g)}{\tilde{\sigma}\lambda_g+\eta_{\pi g}\varphi_\pi}\pi_t - \frac{\tilde{\sigma}\lambda_y}{\tilde{\sigma}\lambda_g+\eta_{\pi g}\varphi_\pi}\tilde{y}_t$*, equilibrium determinacy obtains if and only if*

$$\varphi_\pi > 1. \tag{40}$$

We see that in the current-looking case, the necessary and sufficient condition is for the Taylor rule coefficient on inflation to be greater than one. While this condition also applies to the forward-looking Taylor rule, in the latter case the coefficient on inflation also has to lie below the bound $\varphi_\pi < 1 + 2\frac{\tilde{\sigma}\left[(\kappa_y-\kappa_g)^2+(1+\beta)(\lambda_y+\lambda_g)\right]}{(\kappa_y\lambda_g+\kappa_g\lambda_y)}$ for determinacy. The existence of the upper bound parallels exactly the case of optimal monetary policy in isolation, where inter alia Carlstrom and Fuerst (2001), Lubik and Marzo (2007) derive the parameter restriction $1 < \varphi_\pi < 1 + 2\frac{\tilde{\sigma}(1+\beta)}{\kappa_y}$ as necessary and sufficient condition for determinacy.

Thus, fundamentally, when the monetary authority follows a Taylor rule, the additional of an optimising fiscal authority leaves the established results on the Taylor coefficient intact. The reason for this is that government spending affects demand in essentially the same manner as monetary policy, namely through its effect on consumption. The fiscal demand management simply supplies some of the stabilisation which monetary policy leaves to be undertaken as it is not set optimally but restricted to follow a Taylor rule. As the optimal fiscal policy targeting rule mimics the optimal monetary policy targeting rule (33).

6 Optimal Fiscal and Monetary Policy under Monetary Frictions

The present section returns to the assumption that monetary policy is set optimally. However, while so far the only friction in the model has been that of price-rigidity, we now also allow for the existence of monetary frictions. If monetary frictions exist, then the monetary liabilities of the central bank do facilitate transactions. This is in fact what we observe in actual economies as positive quantities of base money are held by private parties the fact notwithstanding that base money yields a lower turn than other riskless assets even over the very short term.

I heuristically introduce monetary frictions into the benchmark New Keynesian model to examine how this affects the trade-offs between monetary and fiscal policy. To introduce monetary frictions, I follow the so-called money-in-the-utility (MIU) approach which goes back to Sidrauski (1967) and Brock (1974) and is summarised in the context of the New Keynesian model in Woodford (2003). This approach introduces real money balances M_t/P_t as an argument into the utility function, where M_t are holdings of a nominal monetary asset. Hence, the economy is not cashless as in all other sections of this paper. With MIU we obtain two additional first order conditions, $M_t \geq 0$ and $\frac{U_m(C_t,M_t;\xi_t)}{U_c(C_t,M_t;\xi_t)} \leq \frac{\imath_t-\imath_t^m}{1+\imath_t}$, characterising the relative price of real money balances in terms of consumption. For $M_t > 0$ the latter condition holds with equality, i.e.

$$\frac{U_m(C_t, M_t; \xi_t)}{U_c(C_t, M_t; \xi_t)} = \frac{\imath_t - \imath_t^m}{1 + \imath_t}. \tag{41}$$

Under the assumption that consumption and real balances are both normal goods, we can solve (41) for equilibrium real balances

$$\frac{M_t}{P_t} = L(Y_t, \frac{\imath_t - \imath_t^m}{1 + \imath_t}), \tag{42}$$

where the liquidity preference function L increases in output and decreases in the interest rate differential $\frac{\imath_t - \imath_t^m}{1+\imath_t}$. The central bank can still choose freely the non-monetary interest rate $\imath_t$. Furthermore, it can choose freely either the monetary-interest rate $\imath_t^m$ or the money supply, where the respective other variable adjusts to satisfy (42).

As long as money enters utility in additively separable form, the IS relation (19) and the Phillips curve (18) remain unchanged. However, as shown in Woodford (2003) ch.6, due to the monetary frictions the loss function (25) now includes a quadratic term in the interest rate

$$L_t = \pi_t^2 + \lambda_y \tilde{y}_t^2 + \lambda_g \tilde{\mathfrak{g}}_t^2 + \lambda_f (\tilde{\mathfrak{g}}_t - \tilde{y}_t)^2 + \lambda_\imath (\hat{\imath}_t - \hat{\imath}_t^m + \bar{\Psi})^2, \tag{43}$$

with $\lambda_\imath \equiv \frac{\eta_\imath \delta}{\theta V} > 0$, where $V \equiv Y/M$ is the steady state velocity of money and $\eta_\imath > 0$ is the elasticity of money-supply with regard to the interest rate. $\bar{\Psi}$ is the steady state interest differential between nonmonetary and monetary assets. $\hat{\imath}_t^m$ is the interest rate paid on the monetary asset.

We see that setting nominal interest equal to the natural rate of interest, which would close the gaps of inflation, output and government spending (19), creates a loss in (43) and is thus not an optimal response to shocks to the natural rate of interest anymore.

