Does One Soros Make a Difference?
A Theory of Currency Crises with Large and Small Traders*

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Abstract

Do large investors increase the vulnerability of a country to speculative attacks in the foreign exchange markets? To address this issue, we build a model of currency crises where a single large investor and a continuum of small investors independently decide whether to attack a currency based on their private information about fundamentals. Even abstracting from signalling, the presence of the large investor does make all other traders more aggressive in their selling. Relative to the case in which there is no large investor, small investors attack the currency when fundamentals are stronger. Yet, the difference can be small, or non-existent, depending on the relative precision of private information of the small and large investors. Adding signalling makes the influence of the large trader on small traders’ behaviour much stronger.

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1. Introduction

A commonly encountered view among both seasoned market commentators and less experienced observers of the financial markets is that large traders can exercise a disproportionate influence on the likelihood and severity of a financial crisis by fermenting and orchestrating attacks against weakened currency pegs. The famously acrimonious exchange between the financier George Soros and Dr. Mahathir, the prime minister of Malaysia at the height of the Asian crisis is a prominent example in which such views have been aired and debated. The issues raised by this debate are complex, but they deserve systematic investigation.

At one level, the task is one of dissecting the problem in search of the possible mechanisms (if any) that may be at play in which a large trader may exercise such influence on the market outcome. What is it about the large trader that bestows such influence? Is it merely that this trader can bring to bear larger resources and hence take on larger trading positions? What if the information of the large trader is no better than the small traders in the market? Does the large trader still exercise a disproportionate influence? Finally, does it make a difference to the market outcome as to whether the trading position of the large trader is disclosed publicly to the market? If so, does such “transparency” of the trading position enhance financial stability or undermine it? This last question is especially important given the emphasis placed by policy makers on the public disclosures by the major market participants as a way of forestalling future crises.1

The available evidence from empirical studies on this issue is quite controversial. Circumstantial evidence of aggressive trading practices by large traders is presented by the Study Group on Market Dynamics in Small and Medium Sized Economics, set up by the Financial Stability Forum as part of its investigation into the role of hedge funds and other highly leveraged institutions (HLIs) in the market turbulence of 1997/8 (Financial Stability Forum (2000)). This group could not reach consensus on the question of whether the manipulative trading practices of large traders played a pivotal role in the Asian crisis of 1997. Yet, the study’s conclusions stress the possible role of large traders in such episodes. These conclusions differ from those of an earlier study by the IMF (1998), whose analysis was however limited to the events in Asia up to late 1997. This earlier study had expressed a skeptical view on a preminent role of hedge funds in market turbulence — arguing that they were at the rear of the herd of investors rather than in the lead. As regards econometric evidence, Wei and

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1The response of the regulators and official bodies to the financial turbulence of 1998 has been to call for greater public disclosures by banks and hedge funds. The recent document from the Financial Stability Forum (2000) reiterates similar calls by the BIS, IOSCO, and the President’s Working Group. In contrast, the private sector is more ambivalent towards the value of public disclosures. See, for instance, Counterparty Risk Management Policy Group (1999).
Kim (1997) report that, while the trading positions of large participants Granger-cause exchange rate volatility, there is no consistent evidence that they are successful in their trades. This result is confirmed by Corsetti, Pesenti and Roubini (2001), who repeat and extend the analysis using a larger sample including additional crisis episodes. The latter study also provides an extensive survey of the literature.

Against this background, we propose to investigate the role of large traders in a theoretical model of speculative attacks in which a large trader interacts with a continuum of small traders. The large trader is ‘large’ by virtue of the size of the speculative position that he can take on as compared to the small traders. The two types of traders face a monetary authority defending a currency peg, and stand to gain if their attack on the peg is successful, but stand to lose if the attack fails to break the peg. Both types of traders are well informed about the underlying fundamentals, but they are not perfectly informed. Moreover, we allow the possibility that the information precision of one type of trader is higher than another. We can examine the case in which the large trader is better informed than the small trader and contrast this with the case in which small trader is relatively better informed.

To anticipate our main conclusions to these questions, we can summarize our findings as follows.

- As a general rule, the presence of the large trader does increase the incidence of attack against a peg. The reason is not so much that the large trader’s market power manufactures these crises, but rather that the presence of the large trader makes the small traders more aggressive in their trading strategies. In other words, the large trader injects a degree of strategic fragility to the market.

- However, within this broad general finding, it turns out to be important to distinguish the effect of the size of the large trader (the “size effect”) from the degree to which the large trader is better or worse informed relative to the small traders (the “information effect”). For the limiting case when both types of traders have precise information, we have the following conclusions.

  - The information effect is always positive in the sense that when the information of the large trader improves relative to that of the small traders, the incidence of attack increases.

  - In contrast, the size effect need not be positive, locally. That is, when the size of the insider increases marginally, the incidence of attack may decrease. However, globally, the size effect is always positive in the sense that the incidence of attack with a large trader (of whatever size) is always higher than in the absence of a large trader.
Finally, the influence of the large trader is magnified greatly when the large trader’s trading position is revealed to the small traders prior to their trading decisions. Thus, when the large trader moves first, and his position is disclosed publicly to other traders before their trading decisions, the impact of the large trader is that much larger. The reason for this added impact lies in the signalling potential of the large trader’s first move. To the extent that a speculative attack is the resolution of a coordination problem among the traders, the enhanced opportunity to orchestrate a coordinated attack helps to resolve this collective action problem.

The technical and modelling innovations necessary to examine the role of a large trader in a currency attack model deserve some attention by itself. The theoretical framework employed in this paper is an extension of the incomplete information game formulation used in Morris and Shin (1998). In this earlier setting, the argument makes heavy use of the fact that the game is symmetric - that is, all the speculators are identical. This assumption is clearly not available to us in the current setting. It is not at all obvious that the argument used in Morris and Shin (1998) to prove uniqueness of equilibrium is applicable in asymmetric payoffs settings, and one of the contributions of our current paper is to demonstrate that this argument can be used with some modifications.

There is a more subtle, but important theoretical contribution. The incomplete information game approach of Morris and Shin (1998) is an instance of a more general approach to equilibrium selection pioneered by Carlsson and van Damme (1993), in which the type space underlying the game is generated by adding a small amount of noise in the signals of the players concerning some payoff relevant state. Carlsson and van Damme refer to such games as “global games”, and the general class of such games turn out to have a rich and interesting structure. Morris and Shin (2000) discuss some general results and applications. Analysis using global games should be seen as a particular instance of equilibrium selection through perturbations, but it is important to disentangle two distinct sets of results concerning global games. The first question is whether a unique outcome is selected in the game. A second, more subtle, question is whether such a unique outcome depends on the underlying information structure and the structure of the noise in the players’ signals. One of the remarkable results for symmetric binary action global games is that the answer to the second question is ‘no’. In other words, not only is a unique equilibrium selected in the limit as the noise becomes small, but the selected equilibrium is insensitive to the structure of the noise (see Morris and Shin (2000) section 2). However, our second bullet point above points to the fact that, in our model, the structure of the noise does make a difference. The equilibrium outcome depends on whether the large trader is relatively better or worse informed as compared to the small traders. Thus, in our asymmetric global game, although we have a unique equilibrium being selected, this unique equilibrium
depends on the noise structure. It is this latter feature that allows us to draw non-trivial conclusions concerning the economic importance of information. Frankel, Morris and Pauzner (1999) explore the equilibrium selection question in the context of general global games.

Our examination of the sequential move version of the game necessitates a further extension the current state of the art. When moves occur sequentially in which the actions of the early movers are observable to the late movers, herding and signalling effects must be taken into consideration, as well as the usual strategic complementarities. Although a general analysis of sequential move variations of global games is rather intractable, the fact that small traders (individually) are of measure zero in our model allows us to focus attention on the signalling effects of the large trader. This simplifies the analysis sufficiently for us to derive explicit closed form solutions to the game.2

Our paper relates to several strands in the large and growing literature on financial crises in general, and currency crises in particular. While we will not have the space to provide a thorough account of this literature, it can be useful to sketch a short bibliographical note, primarily focused on currency. Early models stressing self-fulfilling features of currency crises are already provided by Flood and Garber (1984) and Obstfeld (1986), but at the time most of the literature followed Krugman (1979), implicitly downplaying the role coordination problems. Grilli (1986) identifies sufficient conditions to rule out multiple equilibria in first-generation models after Krugman (1979). Later on, however, examples of multiplicity in the classical set up are provided by Cavallari and Corsetti (2000) and Flood and Marion (2001). The coordination problem among investors is instead at the heart of most second- and third generation models of currency crises — see for instance Obstfeld (1994) and Chang and Velasco (1999), Furman and Stiglitz (1998), and Krugman (1999c) among many others. Chari and Kehoe (1999) model currency attacks in terms of herding, while Aghion Bacchetta and Banerjee (2001) addresses the issue of monetary policy during financial crises using an agency perspective to the problem.3

The paper is organized as follows. Section 2 lays out the basic framework and establishes two benchmark results in setting the stage for the general analysis. Section 3 characterizes the unique equilibrium in a simultaneous move trading game. Section 4 explores the comparative statics properties of the equilibrium to changes in the traders’ information precision. The focus here is on the interaction between the size of the

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2Dasgupta (1999) has examined some of the issues that arise with many large players and multi-period signalling.

large trader and his information precision. Section 5 investigates the sequential move version of the game. Section 6 concludes.

