Qualitative Voting in Conflict Resolution Situations

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Abstract

Qualitative Voting is an alternative to the Majority Rule that endows agents with a certain number of votes that can be freely distributed between a set of issues that have to be approved or dismissed. Its novelty, and appeal, is that it allows voters to express the intensity of their preferences in a simple manner and with no use of monetary transfers. In Conflict Resolution situations where two parties with opposed preferences need to decide over their discerning views, Qualitative voting allows agents to trade off their voting power across issues and extract gains from their differing relative intensities.

The appealing properties of such voting system may be overcome by the strategic interactions among individuals. In this paper we test its properties using controlled laboratory experiments. We observe that agents learn to play equilibrium and reach the welfare predicted by the theory. The latter is true even when their behaviour is far from equilibrium. This last aspect brings an extra positive aspect to the use of Qualitative Voting in the real world given that deviations from equilibrium do not terribly harm its welfare properties.

JEL Classification: C72, C91, D70, P16
Keywords: Voting Experiments, Alternatives to Majority Rule, Conflict Resolution.

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1 Introduction

It is common to find what we call Conflict Resolution situations where two parties need to agree on multiple divergent issues. An international dispute, a bilateral agreement in arms/pollution reduction, a country having the two chambers governed by opposing parties or a clash between the management and the union of a particular firm, are just a few examples in which such situations may occur.

As is stressed in the negotiation analysis literature (see Keeney and Raiffa 1991), in conflict resolution situations parties need to assess trade-offs in terms of how much to give up in [...] one issue in order to obtain a specified gain in another issue.\(^1\) It is often the case that parties have differing relative intensities, differing attitudes towards risk or differing patience and they can both extract gains from such differences. The negotiation analysis literature has offered few different approaches and techniques to settle such disputes focusing on the candid or strategic behaviour of parties. It has proposed specific algorithms that allow agents to elicit their preferences and subsequently aggregate them (in particular, see Phillips and Bana e Costa 2005 and references therein for references within the operations research literature).

In the present paper we offer a strategic analysis of a situation where an exogenous agenda is given and players need to simultaneously decide over various issues in the absence of monetary transfers. We want to allow agents to be more decisive on those issues they relatively care more about. It is a simple comparative advantage argument: just in the same way that countries should specialise in those products they are relatively more productive at, agents should decide on those issues they relatively feel more intensively about. In this vein, we propose a new mechanism we call Qualitative Voting (QV, hereafter) that allows parties to embed a quality in their vote and express the intensity of their preferences in a simple manner. Ultimately, we want to show under which circumstances the strategic interactions between voters do not undermine the gains from taking into account the intensity of their preferences.

We consider a situation where a public decision needs to be taken over various issues. There are two parties that hold opposing views in every issue (we can think that there is an unmodelled previous stage where unanimous wills are implemented). Generally, parties do not have a common valuation towards each issue, thus there are gains from trade. That is, Pareto improvements are possible because subjects can obtain support towards their most preferred issues by giving up on their least preferred ones. QV endows each opposing party with a given number of votes that need to be simultaneously distributed among the multiple issues\(^2\). Parties are decisive over the issues where they invest more votes than their opponent. In case of ties, a fair coin is tossed. Note that we can draw a parallelism between our setting and an auction where two agents are endowed with token money to bid simultaneously towards various goods.\(^3\)

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\(^1\)Keeney and Raiffa (1991), pg. 131.

\(^2\)QV has the flavour of a scoring rule (especially cumulative voting) though there is a crucial distinction between them. The latter is used to elect one representative out of many; instead QV deals with a situation where there are N offices for which 2 candidates are running and voters can abstain in some elections cumulating their votes for the rest.

\(^3\)Szentes and Rosenthal (2003) analyse a simultaneous auction for three objects in a setting with complete information. As opposed to our setting, they assume that parties have a common valuation towards the objects.
The novelty and appeal of QV lies in its simplicity. It thus seems natural to parallel its theoretical analysis (see HortalaVallve 2006) with the design of a series of experiments to examine its properties. On the one hand, we want to contrast the theoretical predictions with the experimental evidence. And, on the other hand, we want to test whether the use of QV is beneficial to our subjects in terms of achieved welfare. In the present paper we focus on three stylised situations with two, three and six issues.

The theoretical properties of the case with only two issues have been studied in Hortala-Vallve (2006) where the author shows that it is a dominant strategy for any voter to invest all votes in his most preferred issue. This strategy allows parties to be most influential in their most preferred issue and allows the mechanism to reach the only ex-ante optimal incentive compatible allocation. We have not been able to characterise the theoretical solution for the cases with three and six issues. However, in order to frame the results in this paper, we have found their solution computationally. From a theoretical perspective we are dealing with a Colonel Blotto game where parties are not indifferent between winning any of the issues. The literature has remained silent regarding the solution of such games and we are just aware of a recent paper, Kvassov (2003), that analyses equilibria of Blotto games whenever the subjects equally value each issue. Instead, our essential characteristic is precisely that the opposing parties have differing relative intensities (and gains can be extracted from that).

Our first main finding is that subjects learn to play equilibrium behaviour in all three cases. Needless to say, the effect is specially persuasive in the two issues case: subjects play the dominant strategy equilibrium in 40% of the cases in the initial period and do so in almost 90% of the last period cases. The increase of equilibrium play in the cases with 3 and 6 issues is not as sharp but is still noteworthy: there is a 15% improvement during the 50 periods that our subjects played.