The problem for the policymaker is to minimise (43) subject to the Phillips curve (18) and (19) with regard to inflation, output, government spending and nominal interest. The first order conditions for the first three variables remain (27), (28) and (29). The new first order condition for the nominal interest rate is

$$\lambda_\imath (\hat{\imath}_t - \hat{\imath}_t^m + \bar{\Psi}) - \tilde{\sigma}^{-1} \Lambda_t^{is} = 0. \tag{44}$$

We immediately see that the IS relation is a binding constraint on monetary policy as $\Lambda_t^{is} \neq 0$. As a result, combining (28) and (29) with the first order condition of the nominal interest rate (44) delivers $\tilde{\mathfrak{g}}_t \neq 0$. Combining (27) with (28) and (44) as well as (27) with (29) and (44), we can eliminate all Lagrange-multipliers and obtain two optimality conditions in the four endogenous variables. We then can eliminate the interest rate to obtain the optimal targeting condition for the government spending gap, namely $\pi_t = -\frac{\lambda_y}{\kappa_y - \kappa_g}\tilde{y}_t - \frac{\lambda_g}{\kappa_y - \kappa_g}\tilde{\mathfrak{g}}_t$, or equivalently

$$\tilde{\mathfrak{g}}_t = -\frac{\kappa_y - \kappa_g}{\lambda_g}\pi_t - \frac{\lambda_y}{\lambda_g}\tilde{y}_t. \tag{45}$$

This allows us to state the following.

Proposition 7 [Fiscal Targeting Rule - Monetary Frictions] *If, in the New Keynesian model with monetary frictions both fiscal and monetary policy are set optimally in the presence of monetary frictions, fiscal policy is not neutral and the optimal targeting condition for the government spending gap is* $\tilde{\mathfrak{g}}_t = -\frac{\kappa_y - \kappa_g}{\lambda_g}\pi_t - \frac{\lambda_y}{\lambda_g}\tilde{y}_t$. *This optimality condition is identical to (35), the optimal targeting condition for fiscal policy when monetary frictions are absent but the monetary authority follows a forward-looking Taylor rule.*

The properties of (45) are discussed in more detail in section 5.1. Plugging (45) into the combination of (27) with (28) and (44), we obtain the optimal targeting condition for the interest rate

$$\hat{\imath}_t - \hat{\imath}_t^m + \bar{\Psi} = \left(\frac{\kappa_y + \frac{\lambda_y}{\lambda_g}}{\tilde{\sigma}\lambda_\imath}\right)\pi_t + \left(\frac{\lambda_y + \lambda_f + \frac{\lambda_f \lambda_y}{\lambda_g}}{\tilde{\sigma}\lambda_\imath}\right)\tilde{y}_t, \tag{46}$$

where $\left(\frac{\kappa_y + \frac{\lambda_y}{\lambda_g}}{\tilde{\sigma}\lambda_\imath}\right), \left(\frac{\lambda_y + \lambda_f + \frac{\lambda_f \lambda_y}{\lambda_g}}{\tilde{\sigma}\lambda_\imath}\right) > 0$. What matters is thus not only the nominal interest rate itself, but the interest rate differential between nonmonetary and monetary assets, as well as Ψ which is that differential in steady state. However, we can always set the interest on money $\hat{\imath}_t^m = 0$, in which case the money-supply M_t adjusts endogenously to satisfy (42). We thus can summarise the properties of (46) as follows.

Proposition 8 [Interest Rate Targeting Rule - Monetary Frictions] *The nominal interest rate should rise in response to inflation and positive output gaps. Whether the coefficient on inflation and output are smaller or greater than one, depends on the size of the weights on the interest rate relative to those of output and government spending in the loss function.*

The interest rate targeting rule complements the fiscal targeting rule in that it, too, is restrictive in reaction to higher inflation and output gaps. The higher is the weight on interest rate movements in loss function, the less should the interest rate be moved, and thus the higher the relative role of fiscal policy in stabilisation.

Note that if consumption and real balances are non-separable, the interest rate enters not only the loss-function but also the Phillips curve and IS relation. If anything, this will strengthen the stabilisation role for fiscal policy.

In any case, the most important insight is that monetary frictions generate a stabilisation role for fiscal policy in the form of government spending and, by implication, the relative role of fiscal and monetary policy even when both are set optimally. As discussed extensively i.a. in Woodford (2003), when monetary policy is considered by itself, assuming away monetary frictions does often not materially affect conclusions about monetary policy. Here, however, we see that assuming away monetary frictions does dramatically change our conclusion about the role fiscal policy should play. As variations in the nominal interest rate do directly lead to a social loss, demand management through fiscal policy has an important contribution to make. Other things equal, fiscal policy should lean against inflation and be countercyclical relative to output.

7 Conclusion

This paper uses the benchmark New Keynesian model to study under what conditions fiscal policy in terms of government spending should contribute to macroeconomic stabilisation.

