Free entry and social inefficiency

N. Gregory Mankiw*

and

Michael D. Whinston**

Previous articles have noted the possibility of socially inefficient levels of entry in markets in which firms must incur fixed set-up costs upon entry. This article identifies the fundamental and intuitive forces that lie behind these entry biases. If an entrant causes incumbent firms to reduce output, entry is more desirable to the entrant than it is to society. There is therefore a tendency toward excessive entry in homogeneous product markets. The roles of product diversity and the integer constraint on the number of firms are also examined.

1. Introduction

- Economists typically presume that free entry is desirable for social efficiency. As several articles have shown, however, when firms must incur fixed set-up costs upon entry, the number of firms entering a market need not equal the socially desirable number. Spence (1976a) and Dixit and Stiglitz (1977), for example, demonstrate that in a monopolistically competitive market, free entry can result in too little entry relative to the social optimum. In more recent work von Weizsäcker (1980) and Perry (1984) point to a tendency for excessive entry in homogeneous product markets. Nevertheless, despite these findings, many economists continue to hold the presumption that free entry is desirable, in part, it seems, because the fundamental economic forces underlying these various entry biases remain somewhat mysterious.

Our goal is to provide a simple, yet general, exposition of the conditions under which the number of entrants in a free-entry equilibrium is excessive, insufficient, or optimal. Our analysis compares the number of firms that enter a market when there is free entry with the number that would be desired by a social planner who is unable to control the behavior of firms once they are in the market. That is, we consider the second-best problem of choosing the welfare-maximizing number of firms, while we take as given their noncompetitive behavior after entry.

We demonstrate that the crucial conditions for establishing the presence of an entry bias can be stated quite simply in terms of the outcome of the postentry game played by entrants. In contrast to previous work, we do not need to model this postentry game explicitly.

* Harvard University and National Bureau of Economic Research.
** Harvard University.

We would like to thank Franklin Fisher, Alvin Kleovorick, Andreu Mas-Colell, John Riley, Richard Schmalensee, Lawrence Summers, and two referees for helpful comments on earlier drafts of this article.
This approach has two advantages. First, it uncovers the fundamental and intuitive reasons behind the presence of entry biases. Second, it provides a set of properties that can be readily checked for any particular application.

We focus extensively on the central roles played by two aspects of the postentry game in producing an entry bias: imperfect competition and what we call the “business-stealing effect.” The business-stealing effect exists when the equilibrium strategic response of existing firms to new entry results in their having a lower volume of sales—that is, when a new entrant “steals business” from incumbent firms. Put differently, a business-stealing effect is present if the equilibrium output per firm declines as the number of firms grows. Intuitively, it would seem that most markets would be characterized by such an effect.\(^1\) As we shall demonstrate, in the presence of imperfect competition (so that firms do not act as price-takers after entry), the business-stealing effect is a critical determinant of the direction of entry bias.

After formally specifying our model in Section 2, we begin our analysis in Section 3 by considering a homogeneous product market. Ignoring the integer constraint on the number of firms (as has been done in most of the previous literature), we demonstrate that if the postentry price exceeds marginal cost and if a business-stealing effect exists, then free entry leads to excessive entry from a social standpoint (Proposition 1). Intuitively, business stealing by a marginal entrant drives a wedge between the entrant’s evaluation of the desirability of his entry and the planner’s: the marginal entrant’s contribution to social surplus is (except for second-order effects) equal to his profits less the social value of the output lost owing to the output restriction he engenders in other firms. The business-stealing effect therefore makes entry more attractive than is socially warranted.\(^2\) We show by example that the resulting bias can, in fact, be dramatic: the equilibrium number of firms can exceed the socially optimal number by a very large margin.

We then consider the role of the integer constraint on the number of firms. In Proposition 2 we prove the fully rigorous result: Under the conditions described above, the free-entry number of firms can be less than the welfare-maximizing number, but not by more than one firm. This result relates to the common observation that a monopolist does not capture all of the social surplus generated by his product. It is interesting that even in oligopolistic (homogeneous product) markets, this consideration cannot bias entry by more than one firm. Of course, this finding does not imply that the welfare losses associated with such occurrences are unimportant; as we show by example, they can, in fact, be substantial.

