Simple Auctions for Multiple Related Goods*

Paul Klemperer – September 2018

1. Examples of the Problem
2. A Solution – as used by the Bank of England
3. Further Developments, and Other Applications

Appendices: Some Practical Details/A Little Theory

***see paper <u>"Product-Mix Auctions"</u> (2008, updated in 2018) on my website**

1. Examples of the Problem

U.K. got global financial crisis first—
the Northern Rock bank run (Sept. 2007)



U.K. got global financial crisis first…

After Northern Rock bank run (September 2007)
Bank of England urgently wanted to lend money
to commercial banks, building societies, etc.

Before 2007, borrowers had to give the Bank of England govt bonds ("strong" collateral) until their debts paid (cf. a bank owns part of your house until mortgage paid)

In crisis, borrowers had insufficient "strong" collateral
→ Bank of England instead had to accept "toxic" assets ("weak" collateral), so wanted higher interest rates

Which banks should get loans?, At what interest rates?,
What collateral should each bank give?

Governor of Bank of England phones me …

Bank of England's problem

(& others' problem)

Want to buy or sell ***many*** types of objects

e.g., Bank of England wanted to "sell" ***many*** "types"
of loans of money to commercial banks, etc.,
"type" = quality of collateral used by borrower

and want to do this in a single auction:
--so buyers can choose type efficiently;
--so seller can decide best amount of each type to sell;
--to create more competition

but central banks had always auctioned ***one*** type at a time

Why not run a separate auction for each “variety”?

(1) ***Market power:*** too little competition in each auction

(2) ***Bidders have to decide: which auction to enter?***

e.g., potential borrower (commercial bank) might prefer to say:

“will use my poor collateral if price difference < (e.g.)15 basis pts”

(3) ***Auctioneer has to decide: how much to sell in each auction?***

e.g., Central bank might prefer to say:

“will accept more poor collateral if the price difference (interest-rate premium) for accepting poor, instead of good, collateral, is larger”

e.g., will accept 5% of poor if price difference > 5 basis pts

15% - - - - - - - - - - - - - - - - - - > 10 bp,

50% - - - - - - - - - - - - - - - - - - > 15bp, etc

Many related problems

Sale of different issues of government debt

Purchasing related corporate bonds (cf. UK's CBPS)

Swapping blocked accounts for alternative assets (Iceland's plan)

Procuring commitments to build energy capacity,
e.g., nuclear *vs.* gas *vs.* wind

Selling used exercise machines (Oxford student union)

General problem:

Sell multiple varieties of a **Product**
when costs depend on **Mix** of varieties sold

or buy multiple varieties of a **Product**
when benefits depend on **Mix** of varieties bought

→ **I developed: "Product-Mix Auction"**

Why not use Multi-stage Auctions?

e.g., I ran a multi-stage auction (with Ken Binmore)
to sell the UK's 3G mobile-phone licences

1. UK 3G auction pre-specified the numbers of large and small licenses to be sold (2 large & 3 small),
--simple multi-stage auctions *don't* permit the mix of varieties sold to depend upon the bids

2. UK 3G auction took 7 weeks
-- multi-stage auctions take longer, cost everyone more, and so may also reduce participation

3. UK 3G auction raised $34 billion (2½% of GNP)
but other 3G auctions raised *much* less
-- multi-stage auctions can facilitate collusion or predation in some contexts (because bidders can respond to each others' bids)

Why not simultaneously collect bids on all varieties, and afterwards decide how many bids to accept on each?

This *does* permit the mix of varieties sold to depend upon the bids,

but -- can be hard to do rapidly,

-- may create perceptions of favouritism or abuse of discretion.

[also cannot permit bidders to make their bids contingent on relative prices, and so also

-- may mitigate market power less]

→ better to use predetermined rules to set prices and allocations

(although decision-maker can adjust its preferences in the software to explore alternative outcomes, if desired)

2. A Solution – as used by the Bank of England

I developed the Product-Mix Auction after the Governor of Bank of England contacted me in 2007….



We have **(free, open-source) software** for several versions of Product-Mix Auctions*

Product-Mix Auctions

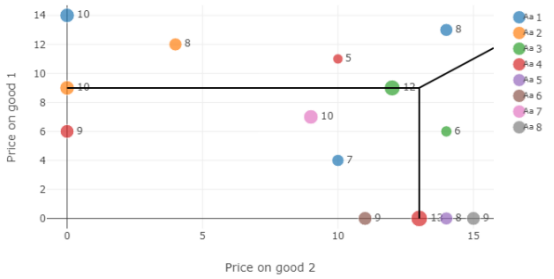
We provide open-source software for three different Product-Mix Auctions. (All Product-Mix Auctions are single-round sealed-bid auctions for multiple units of multiple distinct goods.)

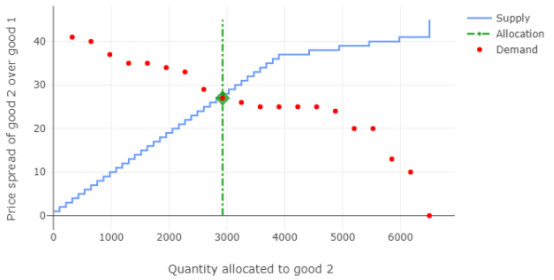
- **Standard Version.**
  - This is a generalisation of the design that is the basis of the "ILTR" auction that is now regularly used by the Bank of England — see paper.
  - It permits flexible buyer and seller preferences over an arbitrary number of goods.
- **Version for Budget-constrained bidders.**
  - This version was originally designed for the Government of Iceland.
  - As in the standard version, bidders simultaneously make sets of bids. In this case, however, each of a bidder's bids can specify a total budget to be spent.
- **"Positive and negative dot-bids" Version.**
  - This version extends the standard version to permit all participants (buyers and seller) to express *any* strong substitute preferences for indivisible goods.

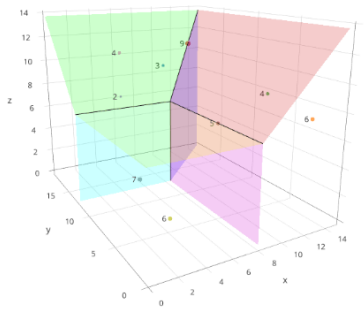
This software is available both as a command-line program `pma`, and a web application `pma-server` (all open-source). The latter provides a single-user interface allowing an auction specification to be constructed in a web browser form. We host the web application here and also provide the code so users can host it locally.

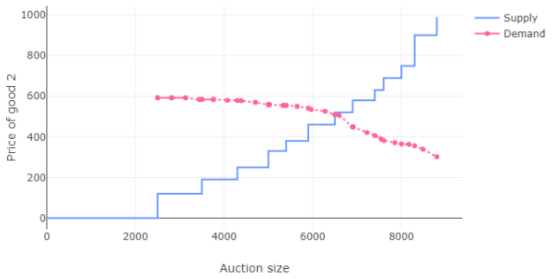
Please contact Elizabeth Baldwin or Paul Klemperer for the password to access this software, or for further information about it; its use is free of charge.

Download software and documentation (requires password)









*developed by Elizabeth Baldwin and Paul Klemperer, with valuable help from Simon Finster, Adam Gundry, and others

Auction input

Supply

Supply Ordering: HorizontalSupply

Supply curves for each good

Supply curve for good 1

Units	Price
30	0

+ Supply step

Supply curve for good 2

Units	Price
30	0

+ Supply step

+ Supply curve

Bidders

Bidder 1

Bid	Units	Price	Price
1	11	17	0

Bidder 2

Bid	Units	Price	Price
1	11	9	0
2	11	0	10

+ Bid

Bidder 3

Bid	Units	Price	Price
1	11	8	0

+ Bid

Bidder 4

Bid	Units	Price	Price
1	11	6	0

Bid	Units	Price	Price
1	11	0	14

+ Bid

Bidder 6

Bid	Units	Price	Price
1	11	0	16

+ Bid

Bidder 7

Bid	Units	Price	Price
1	11	0	17

Auction output

Prices and total allocations

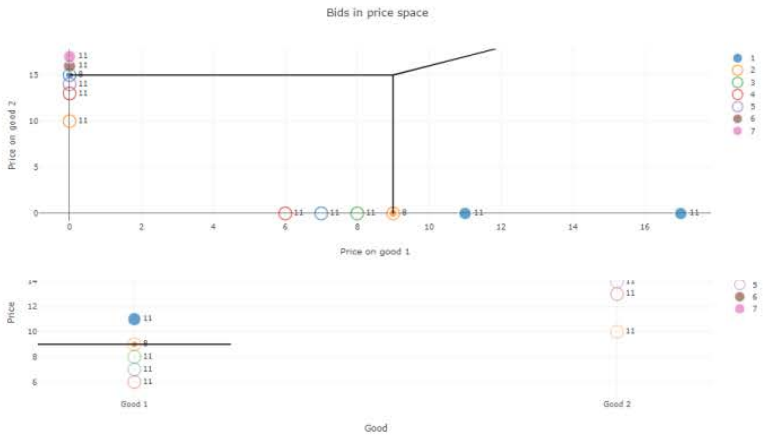
	Good 1	Good 2
Price	9	15
Allocation	30	30

Allocations

Bidder	Good 1	Good 2
1	22	8
2	8	0
6	0	11
7	0	11

Bid allocations

Bids in price space



Supply curve and unadjusted demand* for good 1

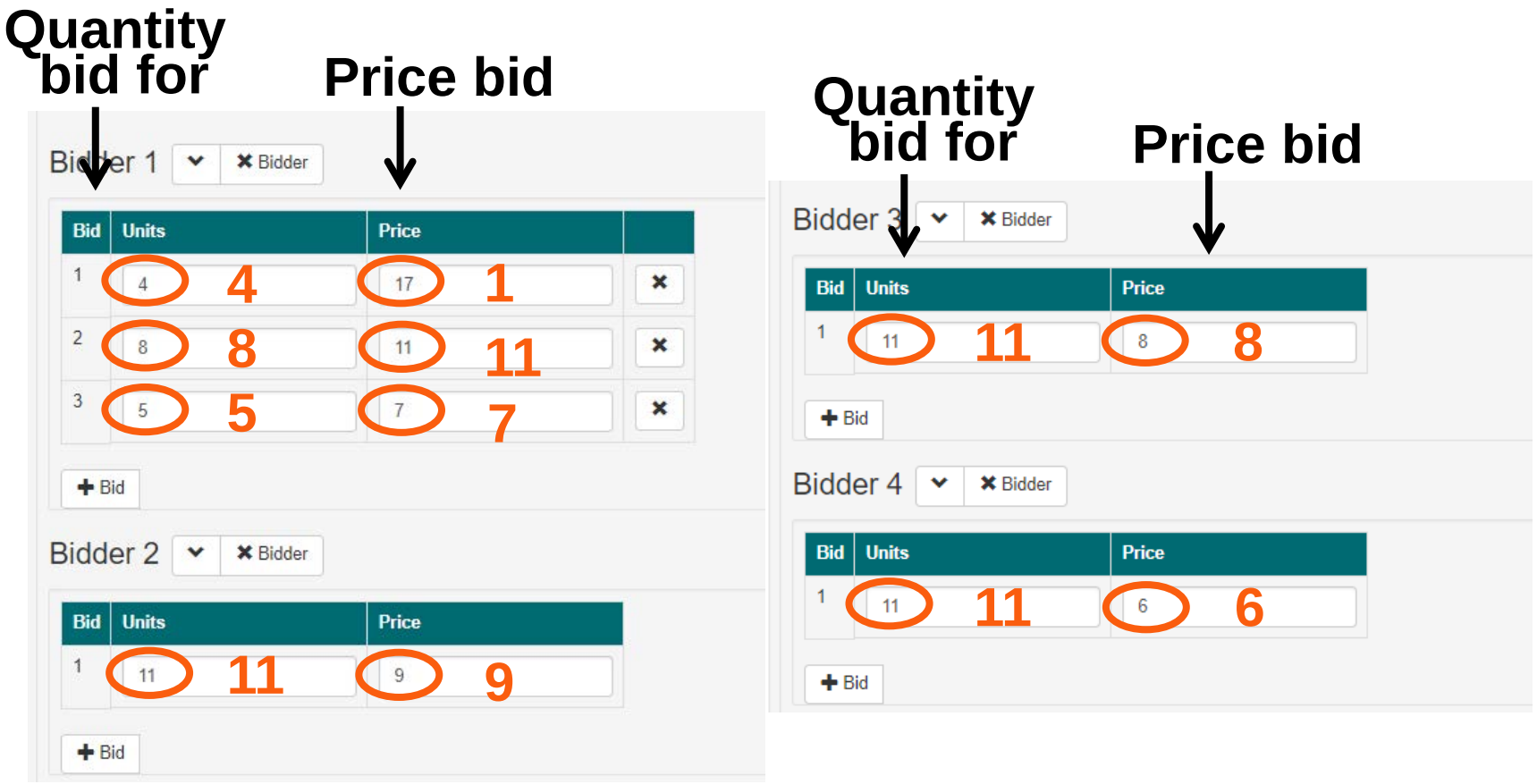


Supply curve and unadjusted demand* for good 2



demoTwoSeparate

Standard single-variety auction
Bidders/bids

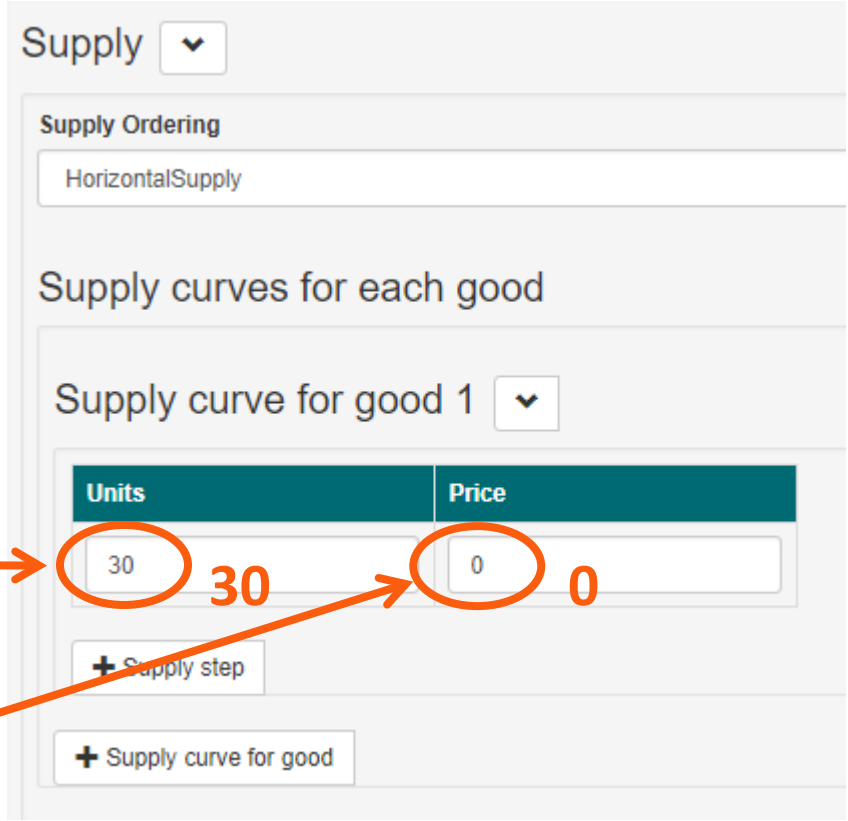


demoSingle

Standard single-variety auction
Supply curves

Auctioneer is willing to sell up to 30 units

with reserve price 0

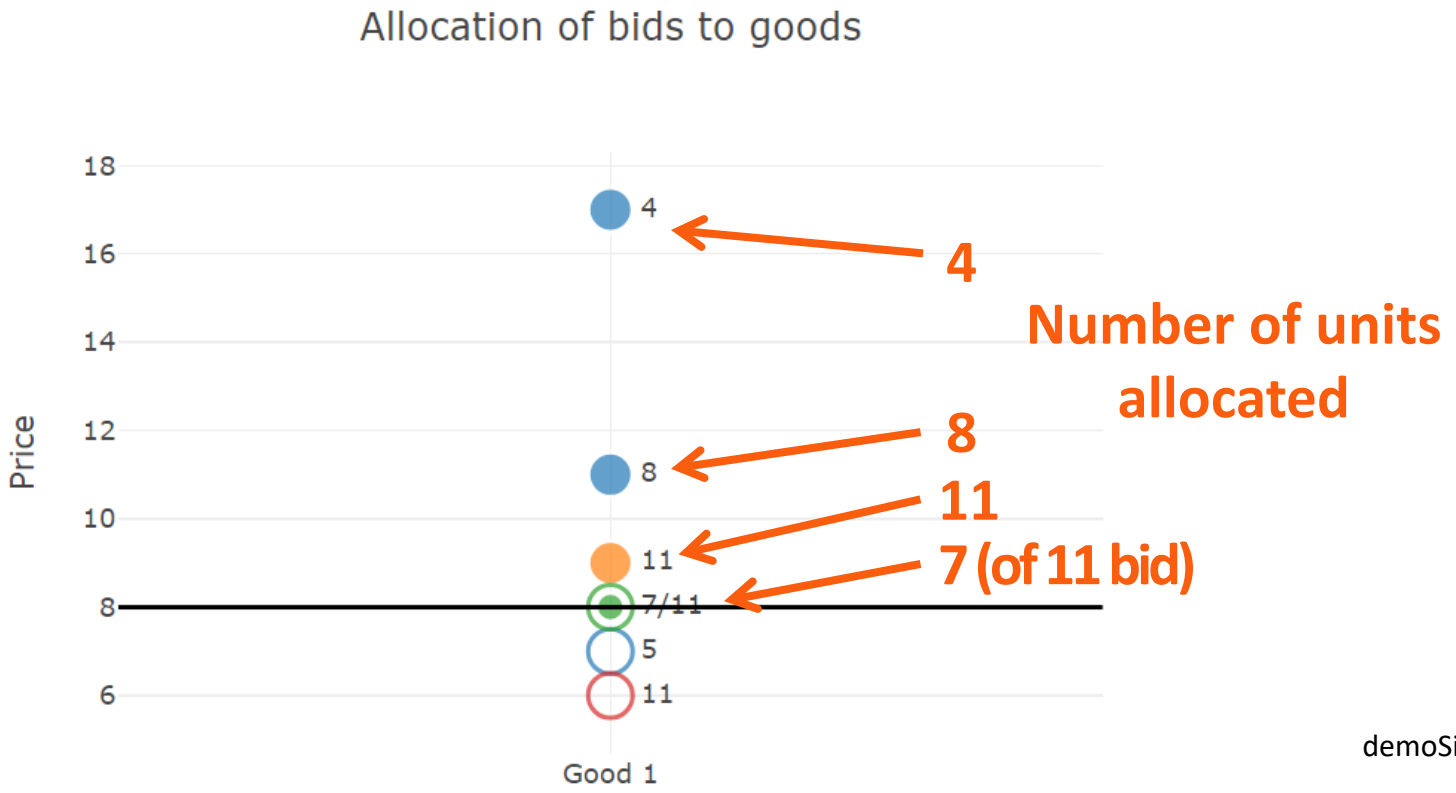


demoSingle

Standard single-variety auction
Allocations of bids to goods

we have a choice of pricing rules…
standard rules are uniform and "pay-as-bid"