It is shown that, if monetary policy is set optimally and not subject to monetary frictions, fiscal policy should be neutral in the sense that government spending has no role to play in macroeconomic stabilisation. However, as soon as monetary policy faces monetary frictions, or is constrained to a Taylor rule, a stabilisation role for government spending re-emerges. Interestingly, the optimal targeting rule for fiscal policy is the same in the latter two cases. The optimal fiscal targeting rule essentially mimics the targeting rule of monetary policy. The government spending gap should be negative in the face of inflation. Furthermore, government spending should be countercyclical in that the government spending gap should be negative when the output gap is positive and vice-versa.

Furthermore, I show that, if fiscal policy follows this targeting rule and monetary policy a Taylor rule, satisfaction of the Taylor principle is necessary and sufficient for equilibrium determinacy. The Taylor principle thus turns out to be robust to the introduction of an optimal stabilising fiscal policy.

The results of this paper are not only informative on the role of fiscal policy itself. Rather, they also give insight into the robustness of results on monetary policy which have been established under the assumption that fiscal policy is exogenous. While the cashless economy without monetary frictions may be a convenient limiting case to study monetary policy on its own, monetary frictions fundamentally change the role of fiscal policy in macroeconomic stabilisation itself, as well as the relative assignment of monetary and fiscal policies.

A Appendix

A.1 Notation

For any variable X_t, let $x_t \equiv \ln X_t$, that is lower case variables denote logs, unless otherwise noted. The steady state of X_t carries no time subscript and is written X. The value of X_t obtaining under flexible prices, is called its natural rate, i.e. X_t^n. In this paper we will make assumptions ensuring that the natural rate is also efficient. The log-deviation of X_t is defined and written as $\hat{x}_t = x_t - x$. Gap variables, in turn, are defined as the difference of the log-deviation of the current rate of X_t from steady state and the log-deviation of the natural rate from steady state, i.e. $\tilde{x}_t = \hat{x}_t - \hat{x}_t^n$.

A.2 Derivation of Loss Function

Following Woodford (2003), the quadratic social loss is obtained as a second order approximation of the household's utility around the efficient non-stochastic steady state.

A second order Taylor expansion of household consumption steady state yields

$$U^c(Y_t - G_t; \xi_t) = u_c Y \left\{ \begin{array}{c} \hat{y}_t + \frac{1}{2}[1 + \frac{u_{cc}Y}{u_c}]\hat{y}_t^2 + \frac{u_{c\xi}\xi}{u_c}\hat{\xi}_t \hat{y}_t \\ -\frac{u_{cc}Y}{u_c}\hat{y}_t\hat{\mathfrak{g}}_t - \left(\hat{\mathfrak{g}}_t + \frac{1}{2}[\frac{Y}{G} - \frac{u_{cc}Y}{u_c}]\hat{\mathfrak{g}}_t^2 + \frac{u_{c\xi}\xi}{u_c}\hat{\xi}_t\hat{\mathfrak{g}}_t\right) \end{array} \right\} + t.i.p. + O\|\theta, \xi\|^3, \tag{47}$$

where $t.i.p.$ stands for 'terms independent of policy'. $O(\|\theta, \xi\|^3)$ is the residual of order three, where $\|\theta, \xi\|$ indicates bounds on the amplitude of the exogenous disturbance and the size of monopolistic distortions for the approximation to be valid.

The approximation to the utility to the agent from the level of government spending is

$$W(G_t; \varepsilon_t) = w_g Y \left\{ \hat{\mathfrak{g}}_t + \frac{1}{2}(\frac{Y}{G} + \frac{w_{gg}Y}{w_g})\hat{\mathfrak{g}}_t^2 + \frac{w_{g\varepsilon}\varepsilon}{w_g}\hat{\varepsilon}_t\hat{\mathfrak{g}}_t \right\} + t.i.p. + O\|\theta, \varepsilon\|^3. \tag{48}$$

Take a second order approximation of the disutility of work of type j, expressed in terms of output through the production function, integrate over j and simplify to obtain

$$V^y(Y_t) = v_y Y \left\{ \begin{array}{c} (1 + \frac{v_{yA}A}{v_y}\hat{a}_t)\hat{y}_t + \frac{1}{2}(1 + \frac{v_{yy}Y}{v_y})\hat{y}_t^2 \\ +\frac{1}{2}(\theta^{-1} + \frac{v_{yy}Y}{v_y})Var_j(\hat{y}_{j,t}) \end{array} \right\} + t.i.p. + O\|\theta, A\|^3. \tag{49}$$

For the efficient steady state (47), (48) and (49) add to

$$U = -v_y Y \left\{ \begin{array}{c} -\frac{1}{2}\frac{u_{cc}Y}{u_c}\hat{y}_t^2 + \frac{1}{2}\frac{v_{yy}Y}{v_y}\hat{y}_t^2 - \frac{1}{2}\left(\frac{u_{cc}Y}{u_c} + \frac{w_{gg}Y}{w_g}\right)\hat{\mathfrak{g}}_t^2 \\ + \left(\frac{v_{yA}A}{v_y}\hat{a}_t - \frac{u_{c\xi}\xi}{u_c}\hat{\xi}_t\right)\hat{y}_t + \left(\frac{u_{c\xi}\xi}{u_c}\hat{\xi}_t - \frac{w_{g\varepsilon}\varepsilon}{w_g}\hat{\varepsilon}_t\right)\hat{\mathfrak{g}}_t \\ + \frac{u_{cc}Y}{u_c}\hat{y}_t\hat{\mathfrak{g}}_t + \frac{1}{2}(\theta^{-1} + \frac{v_{yy}Y}{v_y})Var_j(\hat{y}_{j,t}) \end{array} \right\} + t.i.p. + O\|\theta, \xi, \varepsilon, A\|^3. \tag{50}$$