In Section 4 we examine the implications of product diversity for the nature of entry biases. We show that product differentiation can reverse the tendency toward excessive entry that is found in homogeneous product markets. Intuitively, the presence of heterogeneity introduces another factor that biases entry: the marginal entrant creates surplus by increasing product variety that he does not capture as profits. The product diversity and business-stealing effects thus work in opposite directions. With the functional form (constant elasticity of substitution) and equilibrium notion used by Spence (1976a), this new effect always dominates the business-stealing effect, and the result is insufficient entry. This need not be the case, however, and we give examples in which the contrary result holds with the business-stealing effect always dominating the product diversity effect.

The results of Sections 3 and 4 indicate that when firms must incur fixed set-up costs, the regulation of entry is often desirable. In Section 5 we explore the limiting case in which the fixed cost approaches zero. Simple examples illustrate that the entry bias may get infinitely

---

\(^1\) Seade (1980) examines the conditions under which business stealing does and does not characterize markets operating under conjectural variation equilibrium. Business stealing also relates to the concept of “strategic substitutes” discussed by Bulow, Geanakoplos, and Klemperer (1985).

\(^2\) Spence (1976b, p. 410) suggests the importance of such firm interaction for the social evaluation of entry.
large as the fixed cost grows small: the difference between the equilibrium and optimal number of firms may approach infinity. Nonetheless, we prove that the welfare loss caused by having the wrong number of firms approaches zero if firms act approximately as price-takers as their numbers grow large. Thus, the regulation of entry becomes unnecessary as an industry comes close to the ideal of no fixed costs.

Finally, Section 6 provides concluding remarks.

2. The model

We wish to compare the number of firms that enter a market in a "free-entry equilibrium" with the number that a welfare-maximizing social planner would desire. We model the entry process as a two-stage game. In the first stage a large (infinite) number of identical potential entrants exists, each of whom must decide whether to enter the industry. Should a firm decide to enter, it must incur set-up costs of $K$. Stage two is the production period. In this stage those firms that have entered and have sunk the set-up costs play some oligopoly game, the details of which we do not need to specify. Each entrant possesses a technology given by the cost function $c(q)$, where $c(\cdot)$ is continuous, $c(0) = 0$, $c'(\cdot) \geq 0$, and $c''(\cdot) \geq 0$ for all $q \geq 0$. We assume that the equilibrium in this stage is symmetric, and we define $q_N$ to be the equilibrium output per firm, given that $N$ firms have entered the industry. Typically, $q_N$ is not the competitive output level, because the firms that enter do not act as price-takers, but rather behave strategically. We also define $\pi_N$ as the equilibrium profits per firm, given that $N$ firms have entered.

We denote the free-entry equilibrium number of firms by $N^e$. The necessary and sufficient conditions for $N^e$ to be the free-entry equilibrium number of firms are:

(i) $\pi_{N^e} \geq 0$ and (ii) $\pi_{N^e+1} < 0$. Condition (i) ensures that no firm that has entered would have been better off not entering. Condition (ii) requires that no firm that has not entered would have found it worthwhile to have entered.

We consider the second-best problem faced by a social planner who can control the number of firms that enter the industry but not their oligopolistic behavior once they have entered. As Scherer (1980, p. 525) notes, "Oligopolists refraining from price competition merely because they recognize the likelihood of rival retaliation do not violate the law as long as their decisions are taken independently." Thus, we assume that the social planner cannot affect the market outcome for any given number of firms. In particular, the planner cannot ensure that the active firms behave as price-takers.

Finally, we further suppose that the objective of the planner is to select the number of firms that maximizes the total surplus in the market.

3. Homogeneous product markets

We begin our analysis with consideration of a simple homogeneous product market. Denote the inverse market demand function for the product by $P(Q)$, where $Q$ is aggregate output in the market, and assume that $P'(Q) < 0$ at all $Q$. Given this inverse demand function, we can then define the equilibrium profits per firm when $N$ firms have entered, $\pi_N$, by $\pi_N = P(Nq_N) - c(q_N) - K$ and the socially optimal number of firms, $N^*$, as the number of firms that solves:

$$\max_N W(N) = \int_0^{N^*} P(s)ds - Nc(q_N) - NK.$$

---

3 $N^*$ need not, in general, be unique. Our assumptions below, however, guarantee its uniqueness.

4 We assume that firms do, in fact, enter when they are indifferent about entering.

5 Throughout we assume that a partial equilibrium approach is justified, that is, that income effects can be ignored.
Our basic result characterizes the relationship between \( N^e \) (the free-entry equilibrium number of firms) and \( N^* \) (the socially optimal number of firms) for postentry behavior that satisfies the following three assumptions.

