Afterword

Although auction theory has remained an extremely active area of research since this survey was written in 1999, an introductory survey written today in 2004 would not be very different. The important changes would be in three areas: multi-unit auctions, collusion, and entry, in which much work has been stimulated by the practical concerns and experience arising from the many newly created auction markets (especially the recent government auctions of mobile-phone licenses) that we will discuss in parts C and D.[1]

There has been particular interest in multi-unit auctions of heterogeneous goods, especially in auctions in which there are complementarities between the goods. This work has yielded few definite answers about what mechanisms might be optimal (either revenue maximizing, or socially most efficient), but much effort has focused on what might be practical auction designs.[2]

The most important new design is the Simultaneous Ascending Auction (SAA). This is a fairly natural extension of the basic ascending auction to multiple objects; the bidding remains open on all the objects until no one wants to make any more bids on any object. Some complexity arises from the fact that a bidder may be reluctant to place bids until he sees other players' bids, in order to learn others' valuations. In particular, a bidder may be concerned about the risk of being "stranded" winning an object that he had wanted to win only if he had won other objects which were in fact won by other bidders. So "activity" rules that specify what bids a bidder must make to remain eligible to win objects are necessary to ensure that the bidding proceeds at a reasonable pace.

Although the germ of the SAA idea can perhaps be traced back to Vickrey (1976), it was first developed for practical use by Milgrom, Wilson, and McAfee who proposed the rules that were necessary to make the SAA effective in the context of US radio spectrum auctions. A full description of one version of the SAA is given in section 6.5.2.[3] We also show in that section that

[1] The increased attention to collusion and entry may perhaps have been reinforced by my urgings, in Klemperer (1998, 1999b, 2000b).

[2] If there is no complementarity or substitutability between objects, that is, a bidder's valuation for a bundle of objects just equals the sum of his (private) valuations of the individual objects in the bundle, it is efficient to auction the objects separately, but it may nevertheless increase expected revenue to auction the objects in bundles, see section 1.10.1 of our survey. If no bidder is interested in more than one unit, and units are homogeneous, most standard auctions (whether simultaneous or sequential) are revenue equivalent, see section 1.10.3 of the survey.

[3] The description given there is for a context in which bidders are permitted to win just one object each, so the activity rules are particularly simple.

the SAA is, in theory, an efficient mechanism for the sale of heterogeneous objects when bidders have private values, but want (or are permitted) to win at most a single object each.[4] However, the design has also been used in many other circumstances than these, and we will describe some of its successes and failures in practice in parts C and D.

One reason the SAA was originally proposed for the US radio spectrum auctions was that it was thought it might work well when bidders have complementarities between objects, but this will not be true if the complementarities are sufficiently important. The reason is the one noted above that bidders are required (by the activity rules) to make firm bids on some objects before they know which other objects they will win.[5] Some bidders may therefore end up stuck with objects that are worth very little to them because they failed to win complementary objects (this is called the *exposure problem*), while other bidders may quit the bidding early because of fear of this. Thus inefficiencies are likely. So if complementarities are important, it is natural to use some form of "combinatorial auction" in which a buyer can place bids for one or more packages of items and/or make contingent bids.[6] (A package bid is a single price offered for a set of items; a contingent bid is one that applies only under specified circumstances such as the buyer winning a particular other object; these auctions are called "combinatorial" because the auctioneer must solve a combinatorial optimization problem).

The most famous combinatorial auction is the Vickrey auction, in the general version of which the auctioneer maximizes social surplus and sets prices so that each participant's net profits equal her contribution to social surplus, assuming participants bid truthfully (i.e., the participant pays a price for those items she wins equal to her declared value for those items *less* the total social surplus achieved by the allocation *plus* the social surplus that the auctioneer could have achieved if that participant had not been present). However, as noted in section 1.10.4, a Vickrey auction is usually totally impracticable even in those private-value contexts in which it is, in theory,

[4] Milgrom (2000) gives more detail.

[5] If there is no penalty for withdrawing bids, the bidding process may never end, and there may also be substantially enhanced possibilities of collusion.