To express the terms multiplicative in the shocks,

$$\left(\frac{v_{yA}A}{v_y}\hat{a}_t - \frac{u_{c\xi}\xi}{u_c}\hat{\xi}_t\right)\hat{y}_t + \left(\frac{u_{c\xi}\xi}{u_c}\hat{\xi}_t - \frac{w_{g\varepsilon}\varepsilon}{w_g}\hat{\varepsilon}_t\right)\hat{\mathfrak{g}}_t \tag{51}$$

as natural rates, approximate to first order the steady state relationship $u_c \frac{\theta-1}{\theta} = v_y$, so that

$$\frac{v_{yy}Y}{v_y}\hat{y}_t^n + \frac{v_{yA}A}{v_y}\hat{a}_t = \frac{u_{cc}Y}{u_c}\hat{c}_t^n + \frac{u_{c\xi}\xi}{u_c}\hat{\xi}_t. \tag{52}$$

Approximate the steady state relation of the marginal utilities of consumption and government spending as

$$\frac{u_{cc}Y}{u_c}\hat{c}_t^n + \frac{u_{c\xi}\xi}{u_c}\hat{\xi}_t = \frac{w_{gg}Y}{w_g}\hat{\mathfrak{g}}_t^n + \frac{w_{g\varepsilon}\varepsilon}{w_g}\hat{\varepsilon}_t. \tag{53}$$

Substituting (52) into (51) and using the economy resource constraint, we obtain

$$-\frac{v_{yy}Y}{v_y}\hat{y}_t^n\hat{y}_t + \frac{u_{cc}Y}{u_c}\hat{y}_t^n\hat{y}_t - \frac{u_{cc}Y}{u_c}\hat{\mathfrak{g}}_t^n\hat{y}_t + \left(\frac{u_{c\xi}\xi}{u_c}\hat{\xi}_t - \frac{w_{g\varepsilon}\varepsilon}{w_g}\hat{\varepsilon}_t\right)\hat{\mathfrak{g}}_t.$$

Replacing $\left(\frac{u_{c\xi}\xi}{u_c}\hat{\xi}_t - \frac{w_{g\varepsilon}\varepsilon}{w_g}\hat{\varepsilon}_t\right)$ with (53) and rearranging gives

$$-\frac{v_{yy}Y}{v_y}\hat{y}_t^n\hat{y}_t + \frac{u_{cc}Y}{u_c}(\hat{y}_t^n - \hat{\mathfrak{g}}_t^n)(\hat{y}_t - \hat{\mathfrak{g}}_t) + \frac{w_{gg}Y}{w_g}\hat{\mathfrak{g}}_t^n\hat{\mathfrak{g}}_t. \tag{54}$$

Substituting (54) for (51) into (50), we obtain

$$U = -u_cY\left\{\begin{array}{c} \frac{1}{2}\frac{v_{yy}Y}{v_y}(\hat{y}_t - \hat{y}_t^n)^2 - \frac{1}{2}\frac{u_{cc}Y}{u_c}\left[(\hat{\mathfrak{g}}_t - \hat{\mathfrak{g}}_t^n) - (\hat{y}_t - \hat{y}_t^n)\right]^2 \\ -\frac{1}{2}\frac{w_{gg}Y}{w_g}(\hat{\mathfrak{g}}_t - \hat{\mathfrak{g}}_t^n)^2 + \frac{1}{2}(\theta^{-1} + \frac{v_{yy}Y}{v_y})Var_j(\hat{y}_{j,t}) \end{array}\right\} + t.i.p. + O\|\theta, \xi, \varepsilon, A\|^3. \tag{55}$$

Substituting $Var_j(\hat{y}_{j,t}) = \theta^2 Var_j(\ln p_{j,t})$ and, as shown for Calvo-pricing in proposition 3.5 of chapter 4 of Woodford (2003), $\sum_{t=0}^{\infty}\beta^t Var_j(\ln p_{j,t}) = \frac{\omega}{(1-\omega)(1-\omega\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 + t.i.p. + O\|\theta, \xi, \varepsilon, A\|^3$ into (55), we obtain the aggregate loss function:

$$\sum_{t=0}^{\infty} U(C_t, G_t, Y_t) = -v_yY\left\{\begin{array}{c} \frac{1}{2}\frac{v_{yy}Y}{v_y}(\hat{y}_t - \hat{y}_t^n)^2 \\ -\frac{1}{2}\frac{u_{cc}Y}{u_c}\left[(\hat{\mathfrak{g}}_t - \hat{\mathfrak{g}}_t^n) - (\hat{y}_t - \hat{y}_t^n)\right]^2 \\ -\frac{1}{2}\frac{w_{gg}Y}{w_g}(\hat{\mathfrak{g}}_t - \hat{\mathfrak{g}}_t^n)^2 \\ +\frac{1}{2}\left(\frac{\omega}{(1-\omega)(1-\omega\beta)}\right)(\theta^{-1} + \frac{v_{yy}Y}{v_y})\theta^2\pi_t^2 \end{array}\right\} + O\|\xi, \varepsilon, A\|^3.$$

This can be expressed as (25) in the text.

