Abstract

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Social Value of Public Information

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Keywords: Transparency, disclosures, coordination, overreaction to public information.

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1This paper was previously circulated under the title “The CNBC Effect: Welfare Effects of Public Information”.
“The history of speculative bubbles begins roughly with the advent of newspapers. One can assume that, although the record of these early newspapers is mostly lost, they regularly reported on the first bubble of any consequence, the Dutch tulipmania of the 1630s. Although the news media - newspapers, magazines, and broadcast media, along with their new outlets on the Internet - present themselves as detached observers of market events, they are themselves an integral part of these event. Significant market events generally occur only if there is similar thinking among large groups of people, and the news media are essential vehicles for the spread of ideas.” Shiller (2000).

1. Introduction

For a decision maker facing a choice under uncertainty, greater access to information permits actions that are better suited to the circumstances. Also, to the extent that one decision maker’s choice is made in isolation from others, more information is generally beneficial. This conclusion is unaffected by whether the incremental information is public (shared by everyone) or private (available only to the relevant individual).

How far does this conclusion extend to social contexts where decision makers are interested parties in the actions of others? Public information has attributes that make it a double-edged instrument. On the one hand, it conveys information on the underlying fundamentals, but it also serves as a focal point for the beliefs of the group as a whole. When prevailing conventional wisdom or consensus impinge on people’s decision making process, public information may serve to reinforce
their impact on individual decisions to the detriment of private information. The “sunspots” literature has explored this latter theme by emphasizing the ability of public signals to serve as a coordination device. Even when the signal is ‘extrinsic’ and has no direct bearing on the underlying fundamentals, its very public nature allows full play to self-fulfilling beliefs in determining economic outcomes. Costas Azariadis (1981) and David Cass and Karl Shell (1983) are early references. Michael Woodford (1990) and Peter Howitt and Preston McAfee (1992) bolster the case for sunspot equilibria by showing how they may arise in the context of individual learning, and how they arise from a variety of economic mechanisms.

However, while the extrinsic nature of sunspots allows a clean expression of the coordination role of public information, it fails to do justice to the fact that public information does, in general, convey information on the fundamentals, and that such information will be of value to decision makers. Indeed, for policy makers in a variety of contexts, it is the fundamentals information conveyed by public disclosures that receives all the emphasis. For instance, the proposals to revise the 1988 accord on bank capital adequacy place great emphasis on the disclosures by banks that allow market discipline to operate more effectively (Basel Committee on Banking Supervision (1999b)); it is no less than the third of three “pillars” of the proposed accord. More generally, the policy response to the recent turbulence in international financial markets has been to call for increased transparency through disclosures from governments and other official bodies, as

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2Howitt and McAfee note with irony that William Jevons (1884), who introduced sunspots to economics, very much believed them to be part of the fundamentals of an agricultural economy.
well as from the major market participants (see IMF (1998), Basel Committee (1999a)). Thus, assessing the social value of public information entails recognizing its dual role - of conveying fundamentals information as well as serving as a focal point for beliefs. Our task in this paper is to assess the social value of public information when allowing for this dual role.

Our investigation centers on a model that is reminiscent of Keynes’s beauty contest example, and which also shares key features with the ‘island economy’ model of Robert Lucas (1972, 1973) and Edmund Phelps (1970). A large population of agents have access to public and private information on the underlying fundamentals, and aim to take actions appropriate to the underlying state. But they also engage in a zero-sum race to second-guess the actions of other individuals in which a player’s prize depends on the distance between his own action and the actions of others. The smaller is the distance, the greater is the prize. This imparts a coordination motive to the decision makers as well as the desire to match the fundamentals. When there is perfect information concerning the underlying state, the unique equilibrium in the game between the agents also maximizes social welfare. However, when there is imperfect information, the welfare effects of increased public information are more equivocal. In particular,

- when the agents have no private information - so that the only source of information for the agents is the public information - then greater precision of the public information always increases social welfare.

- However, if the agents have access to some private information, it is not always the case that greater precision of public information is desirable.
Over some ranges, increased precision of public information is detrimental to welfare. Specifically, the greater the precision of the agents’ private information, the more likely it is that increased provision of public information lowers social welfare.

The detrimental effect of public information arises from the fact that the coordination motive entails placing too much weight on the public signal relative to weights that would be used by the social planner. The impact of public information is large, and so is the impact of any noise in the public signal that inevitably creep in. In short, although public information is extremely effective in influencing actions, the danger arises from the fact that it is too effective at doing so. Agents overreact to public information, and thereby magnify the damage done by any noise. Our objective is to show how such ‘overreaction’ need not be predicated on any wishful thinking or irrationality on the part of agents.