[6] Although combinatorial auctions can often reduce inefficiency, they also sometimes have the opposite effect. Just as with ordinary monopolists who bundle products or use non-linear pricing and quantity discounts, a combinatorial auction that permits or requires bids for packages rather than individual objects can result in bundling items that would be more efficiently allocated to different bidders, but may nevertheless increase expected seller revenue. (Thus, for example, if one bidder values objects A and B at x and $2x$ respectively, while a second bidder values A and B at $2x$ and x respectively, and x is known to the bidders but not to the seller, selling the two objects in two separate auctions (either ascending or sealed bid) is efficient, but yields total revenues of just $x + x = 2x$, while selling the two objects as a bundle in a single "combinational" auction obtains revenues of $3x$ (which both bidders will bid) but inefficiently sells both objects to the same winner.)

efficient.[7] There has therefore been considerable renewed interest in alternative combinatorial auction forms. Examples include Bernheim and Whinston's (1986) first-price package auctions (in which bidders submit package bids, the seller selects the combination that maximizes her revenue, and each bidder pays the amount it bid for the package it receives), and ascending package auctions in which bidders raise their offers over a series of rounds, such as Ausubel and Milgrom's (2002) "ascending proxy auction", and Banks, Ledyard, and Porter's (1989) "adaptive user selection mechanism" or AUSM (pronounced "awesome", and developed—as can, of course, be inferred from its name—in California). Milgrom (2004) is an excellent introduction to the state of the art in combinatorial auctions, and to multi-unit auctions more generally.

The practical use of multi-unit auctions has also reinforced the importance of the *demand reduction* problem we noted in section 1.10.4 of our survey, above, that bidders in these auctions, like oligopsonists in other kinds of markets, can often lower the prices they pay by buying fewer units than they actually want (because offering to buy fewer units means the auction closes at lower prices).[8] This behavior can arise even if there is just one "large" bidder who wants more than one unit, and if all bidders bid independently. When there are several "large" bidders, multi-unit auctions also open up the possibility of these bidders coordinating their behavior to reduce their demand in concert. Such "collusion", whether tacit or otherwise, has been a serious problem in multi-unit auctions, especially in ascending designs such as the SAA, as we will emphasize in chapters 3 and 5.

Thus multi-unit auctions have focused renewed attention on the point made in section 1.9 of our survey, above, that "A crucial concern about auctions in practice is the ability of bidders to collude, but the theoretical work on this issue is rather limited." Although the importance of this issue is now generally recognized, and research on it is beginning to develop,[9] this literature remains in its infancy. In particular, as we will emphasize in chapter 4, it is not clear that it has yet taught us very much more than could be gleaned from an intelligent reading of the industrial organization literature.

[7] Policy makers usually find a Vickrey auction very hard to understand and operate; it often results in bidders with high values paying less for objects than bidders who win identical objects but have lower values for them (which seems strange and unfair to many people); it offers unusual opportunities for collusive behavior which are also hard to guard against; and it sometimes yields low revenues. Furthermore, it is not efficient (and may perform very badly) if bidders are risk-averse or have budget constraints or have common-value elements to their valuations.

[8] This problem has been emphasized by Ausubel and Cramton (1998a) and Ausubel (forthcoming).

[9] For example, Menezes (1996), Weber (1997), Engelbrecht-Wiggans and Kahn (1998c), Ausubel and Schwartz (1999), Cramton and Schwartz (2000, 2002), Brusco and Lopomo (2002a), and Grimm, Riedel, and Wolfstetter (2003) all discuss how vulnerable multi-unit ascending auctions are to collusive outcomes. See also Marshall and Meurer (2002).

Finally, and also prompted by practical experience, there is a much greater general understanding than previously that, as emphasized in section 1.8.1 of the survey above, "In practical auction design, persuading bidders to take the time and trouble to enter the contest is a major concern." Although no general principles have yet emerged beyond those in section 1.7 (about asymmetries between bidders) or section 1.8 (explicitly about entry) of the survey, this issue is now an area of active research.

We will give considerable attention to the problems of encouraging entry and discouraging collusion, often in the context of multi-unit auctions, in Parts C and D of this volume.[10]

[10] Klemperer et al. (forthcoming) is a case-study of a multi-unit auction of environmental goods that emphasizes entry issues.