**Assumption 1.** \( Nq_N > \bar{N}q_{\bar{N}} \) for all \( N > \bar{N} \) and \( \lim_{N \to \infty} Nq_N = M < \infty \).

**Assumption 2.** \( q_N < q_{\bar{N}} \) for all \( N > \bar{N} \).

**Assumption 3.** \( P(Nq_N) - c(q_N) \geq 0 \) for all \( N \).

Assumption 1 simply says that (postentry) equilibrium aggregate output rises with the number of firms entering the industry and approaches some finite bound. This assumption guarantees that the free-entry equilibrium number of firms \( (N^e) \) is well defined. Assumption 2 requires that output per firm fall as the number of firms in the industry increases, that is, that a business-stealing effect is present. Finally, Assumption 3 requires that for any number of entrants, the resulting equilibrium price is not below marginal cost.

We begin our analysis by ignoring the integer constraint on the number of firms. Although we do this primarily for expositional purposes, this simplification also facilitates comparisons with previous results in the literature. Thus, we assume that the free-entry equilibrium number of firms exactly satisfies the zero-profit condition,

\[
\pi_{N^e} = 0,
\]

while the socially optimal number of firms exactly satisfies the first-order condition,

\[
W'(N^*) = 0.
\]

In what follows we also assume that \( q_N \) is a differentiable function of \( N \).

**Proposition 1.** Suppose that Assumptions 1–3 hold. Then, when we ignore the integer constraint, the free-entry equilibrium number of firms is not less than the socially optimal number; that is, \( N^e \geq N^* \). Furthermore, if the inequality in Assumption 3 holds strictly, then the equilibrium number of firms strictly exceeds the optimal number, that is, \( N^e > N^* \).

**Proof:** Differentiating the expression for social surplus, \( W(N) \), with respect to the number of firms yields for all \( N \):

\[
W'(N) = P(Nq_N) \left[ N \frac{dq_N}{dN} + q_N \right] - c(q_N) - Nc'(q_N) \frac{dq_N}{dN} - K.
\]

Rearranging terms and recalling the expression for equilibrium profits per firm, \( \pi_N \), yield

\[
W'(N) = \pi_N + N[P(Nq_N) - c'(q_N)] \frac{dq_N}{dN}
\]

for all \( N \). Under Assumptions 2 and 3, the second term on the right-hand side of equation (2) is nonpositive, and it is strictly negative if the inequality in Assumption 3 holds strictly. Therefore, for all \( N \), \( W'(N) \leq \pi_N \). Thus, \( \pi_{N^*} \geq 0 \).

We can now complete the argument by showing that profits per firm fall as \( N \) increases, since then \( \pi_{N^e} = 0 \) implies that \( N^e \geq N^* \). The derivative of \( \pi_N \) with respect to \( N \) is given by

\[
\frac{\partial \pi_N}{\partial N} = \left[ P(Nq_N) - c'(q_N) \right] \frac{dq_N}{dN} + q_N P'(Nq_N) \frac{d(Nq_N)}{dN},
\]

which is negative under our assumptions. Thus, \( N^e \geq N^* \).

Finally, note that if the inequality in Assumption 3 holds strictly, then \( \pi_{N^*} > 0 \), which implies that \( N^e > N^* \). *Q.E.D.*
Equation (2) helps to provide the intuition for this result. The (first-order) change in social welfare attributable to a marginal entrant is composed of two terms. First, the new entrant contributes directly to social surplus through his profits. Second, the entrant causes all existing firms to contract their output levels, which results in an aggregate contraction equal to \( \frac{\partial q_N}{\partial N} \). This causes a reduction in social surplus of
\[
[P(Nq_N - c'(q_N))]N(\frac{\partial q_N}{\partial N} \cdot N).
\]
Thus, the presence of the business-stealing effect drives a wedge between the marginal entrant's evaluation of the desirability of entry and the social planner's. If entry does not change the output of existing firms, then equation (2) reduces to
\[
W'(N) = \pi_N \quad \text{for all } N,
\]
and the level of entry resulting from a situation of free entry is socially efficient.

As a final remark on Proposition 1, note that if firms act as price-takers after entry, then (again when we ignore the integer constraint) free entry results in exactly the socially efficient number of firms, despite the presence of a business-stealing effect. The reason is that the output contraction caused by a marginal entrant no longer has any net social value. We summarize this in the following corollary.