(auctioneer is selling 30 units; reserve price 0)



demoSingle

Standard single-variety Uniform-Price auction

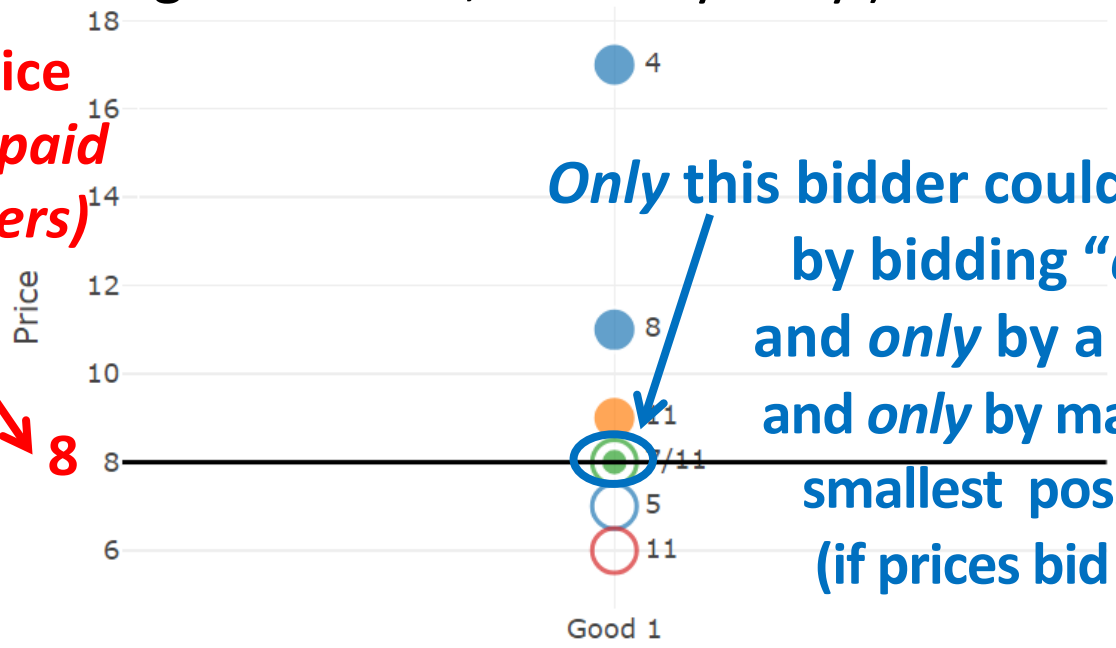
***Importantly*:**

1. Any bidder who bid her actual valuations receives *exactly* what she would have chosen at the auction prices

2. Bidders obviously do best to bid (close to) their actual valuations

("Honest bidding" *is* best policy unless you can affect the prices, which is unlikely if the number of bidders is "not too small". Typically only a rationed bidder can gain from "*dis*honest bidding", and potential gain is small, and very risky.)

Uniform Price
(*Same Price paid by All Winners*)



8

***Only* this bidder could have benefited by bidding "*dis*honestly", and *only* by a small amount, and *only* by making the smallest possible deviation [1] (if prices bid must be integers)**

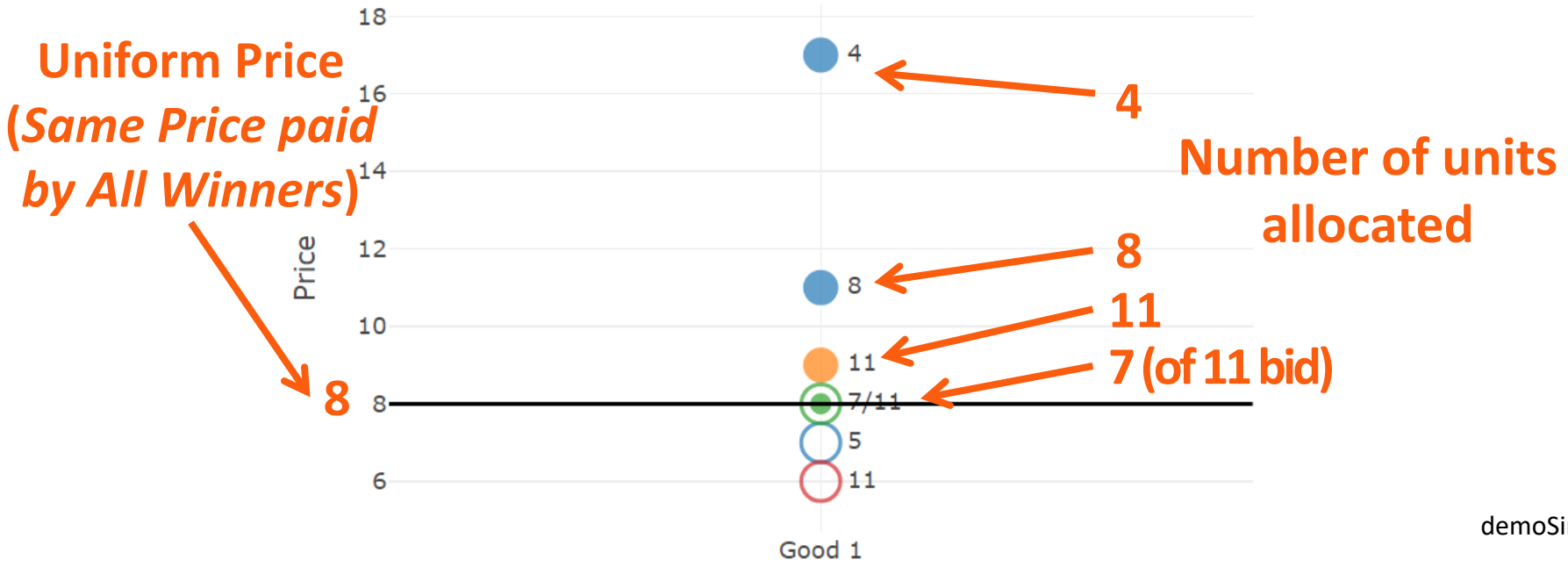
demoSingle

Standard single-variety Uniform-Price auction

Importantly:

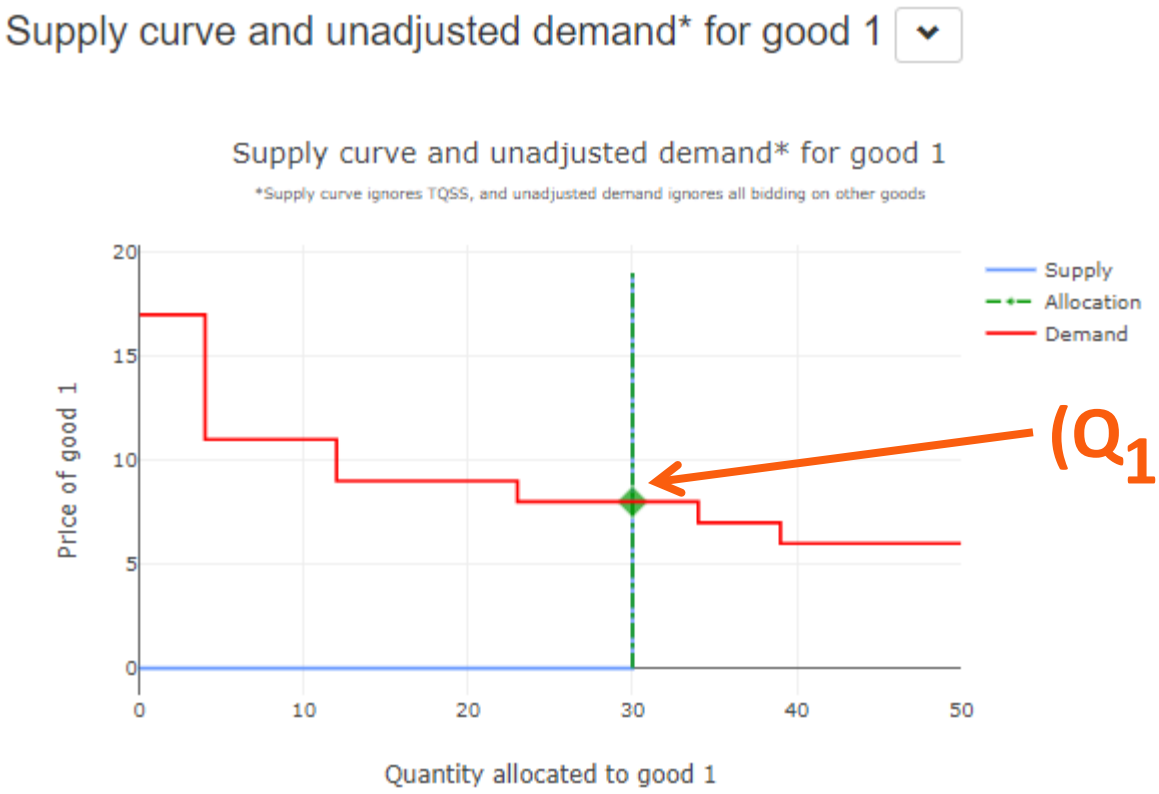
1. Any bidder who bid her actual valuations receives *exactly* what she would have chosen at the auction prices
2. Bidders obviously do best to bid (close to) their actual valuations
3. Bidding is ***Efficient, Informative, and Easy***

Our auctions will preserve *all* these properties



demoSingle

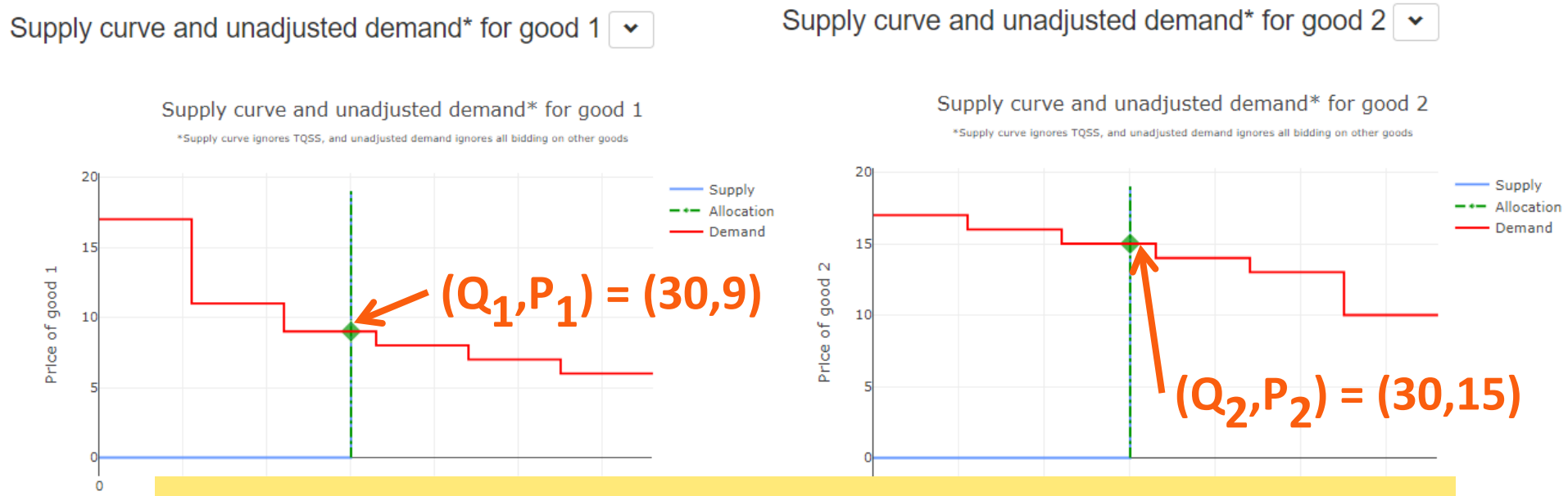
Standard single-variety Uniform-Price auction
Demand and supply



$(Q_1,P_1) = (30,8)$

demoSingle

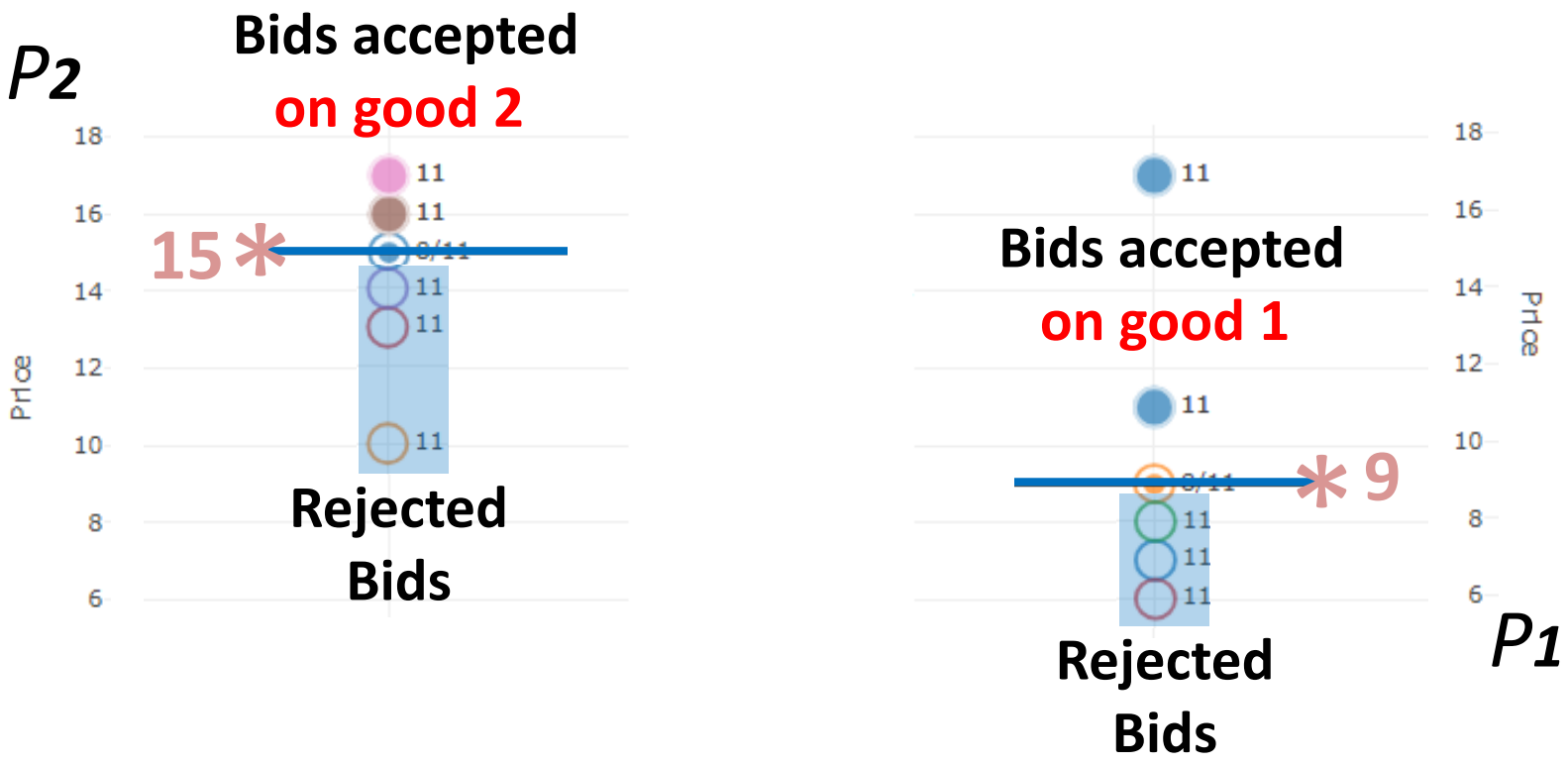
Two separate uniform-price auctions
Demand and supply



e.g., U.S. Federal Reserve's "TSLF"
(Term Securities Lending Facility, 2008-10) did this
[ran auctions for loans against
Schedule 1 collateral in "odd-numbered" weeks, and
Schedule 2 collateral in "even-numbered" weeks]