2. The model

The focus of our analysis is on the mechanism by which a fixed exchange parity is abandoned as a result of a speculative attack on the currency. Consider an economy where the central bank pegs the exchange rate. There is a single “large” trader and a continuum of “small” traders. The distinguishing feature of the large trader is that he has access to a sufficiently large line of credit in the domestic currency to take a short position up to the limit of $\lambda < 1$. In contrast, the set of all small traders taken together have a combined trading limit of $1 - \lambda$.

We envisage the short selling as consisting of borrowing the domestic currency and selling it for dollars. There is a cost to engaging in the short selling, denoted by $t > 0$. The cost $t$ can be viewed largely as consisting of the interest rate differential between the domestic currency and dollars, plus transaction costs. This cost is normalized relative to the other payoffs in the game, so that the payoff to a successful attack on the currency is given by 1, and the payoff from refraining from attack is given by 0. Thus, the net payoff to a successful attack on the currency is $1 - t$, while the payoff to an unsuccessful attack is given by $-t$.

Each trader must decide independently, and (for now) simultaneously whether or not to attack the currency. The strength of the economic fundamentals of this economy are indexed by the random variable $\theta$, which has the (improper) uniform prior over the real line.\footnote{Improper priors allow us to concentrate on the updated beliefs of the traders conditional on their signals without taking into account the information contained in the prior distribution. In any case, our results with the improper prior can be seen as the limiting case as the information in the prior density goes to zero. See Hartigan (1983) for a discussion of improper priors, and Morris and Shin (2000, section 2) for a discussion of the latter point.}

Whether the current exchange rate parity is viable depends on the strength of the economic fundamentals and the incidence of speculative attack against the peg. The incidence of speculative attack is measured by the mass of traders attacking the currency in the foreign exchange market. Denoting by $\ell$ the mass of traders attacking the currency, the currency peg fails if and only if

$$\ell \geq \theta$$

(2.1)

So, when fundamentals are sufficiently strong (i.e. $\theta > 1$) the currency peg is maintained irrespective of the actions of the speculators. When $\theta \leq 0$, the peg is abandoned even in the absence of a speculative attack. The interesting range is the intermediate
case when \( 0 < \theta \leq 1 \). Here, an attack on the currency will bring down the currency provided that the incidence of attack is large enough, but not otherwise. This tripartite classification of fundamentals follows Obstfeld (1996) and Morris and Shin (1998). Although we do not model explicitly the decision of the monetary authorities to relinquish the peg, it may be helpful to keep in mind the example of an economy endowed with a stock of international reserves, where the central bank is willing to defend the exchange rate as long as reserves do not fall below a predetermined critical level. The central bank predetermines this level based on its assessment of the economic fundamentals of the country. The critical level is low when fundamentals are strong (\( \theta \) is high): the central bank is willing to use a large amount of (non-borrowed and borrowed) reserves in defending the exchange rate. Conversely, the critical level is high when fundamentals are weak (\( \theta \) is low). Even a mild speculative attack can convince the central bank to abandon the peg.

### 2.1. Information

Although the traders do not observe the realization of \( \theta \), they receive informative private signals about it. The large trader observes the realization of the random variable

\[
y = \theta + \tau \eta
\]

where \( \tau > 0 \) is a constant and \( \eta \) is a random variable with mean zero, and with smooth symmetric density \( g(\cdot) \). We write \( G(\cdot) \) for the cumulative distribution function for \( g(\cdot) \). Similarly, a typical small trader \( i \) observes

\[
x_i = \theta + \sigma \varepsilon_i
\]

where \( \sigma > 0 \) is a constant and the individual specific noise \( \varepsilon_i \) is distributed according to smooth symmetric density \( f(\cdot) \) (write \( F(\cdot) \) for the c.d.f.) with mean zero. We assume that \( \varepsilon_i \) is i.i.d. across traders, and each is independent of \( \eta \).

A feature already familiar from the discussion of global games in the literature is that even if \( \sigma \) and \( \tau \) become very small, the realization of \( \theta \) will not be common knowledge among the traders. Upon receiving his signal, the representative trader \( i \) can guess the value of \( \theta \), and the distribution of signals reaching the other traders in the economy, as well as of their estimate of \( \theta \). He cannot, however, count on the other traders to know what he knows— and agree with his guesses. The other traders will have to rely exclusively on their own information to form their beliefs. This departure from the assumption of common knowledge of the fundamentals, no matter how small, is key to the results to follow. The relative magnitude of the constants \( \sigma \) and \( \tau \) indexes the relative precision of the information of the two types of traders.

A trader’s strategy is a rule of action which maps each realization of his signal to one of two actions - to attack, or to refrain. We will search for Bayes Nash equilibria
of the game in which, conditional on each trader’s signal, the action prescribed by this trader’s strategy maximizes his conditional expected payoff when all other traders follow their strategies in the equilibrium.

2.2. Two benchmark Cases

Before proceeding to our main task of solving the game outlined above, we present a brief discussion of the coordination problem under two special cases to set a benchmark for our main results. The first is when all traders are small ($\lambda = 0$), the second is when the sole trader is the large trader himself ($\lambda = 1$).

2.2.1. Small traders only

The case when $\lambda = 0$ takes us into the symmetric game case of Morris and Shin (1998). We will conduct the discussion in terms of switching strategies in which traders attack the currency if the signal falls below a critical value $x^*$. We will show later that this is without loss of generality, and that there are no other equilibria in possibly more complex strategies. The unique equilibrium can be characterized by a critical value $\theta^*$ below which the currency will always collapse, and a critical value of the individual signal $x^*$ such that individuals receiving a signal below this value will always attack. To derive these critical values, note first that, if the true state is $\theta$ and traders attack only if they observed a signal below $x^*$, the probability that any particular trader receives a signal below this level is

$$\text{prob}(x_i \leq x^* \mid \theta) = F\left(\frac{x^* - \theta}{\sigma}\right)$$

Since the noise terms $\{\varepsilon_i\}$ are i.i.d., the incidence of attack $\ell$ is equal to this probability. We know that an attack will be successful only if $\ell \geq \theta$. The critical state $\theta^*$ is where this holds with equality. Thus, the first equilibrium condition — a “critical mass condition” — is

$$F\left(\frac{x^* - \theta^*}{\sigma}\right) = \theta^*.$$ (2.5)

Figure 2.1 depicts the incidence of attack as the downward sloping curve $F\left(\frac{x^* - \theta}{\sigma}\right)$. Given $x^*$, any realization of the fundamental $\theta \leq \theta^*$ is associated with a successful speculative attack on the currency.

Second, consider the optimal trigger strategy for an trader receiving a signal $x_i$, given $\theta^*$. The trader has the conditional probability of a successful attack of

$$\text{prob}(\theta \leq \theta^* \mid x_i) = F\left(\frac{\theta^* - x_i}{\sigma}\right).$$ (2.6)
and hence attacks if and only if his expected gross payoff is at least as high as the cost of attack \( t \). As the expected payoff to attacking for a marginal trader receiving a signal \( x^* \) must be 0, the “optimal cutoff” condition for \( x^* \) is

\[
F \left( \frac{x^* - \theta}{\sigma} \right) = t.
\]  

(2.7)

This point \( x^* \) is illustrated in figure 2.1. Solving for the equilibrium entails solving the pair of equations above. Equation (2.7) gives \( \theta^* = x^* + \sigma F^{-1} (t) \); substituting into (2.5) gives \( \theta^* = F (-F^{-1} (t)) = 1 - F (F^{-1} (t)) = 1 - t \). We obtain the following proposition

**Proposition 2.1.** If \( \lambda = 0 \),

\[
\begin{align*}
    x^* &= 1 - t - \sigma F^{-1} (t) \\
    \theta^* &= 1 - t
\end{align*}
\]

The currency will collapse for any realization of the fundamental \( \theta \) smaller than \( 1 - t \), while each individual trader will attack the currency for any realization of his signal below \( 1 - t - \sigma F^{-1} (t) \).\(^5\) Note that this trigger tends to \( 1 - t \) as \( \sigma \to 0 \).

**2.2.2. A single large trader**

We now consider the opposite extreme case of \( \lambda = 1 \), in which there is a single large trader. This reduces the game to a single person decision problem, and implies a trivial

\(^5\) For \( t < 1/2 \), \( F^{-1} (t) \) is a negative number, so that \( x^* > \theta^* \). As \( \sigma \to 0 \), i.e. letting the private signal become arbitrarily precise, the optimal cutoff point will tend to the fundamental threshold, \( x^* \to \theta^* \).
solution to the coordination problem described above. As this single trader controls
the market, there is no need of an equilibrium condition equivalent to the “critical mass
condition” (2.5). The only condition that is relevant for a single large risk-neutral trader
is the “optimal cutoff”: he will attack the currency if and only the expected payoff from
a speculative position is non-negative, that is when

\[ G\left( \frac{1 - y}{\tau} \right) \geq t \]

Thus he attacks if and only if \( y \leq y^* = 1 - \tau G^{-1}(t) \). Note that the trigger \( y \) is smaller
than one, but tends to 1 as \( \tau \to 0 \).

3. Equilibrium with Small and Large Traders

We can now turn to the general case when there are both small and large traders. We
will show that there is a unique, dominance solvable equilibrium in this case in which
both types of traders follow their respective trigger strategies around the critical points
\( x^* \) and \( y^* \). The argument will be presented in two steps. We will first confine our
attention to solving for an equilibrium in trigger strategies, and then proceed to show
that this solution can be obtained by the iterated deletion of strictly interim dominated
strategies.