**Corollary 1.** Suppose that Assumptions 1 and 2 hold and that \( P(Nq_N) - c'(q_N) = 0 \) for all \( N \). Then, when we ignore the integer constraint on the number of firms, the free-entry number of firms exactly equals the socially efficient number (i.e., \( N^* = N^* \)).

Proposition 1 provides very general and intuitive conditions under which free entry leads to excessive entry. Furthermore, as the following examples demonstrate, the bias toward excessive entry can be extremely large.

**Example 1.** Consider a linear market structure in which firms behave as Cournot oligopolists in the production period. That is, suppose that the inverse demand function is given by \( P(Q) = a - bQ \) and that \( c(q) = cq \). It is easy to show that the equilibrium output per firm is given by:
\[
q_N = \left( \frac{1}{N + 1} \right) \left( \frac{a - c}{b} \right).
\]

Using this fact, one can show that the free-entry equilibrium number of firms, \( N^e \), satisfies (when we ignore the integer constraint):
\[
(N^e + 1)^2 = \frac{(a - c)^2}{bK},
\]
while the socially optimal number of firms, \( N^* \), satisfies (when we again ignore the integer constraint):
\[
(N^* + 1)^3 = \frac{(a - c)^2}{bK}.
\]

Thus, in a linear market, if the socially optimal number of firms is three, then seven firms actually enter the market under free entry; if the social optimum has five firms, then thirteen firms enter; and if the social optimum has eight firms, then twenty-six firms enter. Clearly, the bias toward excessive entry under free entry can be very large, although, of course, the

---

6 Note that the second-best social optimum here is first-best.

7 Note, though, that if the inequality in Assumption 2 is reversed—that is, if entry is "business augmenting"—then free entry results in an insufficient level of entry.
difference in the number of firms is not the correct measure of the welfare loss. In this market the government would achieve a welfare improvement by raising the private cost of entry through an entry tax (e.g., a licensing fee). Furthermore, if the government found that artificial restrictions on entry were present in this market, it is possible that their removal would lead to a welfare loss.

**Example 2.** Though it is perhaps easiest to think of \( q_N \) as resulting from noncooperative behavior, our result does not depend upon any such assumption. For example, suppose that in the linear market described above, firms that enter the industry behave as a cartel rather than as Cournot oligopolists. In this case we would have \( Nq_N = (a - c)/2b \) (the monopoly output) for all \( N \). Since aggregate output is invariant to the number of firms in the industry, if the market should exist at all, the social optimum is to have only one firm (so as to pay \( K \) only once). Yet, if the number of firms is continuous, firms enter until all of the collusive monopoly profits are dissipated into set-up costs. The welfare losses caused by free entry in this example are similar to those Posner (1975) describes in his discussion of competition for monopoly positions. In both cases rent-seeking turns monopoly profits into deadweight social losses.

Proposition 1 and the previous two examples indicate that the presence of a business-stealing effect creates a strong bias toward excessive entry. The avoidance, up to this point, of the integer constraint on the number of firms, however, is not satisfactory. In Example 2, for instance, consideration of integer constraints reveals that the level of entry is actually insufficient whenever \( (a - c)^2/4bK \in [2/3, 1] \). In these cases, no firm enters the industry, even though a monopoly is the socially optimal outcome. This occurs because, as is well known, a monopolist does not capture the full social surplus generated by his entry.

The following result explicitly takes account of the integer constraint.

**Proposition 2.** Suppose that Assumptions 1–3 hold. Then \( N^* \geq (N^* - 1) \).

**Proof:** See the Appendix.

Proposition 2 modifies the very strong result of Proposition 1. In particular, in the presence of a business-stealing effect, entry may be insufficient, although never by more than one firm. Although Proposition 2 and our previous examples still suggest a tendency toward excessive entry in homogeneous markets, it is important to recognize that the welfare losses in cases of insufficient entry can be substantial. The following example makes this clear.

**Example 3.** Consider a linear market as above in which a single firm acts as a monopolist but two firms act as Bertrand competitors. For any value of \( K \), duopolists earn negative profits. If \( K \leq (a - c)^2/4b \), a monopolist earns nonnegative profits: hence, \( N^* = 1 \). When

---

8 It is interesting that in this example the welfare loss due to free entry declines as the socially optimal number of firms increases. As a percentage of the total area between the demand and marginal cost curves, the loss is 7.8%, 5.2%, and 3.8% for the three cases. We say more about this below in Section 5.