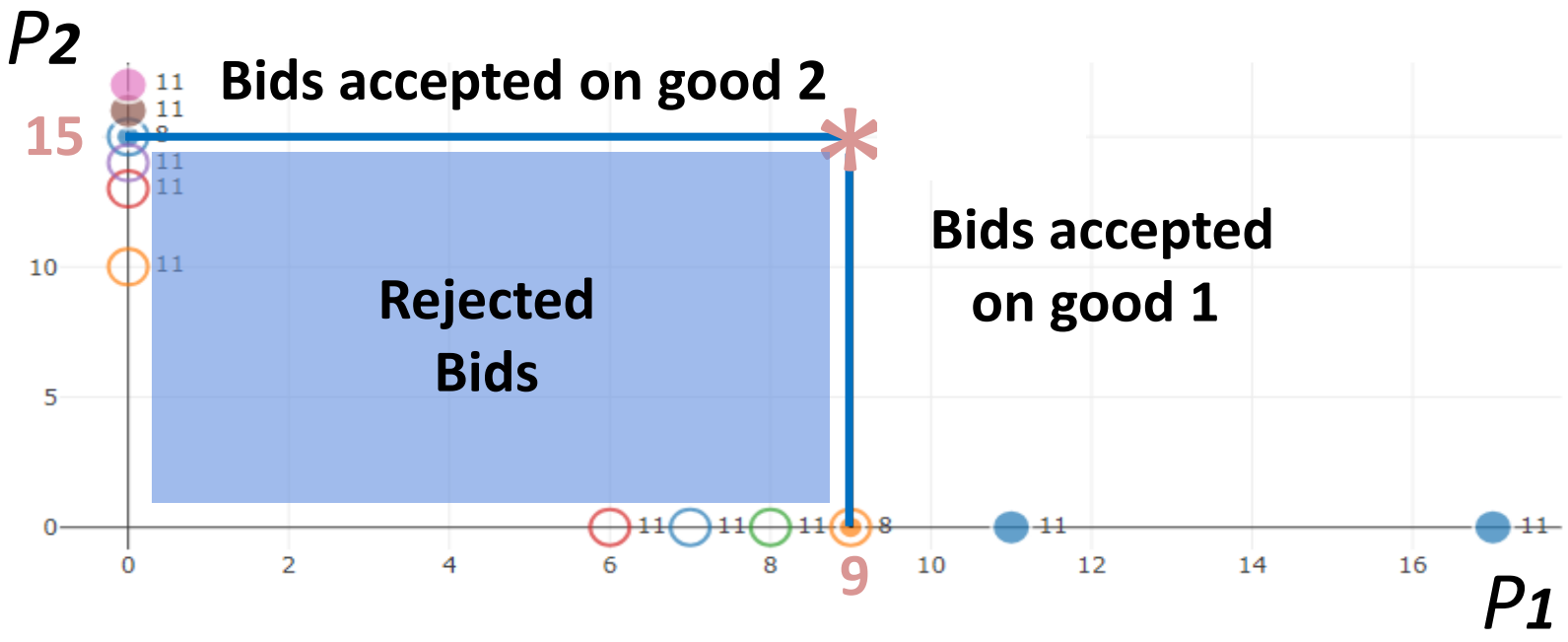
demoTwoSeparate

Two separate uniform-price auctions
Allocations



Prices 9 and 15 (marked *) sell 30 units in each auction

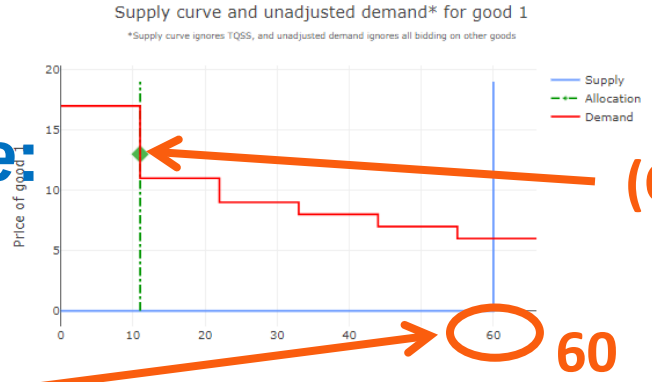
Two separate uniform-price auctions
Showing Allocations in Two Dimensions



Prices 9 and 15 (marked ✱) sell 30 units in each auction

Combining the two auctions, treating the two varieties as identical

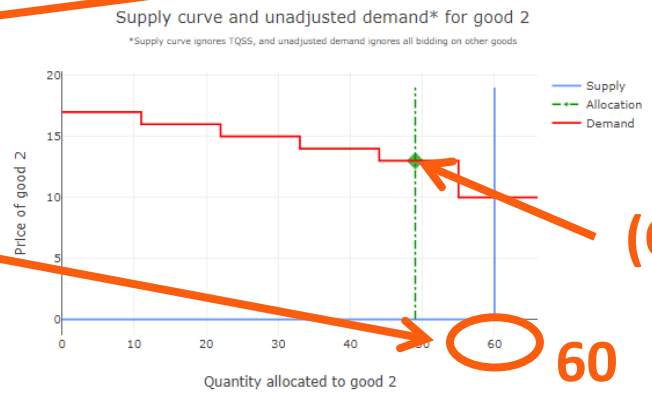
Effectively one single, combined, supply curve:



$(Q_1,P_1) = (11,13)$

Auctioneer is willing to sell up to 60 of good 1,

60

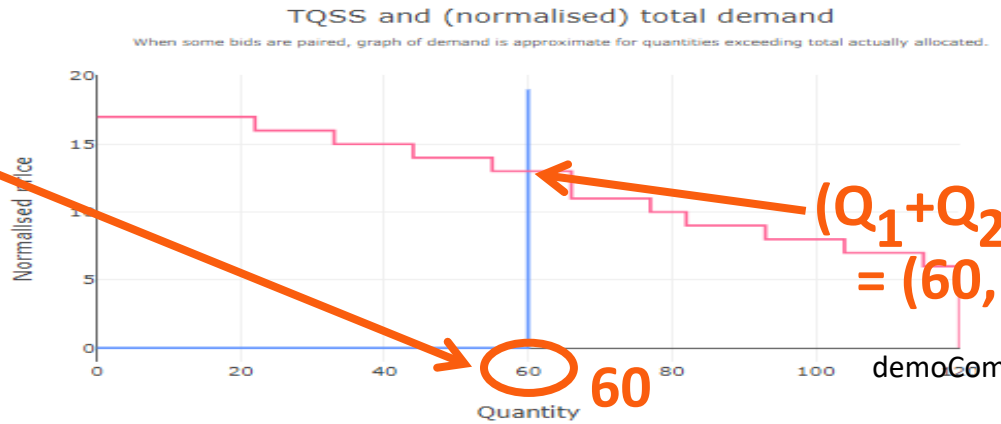


$(Q_2,P_2) = (49,13)$

and up to 60 of good 2

60

but no more than 60 in total (Total Quantity Supply Schedule)

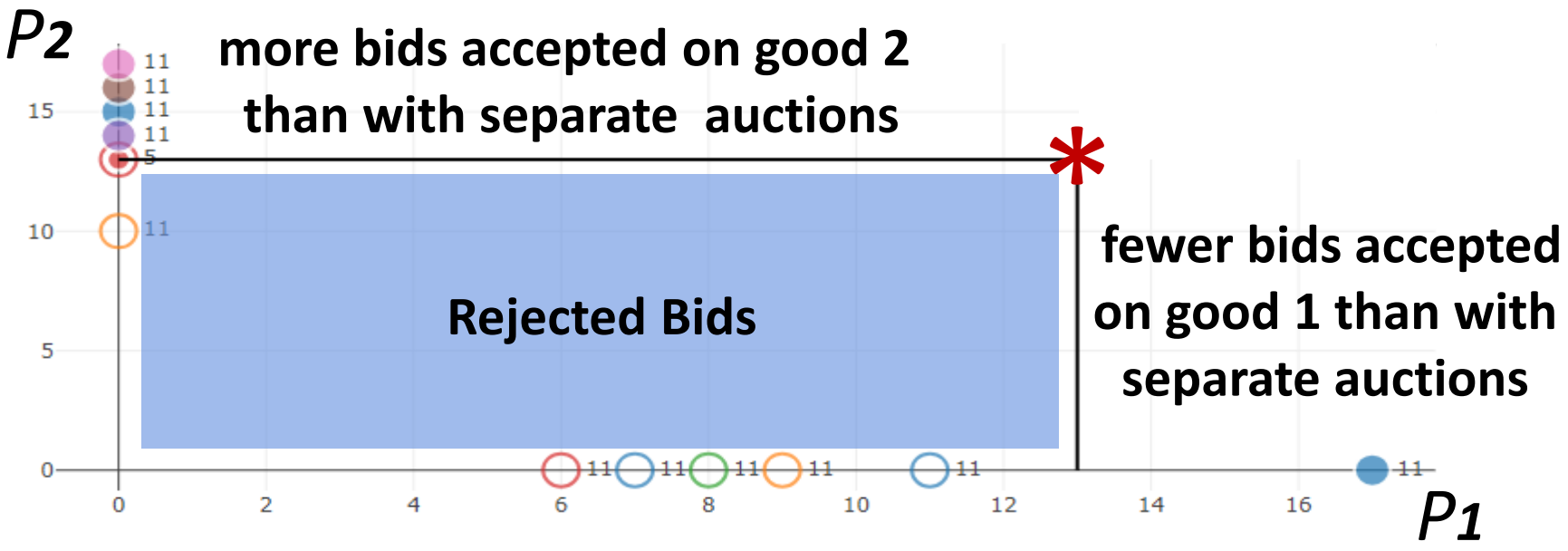


$(Q_1+Q_2,P) = (60,13)$

60

demoCombine

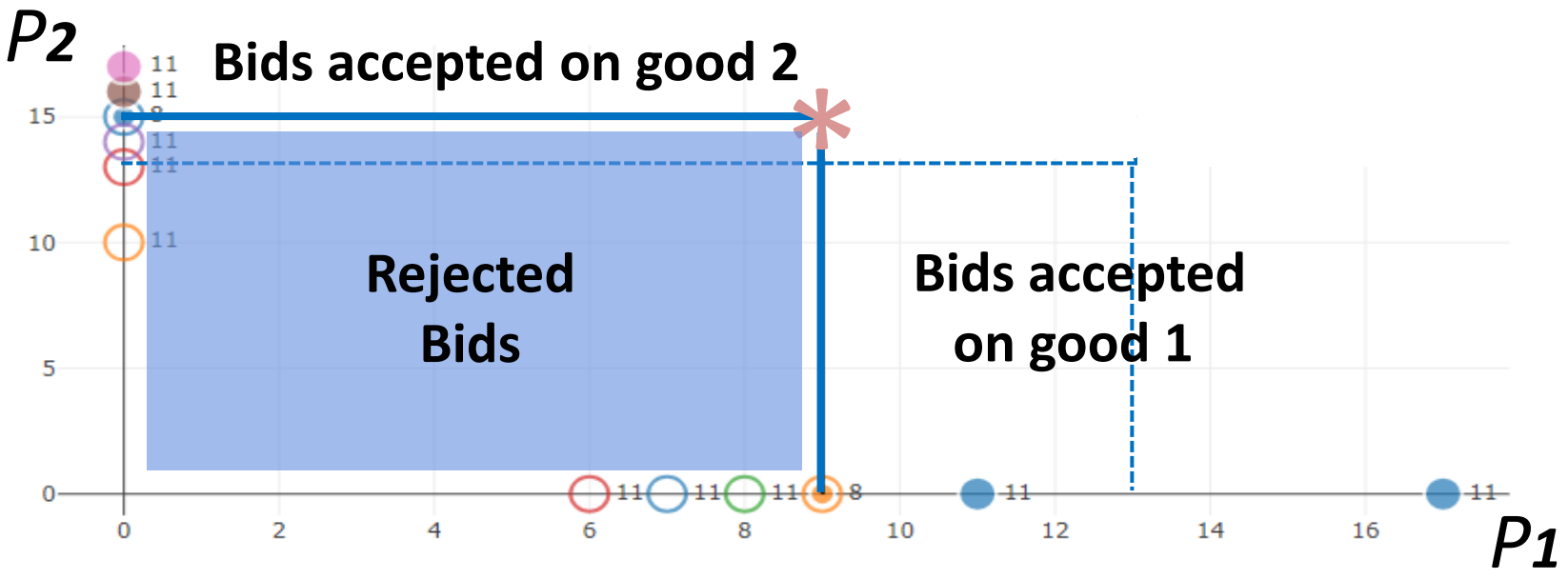
Combining the two auctions: treating the varieties as identical (single supply curve)



Prices of 13 on both goods (marked ✱) sell 60 units in total

demoCombine

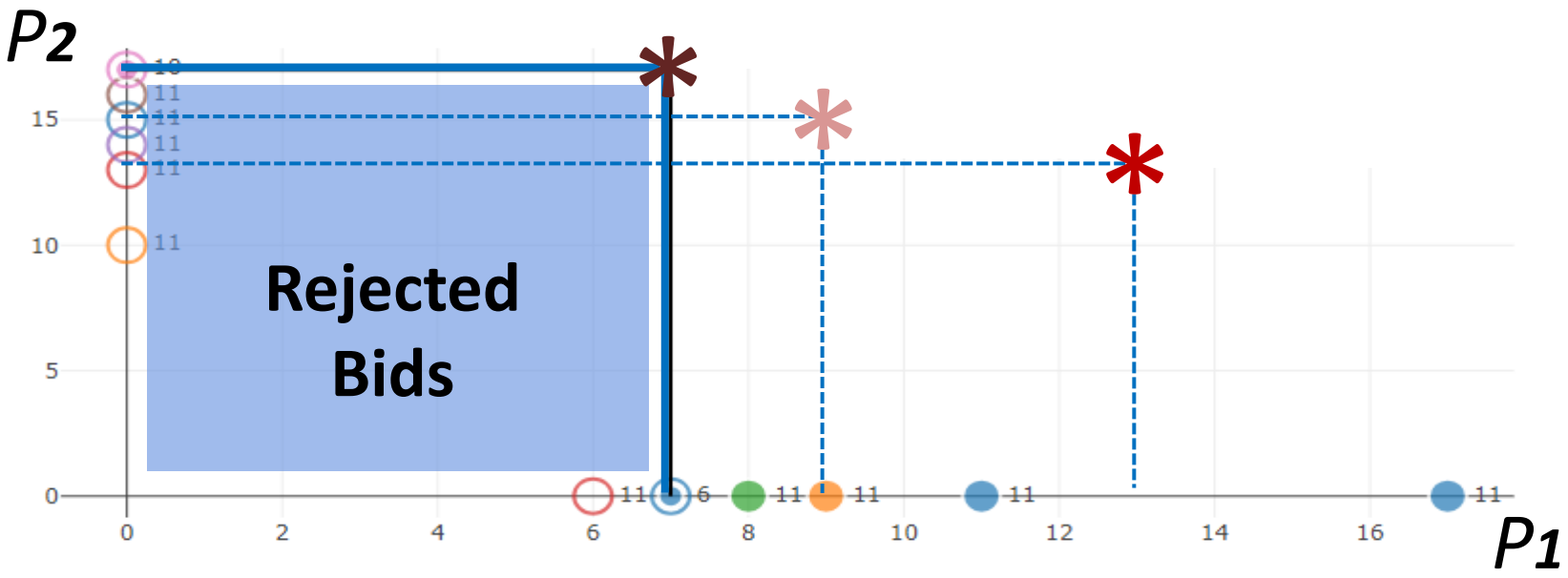
Two separate uniform-price auctions (as before)



Prices 9 and 15 (marked ✳) ***also*** sell 60 units in total (more good 1 than with combined auctions; less good 2)

demoTwoSeparate

another option that sells 60 units in all
(even *lower* price (7) on good 1, and *higher* price (17) on good 2)