The dilemma posed by the potential for overreaction to public information is a familiar one to policy makers that command high visibility in the market. Central bank officials have learned to be wary of public utterances that may unduly influence financial markets, and have developed their own respective strategies for communicating with the market. In formulating their disclosure policies, central banks and government agencies face a number of interrelated issues concerning how much they should disclose, in what form, and how often. Frequent and timely dissemination would aid the decision making process by putting current information at the disposal of all economic agents, but this has to be set against the fact that provisional estimates are likely to be revised with the benefit of hindsight.
By their nature, economic statistics are imperfect measurements of sometimes imprecise concepts, and no government agency or central bank can guarantee flawless information. This raises legitimate concerns about the publication of preliminary or incomplete data, since the benefit of early release may be more than outweighed by the disproportionate impact of any error. This trade-off between timely but noisy information and slow but more accurate information is a familiar theme, as witnessed by the debate in Japan about whether preliminary GDP figures should be published. Australia moved from a monthly calendar in reporting its balance of trade figures to a quarterly calendar because it was felt that the noise in the monthly statistics were injecting too much volatility into the price signals from financial markets\(^3\). The flaws in the United Kingdom’s earnings data have been credited with provoking unjustifiably tight credit conditions in the U.K. in the spring and summer of 1998\(^4\). The challenge for central banks and other official bodies is to strike the right balance between providing timely and frequent information to the private sector so as to allow it to pursue its goals, but to recognize the inherent limitations in any disclosure and to guard against the potential damage done by noise.

Before turning to our analysis, it is important to place our contribution in the broader context of the literature on public information. As well as the sunspots literature already alluded to, there a several bodies of work that should be borne in mind. The literature on herding and information cascades focus on the inefficiencies both in the generation of new information when free-riding decision makers

\(^3\)We are grateful to Philip Lowe for this example.

fail to engage in socially valuable experimentation, and also in the *dissemination* of information when private information fails to find an expression through the actions of decision makers. Abhijit Banerjee (1992) and Sushil Bikhchandani, David Hirshleifer and Ivo Welch (1992) are early references. Henry Cao and David Hirshleifer (2000) develop a model that allows full play to both types of inefficiency. The insights from this literature are complementary to that gained from ours. In both cases, access to noisy public information results in socially valuable private information being lost. However, the mechanisms are very different.

Jack Hirshleifer’s (1971) paper is an instance of how public information may be damaging because it removes insurance possibilities. There is also a large literature in industrial organization and related strategic contexts where the smoothing effects of uncertainty affect players’ actions. When the unique equilibrium is inefficient, the smoothing effect of uncertainty may improve welfare. Siew Hong Teoh (1997) shows an instance of this in a game of voluntary contribution to public goods. Michael Raith (1996) reviews a literature on private and public information in oligopoly. Simon Messner and Xavier Vives (2000) examine the welfare properties of a rational expectations equilibrium in which the price serves as a public signal of the distribution of costs among producers, and show how this information may be detrimental to welfare.

The ‘global games’ literature has examined the impact of public information in binary action coordination games where agents have both private and public signals about some underlying state (see Stephen Morris and Hyun Song Shin (1999, 2000), Christina Metz (2000) and Christian Hellwig (2000)). Here, if private information is sufficiently accurate relative to public information, there is a
unique equilibrium in a setting where multiple equilibria would exist with common knowledge of fundamentals. The comparative statics of the precision of public information reveal complex effects that arises from the interplay between better fundamentals information and shifts in strategic uncertainty. One virtue of the simple model proposed in this paper is that equilibrium is unique irrespective of the parameters, so that we are able to examine the impact of public signals and obtain cleaner welfare implications.

The plan for the rest of the paper is as follows. We introduce our model in the next section, and solve for the unique equilibrium, highlighting along the way the distinctive channels through which public information operates. The core of the paper is section 3, which examines the welfare effects of shifts in the precision of public information. A number of extensions and variations of our model are discussed in section 4, although the details of these extensions are presented separately in an appendix. The purpose of these extensions is both to demonstrate the robustness of our main conclusions to changes in the modelling assumptions, but also to delve deeper into the underlying mechanisms for the theoretical results. We conclude by pursuing some of the policy issues on disclosures further.