A.3 Optimal Policy with Two Targets and Two Instruments

We can remove government spending from the loss function by setting $\lambda_f = \lambda_g = 0$. Then, there are only two targets. Imposing this on the first order conditions (27)-(30), they become in the same order

$$-\pi_t - \Lambda_t^{pc} = 0; \tag{56}$$

$$-\lambda_y\tilde{y}_t + \kappa_y\Lambda_t^{pc} - \Lambda_t^{is} = 0; \tag{57}$$

$$-\kappa_g\Lambda_t^{pc} + \Lambda_t^{is} = 0; \tag{58}$$

$$-\tilde{\sigma}^{-1}\Lambda_t^{is} = 0. \tag{59}$$

We see from (59) that the nominal interest makes the IS relation a non-binding constraint. Imposing (59) on (58), we see that now also the Lagrange-multiplier on the Phillips curve is zero. Imposing (59) and (58) on (56) and (57), we see that inflation and the output gap are zero. Since in this problem social loss can only arise from non-zero inflation or a non-zero output gap, both inflation shocks and shocks to the natural rate of interest can be offset perfectly without creating any loss.

We can impose (56)-(59) on the Phillips curve (18) and the IS relation (19) to solve for the optimal choices of the government spending gap as $\tilde{\mathfrak{g}}_t = \frac{1}{\kappa_g}\mu_t$, while the optimal nominal interest rate is set as $\hat{\imath}_t = \frac{\tilde{\sigma}}{\kappa_g}\mu_t + r_t^n$.

A.4 Optimal Policy under Commitment

Under commitment, the policymakers again maximise (26) but not only over the current values of the endogenous variables but also their previous expectations. Then the respective first order conditions for π_t, $\tilde{y}_t$, $\tilde{\mathfrak{g}}_t$ and $\hat{\imath}_t$ are:

$$-\pi_t - \Lambda_t^{pc} + \Lambda_{t-1}^{pc} = 0; \tag{60}$$

$$-\lambda_y \tilde{y}_t + \lambda_f(\tilde{\mathfrak{g}}_t - \tilde{y}_t) + \kappa_y \Lambda_t^{pc} - \Lambda_t^{is} + \beta^{-1}\Lambda_{t-1}^{is} = 0; \tag{61}$$

$$-\lambda_g \tilde{\mathfrak{g}}_t - \lambda_f(\tilde{\mathfrak{g}}_t - \tilde{y}_t) - \kappa_g \Lambda_t^{pc} + \Lambda_t^{is} - \beta^{-1}\Lambda_{t-1}^{is} = 0; \tag{62}$$

$$-\tilde{\sigma}^{-1}\Lambda_t^{is} = 0. \tag{63}$$

The optimal monetary policy makes the IS relation non-binding in (63), which applies to all periods. Imposing this on (61) as well as (62) and combining the two, we again find, as in Proposition 1, that the government spending gap is optimally set to zero $\tilde{g}_t = 0$. With this in mind, (60)-(62) yields the standard optimal targeting rule for monetary policy under commitment $\pi_t = -\frac{\lambda_f + \lambda_y}{\kappa_y}\Delta\tilde{y}_t$. This is discussed, for instance, in Clarida et al. (1999).

A.5 Determinacy for Forward-Looking Taylor Rule

This section proves Proposition 5.

Proof. For determinacy, the number of eigenvalues of Γ_1 outside the unit circle must equal the number of non-predetermined endogenous variables, Blanchard and Kahn (1980). In this case there are two non-predetermined endogenous variables. According to the Schur Cohn Criterion, LaSalle (1986), both eigenvalues of Γ are outside the unit circle if and only if both of the following two conditions are fulfilled:
Condition (1)

$$|\det(\Gamma)| > 1. \tag{64}$$

Condition (2)

$$|tr(\Gamma)| < 1 + \det(\Gamma). \tag{65}$$

Γ_1 has determinant

$$\det(\Gamma_1^*) = \frac{(\kappa_y - \kappa_g)^2 + \lambda_y + \lambda_g}{\beta(\lambda_y + \lambda_g)} > 0$$

and trace

$$tr(\Gamma_1^*) = \frac{(1-\varphi_\pi)(\kappa_y\lambda_g + \kappa_g\lambda_y) + \tilde{\sigma}(\kappa_y - \kappa_g)^2 + (1+\beta)\tilde{\sigma}(\lambda_y + \lambda_g)}{\beta\tilde{\sigma}(\lambda_y + \lambda_g)}.$$