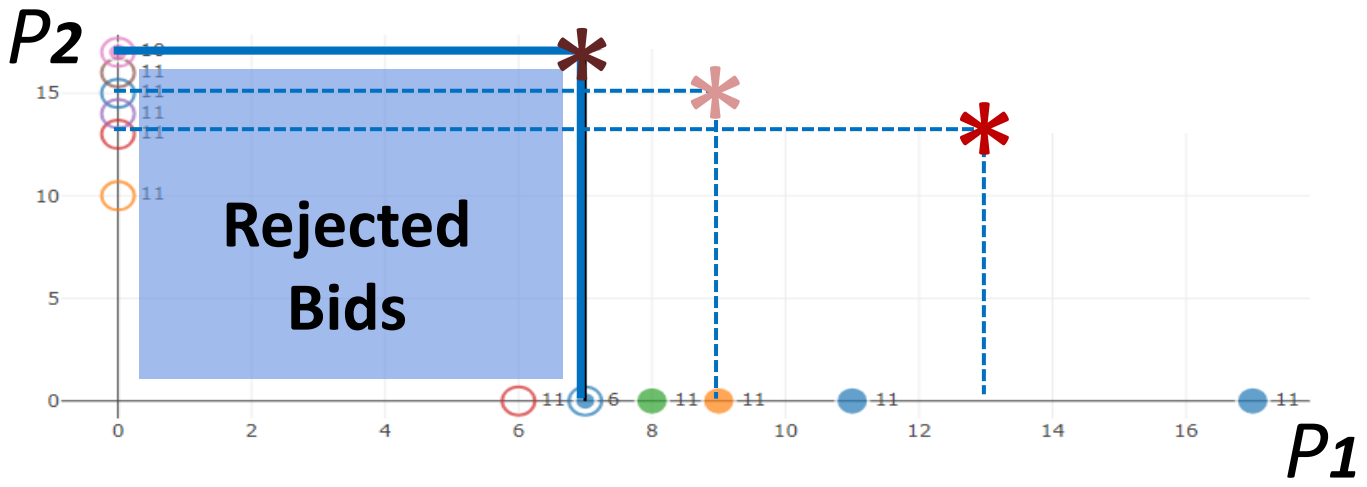


The three price vectors marked ✱, ✱, and ✱ *all* sell 60 units, but with different shares of good 1 and good 2

→ these are all feasible price vectors
if the auctioneer wishes to sell that 60 units in total

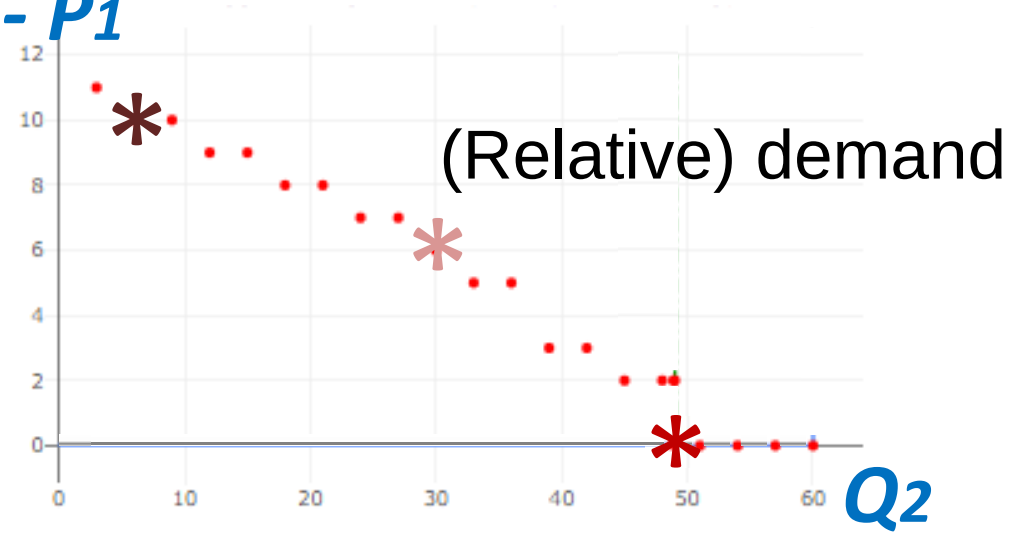
demoCombine4Vert

Constructing the (relative) demand curve



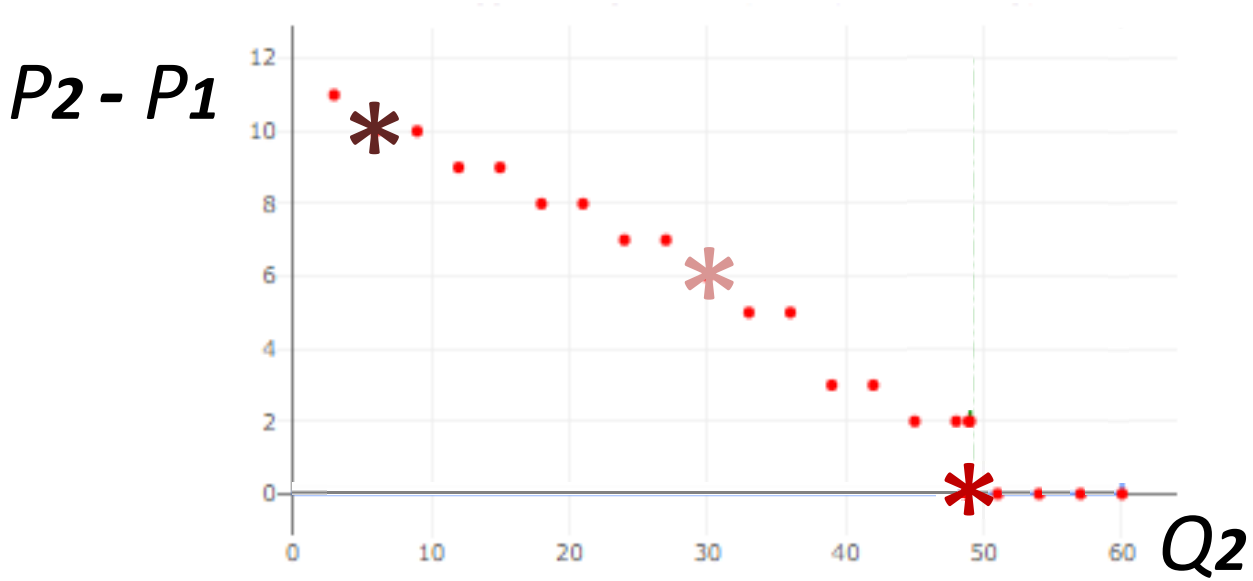
Note new axes: $P_2 - P_1$

the information for this graph is obtained entirely from the graph above (and similar ones)



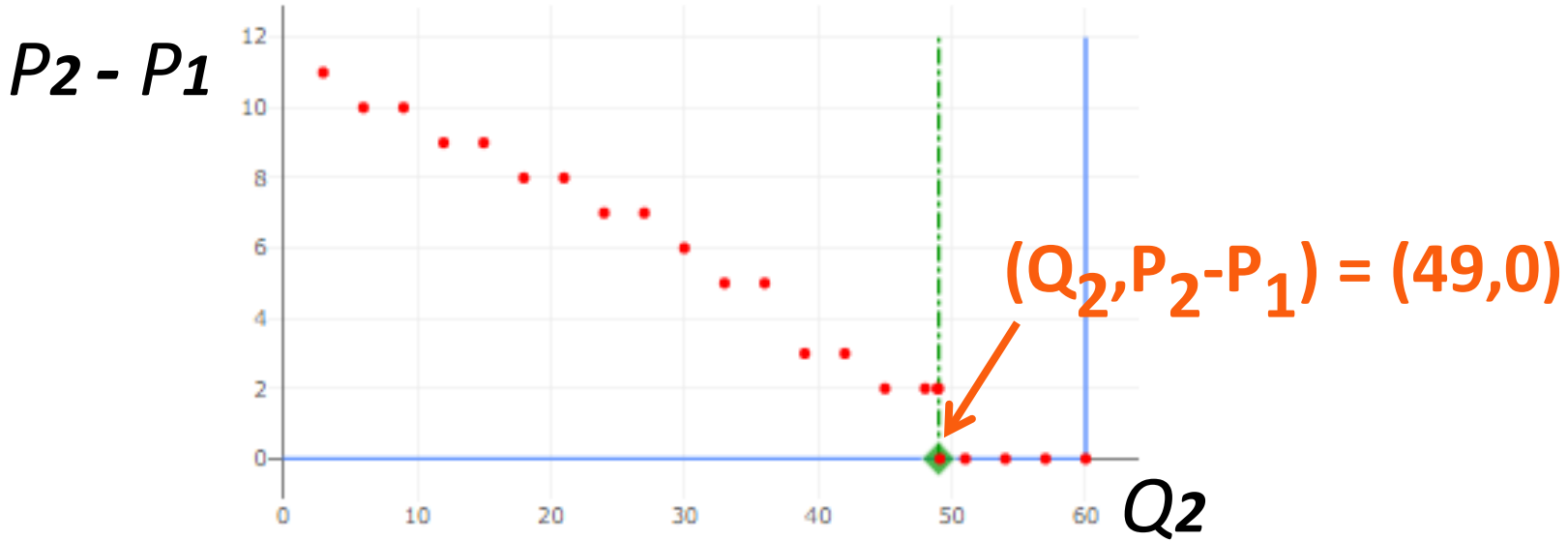
demoCombine4Vert

All options that sell 60 units in all



(Relative) demand

Combining the two auctions
(treating the two varieties as identical)

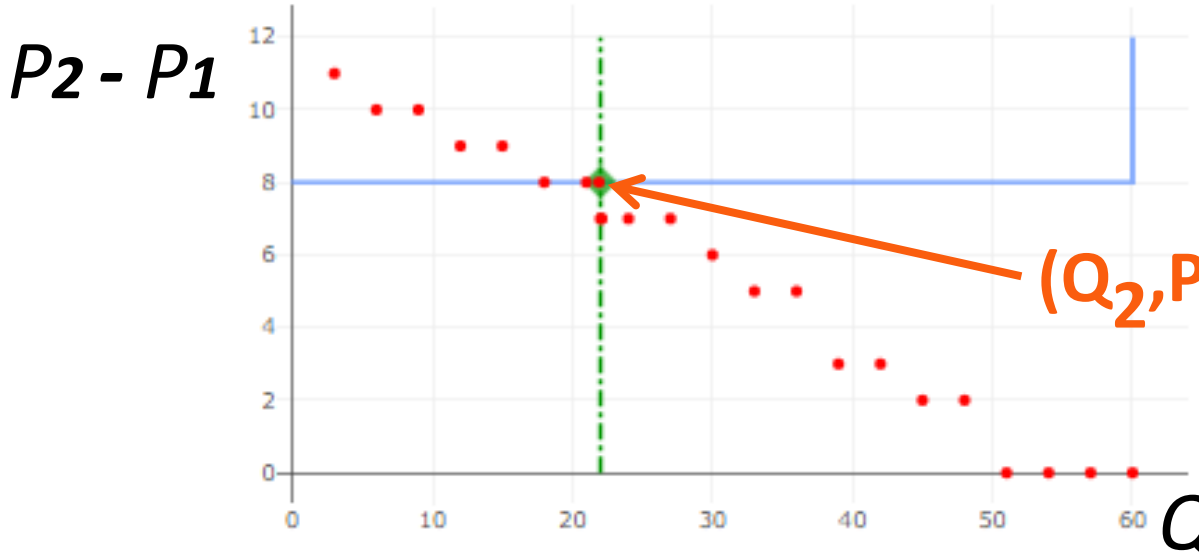


Central banks have often done this

demoCombine4Vert

"Reference-price" auction

(auction varieties simultaneously with pre-specified price difference e.g., $P_2 - P_1 = 8$)



e.g., Bank of Japan, RBA, and U.S. Fed have all done this
but auctioneer

--may not have the information to set "correct" price difference

--may not want to signal its "view" of "correct" price difference

--may want to use the auction to learn about bidders' valuations

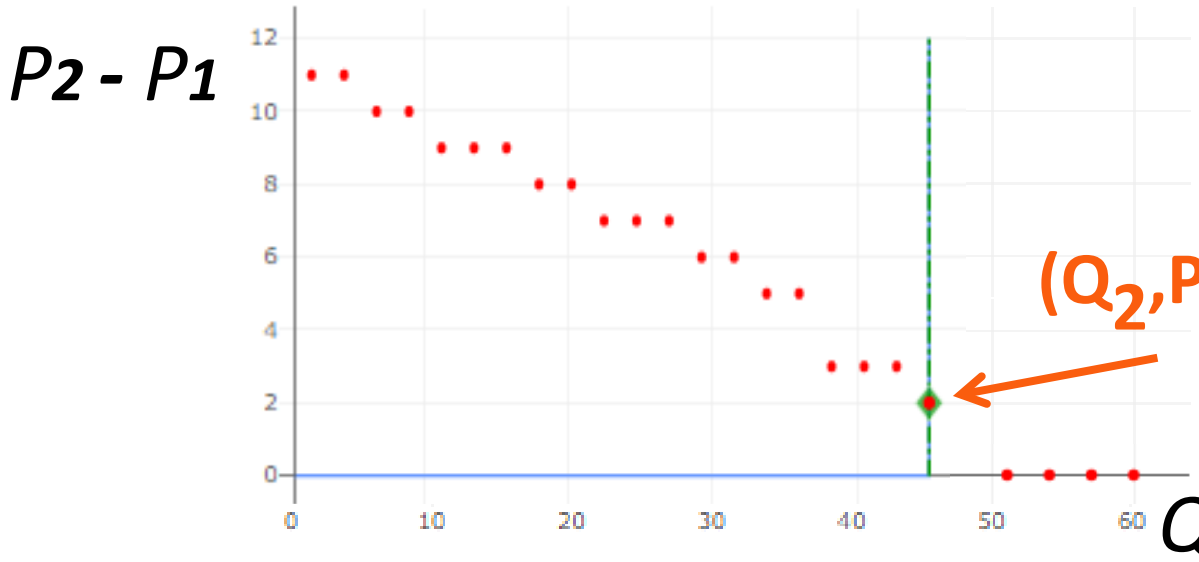
→Bank of England wanted to let market determine price difference

demoCombine3vert

(i.e., let auction determine price difference based on demands)

Constraining maximum quantity of good 2

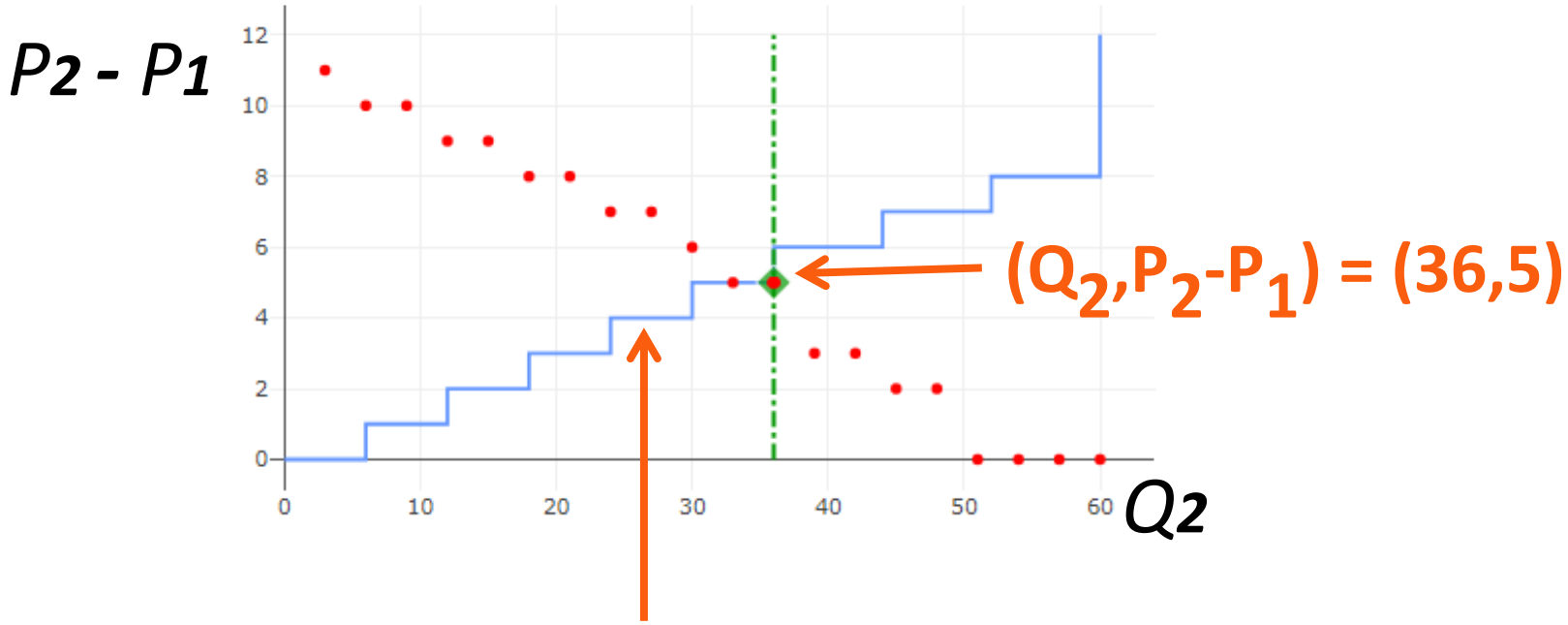
(e.g., $Q_2 \leq 45$)



$(Q_2, P_2 - P_1) = (45, 2)$

demoCombine2vert

Upward sloping supply curve for good 2

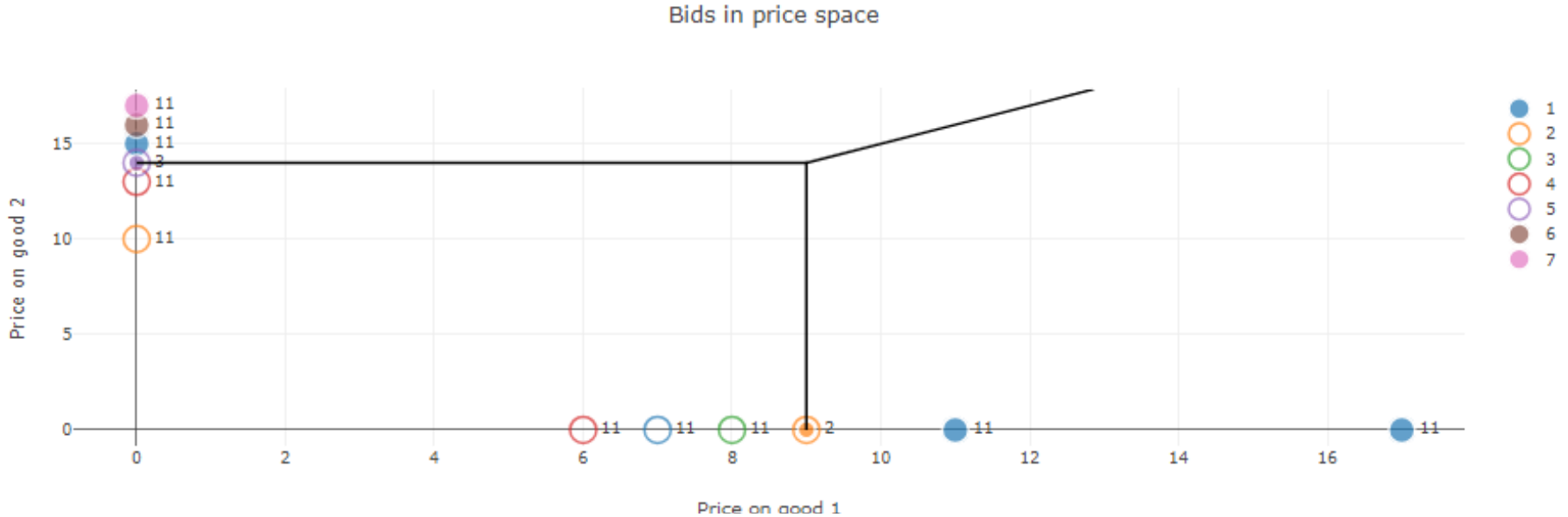


A better approach?