2. Model

Our model is based on game that induces strategic behavior in the spirit of the “beauty contest” example mentioned in Keynes’s General Theory (1936). There is a continuum of agents, indexed by the unit interval $[0, 1]$. Agent $i$ chooses an action $a_i \in \mathbb{R}$, and we write $\mathbf{a}$ for the action profile over all agents. The payoff
function for agent $i$ is given by

$$u_i(a, \theta) \equiv -(1 - r) (a_i - \theta)^2 - r (L_i - \bar{L}) \quad (2.1)$$

where $r$ is a constant, with $0 < r < 1$ and

$$L_i \equiv \int_0^1 (a_j - a_i)^2 \, dj$$
$$\bar{L} \equiv \int_0^1 L_j \, dj$$

The loss function for individual $i$ has two components. The first component is a standard quadratic loss in the distance between the underlying state $\theta$ and his action $a_i$. The second component is the “beauty contest” term. The loss $L_i$ is increasing in the average distance between $i$’s action and the action profile of the whole population. There is an externality in which an individual tries to second-guess the decisions of other individuals in the economy. The parameter $r$ gives the weight on this second-guessing motive. The larger is $r$, the more severe is the externality. Moreover, this spillover effect is socially inefficient in that it is of a zero-sum nature. In the game of second-guessing, the winners gain at the expense of the losers. Social welfare, defined as the (normalized) average of individual utilities is

$$W(a, \theta) \equiv \frac{1}{1 - r} \int_0^1 u_i(a, \theta) \, di$$

$$= - \int_0^1 (a_i - \theta)^2 \, di.$$
his action is determined by the first order condition:

\[ a_i = (1 - r) E_i (\theta) + r E_i (\pi) \]  \hspace{1cm} (2.2)

where \( \bar{a} \) is the average action in the population (i.e., \( \bar{a} = \int_0^1 a_j \, dj \)) and \( E_i (\cdot) \) is the expectation operator for player \( i \). Thus each agent puts positive weight on the expected state and the expected actions of others. Note, however, that if \( \theta \) is common knowledge, the equilibrium entails \( a_i = \theta \) for all \( i \), so that social welfare is maximized at equilibrium. So, when there is perfect information, there is no conflict between individually rational actions and the socially optimal actions. We now examine the case where \( \theta \) is not known with certainty.

2.1. Public Information Benchmark

Consider the case where agents face uncertainty concerning \( \theta \), but they have access to public information. The state \( \theta \) is drawn from an (improper) uniform prior over the real line, but the agents observe a *public signal*

\[ y = \theta + \eta \]  \hspace{1cm} (2.3)

where \( \eta \) is normally distributed, independent of \( \theta \), with mean zero and variance \( \sigma^2 \). The signal \( y \) is ‘public’ in the sense that the actual realization of \( y \) is common knowledge to all agents. They choose their actions after observing the realization of \( y \). The expected payoff of agent \( i \) at the time of decision is then given by the conditional expectation:

\[ E (u_i | y) \]  \hspace{1cm} (2.4)

where \( E (\cdot | y) \) is the common expectation operator. Conditional on \( y \), both agents believe that \( \theta \) is distributed normally with mean \( y \) and variance \( \sigma^2 \). Hence, the
best reply of $i$ is

$$a_i(y) = (1 - r) E(\theta|y) + r \int_0^1 E(a_j|y) \, dj \quad (2.5)$$

where $a_i(y)$ denotes the action taken by agent $i$ as a function of $y$. Since $E(\theta|y) = y$ and since the strategies of both agents are measurable with respect to $y$, we have $E(a_j|y) = a_j(y)$, so that in the unique equilibrium,

$$a_i(y) = y \quad (2.6)$$

for all $i$; expected welfare, conditional on $\theta$, is

$$E(W|\theta) = -E[(y-\theta)^2|\theta]$$

$$= -\sigma_\eta^2$$

Thus, the smaller the noise in the public signal, the higher is social welfare. We will now contrast this with the general case in which agents have private information as well as public information.

2.2. Private and Public Information

Consider now the case where, in addition to the public signal $y$, agent $i$ observes the realization of a private signal:

$$x_i = \theta + \varepsilon_i \quad (2.7)$$

where noise terms $\varepsilon_i$ of the continuum population are normally distributed with zero mean and variance $\sigma_\varepsilon^2$, independent of $\theta$ and $\eta$, so that $E(\varepsilon_i\varepsilon_j) = 0$ for $i \neq j$. 
The private signal of one agent is not observable by the others. This is the sense in which these signals are private.

As before, the agents’ decisions are made after observing the respective realizations of their private signals as well as the realization of the public signal. Denote by

\[ a_i (I_i) \]  

(2.8)

the decision by agent \( i \) as a function of his information set \( I_i \). The information set \( I_i \) consists of the pair \((y, x_i)\) that captures all the information available to \( i \) at the time of decision.\(^5\)

Let us denote by \( \alpha \) the precision of the public information, and denote by \( \beta \) the precision of the private information, where

\[
\begin{aligned}
\alpha &= \frac{1}{\sigma_\eta^2} \\
\beta &= \frac{1}{\sigma_\varepsilon^2}
\end{aligned}
\]  

(2.9)

Then, based on both private and public information, agent \( i \)’s expected value of \( \theta \) is:

\[ E_i (\theta) = \frac{\alpha y + \beta x_i}{\alpha + \beta} \]  

(2.10)

where we have used the shorthand \( E_i (\cdot) \) to denote the conditional expectation \( E (\cdot | I_i) \).