Thus, as the first step let us suppose that the small traders follow the trigger strategy
around \( x^* \). Because there is a continuum of small traders, conditional on \( \theta \), there is
no aggregate uncertainty about the proportion of small traders attacking the currency.
Since \( F\left( \frac{x^* - \theta}{\sigma} \right) \) is the proportion of small traders observing a signal lower than \( x^* \) and
therefore attacking at \( \theta \), an attack by small traders alone is sufficient to break the peg
at \( \theta \) if \( (1 - \lambda) F\left( \frac{x^* - \theta}{\sigma} \right) \geq \theta \). From this, we can define a level of fundamentals below
which an attack by the small traders alone is sufficient to break the peg. Let \( \underline{\theta} \) be
defined by:

\[ (1 - \lambda) F\left( \frac{x^* - \theta}{\sigma} \right) = \underline{\theta} \]  (3.1)

Whenever \( \theta \) is below \( \underline{\theta} \), the attack is successful irrespective of the action of the large
trader. Figure 3.1 depicts the derivation of this critical level. Note that \( \underline{\theta} \) lies between
0 and \( 1 - \lambda \). Clearly \( \underline{\theta} \) is a function of \( x^* \).

Next, we can consider the additional speculative pressure brought by the large
trader. If the small traders follow the trigger strategy around \( x^* \), the incidence of
attack at \( \theta \) attributable to the small traders is \( (1 - \lambda) F\left( \frac{x^* - \theta}{\sigma} \right) \). If the large trader
also chooses to attack, then there is an additional \( \lambda \) to this incidence (see figure 3.1).
Hence, if the large trader participates in the attack, the peg is broken whenever \( \lambda + \)
(1 - \lambda) F\left(\frac{x^* - \theta}{\sigma}\right) \geq \theta. Thus we can define the critical value of the fundamentals at which an attack is successful if and only if the large trader participates in the attack. It is defined by

$$\lambda + (1 - \lambda) F\left(\frac{x^* - \bar{\theta}}{\sigma}\right) = \bar{\theta}$$

(3.2)

As is evident from figure 3.1, \bar{\theta} lies between \underline{\theta} and 1.

Although our notation does not make it explicit, both \underline{\theta} and \bar{\theta} are functions of the switching point \(x^*\). In turn, \(x^*\) will depend on the large trader’s switching point \(y^*\). Our task is to solve these two switching points simultaneously from the respective optimization problems of the traders. A large trader observing signal \(y\) assigns probability \(G\left(\frac{\bar{\theta} - y}{\tau}\right)\) to the event that \(\theta \leq \bar{\theta}\). Since his expected payoff to attacking conditional on \(y\) is \(G\left(\frac{\bar{\theta} - y}{\tau}\right) - t\), his optimal strategy is to attack if and only if \(y \leq y^*\), where \(y^*\) is defined by:

$$G\left(\frac{\bar{\theta} - y^*}{\tau}\right) = t$$

(3.3)

Now consider a small trader. Conditional on signal \(x\), the posterior density over \(\theta\) for this trader is given by

$$\frac{1}{\sigma} f\left(\frac{\theta - x}{\sigma}\right)$$

(3.4)

When \(\theta \leq \underline{\theta}\), the strategies of the small traders are sufficient for a successful attack. When \(\theta \in (\underline{\theta}, \bar{\theta})\) the peg breaks if and only if the large trader attacks, while if \(\theta > \bar{\theta}\), the

Figure 3.1: Incidence of attack at \(\theta\) with both types of traders
peg withstands the attacks, irrespective of the actions of the traders. Thus, the expected payoff to attack conditional on signal $x$ can be written as

$$\frac{1}{\sigma} \int_{-\infty}^{\bar{\theta}} f \left( \frac{\theta - x}{\sigma} \right) d\theta + \frac{1}{\sigma} \int_{\bar{\theta}}^{\theta} f \left( \frac{\theta - x}{\sigma} \right) G \left( \frac{y^* - \theta}{\tau} \right) d\theta$$  \hspace{1cm} (3.5)

The first term is the portion of expected payoff attributable to the region of $\theta$ where $\theta \leq \theta$. The second term is the portion of expected payoff that is attributable to the interval $[\theta, \bar{\theta}]$. Here, one must take into account the fact that the attack is successful if and only if the large trader attacks. The probability that the large trader attacks at $\theta$ given his trigger strategy around $y^*$ is given by $G \left( \frac{y^* - \theta}{\tau} \right)$, so that the payoffs are weighted by this value. Beyond $\bar{\theta}$, the attack is never successful, so that the payoff to attack is zero. Since the cost of attack is $t$, the trigger point $x^*$ for the small trader is defined by the equation:

$$\frac{1}{\sigma} \int_{-\infty}^{\theta} f \left( \frac{\theta - x^*}{\sigma} \right) d\theta + \frac{1}{\sigma} \int_{\theta}^{\bar{\theta}} f \left( \frac{\theta - x^*}{\sigma} \right) G \left( \frac{y^* - \theta}{\tau} \right) d\theta = t$$  \hspace{1cm} (3.6)

There is a unique $x^*$ that solves this equation. To see this, it is helpful to introduce a change of variables in the integrals. Let

$$z \equiv \frac{\theta - x^*}{\sigma}$$  \hspace{1cm} (3.7)

and denote

$$\xi \equiv \frac{\theta - x^*}{\sigma} \quad \text{and} \quad \xi \equiv \frac{\bar{\theta} - x^*}{\sigma}.$$  \hspace{1cm} (3.8)

Then, the conditional expected payoff to attacking given signal $x^*$ is
\[
\frac{1}{\sigma} \int_{-\infty}^{\theta} f \left( \frac{\theta - x^*}{\sigma} \right) d\theta + \frac{1}{\sigma} \int_{\theta}^{\bar{\theta}} f \left( \frac{\theta - x^*}{\sigma} \right) G \left( \frac{y^* - \theta}{\tau} \right) d\theta
\]
\[
= \int_{-\infty}^{\delta} f (z) dz + \int_{\delta}^{\bar{\delta}} f (z) G \left( \frac{\theta - x^* - \sigma z}{\tau} - G^{-1} (t) \right) dz
\]
\[
= \int_{-\infty}^{\delta} f (z) dz + \int_{\bar{\delta}}^{\bar{\theta}} f (z) G \left( \frac{\sigma}{\tau} (\bar{\delta} - z) - G^{-1} (t) \right) dz
\]  

(3.9)

where the third line follows from the fact that

\[
y^* = \bar{\theta} - \tau \sigma^{-1} (t)
\]
\[
= x^* + \sigma \sigma - \tau \sigma^{-1} (t).
\]  

(3.10)

Hence, (3.6) gives:

\[
\int_{-\infty}^{\delta} f (z) dz + \int_{\bar{\delta}}^{\bar{\theta}} f (z) G \left( \frac{\sigma}{\tau} (\bar{\delta} - z) - G^{-1} (t) \right) dz - t = 0
\]  

(3.11)

However, note that both \( \delta \) and \( \bar{\delta} \) are monotonically decreasing in \( x^* \), since

\[
\frac{d \delta}{dx^*} = - \frac{1}{(1 - \lambda) f (\delta) + \sigma} < 0
\]
\[
\frac{d \bar{\delta}}{dx^*} = - \frac{1}{(1 - \lambda) f (\bar{\delta}) + \sigma} < 0
\]

Since the left hand side of (3.11) is strictly increasing in both \( \delta \) and \( \bar{\delta} \), it is strictly decreasing in \( x^* \). For sufficiently small \( x^* \), the left hand side of (3.11) is positive, while for sufficiently large \( x^* \), it is negative. Since the left hand side is continuous in \( x^* \), there is a unique solution to (3.11). Once \( x^* \) is determined, the large trader’s switching point \( y^* \) follows from (3.3).
To this point, we have confined our attention to trigger strategies, and have shown that there is a unique equilibrium within this class of strategies. We can show that confining our attention to trigger strategies is without loss of generality. The trigger equilibrium identified above turns out to be the only set of strategies that survive the iterated elimination of strictly interim dominated strategies. The dominance solvability property is by now well understood for symmetric binary action global games (see Morris and Shin (2000) for sufficient conditions for this property). The contribution here is to show that it also applies in our asymmetric global game. The argument is presented separately in appendix A. We summarize our findings in terms of the following proposition.

**Proposition 3.1.** There is a unique, dominance solvable equilibrium in the game in which the large trader uses the switching strategy around $y^*$ while the small traders uses the switching strategy around $x^*$.

The dominance solvability of our game can be seen as an instance of the conditions identified in the literature on supermodular games that turn out to be sufficient for dominance solvability. See Milgrom and Roberts (1990), Vives (1990) and Morris and Shin (1999, Appendix).

**4. Impact of Large Trader**

Having established the uniqueness of equilibrium, we can now address the main question of whether there is any increased fragility of the peg, and how much of this can be attributed to the large trader. There are two natural questions. Do the small traders become more aggressive sellers when the large trader is in the market? Secondly, does the probability of the peg’s collapse increase when the large trader is in the market? These questions relate to the following comparative statics questions.