--as used by the Bank of England's Product-Mix Auction

demoUpwardSlopingVert

Upward sloping supply curve for good 2
Bids in price space & allocations of bids to goods



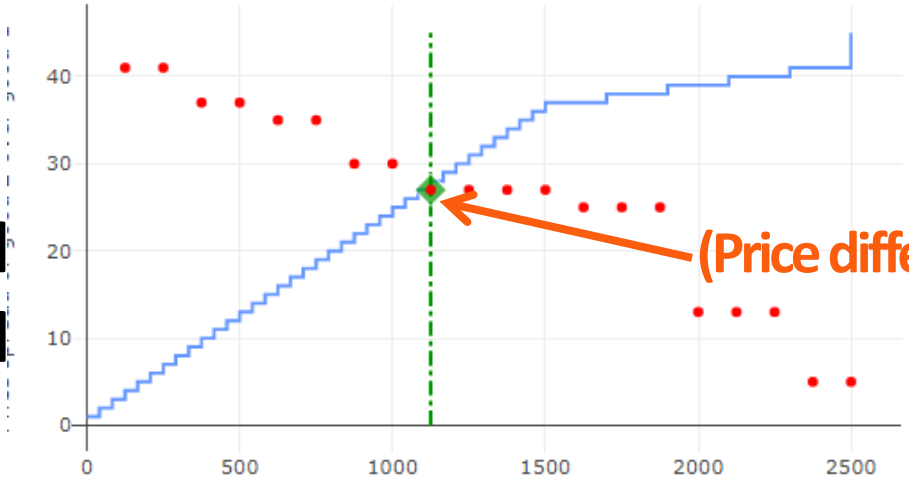


demoUpwardSlopingVert

"Screen Shots" from Bank of England's original Long-Term Repo Product-Mix Auction*

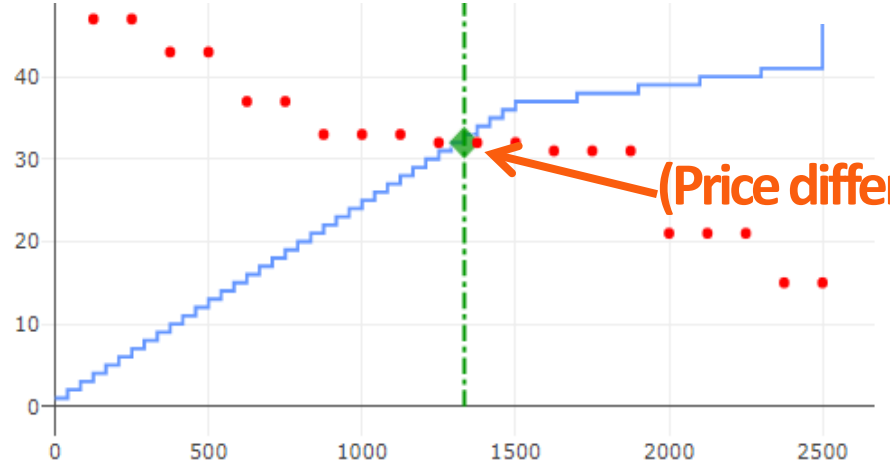
(total allocation capped at £2,500million)

Less stressed demand



(Price difference, % allocated to good 2)
= (27bps, 45%)

More stressed demand



(Price difference, % allocated to good 2)
= (32bps, 53%)

* actual screen shots are confidential

demoBoE/-Stressed

An additional feature

of Bank of England's original Long-Term Repo Product-Mix Auction

What if a bidder can substitute between the two goods?

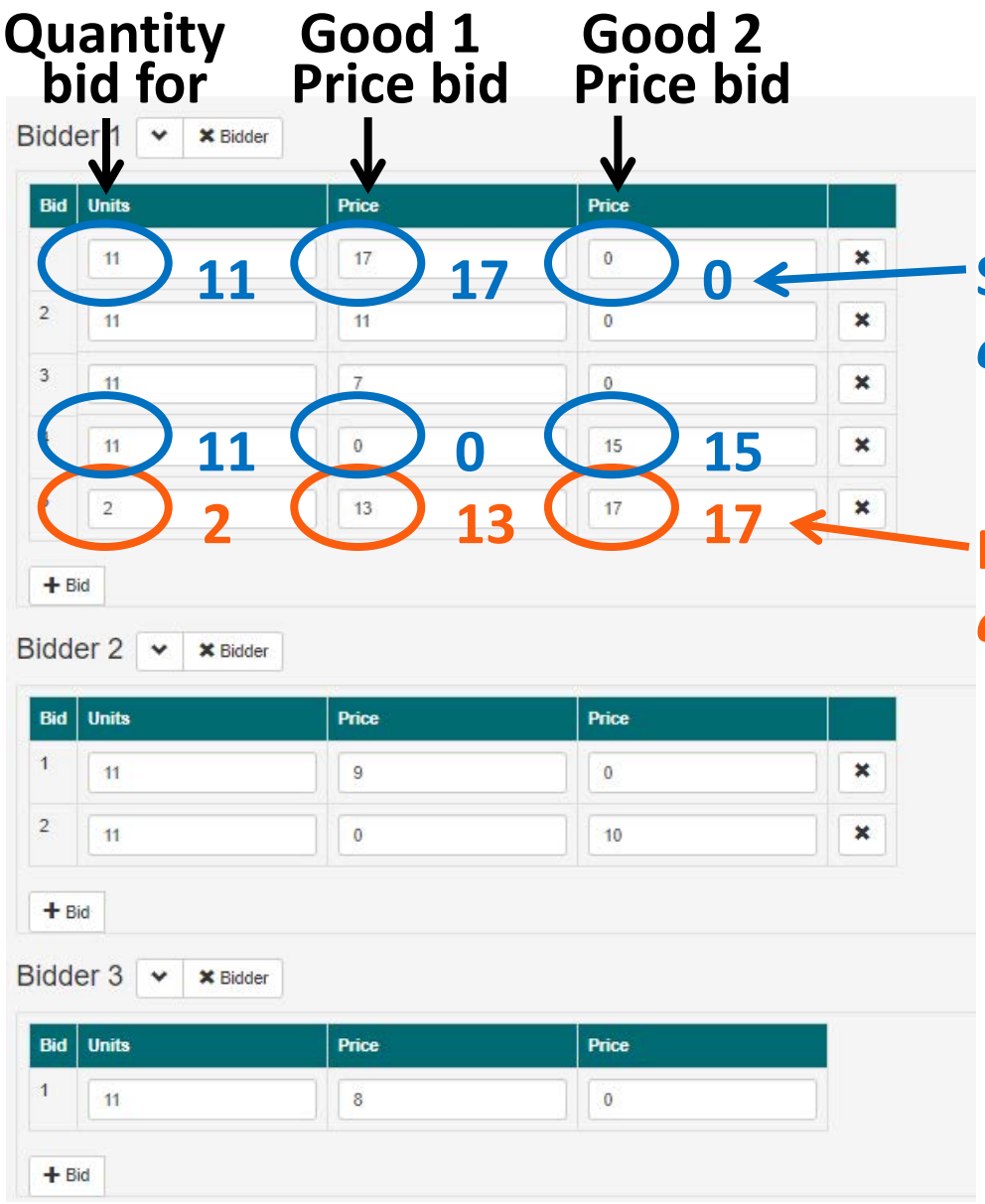
i.e., a bidder has both strong and weak collateral available, and wants to use whichever is "better priced"?

(or in an auction of bonds, a bidder prefers maturity-2 if price difference ≤4bp, but prefers maturity-1 otherwise?)

Answer: allow bidders to make "paired bids" (i.e., "OR" bids)

--a bid can specify a price for both goods;
the bid is allocated ***either*** one good ***or*** the other good (not both)
(it receives whichever good is "best value" for it, or
nothing if both prices are below the auction's prices)

Example of a paired bid

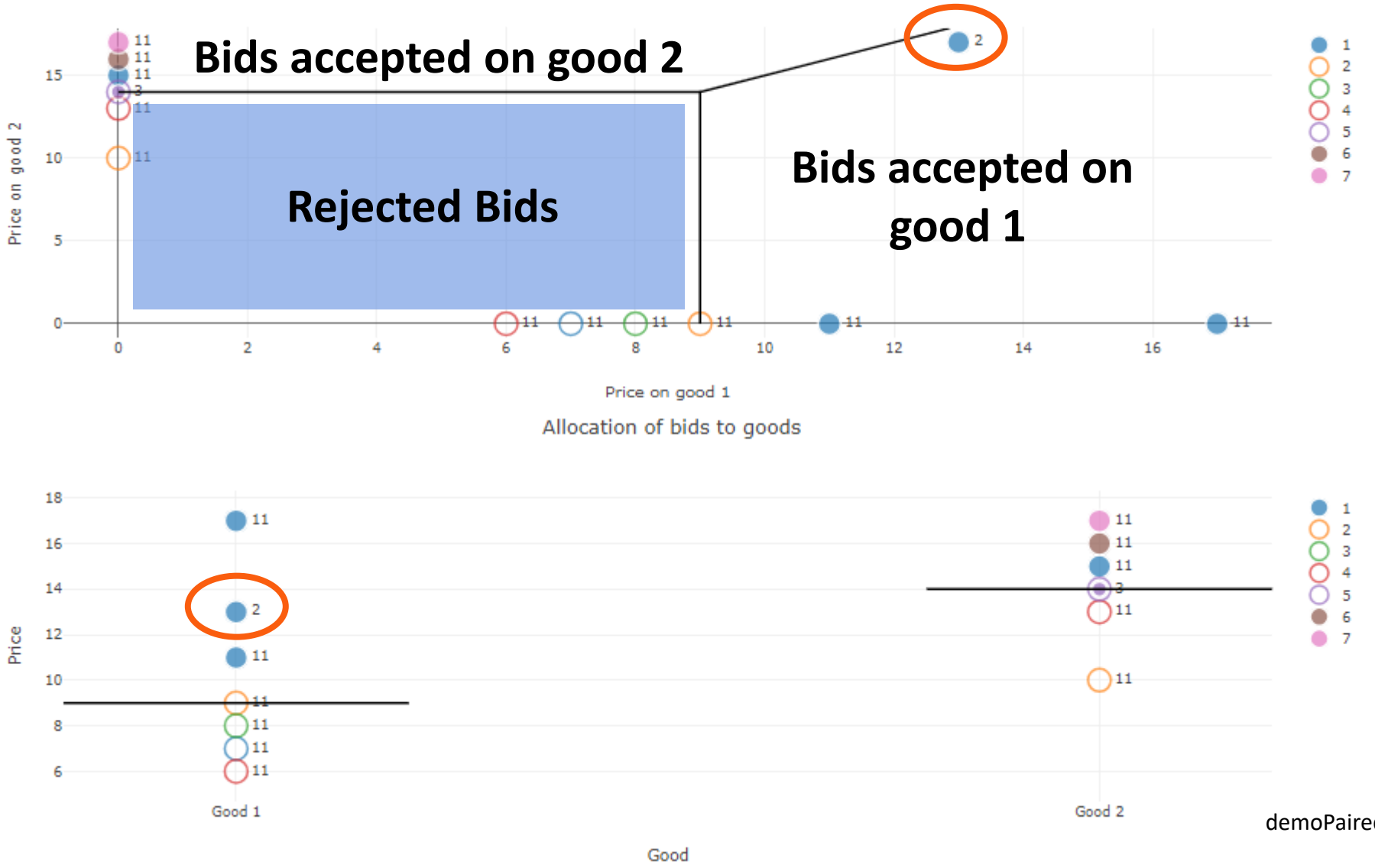


Standard Bid for up to 11 units of *only* good 1 at price up to 17

Paired Bid for up to 2 units of *either* good 1 at price up to 13 *or* good 2 at price up to 17

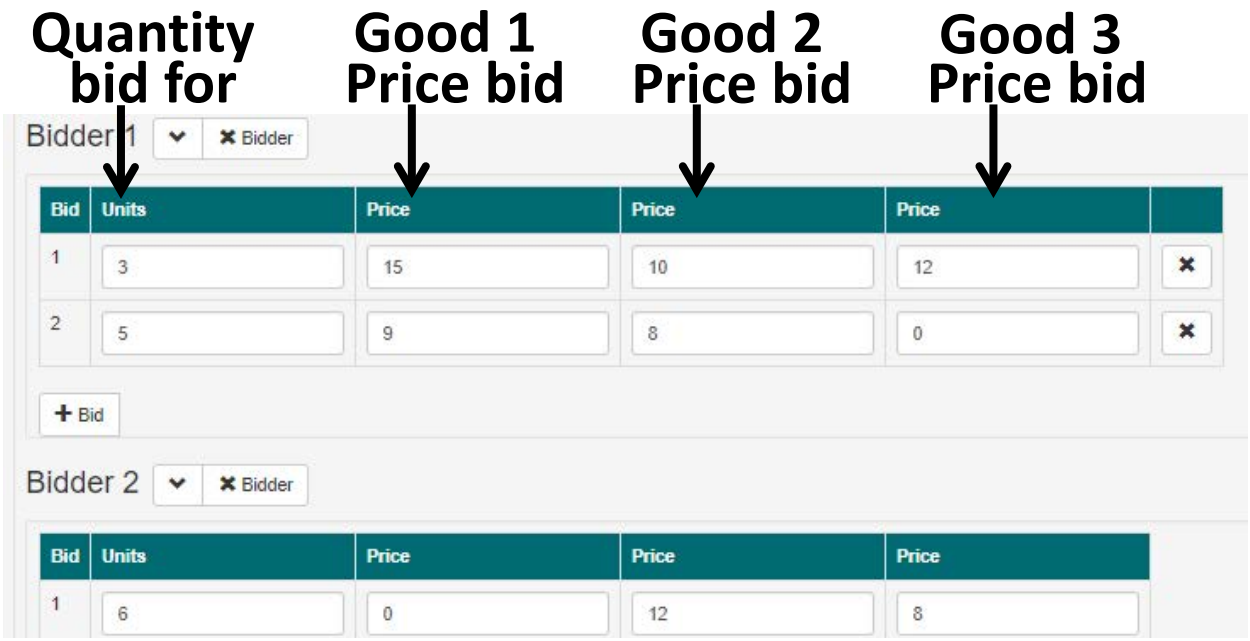
demoPaired

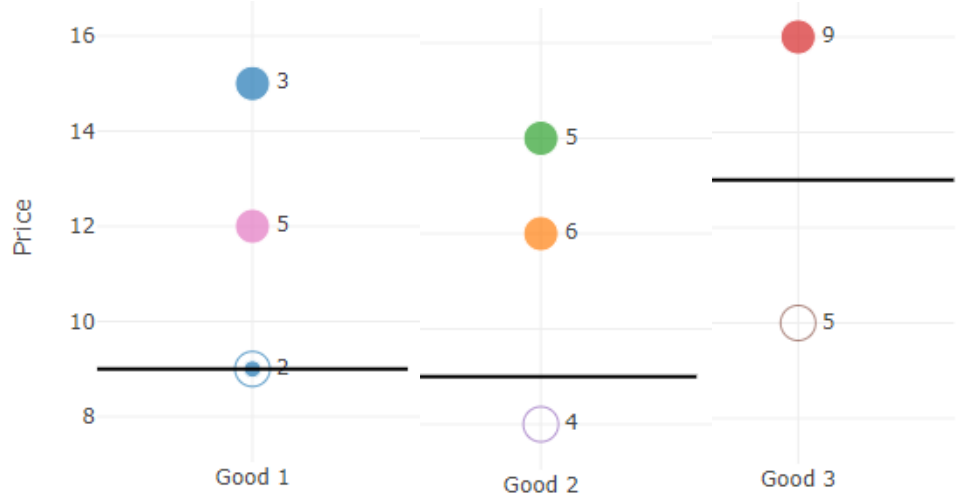
Example of a paired bid at prices (13, 17)
Observe good 1 is better value for this bid at the auction's prices (9, 14) [13-9 > 17-14]