With discount factor $\beta < 1$ (64), is trivially fulfilled:

$$|\det(\Gamma_1^*)| = \left|\frac{1}{\beta} + \frac{(\kappa_y - \kappa_g)^2}{\beta(\lambda_y + \lambda_g)}\right| > 1,$$

(65), requires

$$\left|\frac{(1-\varphi_\pi)(\kappa_y\lambda_g + \kappa_g\lambda_y) + \tilde{\sigma}(\kappa_y - \kappa_g)^2 + (1+\beta)\tilde{\sigma}(\lambda_y + \lambda_g)}{\beta\tilde{\sigma}(\lambda_y + \lambda_g)}\right| < 1 + \frac{(\kappa_y - \kappa_g)^2 + \lambda_y + \lambda_g}{\beta(\lambda_y + \lambda_g)}.$$

As $\beta\tilde{\sigma}(\lambda_y + \lambda_g) > 0$ this simplifies to

$$\left|(1-\varphi_\pi)(\kappa_y\lambda_g + \kappa_g\lambda_y) + \tilde{\sigma}(\kappa_y - \kappa_g)^2 + (1+\beta)\tilde{\sigma}(\lambda_y + \lambda_g)\right| < \tilde{\sigma}(\kappa_y - \kappa_g)^2 + (1+\beta)\tilde{\sigma}(\lambda_y + \lambda_g). \quad (66)$$

There are two cases, depending on whether the expression inside the absolute value brackets is positive or negative. It is positive, call this case (a) if

$$\varphi_\pi < 1 + \frac{\tilde{\sigma}(\kappa_y - \kappa_g)^2 + (1+\beta)\tilde{\sigma}(\lambda_y + \lambda_g)}{(\kappa_y\lambda_g + \kappa_g\lambda_y)}.$$

Then (66) requires $\varphi_\pi > 1$, so that in case (a) (65) is fulfilled for

$$1 < \varphi_\pi < 1 + \frac{\tilde{\sigma}(\kappa_y - \kappa_g)^2 + (1+\beta)\tilde{\sigma}(\lambda_y + \lambda_g)}{(\kappa_y\lambda_g + \kappa_g\lambda_y)}. \quad (67)$$

In case (b)

$$\varphi_\pi > 1 + \frac{\tilde{\sigma}(\kappa_y - \kappa_g)^2 + (1+\beta)\tilde{\sigma}(\lambda_y + \lambda_g)}{(\kappa_y\lambda_g + \kappa_g\lambda_y)}.$$

Then (66) becomes

$$-\left[(1-\varphi_\pi)(\kappa_y\lambda_g + \kappa_g\lambda_y) + \tilde{\sigma}(\kappa_y - \kappa_g)^2 + (1+\beta)\tilde{\sigma}(\lambda_y + \lambda_g)\right] < \tilde{\sigma}(\kappa_y - \kappa_g)^2 + (1+\beta)\tilde{\sigma}(\lambda_y + \lambda_g).$$

This holds when

$$\varphi_\pi < 1 + \frac{2\left[\tilde{\sigma}(\kappa_y - \kappa_g)^2 + (1+\beta)\tilde{\sigma}(\lambda_y + \lambda_g)\right]}{(\kappa_y\lambda_g + \kappa_g\lambda_y)}.$$

Hence, in case (b) (65) is fulfilled for

$$1 + \frac{\tilde{\sigma}(\kappa_y - \kappa_g)^2 + (1+\beta)\tilde{\sigma}(\lambda_y + \lambda_g)}{(\kappa_y\lambda_g + \kappa_g\lambda_y)} < \varphi_\pi < 1 + \frac{2\left[\tilde{\sigma}(\kappa_y - \kappa_g)^2 + (1+\beta)\tilde{\sigma}(\lambda_y + \lambda_g)\right]}{(\kappa_y\lambda_g + \kappa_g\lambda_y)}. \quad (68)$$

Combining (67) and (68), the necessary and sufficient condition for determinacy is (39). □

A.6 Determinacy for Current-Looking Taylor Rule

This section proves Proposition 6.

Proof. We can reduce the four-dimensional system comprising Phillips curve (18), (19), (37) and the Taylor-rule $\hat{\imath}_t = \varphi_\pi \pi_t$ to a two-dimensional system $E_t z_{t+1} = \Gamma_2 z_t + \Psi_2 v_t$, with the vector of endogenous variables $z_t \equiv \begin{bmatrix} \tilde{y}_t & \tilde{\mathfrak{g}}_t \end{bmatrix}'$, and the vector of exogenous disturbances $v_t \equiv \begin{bmatrix} \mu_t & r_t^n \end{bmatrix}'$ and the coefficient matrix

$$\Gamma_2 = [\beta\tilde{\sigma}(\lambda_y + \lambda_g + \eta_{\pi g}\varphi_\pi)]^{-1} \begin{bmatrix} \Gamma_2^{11} & \Gamma_2^{12} \\ \Gamma_2^{21} & \Gamma_2^{22} \end{bmatrix},$$