- How does the switching point $x^*$ for the small traders depend on the presence of the large trader?
- How is the incidence of attack by the small traders at a given state $\theta$ affected by the large trader’s presence?
- How is the probability of the peg’s collapse at a given state $\theta$ affected by the presence of the large trader?
- How is the ex ante probability of the peg’s collapse affected by the large trader?\(^6\)

\(^6\)We are grateful to a referee for encouraging us to address this question.
This final bullet point may seem incongruous when taken at face value, since our model has made use of the assumption that \( \theta \) has an (improper) uniform prior distribution, so that the ex ante expectations are not well defined. However, there is an interpretation of our model that allows us to comment on this issue for general priors over \( \theta \). When the signals received by the traders (small and large) are very precise relative to the information contained in the prior, then a uniform prior over \( \theta \) serves as a good approximation in generating the conditional beliefs of the traders. Then, the equilibrium obtained under the uniform prior assumption will be a good approximation to the true equilibrium. If we can say something about the critical state \( \theta \) at which the peg collapses, then we may give an approximate answer for the ex ante probability of collapse by evaluating the prior distribution \( H(\cdot) \) at this state. We will comment below on one instance when this type of argument can be made.

A more substantial theme in our comparative statics exercise is to disentangle the effects of the size of the large trader (through the size of the trading position that he can amass) from his precision of information relative to small traders. If we can interpret the large trader as a coalition of small traders who pool their resources as well as their information, then it would be natural to assume that the large trader has very much better information as compared to individual small traders\(^7\). There may be other reasons to do with resources that a large player can bring to bear on research or access to contacts in policy circles that makes it more reasonable to assume that the large trader is better informed than the typical small trader. However, there is no reason in principle why the large trader must be better informed. In any case, the separation of the effects of size from that of information is a valuable exercise in understanding the impact of each, and so we will be careful in distinguishing these two effects.

For the purpose of the comparative statics exercise, it may help the reader to gather together and restate the key relationships that determine equilibrium. Using the notation \( \delta = \frac{\theta - \sigma}{\sigma} \), and \( \tilde{\delta} = \frac{\theta - \tilde{\sigma}}{\tilde{\sigma}} \) that we introduced earlier, we restate equations (3.1),

\(^7\)We are grateful to a referee for pointing this out to us.
(3.2), (3.3) and (3.11) as follows.

\[ (1 - \lambda) (1 - F(\delta)) = \theta \]  (4.1)

\[ \lambda + (1 - \lambda) (1 - F(\bar{\delta})) = \bar{\theta} \]  (4.2)

\[ G\left(\frac{\bar{\theta} - y^*}{\tau}\right) = t \]  (4.3)

\[ \int_{-\infty}^{\delta} f(z) \, dz + \int_{\delta}^{\bar{\delta}} f(z) G\left(\frac{\sigma}{\tau}(\delta - z) - G^{-1}(t)\right) \, dz = t \]  (4.4)

These four equations jointly determine the switching points \( x^* \) and \( y^* \) and the critical states \( \theta \) and \( \bar{\theta} \). Obtaining definitive answers to the comparative statics questions can sometimes be difficult for general parameter values, although we will examine a number of simulation exercises below that suggest that the equilibrium behaves in intuitive ways. In contrast to the difficulties for general parameter values, the limiting case where both types of traders have very precise information gives us quite tractable expressions that yield relatively clear cut results. Sometimes, even closed form solutions of the equilibrium are possible.

### 4.1. Comparative Statics in the Limiting Case

Let us examine the properties of the equilibrium in the limiting case where

\[ \sigma \to 0, \quad \tau \to 0, \quad \text{and} \quad \frac{\sigma}{\tau} \to r \]

In other words, both types of traders have precise information, but the noisiness of the small traders’ signals relative to the large trader’s signal tends to \( r \). The case of \( r = 1 \) is an interesting benchmark case, and we will report the result of a simulation exercise later in which we track the properties of the equilibrium as the noise becomes small in the signals of both types of traders. We allow \( r \) to take the value of infinity also, by which we mean that the large trader’s signal is arbitrarily more precise than the small traders’ signals.

One reason for the tractability of the limiting case is that we can identify \( \bar{\theta} \) as the critical state at which the peg fails. That is, the peg fails if and only if \( \theta < \bar{\theta} \). To see this, it is useful to refer to Figure 3.1. The top curve is the incidence of attack when the large trader participates in the attack, while the bottom curve is the incidence of attack without the large trader. However, we can see from equation (4.3) that as \( \tau \to 0 \), we must have \( y^* \to \bar{\theta} \), or else the left hand side of (4.3) will either be zero or one, rather
than being equal to \( t \). Hence, in the limit, the large trader always attacks at states to the left of \( \bar{\theta} \), but refrains from attack at states to the right of \( \bar{\theta} \). In terms of figure 3.1, the total incidence of attack will follow the top curve until \( \bar{\theta} \), but then jump down to the bottom curve thereafter. We can see that the incidence of attack is large enough to break the peg to the left of \( \bar{\theta} \), but is too small to break the peg to the right of \( \bar{\theta} \). Of course, when the small traders also have very precise information, their switching strategies must be such that they attack precisely when the true state is to the left of \( \bar{\theta} \). Thus, in the limit, we must have

\[
x^* = y^* = \bar{\theta}
\]

(4.5)

and the peg fails if and only if \( \theta < \bar{\theta} \). This means that the comparative statics questions raised above in the first three bullet points collapse to a single question of whether the true state \( \theta \) is to the left or right of the critical state \( \bar{\theta} \). Also, the fourth bullet point concerning the ex ante probability of the peg’s failure can be given an approximate answer. Thus, following the earlier discussion, if \( H(\cdot) \) is the prior distribution function for \( \theta \), then the ex ante probability of the peg’s failure is given approximately by \( H(\bar{\theta}) \). Thus, comparative statics on the prior probability of collapse can be reduced to the behaviour of \( \bar{\theta} \). In this sense, the comparative statics questions all hinge on the behaviour of the critical state \( \bar{\theta} \).

In solving for the critical state \( \bar{\theta} \) in the limiting case, it is important to distinguish two cases. Again, it is useful to refer to figure 3.1 to visualize these two cases. As \( \sigma \) becomes small, both curves become steeper, and converge to the respective step function around \( \bar{\theta} \). However, we can distinguish the case when \( \bar{\theta} \leq 1 - \lambda \) from the case where \( \bar{\theta} > 1 - \lambda \). In the former case, both step functions intersect the 45 degree line at \( \bar{\theta} \), so that \( \bar{\theta} = \bar{\theta} \). However, when \( \bar{\theta} > 1 - \lambda \), the lower step function intersects the 45 degree line at its horizontal portion, so that \( \bar{\theta} < \bar{\theta} \). In general, we can characterize the equilibrium value of \( \bar{\theta} \) in the limit as follows.

**Proposition 4.1.** In the limit as \( \sigma \rightarrow 0, \tau \rightarrow 0 \) and \( \frac{\xi}{\tau} \rightarrow r \), the critical state \( \bar{\theta} \) tends to \( \lambda + (1 - \lambda) \left(1 - F(\delta)\right) \), where \( \delta \) falls under two cases. If \( \bar{\theta} > 1 - \lambda \), then \( \delta \) is the unique solution to

\[
\int_{-\infty}^{\delta} f(z) G \left(r \left( \delta - z \right) - G^{-1}(t) \right) dz = t
\]

(4.6)

If \( \bar{\theta} \leq 1 - \lambda \), then \( \delta \) is the unique solution to

\[
\int_{-\infty}^{L} f(z) dz + \int_{L}^{\delta} f(z) G \left(r \left( \delta - z \right) - G^{-1}(t) \right) dz = t
\]

(4.7)

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where

\[ L = F^{-1} \left( F \left( \tilde{\delta} \right) - \frac{\lambda}{1 - \lambda} \right). \]

The proof of this result is given in appendix B. We will also report a numerical example for \( r = 1 \) and where both \( F \) and \( G \) are given by the standard normal. We know from (4.2) that \( \tilde{\theta} = \lambda + (1 - \lambda) \left( 1 - F \left( \tilde{\delta} \right) \right) \), and the main task in the proof is to show that equation (4.4) takes the two cases above when taking into account whether \( \tilde{\theta} \) is smaller or larger than \( 1 - \lambda \). However, this result allows us to give a definitive answer to the question of how the critical state \( \tilde{\theta} \) depends on the relative precision of information between the small and large traders. In both equations (4.6) and (4.7), we see that the left hand side is strictly increasing in both \( \tilde{\delta} \), and the main task in the proof is to show that equation (4.4) takes the two cases above when taking into account whether \( \tilde{\theta} \) is smaller or larger than \( 1 - \lambda \). However, this result allows us to give a definitive answer to the question of how the critical state \( \tilde{\theta} \) depends on the relative precision of information between the small and large traders. In both equations (4.6) and (4.7), we see that the left hand side is strictly increasing in both \( \tilde{\delta} \). Hence, as \( \tilde{\theta} \) increases, \( \tilde{\delta} \) must decrease. Since \( \tilde{\theta} \) tends to \( \lambda + (1 - \lambda) \left( 1 - F \left( \tilde{\delta} \right) \right) \), we have:

**Proposition 4.2.** In the limit as \( \sigma \to 0, \tau \to 0 \) and \( \frac{\xi}{\tau} \to r \), the critical state \( \tilde{\theta} \) is strictly increasing in \( r \).

In other words, when the small traders’ information deteriorates relative to the large trader, the critical state moves up, increasing the incidence of attack and raising the probability of the failure of the peg.

