1. Documentation explaining the calculations for the example in Section 8 (how ERNs would have worked in the crisis) can be found at: http://bit.ly/1QBcg4e

2. A copy of
Bulow, J. and Klemperer, P. (2013). `Market-based bank capital requirements', September 2013 Nuffield College Working Paper 2013-W12, can be found at: www.nuff.ox.ac.uk/users/klemperer/mbbcr.pdf

3. A copy of
Bulow, J. and Klemperer, P. (2014). `Equity recourse notes: creating counter-cyclical bank capital', CEPR Discussion Paper 10213, can be found at: www.nuff.ox.ac.uk/users/klemperer/DP10213.pdf

4. The remainder of this document contains notes by Dr. Antoine Lallour proving the results of appendices C and D by working directly with Appendix A's formula for equity holders' expected value. We are very grateful to Dr. Lallour for these.

**Proof of proposition 1**

This proof computes the shareholders' payoff without and with a new ERN issuance. It then assumes, for the sake of contradiction, that the issuance leaves them worse off. A consequence is that all old ERN holders are also worse off – and thus that value has been destroyed (a contradiction).

Let $V$ denote the state of the world in which the value of the firm, keeping its current funding structure, is $V$.

Keeping the current funding structure, the payoff to shareholders in state $V$ is given by $V_0(V)$ as defined in appendix A.

Let $H_0(V) = V_0((1+x)V)$ denote the payoff to current shareholders in state $V$ if all components of the current balance sheet (funding and assets) are proportionally increased by a fraction $x$. $H_0(V)$and $V_0(V)$are both continuous. $H_0(V)$is everywhere steeper than $V_0(V)$and greater, except at $V = 0$ where they are equal. Thus, for any positive constant $y$, the function $H_0(V-y)$would on its domain either (i) always lie below $V_0(V)$ or (ii) cross $V_0(V)$ only once, from below (i.e. with a steeper slope at the point where they are equal). Another way to express this single-crossing property is to say that their difference $D$ is such that for all $\tilde{v} \geq v$, $D(v) > 0 \Rightarrow D(\tilde{v}) > 0$.

Thus, this property holds in particular for $\tilde{G}_0(V) := H_0(V - D_{N+1})$ where $D_{N+1}$ is the face value of the new ERN.

Since $\frac{1}{\sum_{i=0}^{N+1} S_i}((1+x)V - \sum_{i=j}^{N+1} D_i) \leq V_0(V)$ for all V, this property also holds for

$$G_0(V) = max\left\{\tilde{G}_0(V), \frac{1}{\sum_{i=0}^{N+1} S_i}((1+x)V - \sum_{i=j}^{N+1} D_i)\right\}$$

i.e. this function of V (whose domain is nonnegative real numbers) either (i) always lies below $V_0(V)$ or (ii) crosses $V_0(V)$ only once, from below.

Assume shareholders are worse off with the new issuance than without. This means that

$E[G_0(V) - F_0(V)] < 0$. Because of the single-crossing property, $E[G_0(V) - F_0(V)|V \leq \alpha] < 0$ for all $\alpha$.

We now prove that all old ERN holders are worse off.

Let $G_K(V) := \min(D_K, S_K. G_0(V))$ and $F_K(V) := \min(D_K, S_K. F_0(V))$. Let $V^*$denote the point where $G_0$ and $F_0$ cross. And let $T = G_0(V^*) = F_0(V^*)$. There are two cases.

<u>Case 1</u>: $D_K > T$. In this case, there exist two states $V_1$ and $V_2$ such that $S_K. G_0(V_1) = D_K$ and $S_K. F_0(V_2) = D_K$ and such that we can write:

$$E[G_K(V) - F_K(V)] = S_K. P(V \leq V_1). E[G_0(V) - F_0(V)|V \leq V_1] +$$

$$P(V_1 \leq V \leq V_2). E[S_K. G_0(V) - D_K|V_1 \leq V \leq V_2] +$$

$$P(V > V_2). E[D_K - D_K|V > V_2]$$

The second term is negative (the function is negative point by point). The first one is negative as well (by the property established earlier). Thus, K-ERN holders are worse off.