with

$$\begin{aligned}
\Gamma_2^{11} &= [\tilde{\sigma}(\kappa_y - \kappa_g) + \lambda_g + \eta_{\pi g}\varphi_\pi]\left(\kappa_y + \frac{\lambda_y}{\kappa_y - \kappa_g}\right) - \beta\left(\lambda_g + \eta_{\pi g}\varphi_\pi\right)\left(-\tilde{\sigma} + \frac{\lambda_y \varphi_\pi}{\kappa_y - \kappa_g}\right) \\
\Gamma_2^{12} &= \begin{pmatrix} [\tilde{\sigma}(\kappa_y - \kappa_g) + \lambda_g + \eta_{\pi g}\varphi_\pi]\left(-\kappa_g + \frac{\lambda_g}{\kappa_y - \kappa_g} + \frac{\eta_{\pi g}}{\kappa_y - \kappa_g}\varphi_\pi\right) \\ -\beta\left(\lambda_g + \eta_{\pi g}\varphi_\pi\right)\left(\tilde{\sigma} + \left[\frac{\lambda_g}{\kappa_y - \kappa_g} + \frac{\eta_{\pi g}\varphi_\pi}{\kappa_y - \kappa_g}\right]\varphi_\pi\right) \end{pmatrix} \\
\Gamma_2^{21} &= -[-\tilde{\sigma}(\kappa_y - \kappa_g) + \lambda_y]\left(\kappa_y + \frac{\lambda_y}{\kappa_y - \kappa_g}\right) + \beta\lambda_y\left(-\tilde{\sigma} + \frac{\lambda_y\varphi_\pi}{\kappa_y - \kappa_g}\right) \\
\Gamma_2^{22} &= -[-\tilde{\sigma}(\kappa_y - \kappa_g) + \lambda_y]\left(-\kappa_g + \frac{\lambda_g}{\kappa_y - \kappa_g} + \frac{\eta_{\pi g}}{\kappa_y - \kappa_g}\varphi_\pi\right) + \beta\lambda_y\left(\tilde{\sigma} + \left[\frac{\lambda_g}{\kappa_y - \kappa_g} + \frac{\eta_{\pi g}\varphi_\pi}{\kappa_y - \kappa_g}\right]\varphi_\pi\right).
\end{aligned}$$

As in the previous section we have two non-predetermined variables, both of which need to be outside the unit-circle for determinacy. This again requires (64) and (65) to be fulfilled. Γ_2 has determinant

$$\det(\Gamma_2) = \frac{\tilde{\sigma}(\lambda_y + \lambda_g + \eta_{\pi g}\varphi_\pi) + \tilde{\sigma}(\kappa_y - \kappa_g)^2 + \varphi_\pi\kappa_y(\lambda_g + \eta_{\pi g}\varphi_\pi) + \kappa_g\varphi_\pi\lambda_y}{\beta\tilde{\sigma}(\lambda_y + \lambda_g + \eta_{\pi g}\varphi_\pi)}$$

and trace

$$tr(\Gamma_2) = \frac{(1+\beta)\tilde{\sigma}(\lambda_y + \lambda_g + \eta_{\pi g}\varphi_\pi) + \tilde{\sigma}(\kappa_y - \kappa_g)^2 + \kappa_y(\lambda_g + \eta_{\pi g}\varphi_\pi) + \kappa_g\lambda_y}{\beta\tilde{\sigma}(\lambda_y + \lambda_g + \varphi_\pi\eta_{\pi g})}.$$

Due to $\beta < 1$ (64) is trivially fulfilled:

$$\det(\Gamma_2) = \left|\frac{1}{\beta} + \frac{\tilde{\sigma}(\kappa_y - \kappa_g)^2 + \varphi_\pi\kappa_y(\lambda_g + \eta_{\pi g}\varphi_\pi) + \kappa_g\varphi_\pi\lambda_y}{\beta\tilde{\sigma}(\lambda_y + \lambda_g + \eta_{\pi g}\varphi_\pi)}\right| > 1.$$

Condition 2, (65), takes the form

$$\begin{aligned}
&\left|\frac{(1+\beta)\tilde{\sigma}(\lambda_y + \lambda_g + \eta_{\pi g}\varphi_\pi) + \tilde{\sigma}(\kappa_y - \kappa_g)^2 + \kappa_y(\lambda_g + \eta_{\pi g}\varphi_\pi) + \kappa_g\lambda_y}{\beta\tilde{\sigma}(\lambda_y + \lambda_g + \varphi_\pi\eta_{\pi g})}\right| \\
&\qquad < 1 + \frac{\tilde{\sigma}(\lambda_y + \lambda_g + \eta_{\pi g}\varphi_\pi) + \tilde{\sigma}(\kappa_y - \kappa_g)^2 + \varphi_\pi\kappa_y(\lambda_g + \eta_{\pi g}\varphi_\pi) + \kappa_g\varphi_\pi\lambda_y}{\beta\tilde{\sigma}(\lambda_y + \lambda_g + \eta_{\pi g}\varphi_\pi)}.
\end{aligned}$$

Excluding the odd possibility of a negative coefficient in the Taylor-rule the preceding equation reduces to $(1 - \varphi_\pi)\left[\kappa_y(\lambda_g + \eta_{\pi g}\varphi_\pi) + \kappa_g\lambda_y\right] < 0$, which requires $\varphi_\pi > 1$. □

Bibliography

Benigno, P. and Woodford, M. (2003). Optimal monetary and fiscal policy: A linear-quadratic approach, *in* M. Gertler and K. Rogoff (eds), *NBER Macroeconmics Annual*, MIT Press, pp. 271–333.