Interestingly, it is not always possible to give a definitive answer to the question of whether \( \tilde{\theta} \) is increasing in \( \lambda \) - the size of the large trader. When \( \lambda \) is small, so that \( \tilde{\theta} \leq 1 - \lambda \), we are in the range covered by equation (4.7). The left hand side of this equation is decreasing in \( \lambda \) through its effect on \( L \) so that \( \tilde{\delta} \) is increasing in \( \lambda \). Since \( \tilde{\theta} = \lambda + (1 - \lambda) \left( 1 - F \left( \tilde{\delta} \right) \right) \), the overall effect of \( \lambda \) is given by

\[
\frac{d\tilde{\theta}}{d\lambda} = F \left( \tilde{\delta} \right) - (1 - \lambda) f \left( \tilde{\delta} \right) \frac{d\tilde{\delta}}{d\lambda}
\]

whose sign cannot be tied down definitively. It is only when \( \lambda \) is large (so that \( \tilde{\theta} > 1 - \lambda \)) that we have an unambiguous increase in \( \tilde{\theta} \) as \( \lambda \) increases. This is so, since the left hand side of (4.6) does not depend on \( \lambda \), so that \( \frac{d\tilde{\theta}}{d\lambda} = F \left( \tilde{\delta} \right) > 0 \).

**Proposition 4.3.** In the limit as \( \sigma \to 0, \tau \to 0 \) and \( \frac{\xi}{\tau} \to r \), the critical state \( \tilde{\theta} \) is strictly increasing in \( \lambda \) provided that \( \lambda > 1 - \tilde{\theta} \).

Thus, when we separate the “size effect” of the large trader from the “information effect”, we have the following conclusion. Whereas the incidence of attack on the currency is unambiguously increasing in the information precision of the large trader, the local effect of an increase in the size of the large trader may be negligible or even negative when the large trader is small. However, even though the size effect is ambiguous
locally, we have an argument that shows that it is always positive globally. That is, the critical state $\bar{\theta}$ when $\lambda > 0$ cannot be smaller than the critical state when $\lambda = 0$.

We do this by solving for the critical state in two special cases. The first is when $r \to \infty$ (when the large trader is arbitrarily better informed), and the second is when $r = 0$ (when it is the small traders who are arbitrarily better informed). We know from proposition 4.2 that the solution of $\bar{\theta}$ is monotonic in $r$. Thus, if we can show that the closed form solution to $r = 0$ is non-decreasing in $\lambda$, we will have shown that the size effect is non-negative in a global sense. Both closed form solutions can be obtained as a corollary to proposition 4.1.

First, consider the case where $r \to \infty$. Then, both equations (4.6) and (4.7) become

$$\bar{\delta} \int_{-\infty}^{\infty} f(z) dz = t$$

so that $F(\bar{\delta}) = t$. Since $\bar{\theta} = \lambda + (1 - \lambda) \left(1 - F(\bar{\delta})\right)$, we have

$$\bar{\theta} = \lambda + (1 - \lambda) \left(1 - t\right)$$

Next, consider the case where $r = 0$. Here, we need to keep track of the two cases in proposition 4.1. Equation (4.6) becomes

$$(1 - t) F(\bar{\delta}) = t$$

while (4.7) reduces to

$$F(\bar{\delta}) - t \frac{\lambda}{1 - \lambda} = t$$

so that

$$F(\bar{\delta}) = \begin{cases} \frac{t}{1 - \lambda} & \text{if } \bar{\theta} > 1 - \lambda \\ \frac{1 - t}{1 - \lambda} & \text{if } \bar{\theta} \leq 1 - \lambda \end{cases}$$

Since $\bar{\theta} = \lambda + (1 - \lambda) \left(1 - F(\bar{\delta})\right)$, we have $\bar{\theta} > 1 - \lambda$ if and only if $\lambda \geq \frac{F(\bar{\delta})}{1 + F(\bar{\delta})}$, and

$$F(\bar{\delta}) = \begin{cases} \frac{t}{1 - \lambda} & \text{if } \lambda > t \\ \frac{1 - t}{1 - \lambda} & \text{if } \lambda \leq t \end{cases}$$

Thus, we can obtain the expression for the critical state $\bar{\theta}$ as follows.

$$\bar{\theta} = \begin{cases} \lambda + (1 - \lambda) \left(1 - \frac{t}{1 - \lambda}\right) & \text{if } \lambda > t \\ \frac{1 - t}{1 - \lambda} & \text{if } \lambda \leq t \end{cases}$$

These closed form solutions are presented in the following table, and depicted in figure 4.1.
Limiting properties of the equilibrium:
Equilibrium value of the critical state $\bar{\theta}$
by size and relative precision of the large trader

<table>
<thead>
<tr>
<th>Information precision</th>
<th>$\lambda &gt; t$</th>
<th>$t &gt; \lambda &gt; 0$</th>
<th>$\lambda = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\sigma}{\tau} \to \infty$</td>
<td>$1 - t + \lambda t$</td>
<td>$1 - t + \lambda t$</td>
<td>$1 - t$</td>
</tr>
<tr>
<td>$\frac{\sigma}{\tau} \to 0$</td>
<td>$1 - t + \lambda t - t^2 \frac{1 - \lambda}{1 - t}$</td>
<td>$1 - t$</td>
<td>$1 - t$</td>
</tr>
</tbody>
</table>

Figure 4.1: Critical state $\bar{\theta}$ as a function of $\lambda$

The two closed form solutions define the bounds on the critical state $\bar{\theta}$. We know from proposition 4.2 that the critical state is increasing in $r$, so that the function that maps $\lambda$ to the critical state for general values of $r$ must lie in the triangular region bordered by the two closed form solutions. Note, in particular that for any $r$ and any $\lambda$, the critical state at $\lambda$ is no lower than at zero. Thus, the size effect is positive in a global sense, even if it may fail to be positive locally.

As an illustration of the global size effect, we report below the plot generated by a simulation exercise where $\sigma = 0.01$, $\tau = 0.01$ (so that $r = 1$) and $t = 0.4$. $F$ and $G$ are standard normal.

[Figure 4.2 here]
The dotted lines are the solutions for \( \bar{\theta} \) for the two special cases as shown already in figure 4.1. The solid line is the plot for \( \sigma = \tau = 0.01 \) as \( \lambda \) varies. We can see that the size effect in this case is positive both locally and globally.

4.2. Comparative Statics Away from the Limit

In contrast to the clean comparative statics results in the limiting case, the results away from the limit are not so clear cut. Take, for instance, the question of the probability of the collapse of the peg conditional on some state \( \theta \). By the definition of the critical states \( \bar{\theta} \) and \( \bar{\theta} \), the peg will always fail to the left of \( \bar{\theta} \), never fail to the right of \( \bar{\theta} \), but in the interval in between \( \bar{\theta} \) and \( \bar{\theta} \), failure depends on whether the large trader attacks or not. The probability that the large trader attacks at \( \bar{\theta} \) is given by \( G \left( \frac{\bar{\theta} - \theta}{\tau} \right) \), but since \( y^* = \bar{\theta} - \tau G^{-1} (t) \), the probability of attack is given by \( G \left( \frac{\bar{\theta} - \theta}{\tau} - G^{-1} (t) \right) \). Thus, the probability that the peg will fail at state \( \bar{\theta} \) is

\[
\begin{align*}
1 & \quad \text{if } \theta < \bar{\theta} \\
G \left( \frac{\bar{\theta} - \theta}{\tau} - G^{-1} (t) \right) & \quad \text{if } \bar{\theta} \leq \theta < \bar{\theta} \\
0 & \quad \text{if } \theta \geq \bar{\theta}
\end{align*}
\]

In comparison, we know from the analysis of the case where \( \lambda = 0 \) (i.e. with no large trader) that the peg fails if and only if \( \theta < 1 - t \). Thus, the question of whether the peg is more likely to fail with the large trader depends on the relative sizes of \( \bar{\theta} \) and \( \bar{\theta} \) compared to \( 1 - t \). To address this and other related questions, it is useful to consider bounds on the critical states \( \bar{\theta} \) and \( \bar{\theta} \). In particular, we can show that:

**Proposition 4.4.** For any \( \sigma \) and \( \tau \), and \( \lambda > 0 \), the critical states \( \bar{\theta} \) and \( \bar{\theta} \) satisfy

\[
\begin{align*}
\min \left\{ 1 - t, \lambda + (1 - \lambda) \left( 1 - \frac{t}{1 + t} \right) \right\} & < \bar{\theta} < \lambda + (1 - \lambda) \left( 1 - t \right) \\
\min \left\{ 1 - \lambda - t, (1 - \lambda) \left( 1 - \frac{t}{1 + t} \right) \right\} & \leq \bar{\theta} \leq \min \left\{ 1 - \lambda, \lambda + (1 - \lambda) \left( 1 - t \right) \right\}
\end{align*}
\]

The proof of this result is given in appendix C. For the immediate question at hand concerning the relative sizes of \( \bar{\theta} \) and \( \bar{\theta} \) compared to \( 1 - t \), we note that \( \bar{\theta} > 1 - t \), but it is possible that \( \bar{\theta} < 1 - t \). Thus, in general, we cannot give a definitive answer to whether the presence of the large trader increases the probability of the peg’s collapse. For states \( \theta \in (\bar{\theta}, 1 - t) \) the probability of collapse decreases, but for states \( \theta \in (1 - t, \bar{\theta}) \), the probability of collapse increases. Beyond this, we cannot say anything further. This lack of a definite answer stands in contrast to the limiting case that we examined above.

In order to examine the effect of the large trader on the small traders’ strategies, we examined a number of numerical calculations on the threshold \( x^* \) of the small traders’
switching strategies. We were particularly interested in putting to the test a conjecture
that, in some instances, the presence of the large trader would make the smaller traders
less aggressive than in the case without the large trader.\(^8\) The reasoning is as follows.
The presence of the large trader makes coordination easier and therefore, all else being equal, promotes aggression. However, if the large trader is less well informed than the small traders, his presence may actually make coordination harder because the
correlation between his choice and that of the small traders will be low.