9 Of course, the relevance of the social planning problem we consider depends upon the planner's having no control over this behavior. Some industries, however, seem to behave collusively without any explicit conspiratorial behavior that can be attacked in the courts.

10 In this example free entry results in a 50% welfare loss (again, measured as a percentage of the total area between the demand and marginal cost curves) as \( K \) grows small.

11 It is easy to verify that in Example 2 excessive entry occurs for all parameter values such that \( (a - c)^2/4bK \geq 2 \).

12 Perry (1984) performs numerical simulations assuming constant elasticity cost and demand curves and Cournot equilibrium (his Table 1). His results suggest that the integer constraint is important only if the number of firms in the free-entry equilibrium is small, that is, one or two firms.
$K \leq (a - c)^2 / 8b$, however, duopoly is socially optimal. As $K$ becomes small, the social loss due to one too few firms approaches 25% of the potential surplus in the market (the area between the demand curve and the marginal cost curve).

4. Product diversity

We now explore how the presence of product diversity affects the direction of the entry bias identified in Section 3. Following Spence (1976a), we specify gross consumer benefits to be of the form

$$G[\sum_{i=1}^{\infty} f(q^i)],$$

where $q^i$ is firm $i$'s output level, $f(0) = 0$, $f'(\cdot) > 0$, and $f''(\cdot) \leq 0$ for all $q \geq 0$, and $G'(z) > 0$ and $G''(z) < 0$ for all $z \geq 0$. These assumptions imply that consumers prefer variety and that the outputs of various firms are substitutes for one another.

The objective of the social planner in this market is therefore to select $N$ to solve (here we impose symmetry again):

$$\max_N W(N) = G[Nf(q_N)] - Nc(q_N) - NK.$$  \hfill (7)

To derive equilibrium profits per firm, note that when $N$ firms have entered the industry, consumer maximization implies that each firm’s equilibrium price is exactly equal to $G[Nf(q_N)]f'(q_N)$. We then have

$$\pi_N = G[Nf(q_N)]f'(q_N)q_N - c(q_N) - K.$$  \hfill (8)

To understand the effects of product diversity on entry biases, we again begin by differentiating $W(N)$ (equation (7)). Omitting arguments of some functions for notational simplicity, this yields

$$W'(N) = G \left\{ Nf' \frac{\partial q_N}{\partial N} + f \right\} - c(q_N) - Nc'(q_N) \frac{\partial q_N}{\partial N} - K.$$  \hfill (9)

Adding and subtracting $Gf'q_N$ and rearranging terms give

$$W'(N) = \pi_N + N[Gf' - c'] \frac{\partial q_N}{\partial N} + G[f - f'q_N].$$  \hfill (10)

Equation (10) is the generalization of equation (2) of Section 3 to the case of product diversity. Now there are two different terms that can drive a wedge between the private and social evaluation of marginal entry: the second and third terms on the right-hand side of equation (10). The first of these is identical to that in equation (2) of Section 3: it is negative if the equilibrium price exceeds marginal cost and a business-stealing effect is present. The second term is new and represents the effect of product diversity—it is positive if $f'' < 0$ (consumers prefer variety). More specifically, when consumers have a preference for variety, a marginal entrant, by increasing variety, increases surplus but does not capture this gain in profits. The last term in (10) captures this diversity effect since $Gf'$ is the marginal entrant’s contribution to gross social surplus, while $Gf'q_N$ is his revenue.

Given these two separate effects that work in opposite directions, it is not possible to

---

13 In this example a business-stealing effect only weakly holds, that is, $q_1 = q_2 = (a - c)/2b$. This does not, however, affect the point being made—just suppose, instead, that demand is equal to $(a - c - \epsilon)/b$ for all $P \leq c + \epsilon$, where $\epsilon$ is a small positive number.