\(^5\)The notation in (2.8) makes explicit that the strategy of agent \( i \) in the imperfect information game is a function that maps the information \( I_i \) to the action \( a_i \). For any given strategy, \( a_i \) is therefore a random variable that is measurable on the partition generated by \( I_i \).
2.3. Linear Equilibrium

We will now solve for the unique equilibrium. We do this in two steps. We first solve for a linear equilibrium in which actions are a linear function of signals. We will follow this with a demonstration that this linear equilibrium is the unique equilibrium. Thus, as the first step, suppose that the population as a whole is following a linear strategy of the form

$$a_j(I_j) = \kappa x_j + (1 - \kappa) y.$$ (2.11)

Then agent $i$’s conditional estimate of the average expected action across all agents is:

$$E_i(\bar{\alpha}) = \kappa \left( \frac{\alpha y + \beta x_i}{\alpha + \beta} \right) + (1 - \kappa) y$$

$$= \left( \frac{\kappa \beta}{\alpha + \beta} \right) x_i + \left( 1 - \frac{\kappa \beta}{\alpha + \beta} \right) y$$

Thus agent $i$’s optimal action is

$$a_i(I_i) = (1 - r) E_i(\theta) + r E_i(\bar{\alpha})$$

$$= (1 - r) \left( \frac{\alpha y + \beta x_i}{\alpha + \beta} \right) + r \left( \left( \frac{\kappa \beta}{\alpha + \beta} \right) x_i + \left( 1 - \frac{\kappa \beta}{\alpha + \beta} \right) y \right)$$

$$= \left( \beta \frac{r \kappa + 1 - r}{\alpha + \beta} \right) x_i + \left( 1 - \beta \frac{r \kappa + 1 - r}{\alpha + \beta} \right) y$$

Comparing coefficients in (2.11) and (2.12), we therefore have

$$\kappa = \frac{\beta (r \kappa + 1 - r)}{\alpha + \beta}$$

from which we can solve for $\kappa$. 

$$\kappa = \frac{\beta (1 - r)}{\beta (1 - r) + \alpha}.$$
Thus, the equilibrium action $a_i$ is given by

$$a_i(I_i) = \frac{\alpha y + \beta (1 - r) x_i}{\alpha + \beta (1 - r)}$$  \hfill (2.13)

2.4. Uniqueness of Equilibrium

The argument presented above establishes the existence of a linear equilibrium. We will follow this by showing (through a “brute force” solution method) that the linear equilibrium we have identified is the unique equilibrium. In doing so, we establish the role of higher order expectations in this model. In particular, we note that if someone observes a public signal that is worse than her private signal, then her expectation of others’ expectations of $\theta$ is lower than her expectation of $\theta$, i.e., it is closer to the public signal than her own expectation. This in turn implies that if we look at $n$th order expectations about $\theta$, i.e., someone’s expectation of others’ expectations of others’ expectations of $(n$ times) of $\theta$, then this approaches the public signal as $n$ becomes large. Higher order expectations depend only on public signals.

Recall that player $i$’s best response is to set

$$a_i = (1 - r) E_i(\theta) + r E_i(\bar{a})$$

Substituting and writing $\overline{E}(\theta)$ for the average expectation of $\theta$ across agents we have

$$a_i = (1 - r) E_i(\theta) + (1 - r) r E_i(\overline{E}(\theta)) + (1 - r) r^2 E_i(\overline{E}^2(\theta)) + ....$$

$$= (1 - r) \sum_{k=0}^{\infty} r^k E_i(\overline{E}^k(\theta))$$  \hfill (2.14)
In order to evaluate this expression, and check that the infinite sum is bounded, we must solve explicitly for $E_{i}\left(\bar{E}^{k}(\theta)\right)$. Recall that player $i$'s expected value of $\theta$ is:

$$E_{i}(\theta) = \frac{\alpha y + \beta x_{i}}{\alpha + \beta}$$  \hspace{1cm} (2.15)

Thus the average expectation of $\theta$ across agents is

$$\bar{E}(\theta) = \int_{0}^{1} E_{i}(\theta) \, di = \frac{\alpha y + \beta \theta}{\alpha + \beta}$$