This conjecture is intuitively plausible, and we examined a number of numerical
solutions for the equilibrium threshold \(x^*\) when \(\sigma\) is small but \(\tau\) is large. However, the
simulations have so far proved inconclusive. In those cases where \(\sigma \to 0\), we found
that the terms of order \(\sigma\) are smaller than the error bounds of the simulations, however
accurate. For the various cases we have examined where \(\sigma\) and \(\tau\) are bounded away
from zero, we do not find support for the conjecture - the thresholds \(x^*\) are higher with
the large trader than without. We regard these simulations as being inconclusive, and
so it is an open question whether the conjecture is borne out in a concrete example.

Finally, we report on the numerical calculations of a benchmark case where \(F = \\) \(G = N(0, 1)\), and \(\sigma = \tau\). We plotted the threshold value \(x^*\) for the small traders as
a function of the (common) precision of the signals of the two types of traders when
\(\lambda = 0.5\). The first figure is for \(t = 0.4\), and the second is for \(t = 0.6\). The dotted
line is the threshold without the large trader, while the solid line is the threshold with
the large trader. Whether the curves are upward sloping curves or downward sloping
depends on whether \(x^*\) is positioned the left or right of the critical states \(g\) and \(\bar{g}\). This,
in turn depends on whether \(t\) is less than or greater than 0.5. The two figures reveal
that the threshold with the large trader is everywhere higher than the threshold without.

[Figures 4.3 and 4.4 here]

In gathering together our discussion, the overall conclusion we draw from our anal-
ysis is that both the “size effect” and the “information effect” are important determi-
nants in increasing the probability of collapse. The conclusions are most clear cut in
the limiting cases where both \(\sigma\) and \(\tau\) go to zero, but even away from the limit, numerical
calculations reveal that the equilibrium exhibits both types of effects in a consistent
way. Having said this, we do not doubt that counterexamples can be obtained for
suitably extreme parameter values.

5. Sequential Move Game

An important feature of large traders is their visibility in the market - a feature that is
only captured to a limited extent by our framework so far. Market participants know

\(^8\) This conjecture is due to a referee, and the phrasing of the conjecture is taken verbatim from his/her
report. We record our thanks to this referee for suggesting this conjecture.
the degree of precision of the large trader information, but have no prior information about the exact speculative position of the large trader. In this section, we explore the predictions of our model under a more general assumption regarding observability of actions. Specifically, we let the speculative position taken by any market participant to be observable by the rest of the market. We will see that in equilibrium the large trader will have an incentive to move before the others, so as to maximize his influence.

The analytical framework adopted in this section has essentially the same features of the model presented in Section 2. The main difference is that, instead of analyzing a simultaneous move by all traders, we now allow traders to take a speculative position in either of two periods, 1 and 2, preceding the government decision on the exchange rate. At the beginning of each period, each trader gets a chance to choose an action. However, once he has attacked the currency, he may not do so again and may not reverse his position. So, each trader can choose when, if at all, to attack the currency.

Traders receive their private signal \((x_i \text{ and } y)\) at the beginning of period 1. In addition, traders are now also able to observe at the beginning of period 2, the action choices of other traders in period 1. Thus, traders can learn from the actions of other market participants, and also use their own actions to signal to other traders. We assume that individual small traders ignore the signalling effect of their actions.\(^9\) Payoffs are the same as in section 2, and are realized at the end of period 2. Payoffs do not depend on the timing of traders’ actions, i.e., there are no costs of waiting.\(^10\)

### 5.1. Equilibrium

We begin by making two simple observations about timing incentives in the sequential move game. Small traders will always have an incentive to postpone any action until period 2. Each trader perceives no benefit to signalling, because he believes that he has no power to influence the actions by others by attacking early. On the other hand, he will learn something by waiting to attack: he will find out the large trader’s action and he may learn more about the state of the world. There are no costs of waiting, but there is a weak informational benefit to doing so. So it is a dominant strategy for each small trader to wait to period 2 before deciding whether to attack or not. But if small traders wait until period 2, the large trader knows that in equilibrium he can never learn from the actions of the small traders. On the other hand, he knows that if he attacks in period 1, he will send a signal to the small traders, and thereby influence their actions. In particular, since the large trader is concerned with coordinating his actions with those of the continuum of small traders, he benefits from signalling to the small traders. Thus

\(^9\)Levine and Pesendorfer (1995) and others have provided formal limiting justifications for this standard assumption in continuum player games.

\(^{10}\)Our two period game is best interpreted as a discrete depiction of a continuous time setting, in which the difference between the time periods is very small and represents the time it takes traders to observe and respond to others’ actions.
the large trader has a weak incentive to attack in period 1, if he is ever going to attack. Given that small traders wait until period 2, it is a dominant strategy for the large trader to move early. For these reasons, we assume in the analysis that follows that the large trader moves in period 1 and the small traders move in period 2.

We first characterize trigger equilibria in this game. Suppose that the large trader, acting first, chooses to attack only if his signal is lower than $y^*$. If he does not attack in period 1, small traders that receive a low enough signal may nonetheless attack the currency, thinking that they can bring the currency down alone. This will define a threshold $x^*$ for the signal of small traders, below which these would attack in period 2 even if the large trader has not attacked in period 1. But if the large trader does attack the currency in period 1, then of course this sends a signal to the small traders that (based upon his information) the large trader believes the economy to be weak enough to risk speculating. When the large trader attacks in period 1, small traders would therefore be inclined to attack for a larger range of signals they might receive. This defines a different threshold $\bar{x}^*$ for their signal, where in equilibrium $x^* < \bar{x}^*$. We should note here that these thresholds need not be finite. As shown below, there are situations in which the move by the large trader in period 1 will completely determine the behavior of small traders.

Since traders’ signals are correlated with fundamentals, corresponding to these triggers are critical mass conditions, i.e. threshold levels for the fundamentals below which there will be always a successful attack. As before, we can derive two conditions, depending on whether the large trader participates in the attack, $(\bar{\theta})$, or not $(\underline{\theta})$.

A trigger equilibrium is then a 5-tuple $(y^*, x^*, \bar{x}^*, \bar{\theta}, \underline{\theta})$. The equilibrium conditions described above now become:

- $y^*$ solves the equation
  \[
  \Pr (\theta \leq \bar{\theta} \mid y = y^*) = t \tag{5.1}
  \]

- $x^*$ solves the equation
  \[
  \Pr (\theta \leq \bar{\theta} \mid y > y^* \text{ and } x_i = x^*) = t \tag{5.2}
  \]
  if a solution exists. If the LHS is strictly larger than the RHS for all $x_i, x^* = \infty$. Conversely, if the LHS is strictly smaller than the RHS for all $x_i, x^* = -\infty$.

- $\bar{x}^*$ solves the equation
  \[
  \Pr (\theta \leq \bar{\theta} \mid y \leq y^* \text{ and } x_i = \bar{x}^*) = t \tag{5.3}
  \]
  if a solution exists. If the LHS is strictly larger than the RHS for all $x_i, \bar{x}^* = \infty$. Conversely, if the LHS is strictly smaller than the RHS for all $x_i, \bar{x}^* = -\infty$. 

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\begin{itemize}
\item $\bar{\theta}$ solves the equation
\[
(1 - \lambda) \Pr(x_i \leq \underline{x}^* \mid \theta = \bar{\theta}) = \bar{\theta}
\]
\[(5.4)\]

\item $\bar{\theta}$ solves the equation
\[
\lambda + (1 - \lambda) \Pr(x_i \leq \bar{x}^* \mid \theta = \bar{\theta}) = \bar{\theta}
\]
\[(5.5)\]
\end{itemize}

To solve the model, recall that, in our setting, the information system and the definition of the large trader’s signal implies

\[
y = x_i + \tau \eta - \sigma \varepsilon_i \\
y^* = \bar{\theta} - \tau G^{-1}(t)
\]

Now, consider a small trader’s posterior probability assessment of a successful attack conditional upon observing the large trader attack in period 1 and the signal $x_i$. Using the above expressions, such probability can be expressed as

\[
\Pr(\theta \leq \bar{\theta} \mid y \leq y^*) = \Pr(\varepsilon_i \geq \frac{x_i - \bar{\theta}}{\sigma} \mid \tau \eta - \sigma \varepsilon_i \leq \bar{\theta} - x_i - \tau G^{-1}(t))
\]

We can thus derive $\bar{x}^*$ by solving the following equation

\[
\frac{\Pr(\varepsilon_i \geq \frac{\bar{x}^* - \bar{\theta}}{\sigma}, \tau \eta - \sigma \varepsilon_i \leq \bar{\theta} - \bar{x}^* - \tau G^{-1}(t))}{\Pr(\tau \eta - \sigma \varepsilon_i \leq \bar{\theta} - \bar{x}^* - \tau G^{-1}(t))} = t
\]

\[(5.6)\]

By the same token, $\underline{x}^*$ can be derived by the analogous condition for the case in which the large trader has not attacked the currency in period 1:

\[
\frac{\Pr(\varepsilon_i \geq \frac{\underline{x}^* - \bar{\theta}}{\sigma}, \tau \eta - \sigma \varepsilon_i > \bar{\theta} - \underline{x}^* - \tau G^{-1}(t))}{\Pr(\tau \eta - \sigma \varepsilon_i > \bar{\theta} - \underline{x}^* - \tau G^{-1}(t))} = t
\]

\[(5.7)\]

It is apparent that neither of these equations can be solved in closed form in the general case, without making further parametric assumptions on the distribution functions of the error terms. We therefore resort to two types of analysis. One is to follow the procedure used in section 4, and examine the limiting cases for different relative precisions of the large trader’s information.