demoPaired

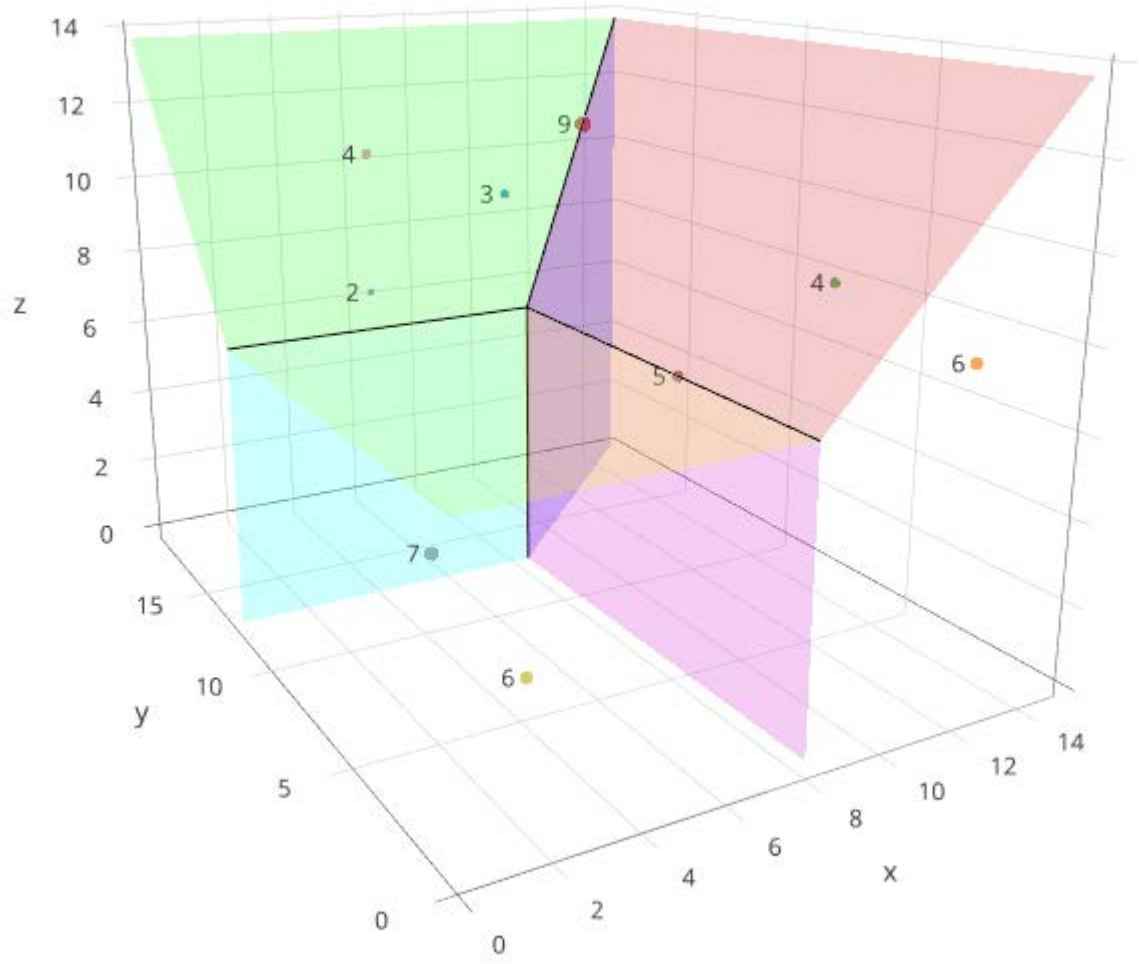
Can have as many different goods as desired





demo3GoodsVert

Three different varieties of goods permits pretty graphs!



demo3GoodsVert

Bank of England's original Product-Mix Auctions extended existing practice by:

1) Combining "standard" auctions
   --so auctioneer can better express its preferences

2) Allowing "paired bids" (i.e., "either/or bids")
   --so bidders can better express their preferences

3. Further Developments, and Other Applications

3.1 Selling Related Bonds

cf. U.K.'s 2016-17
Corporate Bond Purchase Scheme



3.2 Further Development of Bank of England's Long-Term Repo auctions ("ILTRs")

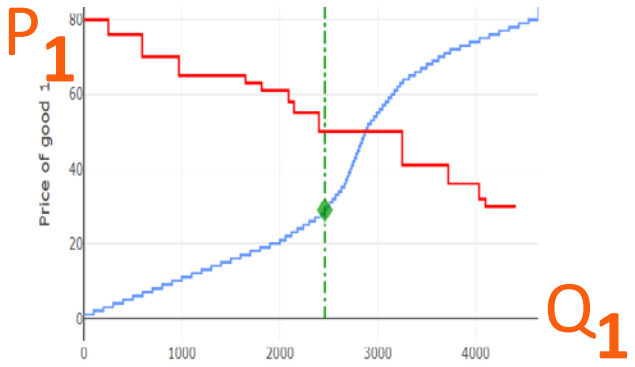
3.3 Iceland's Implementation

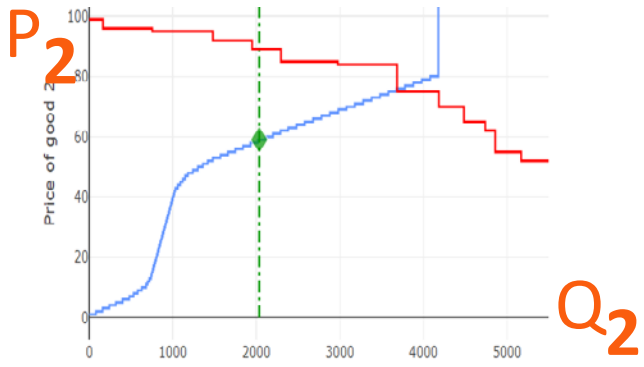


For bonds, may be natural to think of goods "horizontally"

i.e., focus on *absolute* prices of bonds

→ "supply curve" is incremental marginal cost of selling more good ***j*** instead of no sale (a function of Q_j)

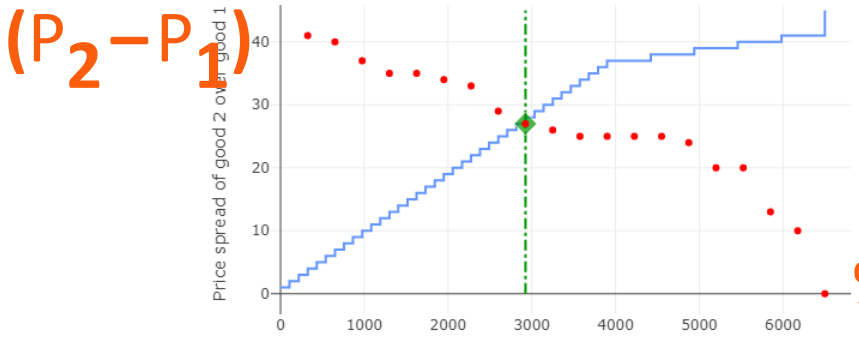




vs. Bank of England's repo auctions think of goods "vertically"

i.e., focus on price *differences* between collateral qualities

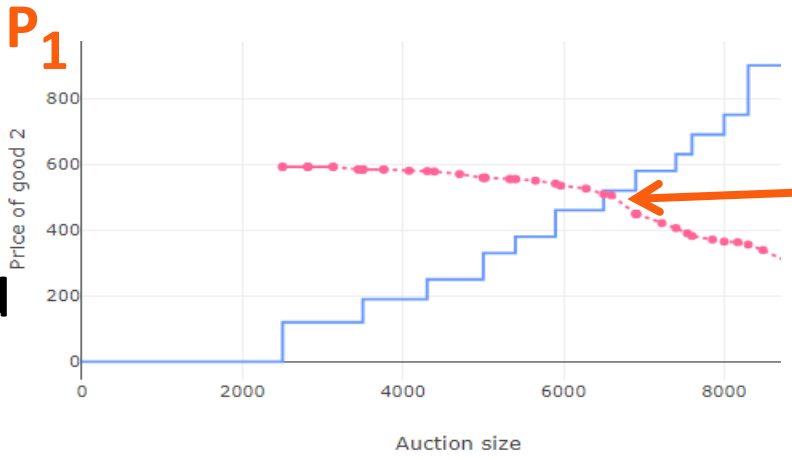
→ "supply curve" is incremental marginal cost of selling more good 2 instead of more good 1 (a function of $Q_2 / (Q_1 + Q_2)$)



3.2 Bank of England's *current* Long-Term Repo auctions*

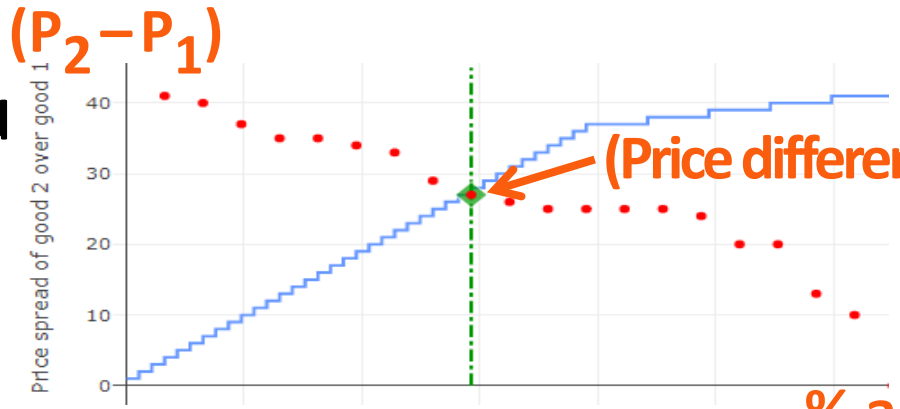
use an *additional* supply curve, e.g.,

Total Supply and Total Demand



→ total supply (Q_1+Q_2) increases if bidding more aggressive

Supply and Demand for good 2



*updated in 2014 (with Elizabeth Baldwin)

3.3 Iceland's problem (2015-16)

(to facilitate exit from capital controls)

Wanted to buy blocked "offshore" accounts
in return for choices of ISK or Euro bonds (or cash)

each bidder (account-owner) has a fixed budget
(i.e., fixed amount of money in her account)

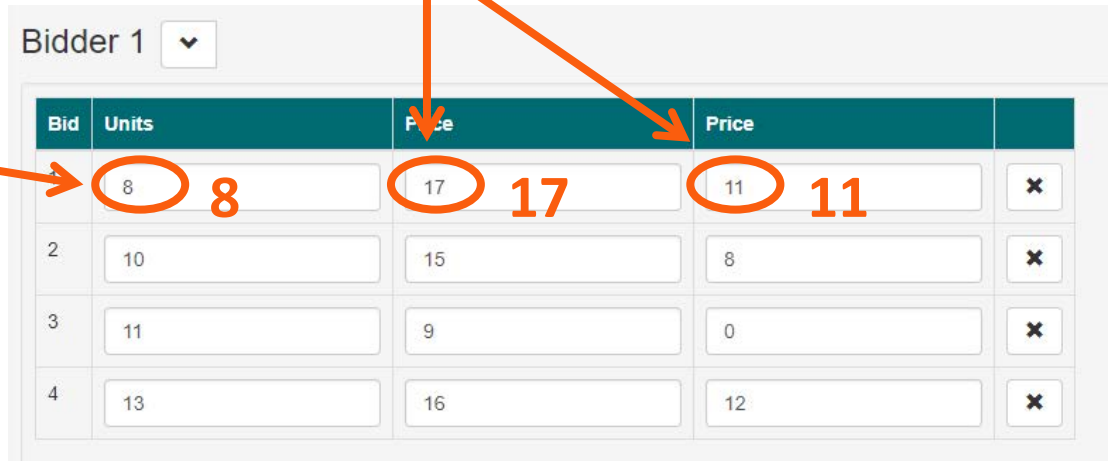
cf. Mexico's "Brady Deal" for debt restructuring
(creditors could choose to exchange their claims
for any combination of three alternatives)

Iceland *vs* Bank of England

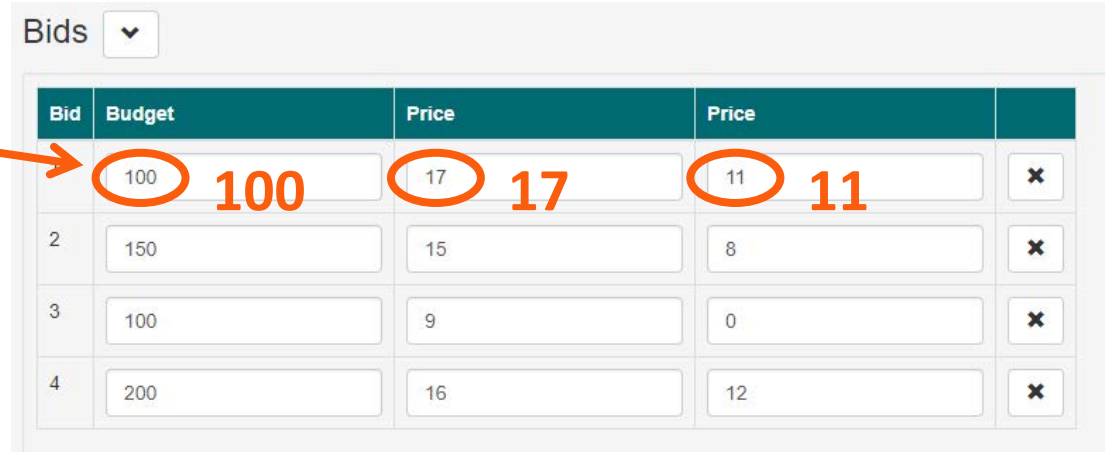
Meaning of bids:

both cases: a bid specifies **maximum price vector** to pay

BoE: also, a **quantity of "product"** to "buy" (i.e., a bid is to receive an amount of loan).



Iceland: also, a **quantity of "money"** (blocked funds) to spend (i.e., a bid is to exchange a quantity of blocked ISK).



Iceland *vs* Bank of England

Choosing between alternatives in "OR" bids:

Both cases: "pay up to V_A for good A, **OR** up to V_B for good B"

BoE: bidder prefers good A if : $\mathbf{V_A - P_A > V_B - P_B}$ (& $P_A < V_A$)

Iceland: bidder who spends budget B
prefers good A if $(V_A - P_A)(B/P_A) > (V_B - P_B)(B/P_B)$ (& $P_A < V_A$)

i.e., bidder prefers good A if : $\mathbf{V_A/P_A > V_B/P_B}$ (& $P_A < V_A$)

Iceland also maximised own surplus *not* "efficiency"

Standard Options

-- maximise (seller's) profit OR maximise efficiency

(Bank of England's repo auctions maximise efficiency
i.e., maximise sum of auctioneer's and bidders' surpluses)

-- uniform pricing OR pay-as-bid (discriminatory) pricing

(Bank of England's repo auctions use uniform pricing)

Pay-as-bid pricing may be more familiar to participants,
but bidders will not necessarily bid (close to) actual valuations,
so *not* as efficient, informative, and easy, as uniform pricing

Lots More!

e.g., can permit ***other forms of preferences***

can have ***Alternative Pricing Rules***

can have ***multiple sellers*** (as well as multiple buyers)
&/or ***traders*** (who can both buy and sell
e.g., a trader could follow the rule:
"switch Shell for BP iff
change in price difference < 2p")

Summary

1. Bidders simultaneously state their preferences (as bids); alternative ways to express these, depending on context.

2. Prices and allocations mean every bidder gets what's best for her, based on her stated preferences.
(Uniform pricing preferable, but pay-your-bid pricing is an option.)