Our second main finding is that the subjects’ realised aggregate welfare matches the one predicted by the theory and almost attains the upper bound imposed by the efficient welfare –the efficient welfare is the one that maximises the sum of utilities. This last fact provides a very supportive argument towards the use of QV as an alternative voting rule given that even when the subjects behaviour is far from equilibrium, the welfare gains we expect from its use are realised. These results contrast with the use of a random mechanism where the achieved welfare is much below the one attained by our subjects. In order to understand the source of these surprisingly good results we simulate the subjects behaviour under alternative voting rules and realise that the efficiency gains arise mainly from playing weakly monotone strategies –i.e. strategies in which more votes are invested in an issue that is valued higher. In our setting, it is evidently suboptimal not to play such strategies given that all issues are symmetric and preferences are uniformly distributed.

Summing up, the efficiency gains arising from subjects being able to freely allocate their voting power across a given set of issues are not overcome by the strategic interactions nor by the complexity of

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4A Colonel Blotto Game can be described by a situation where two colonels are fighting over a few regions and have to decide how to divide their forces; the one with larger forces wins the region and the winner of the battle is the one with the most won territory.

5See also Myerson (1993) and Laslier and Picard (2002) which refer also to the Colonel Blotto Game when analysing the incentives for candidates to create inequalities among voters by making heterogeneous campaign promises.
the game. At the light of the results presented in this paper, QV seems a robust mechanisms to
the subjects’ idiosyncratic behaviour whenever their strategies satisfy the condition that more votes
are invested in those issues they value higher.

The paper is organized as follows. In the remainder of this section we relate our work to the existing
literature. In Section 2 we introduce the theoretical model. In Section 3 we describe the experimental
design and in Section 4 we characterise the theoretical predictions for our specific design. Section 5
analyses the voting behaviour and the welfare achieved by our subjects and in Section 6 we present
further analysis on the subjects behaviour and various robustness checks on our experimental design.
Section 7 concludes.

1.1 Related literature

The literature in conflict resolution is very large and can be found in a variety of fields such as inter-
national relations, game theory, experimental psychology or experimental economics. Nevertheless
most literature focuses on unidimensional situations like the Rubinstein bargaining model (1982),
the ultimatum game (see Thaler 1988 for a review) or the models on arbitration (see for instance
the review by Brams, Kilgour and Merrill 1991). Instead, our model deals with a multidimensional
situation without monetary transfers where not much research has been realised.

Most related to our work is the experimental study of storable votes by Casella, Gelman and Palfrey
(2003). Their mechanism allows voters to abstain and store that vote for further meetings when
the intensity of their preferences may be stronger. The authors show that the subjects’ welfare is
remarkably close to theoretical predictions even when players do not exactly follow the theoretical
equilibrium predictions. This apparent puzzle can be rationalised in our setting given that deviations
from equilibrium do not terribly harm welfare as long as more votes are invested in issues that are
valued higher.

Similarly, Engelmann and Grimm (2006) study the Jackson and Sonnenschein (2005) linking mech-
anism and show that constraining the number of cases where a player can declare a high preference
nearly captures all achievable efficiency gains. The source of the gains we observe in our model is
analogous to theirs.

2 The model

Two subjects have opposing views over N issues that need to be approved or dismissed. Monetary
transfers are not allowed. Each subject privately knows his preferences and the prior distribution
from which they are drawn is common knowledge.

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6When designing our experiment, we had an initial underlying concern regarding its complexity. The experimental
literature on multidimensional voting models offers opposing views on this question. On the one hand, McKelvey and
Ordehook (1981) show that an increase in strategic complexity, through the provision of better information on the
remaining subjects’ preferences, leads subjects to a worse situation. Instead, Yuval (2002) shows that an increase in
complexity, through an increase in the size of the agenda, leads voters to act more sincerely and to a better payoff
overall. We abstract from the former informational effect in this first analysis of QV and do not observe the latter
effect towards playing sincerely when we increase the number of issues.
Voters and issues are denoted $i \in \{1, 2\}$ and $n \in \{1, 2, \ldots, N\}$, respectively. Voter $i$’s valuation towards issue $n$ is denoted $\theta_i^n \in \mathbb{R}^+$. The preference vector of voter $i$ is denoted $\theta^i = (\theta_1^i, \ldots, \theta_N^i) \in \Theta \subseteq \mathbb{R}^N$. Voter $i$’s payoff on issue $n$ is described as follows,

$$\begin{cases} 
\theta_i^n & \text{if his will is implemented in issue } n \\
0 & \text{if his opponent’s will is implemented in issue } n 
\end{cases}$$

The total payoff is the sum of the individual payoffs across the $N$ voting procedures.\(^7\)

Players are endowed with $V$ votes that can be freely distributed between the issues. The action space is the collection of voting profiles:

$$\mathcal{V} := \left\{ (v_1, \ldots, v_N) \in \{0, 1, \ldots, V\}^N : v_1 + \ldots + v_N = V \right\}$$