<u>Case 2</u>: $D_K \leq T$. In this case there is a state $V_1$ such that $S_K. G_0(V_1) = D_K$, and such that we can write:

$$E[G_K(V) - F_K(V)] = P(V \leq V_1). E[\min(S_K. G_0(V), D_K) - F_0(V)|V \leq V_1] +$$

$$P(V > V_1). E[D_K - D_K|V > V_1]$$

$$\leq P(V \leq V_1). S_K. E[G_0(V) - F_0(V)|V \leq V_1] < 0$$

Thus, all current stakeholders would be worse off. But this is clearly a contradiction.

This finishes the proof of proposition 1.

**How much are the gains for current equity holders?**

Using the notation and setup of appendix D, the expected value of equity is given by the following expression:

$$W(x) = S(\infty). E\left[\max\left\{\max_j \frac{1}{\sum_{i=0}^{j} S_i}(\tilde{V}\theta - \sum_{i=j+1}^{N} D_i \; - \; D_{N+1}), \frac{1}{\sum_{i=0}^{N+1} S_i}\tilde{V}\theta\right\}\right]$$

where $\tilde{V} = V + mx$, $D_{N+1} = x$, and $S_{N+1} = \frac{x}{p(x)}$ with for instance $p(x) = 25\%. \frac{W(x)}{S(\infty)}$.

Our goal is to compute the derivative of this expression with respect to $x$, at $x = 0$.

The Leibniz rule implies that our result is the expectation (varying $\theta$) of

$S(\infty).\frac{d}{dx}\max\left\{\max_j \frac{1}{\sum_{i=0}^{j} S_i}(\tilde{V}\theta - \sum_{i=j+1}^{N} D_i \ - \ D_{N+1}), \frac{1}{\sum_{i=0}^{N+1} S_i}\tilde{V}\theta\right\}$ at x=0 (*)

The Leibniz rule applies if there exists an integrable function bounding the absolute value of (*). The expectation of $\theta$ is finite. For now, assume that the function (*) is continuously differentiable at *x*=0 and that it can be bounded by a function proportional to $\theta$.

This problem is similar to finding

$\frac{d}{dx}\max\{f(x,\theta), g(x,\theta)\}$ at x=0, for *f* and *g* continuous and differentiable.

The solution of that secondary problem is $\frac{d}{dx}f(x,\theta)$ if $f(0,\theta) > g(0,\theta)$;

$\frac{d}{dx}g(x,\theta)$ if $g(0,\theta) > f(0,\theta)$;

and whichever of the previous two results is greater, if $f(0,\theta) = g(0,\theta)$.

We can now solve our original problem.

We compute the derivative of the first term in (*) (using the secondary problem repeatedly):

$$S(\infty).\frac{d}{dx}\max_j\left\{\frac{1}{\sum_{i=0}^{j} S_i}(\tilde{V}\theta - \sum_{i=j+1}^{N} D_i \ - \ D_{N+1})\right\} = \frac{S(\infty)}{S(\theta)}[m\theta - 1]$$

Second, we compute the derivative of the second term in (*)

$$S(\infty).\frac{d}{dx}\frac{1}{\sum_{i=0}^{N+1} S_i}\tilde{V}\theta = S(\infty).\frac{d}{dx}\frac{1}{S(0) + {}^{x}/_{p(x)}}(V + mx)\theta = \frac{S(\infty)}{S(0)}\left(m\theta - \frac{V\theta}{p.S(o)}\right)$$

where $p(x)$ is the conversion price. As noted earlier, this price could in theory depend on $x$ – for instance if regulators require that it be set equal to 25% of the current share price. In the right-hand side expression, $p := p(0)$.

Finally, we use the solution of the second problem again and we find that the expression in (*) is equal to

$\frac{S(\infty)}{S(\theta)}[m\theta - 1]$ if $\theta > \hat{\theta}$ ; and

$\frac{S(\infty)}{S(0)}\left(m\theta - \frac{V\theta}{p.S(o)}\right)$ otherwise. This is exactly what appendix D claims.

It is then easy at this point to check that (*) is indeed continuously differentiable at *x*=0 and that it can be bounded by a function proportional to $\theta$, thus completing the proof.