Easy to start simple:
e.g. allow only "single-variety" bids
-- i.e., each bid specifies a price and quantity for just one variety, but each bidder can make multiple bids

More sophisticated: allow "OR" bids
-- i.e., each bid can specify a trade-off between varieties

Software can also be used to develop ideas, and experiment with alternative allocations

"A marvellous application of theoretical economics to a practical problem of vital importance"

-- Mervyn King, then-Governor,
(quoted in *the Economist*)



"A major step forward in practical policies to support financial stability"

-- Paul Fisher, Executive Director



Mark Carney, current-Governor

-- expanded the role of the auctions (in 2013)

-- introduced updated version (more options for auctioneer) (in 2014)



The Product-Mix Auction also has many applications beyond finance …

(I'm happy to offer *pro bono* advice)

APPENDIX 1

Some Practical Details of Bank of England's Indexed Long-Term Repo Auctions

Practical details: Participation

from Bank of England “Red Book” (as at March 2018)
https://www.bankofengland.co.uk/markets/the-sterling-monetary-framework

Banks and building societies

and

“those broker-dealers deemed critical to the stability of the UK financial system (designated investment firms) are eligible to apply for participation”

89 participants in OMOs eligible as of 14, Dec., 2017

Practical details: Collateral Sets

from Bank of England “Red Book”
(please check BoE website for up-to-date details)

Level A: “*high-quality highly liquid sovereign securities”*
[now Canada, France, Germany, Netherlands, UK, US]

Level B: “*high-quality liquid collateral, including other Sovereign”* [Australia, Portugal, etc.], “*supranational, mortgage and corporate bonds”* [e.g., UK AAA RMBS]

Level C: “*less liquid securitisations, own-name securities and portfolios of loans”* [e.g., UK A3/A- RMBS]
(Level C securities must be delivered to BoE in advance)

Practical details: Collateral Sets (at March 2018)

please check BoE website for up-to-date details

e.g., Sovereign and Central Bank debt is

Level A:
Canada, France, Germany, Netherlands, U.K., and U.S.

Level B:
Australia, Austria, Belgium, Denmark, Finland, Ireland, Italy, Japan, Luxembourg, New Zealand, Norway, Portugal, Slovenia, Spain, Sweden, and Switzerland

Practical details: Collateral Sets (at March 2018)

please check BoE website for up-to-date details

Other examples of Level B:

The most senior tranches of prime (ABS)
backed by credit cards issued to prime borrowers,
or by auto loans and certain equipment leases,
or by US student and consumer loans,
of the highest credit quality (broadly equivalent to AAA)
(no unlisted ABS).

Portfolios of senior corporate bonds
of credit quality broadly equivalent to A3/A- or above.

Commercial paper issued by non-financial companies
of credit quality broadly equivalent to A2/P2/F2 or above.

All issued in UK, US or the EEA.

Practical details: Collateral Sets (at March 2018)

please check BoE website for up-to-date details

Other examples of Level B:

FHLMC, FNMC and FHLB Conventional debt security issues of the highest credit quality (broadly equivalent to AAA).

UK and Dutch RMBS most senior tranches of the highest credit quality (broadly equivalent to AAA). (not unlisted RMBS).

UK, French, German and Spanish regulated covered bonds of the highest credit quality (broadly equivalent to AAA).

Practical details: Collateral Sets (at March 2018)

please check BoE website for up-to-date details

Examples of Level C:

UK and EEA RMBS (most senior tranches), and covered bonds,

UK, U.S. and EEA ABS, CMBS, securitised corporate/SME loan portfolios, and securitised corporate bond portfolios,

all of credit quality broadly equivalent to A3/A- or above.

Practical details: Bids

from BoE “SMF Operating Procedures” (as at August 2015)

www.bankofengland.co.uk/markets/Documents/money/documentation/smfopprocs.pdf

Minimum bid size: £5 million; increments of £1 million.

No restriction on the number of bids submitted,

Bids at the clearing spread may be scaled back and rounded down to the nearest £0.1 million

Practical details: Choice of Supply Schedule

from BoE paper (pub. March 2011; on BoE website Aug 2015)
http://www.bankofengland.co.uk/publications/speeches/2011/speech487.pdf

[supply schedule] is not revealed to the market…

[is] continuous but otherwise need not be linear.

Practical details: Choice of supply schedule (cont)

from BoE paper (pub. March 2011; on BoE website Aug 2015)
http://www.bankofengland.co.uk/publications/speeches/2011/speech487.pdf

[schedule] is pinned down by three broad considerations.

1. allocate some funds regularly against wider collateral

2. not undermine incentives to manage liquidity prudently

3. permit an increased allocation against wider collateral to the system in the face of adverse liquidity shocks and the associated heightened demand for liquidity

An announcement (from Bank's website)

Date : Tuesday 11 August 2015 10:00AM

10:00 11/08/15 Announcement of Indexed Long-Term Repo operation
Bids are invited for today's Indexed Long-Term Repo operation

Term	6 months
Date of operation	11/08/2015
Time for submission of bids	10:00 - 10:30
Amount offered	Minimum Stg 5,000mn
Settlement date	13/08/2015
Maturity date	11/02/2016
Auction type	Variable rate tender
Repo type	Indexed to Bank Rate
Allotment method	Uniform price basis
Minimum bid amount	Stg 5mn
Multiples of	Stg 1mn

The minimum amount offered is as set out above; the total amount available can increase to many multiples of this minimum amount.

Example of actual results
(from Bank's website for an early 2-collateral auction --recent auctions have been for 3 collaterals)

10:40 14/12/2010 Results of Long-term Repo operation

03 Month 17/03/2011

Amount on offer	Stg 5,000mn
Total bids received	Stg 7,230mn
Total amount allotted	Stg 5,000mn
Cover Ratio	1.45
Stop-out spread	20bps

Collateral set summary	Narrow	Wide
Bids received	Stg 6,445mn	Stg 785mn
Amount allotted	Stg 4,238mn	Stg 762mn
Cover ratio	1.29	0.16
Percentage allotted	84.76	15.24
Clearing spread above Bank Rate	5bps	25bps

Repos will be indexed to Bank Rate

Summary of 1st year's results: Range (average)

	3-month	6-month
Amount offered	£5,000m	£2,500m
Number of operations	8	4
Cover ratio	71-154% (121)	86-229% (150)
Strong clearing price	0-5bps (2)	0-2bps (1)
Weak clearing price	19-26bps (24)	16-52bps (42)
Stop-out spread	18-26bps (22)	16-52bps (41)
Allocated to Strong	54-89% (80)	32-93% (69)
Allocated to Weak	5-18% (13)	1-33% (21)

4 auctions were uncovered
the same number were unallocated

APPENDIX 2

Geometric Representation of Bids, and Bidding Languages

For more detail see paper **“Product-Mix Auctions”**
original 2008 paper, **updated in 2018**, on my website

and see also videos on my website:

5-minute film published by the Guardian newspaper (2013)

Auction Design in the Financial Crisis (2017)

My Favourite Geometry, and its use in Auction Design (2017)
(for an academic audience)

Product-Mix Auctions*

1. Bidders simultaneously state their preferences (as "bids"); alternative ways to express these, depending on context.

2. Implement Competitive Equilibrium allocation consistent with stated preferences ("bids")

(Basic version uses (lowest) competitive equilibrium prices, but pay-your-bid pricing is an option.)

Auction will work well if ***bidders*** bid (approx.) true values

--auctioneer need ***not*** state her actual preferences
(Bank of England does in its implementation;
Icelandic government planned not to in its)

***see 2008 paper, updated in 2018, on my website**

Product-Mix Auctions

1. Bidders simultaneously state their preferences (as "bids"); alternative ways to express these, depending on context.

Bids can be understood, explained, and analysed *geometrically.*

2. Implement Competitive Equilibrium allocation consistent with stated preferences ("bids")

geometric language is:

--easy for bidders to understand, and use,
and bidders can express their preferences accurately

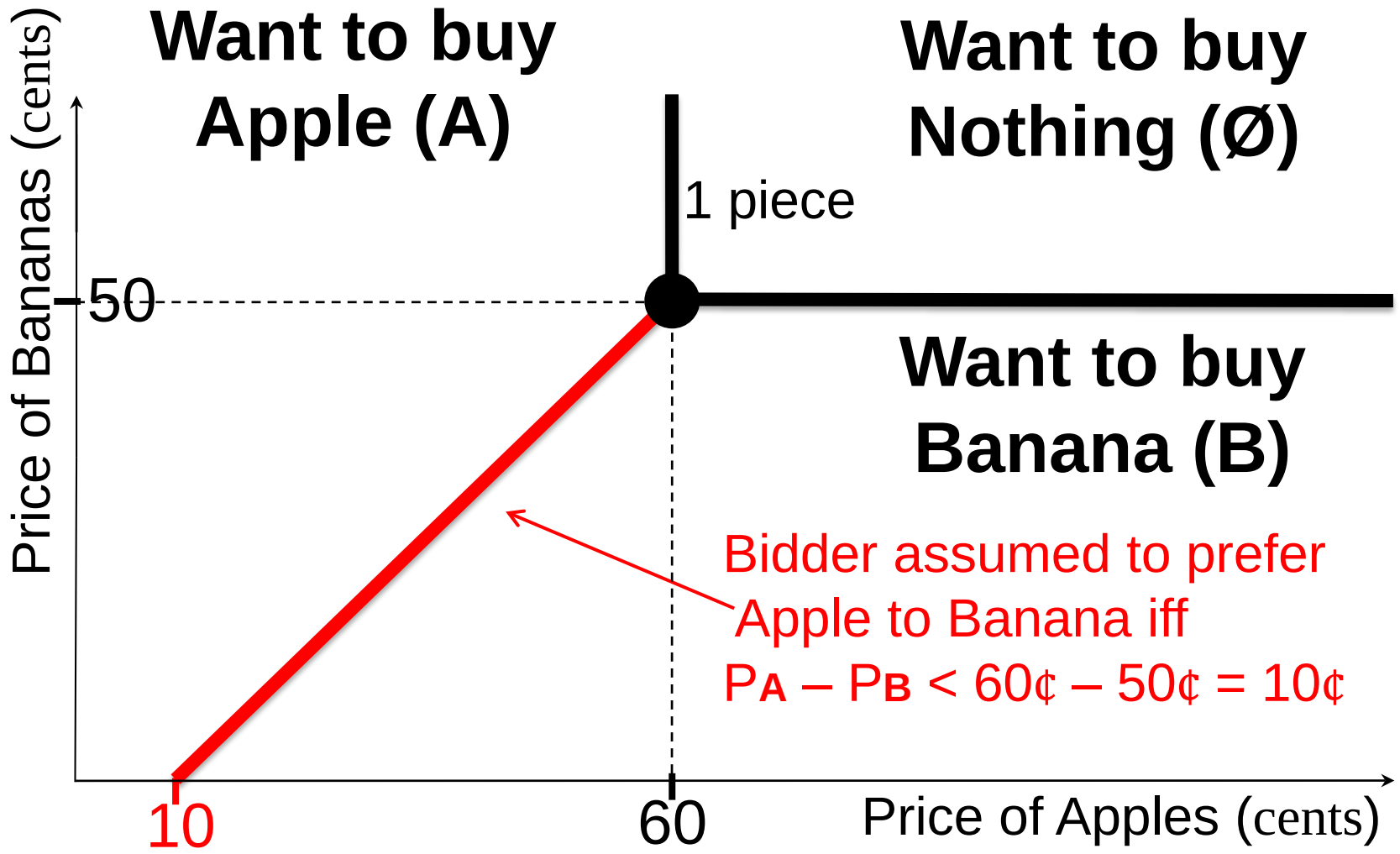
--easy for auctioneer to understand, and trust,
and auctioneer can find competitive equilibrium

--give bidders incentive to state their true preferences
(or near enough), and is robust against other manipulation

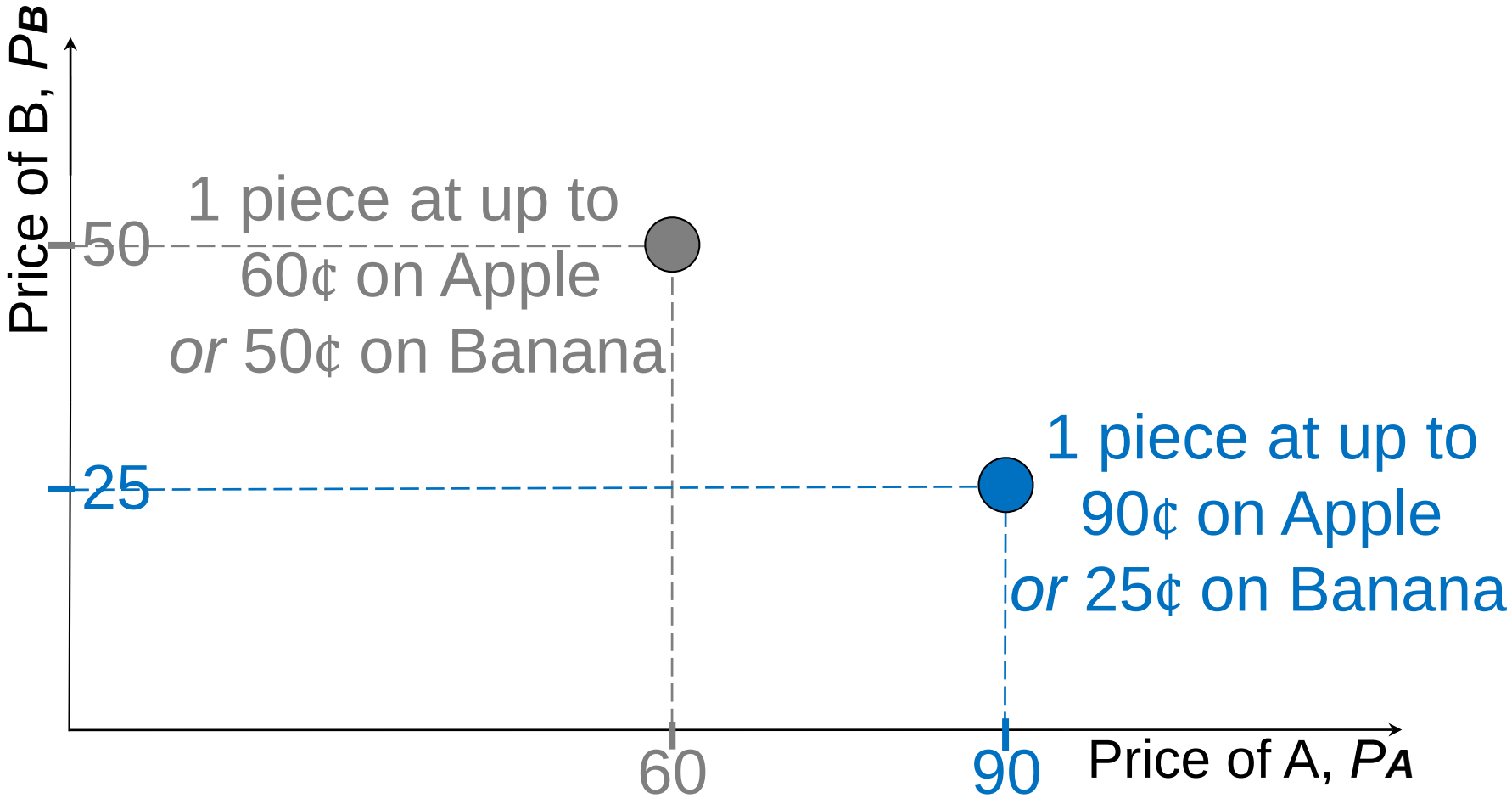
Example of a bid for (at most) a single piece of fruit:
this bid says “willing to pay up to 60¢ for an Apple,
I have no interest in a Banana”



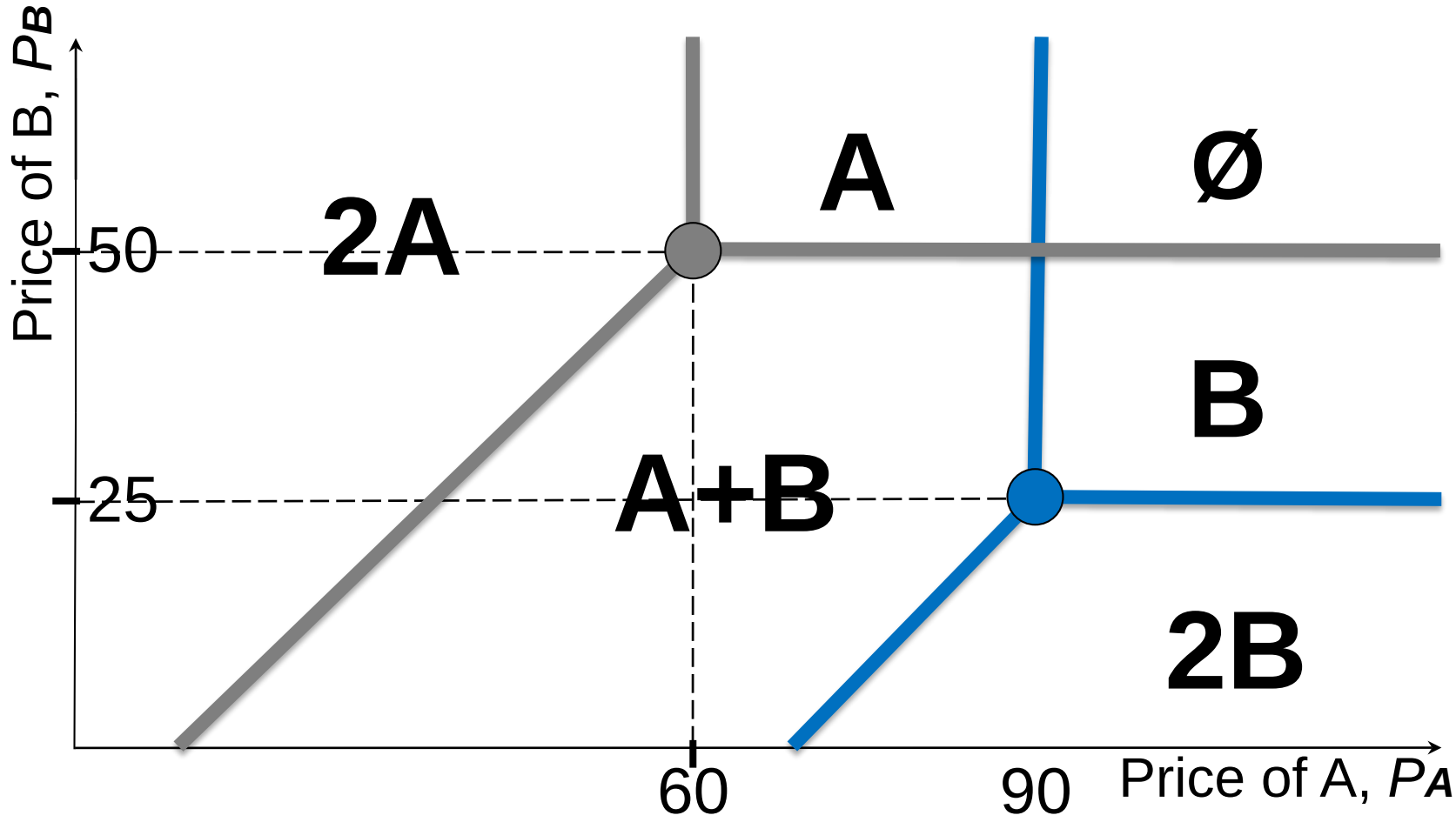
Example of a bid for (at most) a single piece of fruit:
this bid says “willing to pay up to 60¢ for an Apple,
or up to 50¢ for a Banana”