Now player $i$'s expectation of the average expectation of $\theta$ across agents is

$$E_{i}(\bar{E}(\theta)) = E_{i}\left(\frac{\alpha y + \beta \theta}{\alpha + \beta}\right) = \frac{\alpha y + \beta \left(\frac{\alpha y + \beta x_{i}}{\alpha + \beta}\right)}{\alpha + \beta} = \frac{((\alpha + \beta)^{2} - \beta^{2}) y + \beta^{2} x_{i}}{(\alpha + \beta)^{2}}$$

and the average expectation of the average expectation of $\theta$ is

$$\bar{E}^{2}(\theta) = \bar{E}(\bar{E}(\theta)) = \frac{((\alpha + \beta)^{2} - \beta^{2}) y + \beta^{2} \theta}{(\alpha + \beta)^{2}}$$

More generally, we have the following lemma.

**Lemma 2.1.** For any $k$, $\bar{E}^{k}(\theta) = (1 - \mu^{k}) y + \mu^{k} \theta$ and $E_{i}\left(\bar{E}^{k}(\theta)\right) = (1 - \mu^{k+1}) y + \mu^{k+1} x_{i}$ where $\mu = \beta/(\alpha + \beta)$.

The proof is by induction on $k$. We know from (2.15) that the lemma holds for $k = 1$. Suppose that it holds for $k - 1$. Then,

$$E_{i}\left(\bar{E}^{k-1}(\theta)\right) = (1 - \mu^{k}) y + \mu^{k} x_{i};$$
so

\[ \bar{E}^k(\theta) = (1 - \mu^k) y + \mu^k \theta \]

and

\[
E_i \left( \bar{E}^k(\theta) \right) = (1 - \mu^k) y + \mu^k \left( \frac{\alpha y + \beta x_i}{\alpha + \beta} \right) \\
= (1 - \mu^{k+1}) y + \mu^{k+1} x_i
\]

which proves lemma 2.1. Now substituting the expression from lemma 2.1 into equation (2.14), we obtain

\[
a_i = (1 - r) \sum_{k=0}^{\infty} r^k \left[ (1 - \mu^{k+1}) y + \mu^{k+1} x_i \right] \\
= \left( 1 - \mu \frac{(1 - r)}{1 - r \mu} \right) y + \left( \frac{\mu (1 - r)}{1 - r \mu} \right) x_i \\
= \frac{\alpha y + \beta (1 - r) x_i}{\alpha + \beta (1 - r)}
\]

This is exactly the unique linear equilibrium we identified earlier.

2.5. Lucas-Phelps Island Economy Model

We conclude this section by drawing out the parallels between the equilibrium in our model and features of the celebrated Lucas-Phelps ‘island economy’ model\(^6\).

To do this, let each index \( i \) refer an island whose supply \( y_i^s \) of the single consumption good is given by

\[
y_i^s = b \left( a_i - E_i(\bar{a}) \right)
\]

\(^6\)We are indebted to Tom Sargent and to the editor Preston McAfee for pointing out this connection.
where $a_i$ is the price on island $i$, $\bar{a}$ is the average price across all islands, and $b > 0$ is a supply parameter. The demand $y_i^d$ on island $i$ is given by

$$y_i^d = c (E_i (\theta) - a_i)$$

where $\theta$ is the money supply and $c > 0$ is the slope parameter for demand. Market clearing then implies

$$a_i = (1 - r) E_i (\theta) + r E_i (\bar{a})$$

where $r = b/ (b + c)$, so that we have the equation (2.2) that characterizes equilibrium in the beauty contest model. A question that arises in this context is how the profile of prices $\{a_i\}$ across the economy are affected by the shifts in information on money supply. Does greater information precision on the money supply $\theta$ mean that the prices $\{a_i\}$ are tied closer to the fundamentals $\theta$? Phelps (1983) posed this question in the context of an economy in which the central bank is determined to combat the inflation expectations of the private sector agents, and noted that the answer depends on subtle ways on the interaction of beliefs between agents. Our analysis below may be regarded as a formal solution of the original problem posed by Phelps in his 1983 paper.

A disanalogy between our model and the island economy is that there is no clear formal counterpart to the social welfare function in the latter. For this reason, we prefer to conduct our main analysis within the terms of reference of the beauty contest. Nevertheless, even without a formal welfare criterion, the distance between the set of prices $\{a_i\}$ across islands and the underlying fundamentals given by $\theta$ (the money supply) will be of some interest. When this
distance is written as
\[ \int_0^1 (a_i - \theta)^2 \, di \]
then the results that follow in the next section on the welfare effects of public
information have a direct bearing on the question of what effect greater public
information on the money supply has on the tightness of the relationship between
prices and money supply.