The mechanism QV allows each agent to implement his will in issues where he invests more votes than his opponent. Ties are broken with the toss of a fair coin. That is,

$$\begin{cases} 
v_1^n > v_2^n & \Rightarrow \text{Voter 1 decides on issue } n \\
v_1^n < v_2^n & \Rightarrow \text{Voter 2 decides on issue } n \\
v_1^n = v_2^n & \Rightarrow \text{Each voter decides on issue } n \text{ with probability } \frac{1}{2}.
\end{cases}$$

### 3 Experimental design

We run a total of 3 sessions with 18 subjects per session. Students were recruited through the online recruitment system ORSEE (Greiner 2004) and the experiment took place on networked personal computers in the LEEX at Universitat Pompeu Fabra in April 2006. The experiment was programmed and conducted with the software z-Tree (Fischbacher 1999).\(^8\)

The procedure is kept the same throughout all sessions. Instructions (see Appendix) were read aloud and questions are answered in private. Students are asked to answer a questionnaire to check their full understanding of the experimental design (if any of their answers is wrong the experimenter refers privately to the section of the instructions where the correct answer is provided). Students are isolated and are not allowed to communicate. At each period subjects are randomly matched in groups of two.\(^9\) The sessions consist of 50 periods. The only difference across sessions is that players are voting towards a different number of issues (2, 3 or 6).

Preferences are induced by assigning a payment in terms of cents of euro to each of the issues. Payments are drawn from a uniform distribution of vectors with elements being positive multiples

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\(^7\)The definition of payoffs is implicitly assuming that issues are independently valued. That is, there are no complementarities between them. Provided that issues are independently valued, results can be extended to any linear transformation of the payoffs.

\(^8\)The data and programme code for the experiment are available upon request.

\(^9\)We partition the subjects into three sets of six players so as to obtain three independent observations. We have analysed each observation separately and we see no remarkable difference between them. Thus, in the subsequent analysis, we merge the data.
of 50, adding up to 600 and such that no issue is valued zero. The purpose of the constant sum in the subjects’ payments is threefold. First, we ensure that the occurrences are comparable and avoid framing effects. Second, we avoid issues on interpersonals comparisons of utility and consider all subjects equally by normalising their preferences. And third, we introduce a constraint imposed by incentive compatibility into the computation of the efficient outcome, i.e. the outcome that implements the will of the subject that holds stronger preferences on each issue. The following example shows that normalised preferences (so that their sum is constant) implies that the efficient outcome is closer to what is achievable.

Example: Consider a conflict resolution situation with two issues and imagine that the two players’ payments/preferences profiles are : (90, 60) and (10, 40). The efficient allocation requires both issues to be assigned to player 1 and the efficient utilitarian social welfare is 150. Nevertheless, approving both issues is not incentive compatible and the implementable allocation that maximises the sum of the voter’s utilities consists of assigning the first good to the first player and the second good to the second player (i.e. it requires not undertaking interpersonal comparisons of utility and considering instead the relative intensity of each player). When we normalise we have that the payments are (60, 40) and (20, 80) and the efficient outcome is now implementable.

At each period, players are announced their payments and are asked to cast 6 votes among the issues. Given the payment profiles defined above, we are implicitly assuming that the set of payments is finer than the set of voting profiles. This is done to capture a realistic feature of the mechanism QV with discrete votes where actual preferences are, in general, richer than the action profile.

The program computes the outcome and the payment of each period after all subjects cast their votes. Each subject receives the following information on each of the issues at the end of each period: (i) his payment; (ii) his vote; and (iii) his opponent’s vote. Finally, he is also announced his payoff for that period. The final payment of the session is computed by adding the payoff obtained in three (randomly selected) periods. At the end of each session participants are asked to fill in a questionnaire on the computer and are given in private their final payment. The average payoff is 14.36 euros. Session length, including waiting time and payment, is around an hour and a half.

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10 For example (300,300), (100,500), (500,100) or (550,50) are all equally likely.
11 Framing effects imply that voters may behave differently when they are assigned payments (1,2) or (200,400). We want to abstract from such framing issues which have been broadly analysed in many different settings -see the seminal reference Kahneman and Tversky (1983).
12 We should stress however that the chosen parameter values (i.e. number of issues, grid size on the induced payments and number of votes) were chosen to offer a coherent and comparable set of experiments among the parameters that induced a game that was computationally solvable. What would happen when we vary such parameters in interesting ways (e.g. increasing the number of votes or the grid size) remains an open question. We believe that these questions rely on characterising the equilibrium of such games theoretically and hence remain out of the scope of the present paper.
4 Theoretical predictions

The following tables list the payment profiles that were induced in each session (ordered in decreasing order) together with the equilibrium action for each type of subject.

<table>
<thead>
<tr>
<th>2 issues</th>
<th>3 issues</th>
<th>6 issues</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>$\theta_2$</td>
<td>$v_1$</td>
</tr>
<tr>
<td>300</td>
<td>300</td>
<td>3</td>
</tr>
<tr>
<td>350</td>
<td>250</td>
<td>6</td>
</tr>
<tr>
<td>400</td>
<td>200</td>
<td>6</td>
</tr>
<tr>
<td>450</td>
<td>150</td>
<td>6</td>
</tr>
<tr>
<td>500</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>550</td>
<td>50</td>
<td>6</td>
</tr>
<tr>
<td>350</td>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>350</td>
<td>200</td>
<td>50</td>
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<tr>
<td>400</td>
<td>100</td>
<td>100</td>
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<td>400</td>
<td>150</td>
<td>50</td>
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<tr>
<td>450</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>500</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

Utility vectors and the corresponding nash equilibria for each of the 2, 3, and 6 issues games.