Blanchard, O. and Kahn, C. (1980). The solution of linear difference models under rational expectations, *Econometrica* **48**(5): 1305–1311.

Brock, W. (1974). Money and growth: The case of long-run perfect foresight, *International Economic Review* **15**(3): 750–777.

Calvo, G. (1983). Staggered prices in a utility maximizing framework, *Journal of Monetary Economics* **12**(3): 383–398.

Carlstrom, C. and Fuerst, T. (2001). Timing and real indeterminacy in monetary models, *Journal of Monetary Economics* **47**(2): 285–298.

Christiano, L., Eichenbaum, M. and Evans, C. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy, *Journal of Political Economy* **113**(1): 1–45.

Clarida, R., Galí, J. and Gertler, M. (1999). The science of monetary policy: A New Keynesian perspective, *Journal of Economic Literature* **37**(4): 1661–1707.

Correia, I., Nicolini, J. and Teles, P. (2008). Optimal fiscal and monetary policy: Equivalence results, *Journal of Political Economy* **116**(1): 141–170.

Dixit, A. and Lambertini, L. (2003). Interactions of commitment and discretion in monetary and fiscal policies, *American Economic Review* **93**(5): 1522–1542.

Dixit, A. and Stiglitz, J. (1977). Monopolistic competition and optimum product diversity, *American Economic Review* **67**(3): 297–308.

Eser, F. (2006). Fiscal stabilisation in a New Keynesian model, *MPhil Thesis*, University of Oxford, available at http://fabian.eser.googlepages.com.

Eser, F., Leith, C. and Wren-Lewis, S. (2009). When is monetary policy all we need?, *Working Paper 430*, Department of Economics, University of Oxford.

Evans, G., Guse, E. and Honkapohja, S. (2008). Liquidity traps, learning and stagnation, *European Economic Review* **52**(8): 1438–1463.

Galí, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton University Press.

Galí, J. and Monacelli, T. (2008). Optimal monetary and fiscal policy in a currency union, *Journal of International Economics* **76**: 116–132.

Gnocchi, S. (2007). Discretionary fiscal policy and optimal monetary policy in a currency area, *mimeo*, Universitat Autònoma de Barcelona.

Hovath, M. (2008). Simple monetary-fiscal targeting rules, *Working Paper 08/01*, CDMA.

Jensen, H. (2002). Monetary policy frameworks and real equilibrium determinacy, *mimeo*, University of Copenhagen.

Kirsanova, T., Leith, C. and Wren-Lewis, S. (2009). Monetary and fiscal policy interaction: the curent consensus assignment in the light of recent developments, *Economic Journal* **119**(541): F482–F496.

Kirsanova, T., Satchi, M., Vines, D. and Wren-Lewis, S. (2007). Optimal fiscal policy rules in a monetary union, *Journal of Money, Credit, and Banking* **39**: 1759–1784.

LaSalle, J. (1986). *The Stability and Control of Discrete Processes*, Springer Verlag.

Lubik, T. and Marzo, M. (2007). An inventory of simple monetary rules in a New Keynesian macroeconomic model, *International Review of Economics and Finance* **16**(1): 15–36.

Orphanides, A. (2007). Taylor rules, *FEDS Working Paper 2007-18*, Federal Reserve Board.

Schmitt-Grohé, S. and Uribe, M. (2005). Optimal fiscal and monetary policy in a medium-scale macroeconomic model, *in* M. Gertler and K. Rogoff (eds), *NBER Macroeconomics Annual*, MIT Press, pp. 383–425.

Schmitt-Grohé, S. and Uribe, M. (2007). Optimal simple and implementable monetary and fiscal rules, *Journal of Monetary Economics* **54**: 1702–1725.

Sidrauski, M. (1967). Rational choice and patterns of growth in monetary economics, *American Economic Review* **57**(2): 534–544.

Smets, F. and Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the Euro Area, *Journal of European Economic Association* **1**(5): 1123–1175.

Smets, F. and Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach, *American Economic Review* **97**(3): 586–606.

Svensson, L. (1999). Inflation targeting as a monetary policy rule, *Journal of Monetary Economics* **43**: 607–654.

Svensson, L. (2003). What is wrong with Taylor rules? Using judgment in monetary policy through targeting rules, *Journal of Economic Literature* **XLI**: 426–477.

Taylor, J. (1993). Discretion versus policy rules in practice, *Carnegie-Rochester Conference Series on Public Policy* **39**: 195–214.

Tinbergen, J. (1952). *On the Theory of Economic Policy*, 2nd edn, North-Holland, Amsterdam.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, Princeton and Oxford.