3. Welfare Effect of Public Information

We are now ready to address the main question of the paper. How is welfare
affected by the precisions of the agents’ signals? In particular, will welfare be
increasing in the precision \( \alpha \) of the public signal? From the solution for \( a_i \), we
can solve for the equilibrium strategies in terms of the basic random variables \( \theta \),
\( \eta \) and \( \{ \varepsilon_i \} \).

\[ a_i = \theta + \frac{\alpha \eta + \beta (1 - r) \varepsilon_i}{\alpha + \beta (1 - r)} \]  

(3.1)
If \( r = 0 \), the two types of noise (private and public) would be given weights that
are commensurate with their precision. That is, \( \eta \) would be given weight equal to
its relative precision \( \alpha / (\alpha + \beta) \) while \( \varepsilon_i \) would be given weight equal to its relative
precision \( \beta / (\alpha + \beta) \). However, the weights in (3.1) deviate from this. The noise
in the public signal is given relatively more weight, and the noise in the private
signal is given relatively less weight. This feature reflects the coordination motive
of the agents, and reflects the disproportionate influence of the public signal in
influencing the agents’ actions. The magnitude of this effect is greater when \( r \) is
large. What effect does this have on welfare? Expected welfare at $\theta$ is given by

$$E[W(a, \theta)|\theta] = \frac{-\alpha^2 E(\eta^2) + \beta^2 (1 - r)^2 [E(\varepsilon_i^2)]}{(\alpha + \beta (1 - r))^2}$$

$$= \frac{-\alpha + \beta (1 - r)^2}{(\alpha + \beta (1 - r))^2}$$ (3.2)

By examining (3.2), we can answer the comparative statics questions concerning the effect of increased precision of private and public information.

Welfare is always increasing in the precision of the private signals. We can see this by differentiating (3.2) with respect to $\beta$, the precision of the private signals. We have:

$$\frac{\partial E(W|\theta)}{\partial \beta} = \frac{(1 - r) ((1 + r) \alpha + (1 - r)^2 \beta)}{(\alpha + \beta (1 - r))^3} > 0$$ (3.3)

Thus, increased precision of private information enhances welfare unambiguously.

The same cannot be said of the effect of increased precision of the public signal. The derivative of (3.2) with respect to $\alpha$ is:

$$\frac{\partial E(W|\theta)}{\partial \alpha} = \frac{\alpha - (2r - 1)(1 - r) \beta}{(\alpha + \beta (1 - r))^3}$$ (3.4)

so that

$$\frac{\partial E(W|\theta)}{\partial \alpha} \geq 0 \text{ if and only if } \frac{\beta}{\alpha} \leq \frac{1}{(2r - 1)(1 - r)}$$ (3.5)

When $r > 0.5$, there are ranges of the parameters where increased precision of public information is detrimental to welfare. Increased precision of public information is beneficial only when the private information of the agents is not very precise. If the agents have access to very precise information (so that $\beta$ is high), then any increase in the precision of the public information will be harmful. Thus,
as a rule of thumb, when the private sector agents are already very well informed, the official sector would be well advised not to make public any more information, unless they could be confident that they can provide public information of very great precision. If a social planner were choosing ex ante the optimal precision of public information and increasing the precision of public information is costly, then corner solutions at $\alpha = 0$ may be common.

Even if greater precision of public information can be obtained relatively cheaply, there may be technical constraints in achieving precision beyond some upper bound. For instance, the social planner may be restricted to choosing $\alpha$ from some given interval $[0, \bar{\alpha}]$. In this case, even if the choice of $\alpha$ entails no costs, we will see a “bang-bang” solution to the choice of optimal $\alpha$ in which the social optimum entails either providing no public information at all (i.e. setting $\alpha = 0$), or providing the maximum feasible amount of public information (i.e. setting $\alpha = \bar{\alpha}$). The better informed is the private sector, the higher is the hurdle rate of precision of public information that would make it welfare enhancing.

Figure 3.1 illustrates the social welfare contours in $(\alpha, \beta)$-space. The curves are the set of points that satisfy $E(W|\theta) = C$, for constants $C$. As can be seen from figure 3.1, when $\beta > \alpha/[(2r - 1)(1 - r)]$, the social welfare contours are upward sloping, indicating that welfare is decreasing in the precision of public information.

What is the intuition for this result? Observe that equation (2.13) can be re-written as

$$a_i = \frac{\alpha y + \beta (1 - r) x_i}{\alpha + \beta (1 - r)}$$
This equation shows well the added impact of public information in determining the actions of the agents. In addition to its role in forming the conditional expectation of $\theta$, there is an additional (positive) term involving the public signal $y$, while there is a corresponding negative term involving the private signal $x_i$. Thus, the agents “overreact” to the public signal while suppressing the information content of the private signal. The impact of the noise $\eta$ in the public signal is given more of an impact in the agents’ decisions than it deserves.