14 Note that demand substitutability is neither a necessary nor a sufficient condition for the presence of a business-stealing effect.
sign unambiguously the direction of the entry bias: entry can be excessive, insufficient, or even optimal.\textsuperscript{15} Spence (1976a) demonstrates that if \( f(q) = q^\beta \) (\( \beta < 1 \)) and if firms choose quantities and act as price-takers with respect to the "market price" \( G' \) (that is, they take \( G' \) as given when choosing their quantities), then free entry results in insufficient entry. In fact, it is not difficult to show that under this set of assumptions

\[
N[Gf' - c'] \frac{\partial q_N}{\partial N} + G[f - f'q_N] > 0
\]

for all \( N \): the product diversity term always dominates the business-stealing effect. Thus, by equation (10), \( W'(N) > \pi_N \) for all \( N \), so that \( N^c < N^* \).\textsuperscript{16}

This direction of entry bias need not always hold, however. Koenker and Perry (1981), for example, show that if one replaces Spence’s postentry price-taking assumption with a postentry conjectural variation model (with the functional form of \( f \) unchanged), then there exist ranges of values for \( \beta \) and the conjectural variation such that Spence’s result is reversed: excessive entry occurs.

Using equation (10), we can also find other functional forms such that entry is excessive even under Spence’s price-taking assumption. In Mankiw and Whinston (1985), for example, we demonstrate that if

\[
G(z) = -\frac{1}{z}
\]

\[
f(q) = aq - \left(\frac{b}{2}\right)q^2
\]

\[
c(q) = cq; \quad c > \left(\frac{a}{2}\right),
\]

then the business-stealing effect always dominates—that is,

\[
N[Gf' - c'] \frac{\partial q_N}{\partial N} + G[f - f'q_N] < 0
\]

for all \( N \). Thus, here we find that \( N^c > N^* \).

One can also explicitly recognize the integer constraint here as we did in Section 3. Such considerations do not affect the basic points made above regarding the effects of product heterogeneity.\textsuperscript{17}

5. Small set-up costs and the regulation of entry

- The results of the previous two sections indicate that when firms must incur fixed set-up costs upon entry, free entry typically does not result in the socially optimal number of firms. One might hope, however, that as set-up costs become small, the regulation of entry becomes unimportant. For instance, in the linear Cournot example of Section 3, the welfare loss due to free entry falls as the set-up cost falls, even though the excessive number of firms grows infinitely large (see footnote 8).

The cartel example (Example 2) of Section 3 shows, however, that some further qualification is necessary if we wish to obtain such a limiting result. In that example the loss due to free entry does not fall to zero as the set-up cost falls, since firms enter until all

\textsuperscript{15} Note that a business-augmenting effect would reverse the sign of \( N[Gf' - c']\partial q_N/\partial N \) in equation (10), and it would lead to insufficient entry.

\textsuperscript{16} This assumes that \( N^c \) is well defined. A sufficient condition for this is that \( \partial [Gf(q_o)]f'(q_o)/\partial N < 0 \).

\textsuperscript{17} The interested reader is referred to Mankiw and Whinston (1984).
monopoly rents are dissipated. To obtain a limiting result, we assume that as the number of entrants grows large, firms come to act approximately as price-takers.

Let \( N^*_K \) and \( N^e_K \) denote, respectively, the socially optimal and free-entry number of firms when set-up costs are \( K \). Denote the associated levels of welfare by \( W^*_K \) and \( W^e_K \), respectively. The following proposition examines the difference between the social optimum and free-entry welfare as \( K \) approaches zero.

**Proposition 3.** Suppose that Assumption 2 and the following assumptions hold.

**Assumption 1a.** \( G[Nf(q_0)]f'(q_0) < G[\tilde{N}f(q_0)]f'(q_0) \) for all \( N > \tilde{N} \) and \( \lim_{K \to \infty} Nf(q_0) = M < \infty \).

**Assumption 3a.** \( G[Nf(q_0)]f'(q_0) - c'(q_0) > 0 \) for all \( N \).

**Assumption 4.** \( \lim_{N \to \infty} G[Nf(q_n)]f'(q_n) - c'(q_n) = 0 \).

Then the welfare loss due to free entry goes to zero as \( K \) gets small; that is, \( \lim_{K \to 0} (W^*_K - W^e_K) = 0 \).

Assumptions 1a and 3a are the natural generalizations of Assumptions 1 and 3 to a heterogeneous product setting.\(^\text{18}\) Assumption 4 requires that price approach marginal cost as the number of firms grows infinitely large. As we now demonstrate, under these four assumptions, the welfare loss due to free entry disappears as the set-up cost \( K \) approaches zero.

**Proof.** Define \( W^* = \lim_{N \to \infty} G[Nf(q_N)] - Nc(q_N) \). It is not difficult to show that, under our assumptions, \( W^* \) is the socially optimal level of welfare when the set-up cost is zero, and further, that this level is finite.\(^\text{19}\) Then the following set of inequalities clearly must hold for any \( K \):

\[
\infty > W^* \geq W^e_K \geq W^e_K.
\]

The result can therefore be established by showing that \( \lim_{K \to 0} W^e_K = W^* \).