Before we do so, however, we report the results of some numerical calculations on the critical states $\bar{\theta}$ and $\bar{\theta}$. These critical states take on added significance in the sequential version of our game, since the action of the large trader is observed by the
small traders. Below we report the plots for the critical states $\theta$ and $\bar{\theta}$ for a variety of parameter combinations. As before, $F$ and $G$ are standard normal.

[Figures 5.1 to 5.4 here]

In all cases, the numerical plots deliver intuitive answers. As the precision of the large trader’s information becomes improves (so that we move left in all the plots), we can see that the upper critical state $\bar{\theta}$ increases, while the lower critical state $\theta$ falls. This implies that the pivotal influence of the large trader is greater when his information becomes more precise. Note, in particular, that $\bar{\theta}$ approaches 1 when the large trader’s signal becomes more precise. In other words, when $\tau$ is small, the large trader’s action precipitates the attack whenever the peg can be broken.

This and other properties of the sequential game can be examined by analysing the limiting properties of the equilibrium.

5.2. Comparative Statics in the Limit

We now discuss the limiting properties of the model allowing for differences in the information precision across traders of different size. We consider first the case of a large trader who is arbitrarily better informed than small traders. The following proposition summarizes our result.

Proposition 5.1. As $\frac{\sigma}{\xi} \to \infty$, there is a unique trigger equilibrium in $\Gamma$, with

\[
\begin{align*}
\frac{1 - y^*}{\tau} & \to G^{-1}(t) \\
\bar{x}^* & \to \infty \\
x_+^* & \to -\infty \\
\bar{\theta} & \to 1 \\
\theta & \to 0
\end{align*}
\]

Proof: We first rewrite equation (5.6) as

\[
\frac{\Pr \left( \varepsilon_i \geq \frac{x^* - \bar{\theta}}{\sigma}, \frac{\tau}{\sigma} \eta - \varepsilon_i \leq \frac{\bar{\theta} - x^*}{\sigma} - \frac{\tau}{\sigma} G^{-1}(t) \right)}{\Pr \left( \frac{\tau}{\sigma} \eta - \varepsilon_i \leq \frac{\bar{\theta} - x^*}{\sigma} - \frac{\tau}{\sigma} G^{-1}(t) \right)} = t
\]

Taking the limit as $\frac{\tau}{\sigma} \to 0$, the LHS tends to

\[
\frac{\Pr \left( \varepsilon_i \geq \frac{x^* - \bar{\theta}}{\sigma}, -\varepsilon_i \leq \frac{\bar{\theta} - x^*}{\sigma} \right)}{\Pr \left( -\varepsilon_i \leq \frac{\bar{\theta} - x^*}{\sigma} \right)}
\]
which is equal to \( \frac{1}{c_1} \). Thus, in the limit there is no solution to the above equation. Since \( t < 1 \), we use the definition of \( \bar{x}^* \) to set \( \bar{x}^* = \infty \). We can then substitute \( \bar{x}^* \) into equation (5.5) to derive \( \bar{\theta} = 1 \). Symmetric arguments establish that \( \bar{x}^* = -\infty \) and \( \bar{\theta} = 0 \). Thus using the definition of \( y^* \), we get \( y^* = 1 - \tau G^{-1}(t) \).

In words, this result says that, when the large trader is arbitrarily better informed than the small traders, they follow him blindly, and therefore, he completely internalizes the payoff externality in the currency market. This type of equilibrium corresponds to the strong herding equilibrium in Dasgupta (1999), where all the followers ignore their information completely.

This result implies that, when actions are observable, a relatively well-informed large trader can (but not always will) make small traders either extremely aggressive in selling a currency, or not at all aggressive. His influence in this case is much larger (as should be true, intuitively), in comparison to the case of a simultaneous move game, analyzed in the previous section.

Notably, the size of the large trader never appears in the expressions that define the unique trigger equilibrium. The distinctive feature of a large trader is that he does not ignore the signalling effect of his actions. What emerges from our result is that, when he is significantly better informed than the small traders, his absolute size is irrelevant.

The following proposition states our results corresponding to the case in which the large trader is less precisely informed than the rest of the market.

**Proposition 5.2.** As \( \frac{\sigma}{\tau} \to 0 \) there is a unique trigger equilibrium, with

\[
\frac{\lambda + (1 - \lambda)(1-t) - y^*}{\sigma} \to G^{-1}(t)
\]

\[
\frac{\lambda + (1 - \lambda)(1-t) - \bar{x}^*}{\sigma} \to F^{-1}(t)
\]

\[
\frac{(1 - \lambda)(1-t) - \bar{x}^*}{\sigma} \to F^{-1}(t)
\]

\[
\bar{\theta} \to \lambda + (1-\lambda)(1-t)
\]

\[
\bar{\theta} \to (1-\lambda)(1-t)
\]

**Proof:** Rewrite equation (5.6) and taking limits as \( \frac{\sigma}{\tau} \to 0 \), we get

\[
\Pr \left( \varepsilon_i \geq \frac{\bar{x}^* - \bar{\theta}}{\sigma}, \eta \leq \frac{\bar{\theta} - \bar{x}^*}{\tau} - G^{-1}(t) \right) = t
\]

which, given independence of \( \varepsilon_i \) and \( \tau \) implies that

\[
\bar{x}^* = \bar{\theta} - \sigma F^{-1}(t)
\]
Combining with equation (5.5) we get

\[
\bar{\theta} \rightarrow \lambda + (1 - \lambda) \, (1 - t)
\]

Thus

\[
x^* \rightarrow \lambda + (1 - \lambda) \, (1 - t) - \sigma F^{-1}(t)
\]

The remaining quantities are then uniquely defined.

In words, this proposition means that even a relatively uninformed large trader attempts to influence the market. However, since he does not have any informational signalling ability, his actions affect the equilibrium outcome of the game only inasmuch as his size is relevant. Intuitively, as his signal is quite noisy, he cannot reduce the small traders’ uncertainty about the fundamental. By moving first, however, he can eliminate uncertainty about his action. If, in addition, we suppose that \(\sigma \rightarrow 0\), then

\[
x^* \rightarrow 1 - t + \lambda t
\]

Observe that as \(\lambda \rightarrow 0\), the equilibrium triggers converge exactly to the case in which the large trader does not exist.

### 5.3. A synthesis of our results

We are now in the position to offer a complete overview of our results, and reach some conclusions about the role of a large trader in a currency crisis. As explained in the introduction, there are three main elements in our theory: size, information precision and signalling.

Focusing on the limiting properties of our equilibria, the following table presents the equilibrium value of the trigger for small traders in the different cases discussed above.

<table>
<thead>
<tr>
<th>Limiting properties of equilibria</th>
<th>Equilibrium trigger for small traders by relative precision of information</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Large trader is:</strong></td>
<td><strong>informed</strong></td>
</tr>
<tr>
<td>((\frac{\sigma}{\sigma} \rightarrow 0, \sigma \rightarrow 0))</td>
<td>((\frac{\sigma}{\sigma} \rightarrow \infty, \sigma \rightarrow 0))</td>
</tr>
<tr>
<td>Actions are:</td>
<td></td>
</tr>
<tr>
<td>unobservable</td>
<td>(x^* = 1 - t + \lambda t)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>observable</td>
<td>(\bar{x}^* = \infty)</td>
</tr>
<tr>
<td></td>
<td>(x^* = -\infty)</td>
</tr>
</tbody>
</table>

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In each column of the table, the two thresholds $\bar{x}^*$ and $\underline{x}^*$ in the game where actions are observable are higher and lower, respectively, than the corresponding threshold $x^*$ derived in our game with unobservable action. In other words, regardless of the relative precision of information, a large trader can have a much larger influence in the market if he is able to signal to small traders.

As discussed above, the size of the large trader is irrelevant in the sequential move game when the large trader is relatively well informed – this case corresponds to the bottom left cell of the table. What matter here is not the size per se, but the signalling ability associated with size. Conversely, size matters in all other cases.

Reading the entries on the main diagonal of the table, observe that the critical signal $(x^*)$ in the unobservable action, information larger trader case is equal to critical signal contingent on the larger trader have attacked $(\bar{x}^*)$ in the observable action, uninformed large trader case. This equality provides an interesting link across the two games. When actions are not observable, small traders do not expect a better informed trader to “add noise” to the game. Their problem is to estimate the fundamental as well as possible, given their own signal. When actions are observable, the potential noise added to the game by a relatively uninformed large trader is eliminated by his moving first. So, also in this case, the problem of the smaller traders is the same as above, i.e. to estimate the fundamental as well as possible given their own information.

6. Concluding Remarks

Economists and policy makers have long debated whether speculation, especially speculation by large traders, is destabilizing. In our model, a large trader in the market may exacerbate a crisis, and render small traders more aggressive. Figure 4.1 illustrates this well. The small traders’ trading strategies as defined by the switching point $x^*$ become more aggressive as the size of the large trader increases. However, the relative precision of the information available to the traders affects this conclusion. If the large trader is less well informed than the small traders, this effect may be quite small. Finally, the influence of the large trader is magnified greatly if the large trader’s trading position is publicly revealed to the other traders, although this result also must be qualified by the relative precision of information of the two types of traders.