Table 1

Whenever parties vote over 2 issues, they should invest all votes in their most preferred issue—it is a dominant strategy to do so. The intuition behind this result is very simple and relies on the fact that there are only three implementable outcomes: ties on both issues, and two outcomes where each voter decides on a different issue. A subject that is not indifferent between both issues likes best the outcome where he decides on his most preferred issue. Therefore, his action is driven by maximising the likelihood of deciding over such issue. In other words, he invests all votes in his most preferred issue. Subjects that equally value both issues are indifferent between any outcome but whenever they equally split their votes among both issues they allow non-indifferent voters to implement their preferred outcome and allow QV to reach the only ex-ante optimal allocation.

As noted in the introduction, the difficulties of theoretically finding the equilibria in the cases with 3 and 6 issues have led us to characterise them computationally. In both cases, equilibrium behaviour has very appealing properties and does not seem to contradict our intuition. Indifferent voters (those that equally like any of the issues) evenly distribute their voting power and extreme voters intensify their voting power on their most preferred issue. The case with six issues is specially interesting given that the type with most extreme preferences is not investing all his votes in his most preferred issue in equilibrium. This occurs because it is very unlikely that anyone’s opponent most prefers the

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13 See Hortala-Vallve (2006) for a formal proof of such result.
14 The C++ programme code is available upon request.
15 Note that some of the types are playing mixed strategies given that they invest a different number of votes in issues they equally value. See for instance the player with preferences (250, 100, 100, 50, 50, 50). For simplicity, our algorithm assumed that such types mixed uniformly across the available strategies. This assumption implies that all issues are ex-ante symmetrical.
same issue. Therefore, voters gain from diversifying their voting power and increasing their chances of winning more than one issue.

5 Results

We can now proceed to analyse the data of our experiment. We first want to highlight the main results regarding the subjects’ behaviour. Secondly, we characterise the welfare achieved by the use of QV. In the following Section we show some secondary results regarding the subjects’ behaviour and the robustness of our experimental design.

5.1 Individual behaviour

Claim 1 There is convergence towards playing equilibrium strategies.

Figure 1 depicts the fraction of subjects that play equilibrium strategies over time (averaged every 5 periods). We can see that subjects learn to play equilibrium strategies. The effect is specially relevant for the case with 2 issues where we have an equilibrium in dominant strategies. In the first period, just below a 40% of the subjects play equilibrium strategies. This contrasts with almost a 90% playing such strategies in the last period. Even in the cases with 3 and 6 issues (where equilibrium strategies are not so easy to compute) we observe an increase in the fraction of subjects playing equilibrium strategies.

![Figure 1: Fraction of subjects playing equilibrium strategies averaged every 5 periods.](image)

Table 2 shows the statistical significance of the results depicted in Figure 1. We see that the probability of playing equilibrium strategies depends positively on the square root of the period number (the coefficient is always positive and significantly different from zero at the 1% significance level). In the case with two issues, there is a high rate of convergence towards playing equilibrium strategies, and most of the convergence takes place in the initial 15 periods, where the percentage of people playing equilibrium strategies grows from 47% to 83%, reaching its maximum during periods.
42 and 43, where all subjects play the strategy predicted by the theory. The growth is much more subtle for the cases with 3 and 6 issues as can be seen by the magnitude of the estimated coefficient. In the former case, the maximum percentage was achieved in periods 20, 25 and 40 where 10 out of 18 subjects played equilibrium strategies (a 55.55%). In the latter case, the maximum was achieved in period 27 where 14 out of 18 subjects played equilibrium strategies (a 77.77%).

Driven by the fact that learning mainly occurs in the first periods we regressed the fraction of subjects playing equilibrium strategies on the square root of the period number (results are robust but slightly weaker when we consider the period number rather than its square root).

<table>
<thead>
<tr>
<th>Number of Issues</th>
<th>Var</th>
<th>Coef</th>
<th>St.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>period sqrt</td>
<td>.49</td>
<td>.056</td>
</tr>
<tr>
<td>3</td>
<td>period sqrt</td>
<td>.16</td>
<td>.044</td>
</tr>
<tr>
<td>6</td>
<td>period sqrt</td>
<td>.12</td>
<td>.041</td>
</tr>
</tbody>
</table>

Logit regression of the probability of playing equilibrium strategies on the square root of the period number.

All coefficients are significant at the 1% level.

Table 2

The convergence in the case with 2 issues is driven by the behaviour of less extreme players. The most extreme voters have a high propensity to play the equilibrium strategy (invest all votes on the most preferred issue) since the very beginning. This is captured in Figure 2 where we observe learning towards playing equilibrium strategies mainly among the subjects that have non-extreme preferences (i.e. those that value their most preferred issue 350 or 400). This effect is not observed in the 3 and 6 issues cases where there is no dominant strategy equilibrium.