First, we note that \( N^e_K \to \infty \) as \( K \to 0 \). To see this, suppose instead that \( N^e_K \to (\tilde{N} - 1) < \infty \). Then, since

\[
G[\tilde{N}f(q_\tilde{N})]f'(q_\tilde{N})q_\tilde{N} - c(q_\tilde{N}) \geq G[Nf(q_0)]f'(q_0)q_0 - c'(q_0)q_0
\]

\[
= \{G[\tilde{N}f(q_\tilde{N})]f'(q_\tilde{N}) - c'(q_\tilde{N})\}q_\tilde{N} > 0,
\]

there exists a \( \tilde{K} > 0 \) small enough that entry by an \( \tilde{N} \)th firm would be profitable.

Since \( N^e_K \to \infty \), it is clear from the definition of \( W^e_K \) that \( W^e_K \to W^* \) if and only if \( N^e_K \to K \). Now, by the definition of \( N^e_K \) we know that

\[
G[N^e_Kf(q_N^e)]f'(q_N^e)q_N^e - c(q_N^e) \geq K
\]

so that (when we ignore the subscript of \( N^e_K \) for notational simplicity):

\[
G[N^e_f(q_N)]f'(q_N^e)N^e_N^e - N^e_e c(q_N^e) \geq N^e_K
\]

or

\[
\left\{ G[N^e_f(q_N)]f'(q_N^e) - \frac{c(q_N^e)}{q_N^e} \right\} N^e_N^e \geq N^e_K.
\]

---

\(^\text{18}\) Note the lim \( Nq_N < \infty \) implies that lim \( Nf(q_0) < \infty \) as long as \( f'(0) < \infty \).

\(^\text{19}\) The first part of this claim is established by showing that \( \{G[Nf(q_0)] - Nc(q_0)\} \) is increasing in \( N \), while the latter part follows from the assumption that \( \lim_{N \to \infty} Nf(q_0) = M < \infty \).
But, since \( N_K^* \rightarrow \infty \), it must be true that \( q_{N_K^*} \rightarrow 0 \) since \( Nf(q_N) \rightarrow M < \infty \). Thus, \( c(q_{N^e})/q_{N^e} \rightarrow c'(0) \). But, \( \lim_{N \rightarrow \infty} c'(q_N) = c'(0) \). Thus,

\[
\lim_{N \rightarrow \infty} \left\{ G[Nf(q_{N^e})]f'(q_{N^e}) - \frac{c(q_{N^e})}{q_{N^e}} \right\} = \lim_{N \rightarrow \infty} \left\{ G[Nf(q_{N^e})]f'(q_{N^e}) - c'(q_{N^e}) \right\} = 0.
\]

Finally, \( N^e f(q_{N^e}) \rightarrow M < \infty \) implies that \( N^e q_{N^e} \) is also bounded under our assumptions since \( f(q_{N^e}) \geq f'(q_{N^e})q_{N^e} \) and \( f'(0) > 0 \). Thus, \( \lim_{K \rightarrow 0} N_K^* K = 0. Q.E.D. \)

The idea behind the proof is as follows. First, as \( K \) approaches zero, in both the free-entry equilibrium and the optimum, \( N \) approaches infinity, and thus price approaches marginal cost. The only welfare difference between the equilibrium and the optimum might be excessive entry, that is, \( (N^e - N^o)K \). But since industry output is bounded and price is approaching cost, operating profits (and thus \( N^e K \)) are approaching zero. Thus, the welfare cost of excessive entry must be approaching zero as well.

The assumption that price strictly exceeds marginal cost for all finite \( N \) is important for this result. To see this, consider again Example 3 of Section 3. There, price falls to marginal cost for \( N \geq 2 \), and as \( K \) grows small, the loss due to free entry persists.