Perhaps a more illuminating intuition is obtained by considering the role of higher order expectations in our model. This intuition also brings out well the unease expressed by Phelps (1983) (justified, it turns out) concerning the overly
simplistic treatment of iterated expectations. The key to our result is the fact that the ‘average expectations’ operator $\bar{E}(\cdot)$ violates the ‘law’ of iterated expectations. In our model,

$$\bar{E}(\theta) \neq \bar{E}(\bar{E}(\theta)) \quad \text{and} \quad E_i(\bar{E}(\theta)) \neq E_i(\theta)$$

These properties are key, since if we had equality between $\bar{E}(\theta)$ and $\bar{E}(\bar{E}(\theta))$ and between $E_i(\bar{E}(\theta))$ and $\bar{E}(\theta)$ then all higher order level expectations collapse to the first order, so that (2.14) would become

$$E_i(\bar{E}(\theta)) (1 - r) \sum_{k=0}^{\infty} r^k = E_i(\bar{E}(\theta)) = E_i(\theta)$$

which coincides with the socially efficient action. Thus, it is this failure of the law of iterated expectations for the expectations operators that injects genuine strategic uncertainty into the problem, and which entails the overreaction to public information. The importance of shared knowledge in the promulgation of policy was emphasized by Phelps in his 1983 paper, and our results could be seen as giving this assertion formal backing. Arguably, the role of shared knowledge goes far beyond economics. Michael Chwe (2001) argues for the importance of shared knowledge in a wide variety of social settings. For example, he documents the high per unit cost of reaching a viewer when the audience is large, and shows that goods that have a prominent ‘social’ dimension are more likely to receive the benefit of such high cost advertising.

Having established the possibility that public information may be detrimental, we now address a number of extensions and variations of our model. The purpose
is both to gauge the robustness of our conclusions, and also to delve deeper into the results.

4. Extensions and Variations

The linear-normal solution of our model is an attractively simple illustration of our main ideas, but the general conclusions are robust to alternative specifications. In the appendix to this paper, which is intended solely for reference on the AER website, we illustrate several extensions and variations. The first example is for a model where signals have two realizations, in which we show overreaction to public information relative to the welfare benchmark. Indeed, the key result that increasingly higher order expectations of a random variable converges to the expectation with respect to public information only is a robust feature of differential information economies (see Samet (1998)). Thus, the neither the normality, nor the improper prior is essential for our results.

A more immediate question is how our results vary with alternative specification of the payoffs. In our model, the overreaction to public information arises from the positive spillover effects of individual actions. What if actions were strategic substitutes, rather than strategic complements? The solution for the unique equilibrium can be obtained from the same methods used above. Suppose that the best reply function for $i$ is given by

$$a_i = E_i(\theta) - \rho E_i(\bar{a})$$

for some constant $\rho > 0$. Then the unique linear equilibrium can be obtained
from the method of comparing coefficients to yield

\[
a_i = \frac{1}{1 + \rho} \left( \frac{\beta (1 + \rho)}{\alpha + \beta (1 + \rho)} x_i + \frac{\alpha}{\alpha + \beta (1 + \rho)} y \right)
\] (4.1)

The symmetric information benchmark solution is when \( a_i = a_j \) for all \( i, j \) which gives \( a_i = E_i(\bar{a}) \), so that

\[
a_i = \frac{1}{1 + \rho} \left( \frac{\beta x_i + \alpha y}{\beta + \alpha} \right)
\] (4.2)

Comparing (4.1) and (4.2), the introduction of strategic substitutability implies that agents now overreact to private information \( x_i \) relative to the symmetric information benchmark. Players accentuate their differences in order to avoid playing similar actions to other players.

An early paper by Roy Radner (1962) gives a nice instance of this\(^7\). He examines the problem where two members of a team aim to minimize the loss function

\[
(a_1 - \theta)^2 + (a_2 - \theta)^2 + 2q (a_1 - \theta) (a_2 - \theta)
\]

In other words, the loss is increasing in the product of the two errors. This gives rise to strategic substitutability between the two team members, so that the optimal decisions put less weight on the public information and more weight on the private information as compared to the individual decision. The choice of output in a Cournot model examined by Robert Townsend (1978) also falls into this category of strategic substitutes.