![Figure 2: Relative frequency of subjects playing equilibrium strategies.](image)

The results above are robust to considering the distance between the played strategy and the one predicted by the theory instead of the frequency with which subjects play equilibrium strategies.\textsuperscript{16}

\textsuperscript{16} We have computed the distance between the cast voting profile and the one predicted by theory in two different ways: (1) we have added the absolute value of the difference between each coordinate (norm sub one); and (2) we have
5.2 Welfare

Embedded in the proposal of a new and simple voting rule is the belief that it should attain a better outcome for the parties involved in its use. In this section we compare the welfare achieved by our subjects to the one they would have obtained if they played the theoretical solution and the efficient one. The efficient welfare constitutes an upper bound and is achieved by the outcome that maximises the sum of utilities of both conflicting parties. Note that the efficient welfare does not usually coincide with the welfare achieved in equilibrium (we subsequently call this welfare the *Equilibrium Welfare*). For instance, whenever two subjects have the payments/preferences \((50,550)\) and \((250,350)\), equilibrium strategies lead to ties in both issues and a total welfare of 600. Instead, the efficient welfare requires the first (second) subject deciding on the second (first) issue and it yields a total welfare of 900. In order to frame the results we consider that our baseline welfare is the one where no agreement is reached in any decision and the mechanism simply plays a fair lottery in each issue (i.e. a welfare of 300 per subject in all cases). This is the outcome that would have obtained if players used Majority Rule. This bound coincides with the expected welfare that subjects would attain if they played randomly. That is, if they selected any of the voting profiles with equal probability independently of their announced payments.\(^{17}\)

We define the *score* that a particular voting rule (or aggregating device) achieves by normalising its achieved welfare with the upper bound imposed by the efficient welfare and considering the baseline welfare of 300. That is, given a voting rule that achieves a welfare \(W\), its score is defined as follows

\[
score := \frac{W - 300}{EfficientWelfare - 300}.
\]

**Claim 2** Subjects’ achieved score is above 80 in the case with 2 issues and just below 90 in the cases with 3 and 6 issues.

<table>
<thead>
<tr>
<th></th>
<th>2 issues</th>
<th>3 issues</th>
<th>6 issues</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Realised Welfare</strong></td>
<td>371.05</td>
<td>83</td>
<td>388.75</td>
</tr>
<tr>
<td><strong>Equilibrium Welfare</strong></td>
<td>370.66</td>
<td>82</td>
<td>396.64</td>
</tr>
<tr>
<td><strong>Efficient Welfare</strong></td>
<td>385.61</td>
<td>100</td>
<td>399.89</td>
</tr>
</tbody>
</table>

Realised welfare versus the efficient and equilibrium ones.

**Table 3**

Table 3 captures the average welfare in each of the three cases studied. The most relevant aspect computed the minimum angle between both vectors. Results are preserved under both specifications

\(^{17}\)Whenever subjects play randomly there is a probability \(p \in (0,1)\) that ties occur in any issue and there is a probability \((1-p)/2\) that any player decides in any issue. The expected payoff is

\[
\sum_{n=1...N} \left( \frac{1-p}{2} \cdot 1 + p \cdot \frac{1}{2} + \frac{1-p}{2} \cdot 0 \right) \theta_n = 300.
\]
from our experimental data is precisely that the realised welfare is almost reaching the efficient one. This is indeed a very good property of QV which, together with its simplicity as a voting rule, supports its use in real world situations. The Figure below captures the temporal welfare evolution for each of our cases.

![Figure 3: Time evolution of the aggregated welfare averaged every 5 periods.](image)

The realised welfare is always very close to the theoretical one. This is quite surprising given that we have seen in the previous Section that, even though there is learning towards playing equilibrium strategies, in the cases with 3 and 6 issues more than half of the subjects fail to play them. Moreover, the realised welfare in the case of two issues is above the equilibrium. This puzzling feature comes, in the case with 2 issues, from the fact that players that do not play equilibrium strategies play a strategy that is close to the truthful or sincere one where their vote ratio matches their relative intensities and hence allow the implementation of the efficient outcome. That is, it allows the subject with highest relative intensity to decide in each issue. In the Section below we stress some additional remarks regarding the subjects’ behaviour in the case with 2 issues and we analyse in detail the source of the surprisingly positive results for the cases with 3 and 6 issues.

### 6 Further results

#### 6.1 The case with two issues

The case with two issues is specially appealing given that preferences are essentially unidimensional. This allows us a throughout study of the subjects behaviour as well as a very clear graphic representation. Figure 4 draws attention to four specially selected subjects (called A,B,C and D). We map their payment in the first issue into their vote in that issue. In order to have meaningful strategy mappings, we aggregate each individual’s behaviour in groups of ten periods.

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18 An analogous result is found in Casella, Gelman and Palfrey (2004) in a slightly different setting.
A set of observed actions along the diagonal captures a truthful voting behaviour where the subject matches his relative intensity to the vote ratio. Instead, a step function depicts the equilibrium strategy where zero votes are invested in the first issue when the valuation is below 300 and 6 when it is above 300. Subject A represents the most commonly observed subject. He starts playing a truthful strategy but, over time, learns to play the equilibrium. Subject B, instead, plays the equilibrium strategy since the very first periods. Subject C is learning to play the equilibrium strategy (as subject A) but in the first periods he splits his votes evenly regardless of his valuation. Finally, subject D starts playing a quite random strategy and moves towards a truthful strategy.