Both Hart (1979) and Novshek (1980) prove results that are similar in spirit to Proposition 3. Both of these articles show that as the efficient scale of firms grows small (in Hart's case this is done in \( per \) \( capita \) terms), free-entry Cournot equilibrium converges to a first-best allocation.\(^20\) There are two basic differences between our result and theirs. First, and most important, although these authors assume Cournot behavior and derive that price approaches marginal cost, we assume that pricing becomes competitive as the number of firms grows, but we do not need to be specific about the nature of postentry interaction. Second, we specify a two-stage entry process as opposed to the simultaneous (Cournot) entry-quantity decision posited in their models.\(^21\)

### 6. Conclusion

- Economists have long believed that unencumbered entry is desirable for social efficiency. This view has persisted despite the illustration in several recent articles of the inefficiencies that can arise from free entry in the presence of fixed set-up costs. In this article we have attempted to elucidate the fundamental and intuitive forces that lie behind these entry biases.

In homogeneous product markets the existence of imperfect competition and a business-stealing effect always creates a bias toward excessive entry: when we ignore the integer constraint on the number of firms, marginal entry is more desirable to the entrant than it is to society because of the output reduction entry causes in other firms. Hence, in a homogeneous market entry restrictions are often socially desirable, although, as we show, they become unnecessary as the fixed set-up cost becomes small.

The introduction of product diversity, however, can reverse this bias toward excessive entry. Intuitively, a marginal entrant adds to variety, but does not capture the resulting gain in social surplus as profits. Hence, in heterogeneous product markets the direction of any entry bias is generally unclear, although efficient levels of entry remain an unlikely occurrence.

---

\(^{20}\) We would like to thank Andreu Mas-Colell for helpful discussion regarding these issues.

\(^{21}\) A third difference actually exists: as \( K \) grows small in our model, minimum average cost falls. In contrast, in Hart (1979) and Novshek (1980) the limiting process keeps minimum average cost unchanged (in \( per \) \( capita \) units in Hart's case). It is not difficult, however, to adapt Novshek's argument to the case of a limiting process of our type. We would like to thank a referee for calling our attention to this fact.
Appendix

The proof of Proposition 2 follows.

Proof of Proposition 2. The result is trivial for $N^* \leq 1$. Suppose now that $N^* > 1$. We first show that profits per firm are nonnegative when $(N^* - 1)$ firms are in the industry, i.e., that $\pi_{N^*-1} \geq 0$. To see this, note first that by the definition of $N^*$ we have $W(N^*) \geq W(N^* - 1)$, or

$$\int_{Q_{N^*-1}}^{Q_{N^*}} P(s)ds - N^*c(q_{N^*}) + (N^* - 1)c(q_{N^*-1}) \geq K,$$

(A1)

where $Q_N = Nq_N$. We can rearrange (A1) to yield

$$\pi_{N^*-1} \geq P(Q_{N^*})q_{N^*-1} - \int_{Q_{N^*-1}}^{Q_{N^*}} P(s)ds + N^*[c(q_{N^*}) - c(q_{N^*-1})].$$

(A2)

Since $P(\cdot) < 0$ and Assumption 1 holds, this implies that

$$\pi_{N^*-1} \geq P(Q_{N^*-1})[q_{N^*-1} - Q_{N^*} - Q_{N^*-1}] + N^*[c(q_{N^*}) - c(q_{N^*-1})].$$

(A3)

Since $c'(\cdot) \geq 0$, we know that

$$c'(q_{N^*-1})[q_{N^*} - q_{N^*-1}] \leq c(q_{N^*}) - c(q_{N^*-1}).$$

(A4)

Substituting (A4) into (A3) yields

$$\pi_{N^*-1} \geq [P(Q_{N^*}) - c'(q_{N^*})][N^*(q_{N^*} - q_{N^*-1})].$$

(A5)

But by Assumption 3 the first bracketed expression on the right-hand side is nonnegative, and by Assumption 2 the second bracketed expression is also nonnegative. Thus, $\pi_{N^*-1} \geq 0$.

If $\pi_N$ is decreasing in $N$, then $\pi_{N^*-1} \geq 0$ implies that $N^* \geq N^* - 1$. To see that this is the case, note that

$$\pi_N - \pi_{N-1} = [P(Q_N)q_N - c(q_N)] - [P(Q_{N-1})q_{N-1} - c(q_{N-1})]$$

$$\leq P(Q_{N-1})[q_N - q_{N-1}] - [c(q_N) - c(q_{N-1})]$$

$$\leq [P(Q_{N-1}) - c'(q_{N-1})](q_N - q_{N-1}),$$

(A6)

(A7)

(A8)

where the first inequality follows from Assumption 1 and the second by applying equation (A4). Thus, Assumptions 2 and 3 imply that $\pi_{N^*-1} \geq \pi_N$ for all $N$. Q.E.D.

References