Crucial to our conclusion is the assumption that the large trader stands to gain in the event of the devaluation. This may not be an assumption that is widely accepted. If the large trader is an investor with a substantial holding of assets denominated in the currency under attack (say, a U.S. pension fund with equity holdings in the target country), he may prefer that an attack not occur, even though, if he thinks the attack is sufficiently likely he will join the attack. In such a case, the presence of a large trader will have the opposite effect, making attacks less likely. This points to the importance of understanding the initial portfolio positions of the traders in such instances.
Our analysis also abstracted from a large trader’s incentive to take a position discreetly in order to avoid adverse price movements. If this effect were important, a trader would have an incentive to delay announcing his position until it is fully established. But even once a trader has established his position, he may prefer to avoid public disclosures when he is holding a highly leveraged portfolio in possibly illiquid instruments. One of the motivations for the call for greater public disclosures by banks and hedge funds (see Financial Stability Forum (2000)) is the idea that if leveraged institutions know that their trading positions are to be revealed publicly, they would be wary of taking on large speculative positions. The recent decisions by several well known fund managers (Mr. Soros being one of them) to discontinue their ‘macro hedge fund’ activities raise deeper questions concerning the trade-off between the sorts of mechanisms outlined in our model against the diseconomies of scale that arise due to the illiquidity of certain markets. It is perhaps not a coincidence that the closure of such macro hedge funds comes at a time when many governments have stopped pursuing currency pegs and other asset price stabilization policies.

**APPENDIX A**

In this appendix, we show that the unique equilibrium in switching strategies can be obtained by the iterated deletion of strictly dominated strategies.

Consider the expected payoff to attacking the peg for a small trader conditional on signal \( x \) when all other small traders follow the switching strategy around \( \hat{x} \) and when the large trader plays his best response against this switching strategy (which is to switch at \( y(\hat{x}), \) obtained from (3.3)). Denote this expected payoff by \( u(x, \hat{x}) \). It is given by

\[
\begin{align*}
u(x, \hat{x}) &= \frac{1}{\sigma} \int_{-\infty}^{\hat{x}} f \left( \frac{\theta - x}{\sigma} \right) d\theta + \frac{1}{\sigma} \int_{\hat{x}}^{\bar{\theta}(\hat{x})} f \left( \frac{\theta - x}{\sigma} \right) G \left( \frac{y(\hat{x}) - \theta}{\tau} \right) d\theta 
\end{align*}
\]

where \( \bar{\theta}(\hat{x}) \) indicates the value of \( \theta \) when small traders follow the \( \hat{x} \)-switching strategy. \( \bar{\theta}(\hat{x}) \) is defined analogously. We allow \( \hat{x} \) to take the values \(-\infty \) and \( \infty \) also, by which we mean that the small traders never and always attack, respectively. Note that \( u(\cdot, \cdot) \) is decreasing in its first argument and increasing in its second.

For sufficiently low values of \( x \), attacking the currency is a dominant action for a small trader, irrespective of the actions of the other traders, small or large. Denote by \( x_D \) the threshold value of \( x \) below which it is a dominant action to attack the currency for the small trader. All traders realize this, and rule out any strategy for the small traders which refrain from attacking below \( x_D \). But then, refraining from attacking...
cannot be rational for a small trader whenever one’s signal is below $x_1$ where $x_1$ solves
\[ u(x_1, x_0) = t \]  
(6.2)

This is so, since the switching strategy around $x_1$ is the best reply to the switching strategy around $x_0$, and even the most cautious small trader (in the sense that he assumes the worst concerning the possibility of a successful attack) believes that the incidence of attack is higher than that implied by the switching strategy around $x_0$ and the large trader’s best reply $y(x_0)$. Since the payoff to attacking is increasing in the incidence of attack by the other traders, any strategy that refrains from attacking for signals lower than $x_1$ is dominated. Thus, after two rounds of deletion of dominated strategies, any strategy for a small trader that refrains from attack for signals lower than $x_1$ is eliminated. Proceeding in this way, one generates the increasing sequence:

\[ x_0 < x_1 < x_2 < \cdots < x_k < \cdots \]  
(6.3)

where any strategy that refrains from attacking for signal $x < x_1$ does not survive $k + 1$ rounds of deletion of dominated strategies. The sequence is increasing since $u(\cdot, \cdot)$ is decreasing in its first argument, and increasing in its second. The smallest solution $x$ to the equation $u(x, x) = t$ is the least upper bound of this sequence, and hence its limit. Any strategy that refrains from attacking for signal lower than $x$ does not survive iterated dominance.

Conversely, if $x$ is the largest solution to $u(x, x) = t$, there is an exactly analogous argument from “above”, which demonstrates that a strategy that attacks for signals larger than $x$ does not survive iterated dominance. But if there is a unique solution to $u(x, x) = t$, then the smallest solution just is the largest solution. There is precisely one strategy remaining after eliminating all iteratively dominated strategies. Needless to say, this also implies that this strategy is the only equilibrium strategy. This completes the argument.

**APPENDIX B**

In this appendix, we give a proof of proposition 4.1. First, suppose that $\lim \underline{\theta} < \lim \overline{\theta}$ (so that $\lim \overline{\theta} \geq 1 - \lambda$). Since $x^* \to \overline{\theta}$, we must have

\[ \frac{\theta - x^*}{\sigma} \to -\infty \]

In other words, $\delta \to -\infty$. Thus, in the limit, equation (4.4) can written as (4.6). Now, consider the case where $\lim \underline{\theta} = \lim \overline{\theta}$. This is the case where $\delta$ is finite. From (4.1) and (4.2) we have

\[ (1 - \lambda) (1 - F(\delta)) = \lambda + (1 - \lambda) \left( 1 - F(\delta) \right) \]
implying that \( F(\delta) - F(\bar{\delta}) = \frac{\lambda}{1-\lambda}, \) which in turn means that

\[
\delta = F^{-1} \left( F(\bar{\delta}) - \frac{\lambda}{1-\lambda} \right) \tag{6.4}
\]

Equation (4.4) in the limiting case then becomes (4.7). In both (4.6) and (4.7), the left hand side is strictly increasing in \( \delta, \) and there is a unique value of \( \delta \) that solves both equations. Then, the proposition follows from (4.2).

APPENDIX C

In this appendix, we give a proof of proposition 4.4. The bounds can be obtained by manipulating the four equations, (4.1) through (4.4). Let us use the notation: \( \bar{\xi} = F(\delta) \) and \( \xi = F(\bar{\delta}). \) From (4.1) and (4.2),

\[
\overline{\theta} - \theta = \lambda + (1 - \lambda) \left( F\left( \frac{x^* - \overline{\theta}}{\sigma} \right) - F\left( \frac{x^* - \theta}{\sigma} \right) \right) \\
= \lambda + (1 - \lambda) \left( 1 - F\left( \frac{x^* - \overline{\theta}}{\sigma} \right) - 1 + F\left( \frac{\theta - x^*}{\sigma} \right) \right) \\
= \lambda - (1 - \lambda) (\overline{\xi} - \xi).
\]

This implies that \( \overline{\theta} > \theta \) and \( \overline{\xi} > \xi. \) Re-arranging, this implies

\[
\overline{\xi} - \xi < \frac{\lambda}{1 - \lambda}. \tag{6.5}
\]

By (4.3),

\[
y^* = \overline{\theta} - \tau G^{-1}(t)
\]

Substituting into (4.4), we obtain

\[
\frac{1}{\sigma} \int_{-\infty}^{\theta} f\left( \frac{\theta - x^*}{\sigma} \right) d\theta + \frac{1}{\sigma} \int_{\theta}^{\overline{\theta}} f\left( \frac{\theta - x^*}{\sigma} \right) G\left( \frac{\overline{\theta} - \theta}{\tau} - G^{-1}(t) \right) d\theta = t \tag{6.6}
\]

Observe that for all \( \theta \in (\overline{\theta}, \bar{\theta}), \)

\[
1 - t < G\left( \frac{\overline{\theta} - \theta}{\tau} - G^{-1}(t) \right) < 1
\]
and that

$$\frac{1}{\sigma} \int_{-\infty}^{\eta} \frac{f \left( \frac{\theta - x^*}{\sigma} \right)}{d\theta} = F(\xi)$$

so the left hand side of equation (6.6) is strictly less than $\xi$ and is strictly more than $(1 - t)\xi$ and $\xi - t [\xi - \xi]$. By (6.5), this latter expression is strictly more than $\xi - \frac{\lambda t}{1 - t}$.

Now equation (6.6) and the upper and lower bounds on the left hand side of (6.6) imply

$$t < \xi < \min \left\{ \frac{t}{1 - t}, \frac{t}{1 - \lambda} \right\}.$$  

The proposition follows from re-arranging the characterizing these characterizing equations.

References


Figure 4.2: $\bar{\theta}$ at $r = 1$ as a function of $\lambda$; $t = 0.4$, $\sigma = \tau = 0.01$
Figure 4.3: $x^*$ with and without the large trader; $t = 0.4$, $\lambda = 0.5$

Figure 4.4: $x^*$ with and without the large trader; $t = 0.6$, $\lambda = 0.5$
Figure 5.1: $\tilde{\theta}$; $t = 0.4$, $\lambda = 0.5$, $\sigma = 1.0$

Figure 5.2: $\theta$; $t = 0.4$, $\lambda = 0.5$, $\sigma = 1.0$
Figure 5.3: $\bar{\theta}$ ; $t = 0.4$, $\lambda = 0.1$, $\sigma = 1.0$

Figure 5.4: $\underline{\theta}$ ; $t = 0.4$, $\lambda = 0.1$, $\sigma = 1.0$