We also want to analyse the behaviour of subjects that equally value both issues and hence are indifferent among playing any strategy. The way they play has a relevant effect in our welfare analysis: only when they split their votes they allow QV to reach the ex-ante optimal incentive compatible allocation (i.e. allows the opponent to decide on his most preferred issue). Figure 5 shows that more than half of the indifferent subjects split evenly their voting power. However, around a third of the subjects extreme their behaviour and, unfortunately, this behaviour does not change through time.

6.2 Elementary errors

In our experimental design, all issues are identical thus subjects should never invest fewer votes in an issue they value higher. Whenever the opposite occurs we say that they incur an Elementary Error. We understand the existence of such errors as a sign that subjects may not have perfectly understood the functioning of the experiment or did not figure out the rationale of our voting rule. To our relieve, we see that there were only 2% of errors and a 60% of them were made by only four
subjects. Moreover, their presence decreases over time. The table below summarises the presence of such errors by groups of ten periods.

<table>
<thead>
<tr>
<th>Nber of issues</th>
<th>Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1-10</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

Number of elementary errors by the number of issue and groups of ten periods (each cell corresponds to 180 observations).

Table 4

6.3 Alternative voting rules

In Section 5.2 we have seen that the welfare achieved by our subjects is very close from the equilibrium and efficient ones even though subjects are far from playing equilibrium strategies. We have already seen that the two issues case (where the realised welfare outperforms the equilibrium one) is mainly driven by some players deviating from equilibrium strategies and hence allowing more intense players to decide on their most preferred issue.

In order to understand the surprisingly good results in terms of welfare we compute the welfare that subjects would have obtained if they randomly chose their strategies while still keeping the condition that they always invest more votes in an issue they value higher –i.e. their behaviour does not display Elementary Errors. Table 4 shows the average welfare that players would have obtained
in case they were randomising.

<table>
<thead>
<tr>
<th></th>
<th>2 issues</th>
<th>3 issues</th>
<th>6 issues</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>score</td>
<td>score</td>
<td>score</td>
</tr>
<tr>
<td>Random-Action-Without-EE Welfare</td>
<td>360.81</td>
<td>371.44</td>
<td>368.73</td>
</tr>
<tr>
<td>Welfare</td>
<td>71</td>
<td>72</td>
<td>75</td>
</tr>
</tbody>
</table>

Averaged welfare if subjects uniformly randomise among the voting profiles without Elementary Errors. The results show the average welfare among 50 independent realisations (results remain unchanged when we increase the number of realisations).

**Table 5**

Table 5 shows the impressive good results in terms of welfare that a random action (without Elementary Errors) achieves. This brings a very supportive argument on the use of QV in Conflict Resolution situations. Subjects only need to order their voting profiles according to their preferences so that they never cast more votes in an issue they value higher. By doing so their expected welfare is above the 92% of the efficient welfare in all cases with 2, 3 or 6 issues.

Given the results above, we may also think of another rule where agents do not specify their relative intensities but simply declare their ordinal preference across issues (in case of indifference we assume that subjects play a uniform mixed strategy). Each issue would then be decided by the subject that ranks it higher.19

<table>
<thead>
<tr>
<th></th>
<th>2 issues</th>
<th>3 issues</th>
<th>6 issues</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>score</td>
<td>score</td>
<td>score</td>
</tr>
<tr>
<td>Ordinal-Preferences Welfare</td>
<td>365.28</td>
<td>381.83</td>
<td>384.79</td>
</tr>
<tr>
<td>Welfare</td>
<td>76</td>
<td>82</td>
<td>92</td>
</tr>
</tbody>
</table>

Averaged welfare if subjects only declared their ordinal preference.

**Table 6**

Following the results we observed in Table 5, Table 6 shows the good results that subjects achieve if they sincerely declared their ordinal ranking. The welfare our agents achieve under this latter rule is still below the equilibrium welfare but, in the case with 6 issues, it is almost the same. This occurs given that the set of ordered preferences in the case of six issues is much richer than the set of voting profiles with 6 votes and six issues (720 versus 462).

Note also, that the similarity in welfare between the ordered and equilibrium cases is mainly driven by the uniform distribution across preference profiles; the increased freedom that QV induces in the available voting strategies implies that voters with more polarised preferences would be able to better express their preferences and hence may see their welfare improve more dramatically.

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19 This mechanism has a flavour of the scoring rule called Borda Count (where agents give one point to their least preferred option, 2 points to the second least preferred, etc.). The differences between a scoring and QV are highlighted in footnote 2.
6.4 Best response to observed behaviour

Finally, we have checked whether our subjects’ behaviour could be explained by best responding the observed behaviour of their opponents. As a way to approach such behaviour we have computed the times that each ordered voting profile was played in each set of ten periods. We have then computed the best response to such frequency of play by considering all possible permutations of each voting profile (so that each issue is symmetrically treated and the exposition of all possible strategies can be handled easily).

The case with two issues is trivial given that there is a unique dominant strategy and hence the best response to any observed behaviour is always the one prescribed by our equilibrium. Therefore, hereafter we focus on the cases with 3 and 6 issues.

The following table lists the frequencies of each voting profile in the case with 3 issues.