As well as alternative payoff functions for the individual players, we could also consider alternative forms of the welfare function. For instance, if we pursue our

\(^7\)We are grateful to Takashi Ui for this reference. Ui (2001) shows that Radner's model as well as our own model here can be analysed as Bayesian potential games.
macroeconomic interpretation of the model as the interaction between a central bank and the private sector agents, one natural way to formulate the principal’s objective function is in terms of the deviation of the aggregate level of activity from the true state $\theta$. Consider a finite player version of our framework where the principal’s objective is to minimize

$$\left( \frac{1}{n} \sum_j a_j - \theta \right)^2$$

so that the objective for the principal is to set the average action as close as possible to $\theta$. Suppose that all agents follow a linear strategy and set their action according to

$$a_i = \kappa x_i + (1 - \kappa) y$$

where $y$ is the public signal, and $x_i$ is $i$’s private signal. Then the expected loss for the principal at $\theta$ is

$$E \left( \left( \frac{\kappa}{n} \sum_{j=1}^n x_j + (1 - \kappa) y - \theta \right)^2 \bigg| \theta \right)$$

$$= E \left( \left( \frac{\kappa}{n} \sum_{j=1}^n \epsilon_j + (1 - \kappa) \eta \right)^2 \bigg| \theta \right)$$

$$= \frac{\kappa^2}{n\beta} + \frac{(1 - \kappa)^2}{\alpha}$$

The value of $\kappa$ that minimizes the principal’s loss is

$$\kappa = \frac{n\beta}{\alpha + n\beta}$$

Note that when $n$ is large, the principal would like the agents to put small weight on the public signal, and base their decision largely on the private signal. Whereas
the noise terms $\{\varepsilon_i\}$ in the private signals of the agents tend to cancel each other out, the noise term $\eta$ in the public signal remains in place irrespective of the number of agents. Thus, if the welfare function places weight on some aggregate activity variable, the overweighting of the public signal by the agents would cause an even greater social welfare loss. This example is clearly rather simplistic in the way that it exploits the i.i.d. nature of noise terms. More realistically, we might expect that private signals have shared raw ingredients across the population that impart complex correlation structures across private signals. As a simple example, private signals that have the structure $x_i = \theta + \xi + \varepsilon_i$, where $\xi$ is a common noise term that enters into all players’ private signals will impart correlations into the private signals, even if we condition on the true state $\theta$. In the appendix we explore two issues in some detail.

- We explore alternative specifications of the welfare function and determine conditions that give rise to the result that greater public information is welfare decreasing.

- We present a general analysis of the two player version of our model where the players can observe many signals, where the signals are multivariate normal with a general correlation structure. In this context, we show that correlated noise terms give rise to qualitatively similar effects as in the benchmark model.
5. Concluding Remarks and Discussion

Public information has attributes that make it a double-edged instrument for public policy. Whilst it is very effective at influencing the actions of agents whose actions are strategic complements, the trouble is that it is too effective in doing so. Agents overreact to public information, and hence any unwarranted public news or mistaken disclosure may cause great damage.

Commentators such as Paul Krugman (2001) have raised the possibility that the parameter $r$ in our model - indicating the strength of the strategic motive - may have become larger in recent years. Commenting on the recent downturn in economic activity in the United States, he suggests that

“firms making investment decisions are starting to emulate the hair-trigger behavior of financial investors. That means a growing part of the economy may be starting to act like a financial market, with all that implies - like the potential for bubbles and panics. One could argue that far from making the economy more stable, the rapid responses of today’s corporations make their investment in equipment and software vulnerable to the kind of self-fulfilling pessimism that used to be possible only for investment in paper assets.”

In terms of the framework of our paper, the increased vulnerability mentioned by Krugman is an entirely rational response by individual actors, but is socially inefficient.

The challenge for central banks and other public organizations is to strike the right balance between providing timely and frequent information to the private
sector so as to allow it to pursue its goals, but to recognize the inherent limitations in any disclosure and to guard against the potential damage done by noise. This is a difficult balancing act at the best of times, but this task is likely to become even harder. As central banks’ activities impinge more and more on the actions of market participants, the latter have reciprocated by stepping up their surveillance of central banks’ activities and pronouncements. The intense spotlight trained on the fledgling European Central Bank and the ECB’s delicate relationship with the press and private sector market participants illustrate well the difficulties faced by policy makers.

In the highly sensitized world of today’s financial markets populated with Fed watchers, economic analysts, and other commentators of the economic scene, disclosure policy assumes great importance. Our results suggest that private sources of information may actually crowd out the public information by rendering the public information detrimental to the policy maker’s goals. The heightened sensitivities of the market could magnify any noise in the public information to such a large extent that public information ends up by causing more harm than good. If the information provider anticipates this effect, then the consequence of the heightened sensitivities of the market is to push it into reducing the precision of the public signal. In effect, private and public information end up being substitutes, rather than being cumulative.

References