<table>
<thead>
<tr>
<th>Periods</th>
<th>Played Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(222) (321) (330) (410) (420) (510) (600)</td>
</tr>
<tr>
<td>1-10</td>
<td>12 54 26 6 50 18 14</td>
</tr>
<tr>
<td>11-20</td>
<td>6 20 32 13 38 40 31</td>
</tr>
<tr>
<td>21-30</td>
<td>3 27 22 19 42 35 32</td>
</tr>
<tr>
<td>31-40</td>
<td>9 23 18 9 43 42 36</td>
</tr>
<tr>
<td>41-50</td>
<td>11 12 13 15 49 49 31</td>
</tr>
</tbody>
</table>

Frequency of play every ten periods in the case with 3 issues.

<table>
<thead>
<tr>
<th>periods</th>
<th>payments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>200 250 250 300 300 300 350 350 400 400 450 500</td>
</tr>
<tr>
<td>1-10</td>
<td>(330) (330) (330) (420) (330) (420) (420) (510) (420) (510) (600)</td>
</tr>
<tr>
<td>11-20</td>
<td>(222) (411) (330) (411) (420) (420) (510) (420) (600) (510) (600)</td>
</tr>
<tr>
<td>21-30</td>
<td>(222) (321) (330) (411) (420) (420) (510) (420) (600) (510) (600)</td>
</tr>
<tr>
<td>31-40</td>
<td>(222) (321) (330) (411) (420) (330) (510) (420) (600) (510) (600)</td>
</tr>
<tr>
<td>41-50</td>
<td>(222) (321) (330) (411) (420) (330) (510) (510) (600) (510) (600)</td>
</tr>
</tbody>
</table>

Best response to the accumulated observed behaviour every ten periods in the case with 3 issues.

Each column represents each type of player and the values inside the table indicate the best response.

Table 8

We have checked the robustness of our predictions by considering different specifications where we considered sets of 5 or 1 period(s). Similarly, we have also considered the accumulated frequencies since the first period and all results remain unchanged.
The table shows that the best response to the observed behaviour in the case with 3 issues converges to the equilibrium one for each player type. Note that the only changes we observe in terms of the optimal best response to the observed behaviour is that the players that are most indifferent between the issues diversify their voting power over time and, instead, the most extreme players, intensify it over time. Both moves respond to the fact that the played actions show a slight movement towards intensifying the voting power (that is, the voting profiles to the right of the first matrix of values seem to increase their preference over time). In the light of these results, we can refer the analysis of the best response to the observed behaviour to the one undertaken in Section 5.1. We do not write the corresponding table for the case with 6 issues because the best response to the observed behaviour in each group of ten periods coincides with the equilibrium strategy (see Section 4).

Once again, these results shows the robustness of our experimental design and the fact that the behaviour we used as our benchmark one does not only constitute an equilibrium but it also best approaches the best response to the subjects behaviour (even when they are far from playing such strategy). Finally we conclude that the subjects’ behaviour cannot be better explained by best responding their opponents’ actions rather than by following the equilibrium predictions.

7 Conclusion

This paper is the first contribution to the experimental analysis of the voting rule QV. We have centered our analysis in the case of conflict resolution situations. That is, situations where two parties need to decide over N divergent issues. In that case QV shows all its strength allowing voters to trade-off their voting power hence be more decisive on those issues they relatively care more about. The experiment shows convergence towards equilibrium behaviour.

Moreover, the welfare obtained is very close from the efficient one regardless of the existence of many subjects not playing according to equilibrium predictions. These results lead to a very positive judgement of QV in the sense that, even when they do not play according to what is predicted by the theory, the accrued welfare is very close from the efficient one.
References


INSTRUCCIONES

Gracias por participar en este experimento y contribuir con un proyecto de investigación para el Departamento de Economía. La cantidad de dinero que ganes en el juego se te pagará en privado al finalizar el experimento. Desde ahora y hasta el final del experimento no está permitido comunicarse con los otros participantes. Si tienes alguna pregunta, por favor levanta la mano y uno de los instructores contestará a tus preguntas en privado. ¡Por favor, no preguntes en voz alta!

Este experimento consiste de 50 periodos. Las reglas son las mismas para todos los participantes y en todos los periodos. Al principio de cada periodo se te asignará aleatoriamente un participante con el que interactuarás. Ninguno de vosotros sabrá quien es el participante con el que interactúa en cada periodo.

Tú y el otro participante votaréis simultáneamente sobre un grupo de tres cuestiones. Cada cuestión tiene tres posibles resultados: 1) la ganas tú y él la pierde; 2) la pierdes tú y él la gana; y finalmente, 3) empatás. El resultado de estas votaciones determinarán los beneficios que tú y el otro participante tendréis en cada periodo. Recuerda que el participante con el que interactuarás está decidido aleatoriamente en cada periodo.

1. Información al principio de cada periodo

Al principio de cada periodo se te indicarán tus “valoraciones” por cada cuestión. Sólo conocerás tus valoraciones. La valoración de cada cuestión indica cuánto puedes ganar en caso de que ganes esa cuestión. Estas valoraciones están expresadas en términos de céntimos de euro.

Las valoraciones posibles para las tres cuestiones son las de la siguiente tabla, con todas sus posibles permutaciones; es decir, si miramos la primera fila, también es posible que, por ejemplo, las valoraciones de las Cuestiones 1, 2 y 3 sean 50, 500 y 50 respectivamente, en vez de 500, 50 y 50. Como ves las valoraciones son múltiplos de 50, y la suma de ellas siempre es 600.

<table>
<thead>
<tr>
<th>Cuestión 1</th>
<th>Cuestión 2</th>
<th>Cuestión 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>450</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>400</td>
<td>150</td>
<td>50</td>
</tr>
<tr>
<td>400</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>350</td>
<td>200</td>
<td>50</td>
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<tr>
<td>350</td>
<td>150</td>
<td>100</td>
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<td>300</td>
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<td>300</td>
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<tr>
<td>250</td>
<td>250</td>
<td>100</td>
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<tr>
<td>250</td>
<td>200</td>
<td>150</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>
Las valoraciones de cada uno de los participantes han sido elegidas de forma aleatoria e independiente por el ordenador. Todas las posibles combinaciones de valoraciones tienen la misma probabilidad. Las distribuciones de las valoraciones de cada participante no tienen porque ser iguales; es más, normalmente no lo serán.

2. Votación

En cada periodo tendrás 6 votos que deberás distribuir entre las distintas cuestiones. Después de hacerlo deberás pulsar el botón “OK”. El participante con el que estás emparejado en cada periodo tiene la misma cantidad de votos.

3. Resultado de la votación

El resultado de cada votación se determinará según la siguiente regla: si el número de votos que tú has asignado a una cuestión es

- … mayor que el nº de votos del otro votante, ganarás esa cuestión.
- … menor que el nº de votos del otro votante, perderás esa cuestión.
- … igual que el nº de votos del otro votante, empatarás.

Por ejemplo, si votáis de la siguiente forma:

<table>
<thead>
<tr>
<th>Cuestión 1</th>
<th>Cuestión 2</th>
<th>Cuestión 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tus votos</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Sus votos</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

Ganarás la cuestión 1, ya que has puesto más votos (3) que él (0) en esa cuestión; perderás la cuestión 2 ya que has puesto menos votos (1) que él (4); y empatarás la cuestión 3 ya habéis puesto el mismo número de votos (2) en esa cuestión.

4. Beneficios por periodo

En cada periodo, tus beneficios serán igual a la suma de las valoraciones de todas las cuestiones que hayas ganado más la mitad de las que hayas empatado. Si, por ejemplo, tus valoraciones fueran 350, 100 y 150 para las cuestiones 1, 2 y 3 respectivamente, y tu y el participante con el que interactúas votarais como en el ejemplo del apartado 3, tus beneficios en este grupo de cuestiones serían la suma de: 350 por la cuestión 1, 0 por la cuestión 2, y 75 (la mitad de 150) por la cuestión 3. Como no sabes las valoraciones del otro, no sabrás los beneficios del otro.
5. Información al final de cada periodo

Al final de cada periodo, como puedes ver en el dibujo anterior, recibirás la siguiente información:

- Tus pagos en cada cuestión
- Tus votos en cada cuestión
- Los votos en cada cuestión del participante con quien interactúas
- Las cuestiones que tú ganas, empatas y pierdes
- Tus beneficios

6. Pagos finales

Después del último periodo, el ordenador seleccionará al azar tres periodos y se te pagará la suma de los beneficios que hayas obtenido en estos tres periodos elegidos. Adicionalmente, recibirás 3 euros por haber participado en el experimento.
7. Cuestionario

1. Haz un círculo por la respuesta correcta. Cuando tienes que votar...

- ¿Conoces tus valoraciones? SI  NO
- ¿Conoces las valoraciones del otro? SI  NO
- ¿Tus valoraciones y sus valoraciones pueden ser distintas? SI  NO
- ¿Estás informado de la identidad del participante con el que estás emparejado? SI  NO

Imagina que tienes las siguientes valoraciones, y que tú y el otro participante con el que estás emparejado votáis como la siguiente tabla indica:

<table>
<thead>
<tr>
<th></th>
<th>Cuestión 1</th>
<th>Cuestión 2</th>
<th>Cuestión 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tus valoraciones</td>
<td>150</td>
<td>250</td>
<td>200</td>
</tr>
<tr>
<td>Tus votos</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Sus votos</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

2. ¿Quién gana la cuestión 1?   TU    ÉL    EMPATÁIS
3. ¿Quién gana la cuestión 2?   TU    ÉL    EMPATÁIS
4. ¿Quién gana la cuestión 3?   TU    ÉL    EMPATÁIS

5. ¿Cuánto ganas por la cuestión 1?  ___________
6. ¿Cuánto ganas por la cuestión 2?  ___________
7. ¿Cuánto ganas por la cuestión 3?  ___________

8. ¿Cuál es tu beneficio en este periodo?  ___________

9. ¿Cuántos periodos determinarán tus beneficios?

10. Las valoraciones (50, 500, 50), (500, 50, 50), (200, 200, 200) y (200, 250, 150) tienen la misma probabilidad.
    - Cierto
    - Falso

11. En todos los periodos estás emparejado con la misma persona.
    - Cierto
    - Falso

12. Estás informado de la identidad del participante con el que estás emparejado.
    - Cierto
    - Falso