Example of bids for (up to) ***two*** pieces of fruit:
(the second is for my wife, who has an even stronger relative preference for apples)

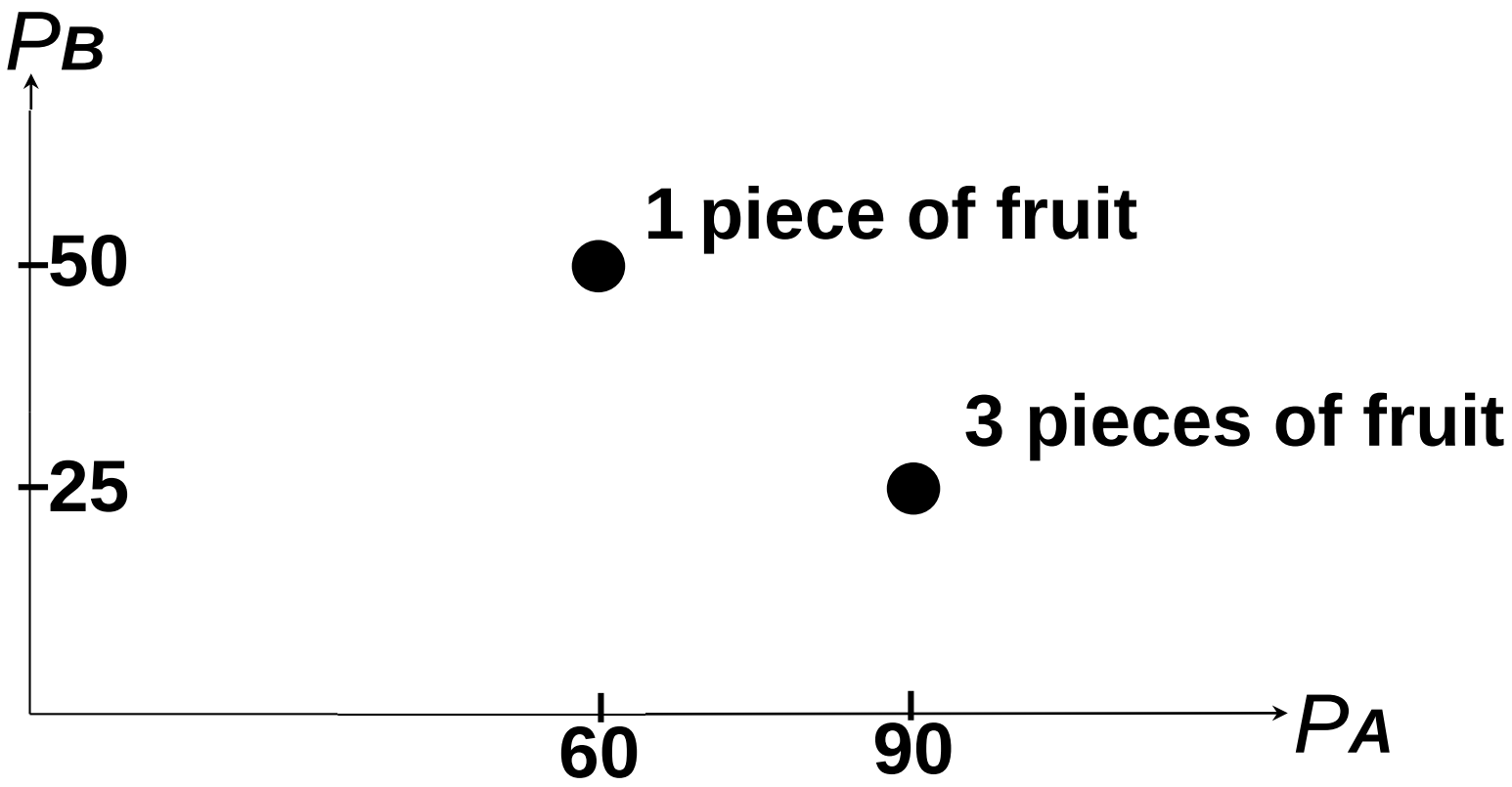


Example of bids for (up to) ***two*** pieces of fruit:
These bids will now results in these purchases,
as function of the prices set by the auction



Bank of England's (original) bidding language

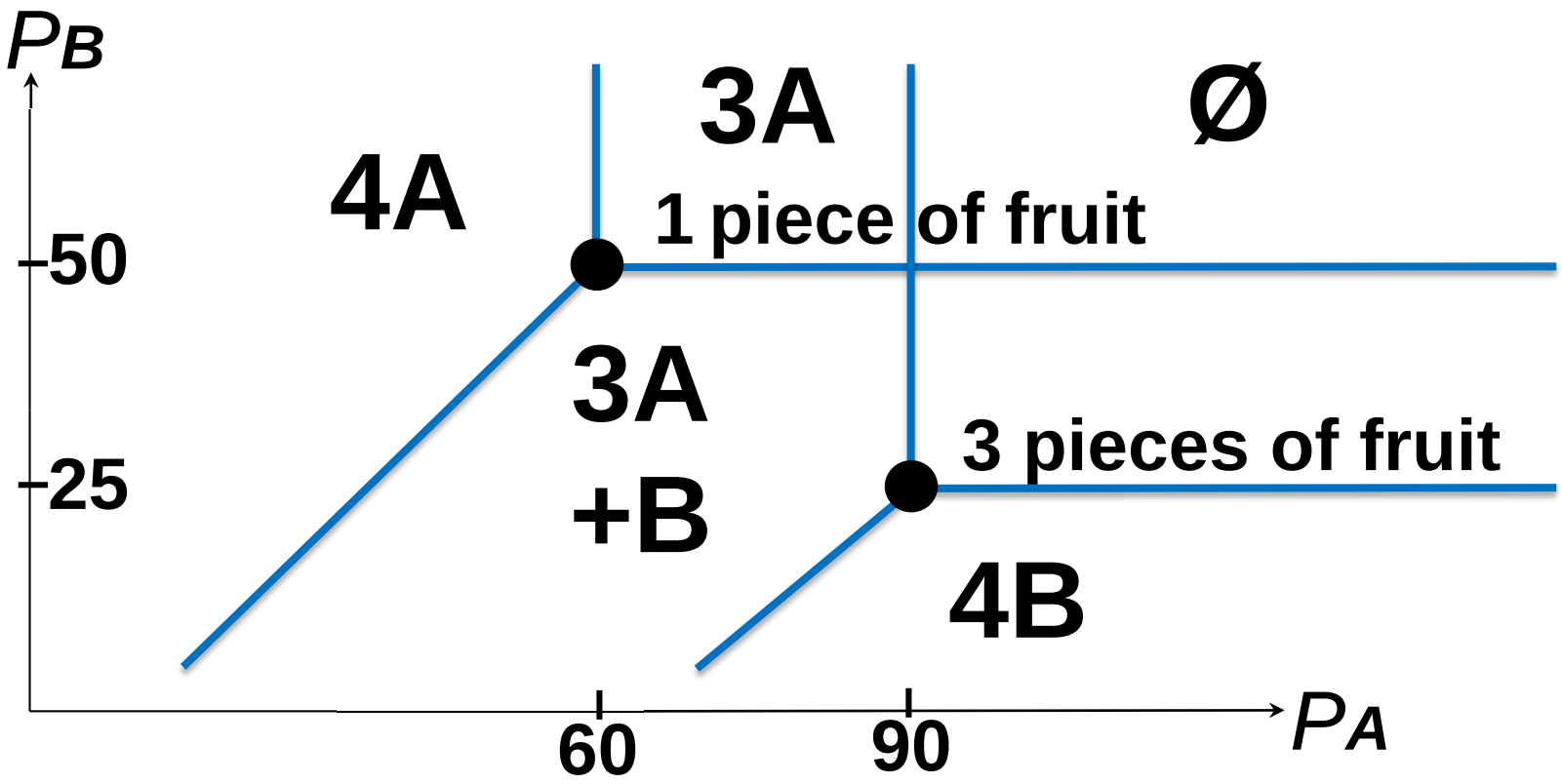
An *ordinary* bid is a list (of any length) of price vectors, and an associated quantity for each price vector, e.g., (60,50; 1), (90,25; 3), ….



Bank of England's (original) bidding language

The *ordinary* bids (60,50; 1), (90,25; 3), together result in these purchases (as a function of the prices (P_A, P_B) set by the auction).

(*note*: from auctioneer's point of view, doesn't matter which bids come from which bidder(s))



ordinary dot-bids express 1:1 trade-offs

--(more than) enough for bidders in Bank of England's auction

e.g., bidder *might* want:

{ **£1bn** loan using Type A collateral if r<3%,
OR **£1bn** loan using Type B collateral if r<5%,
OR neither if both interest rates too high. }

(and may want to make several such bids at different price vectors)

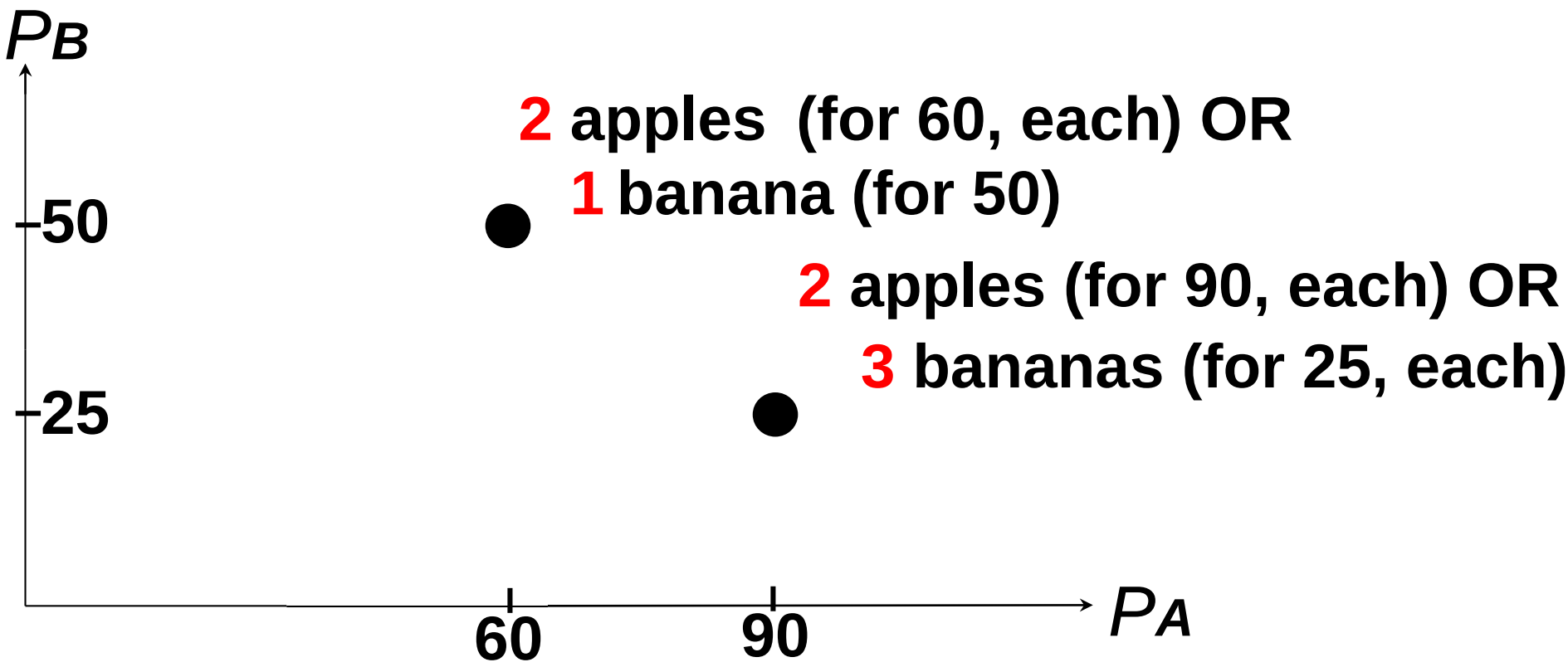
Less likely to want :

{ **£1bn** loan using Type A collateral if r<3%,
OR **£2bn** loan using Type B collateral if r<5%,
OR neither if both interest rates too high. }

*But if bidders **do** want to express these preferences,*
we can generalise the bidding language …

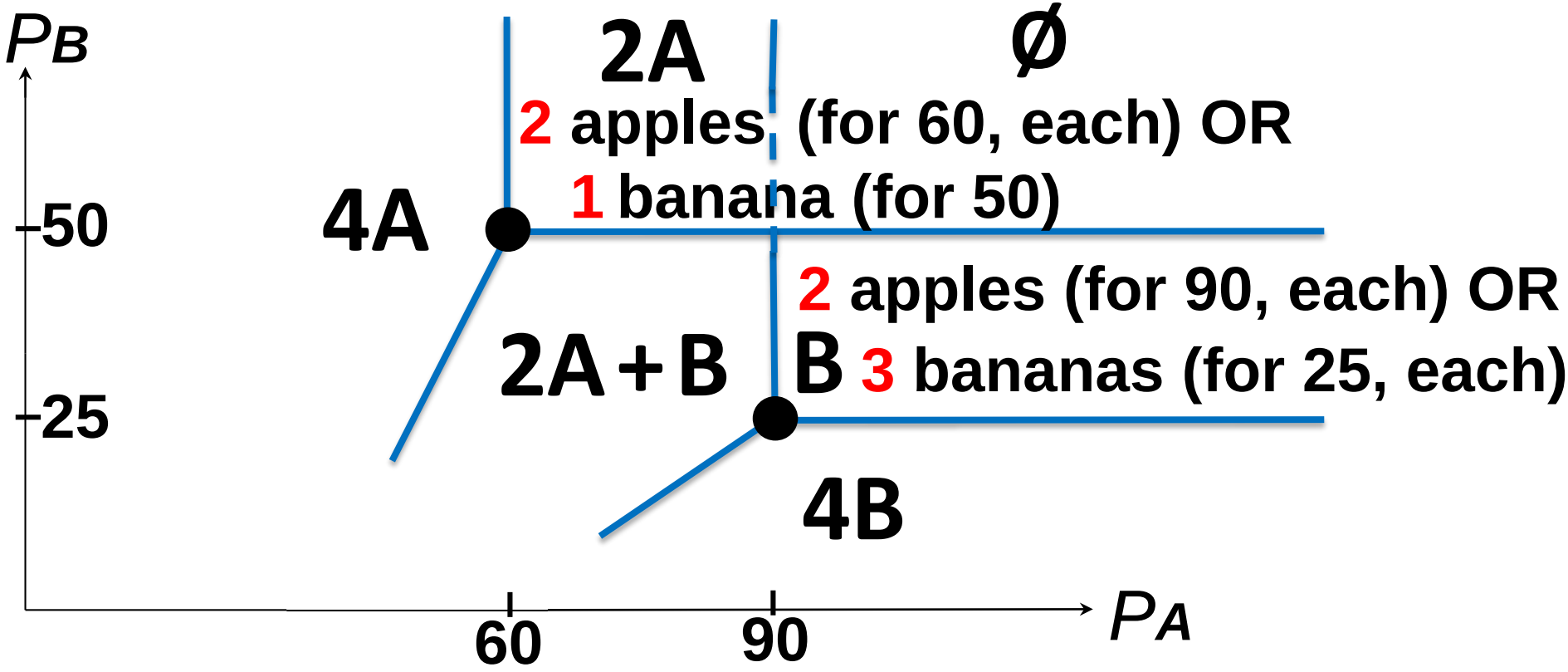
Generalised Bank of England bidding language

A *generalised* bid is a list (of any length) of price vectors, and an associated **quantity vector** for each price vector, e.g., (60,50; **2,1**), (90,25; **2,3**), ….



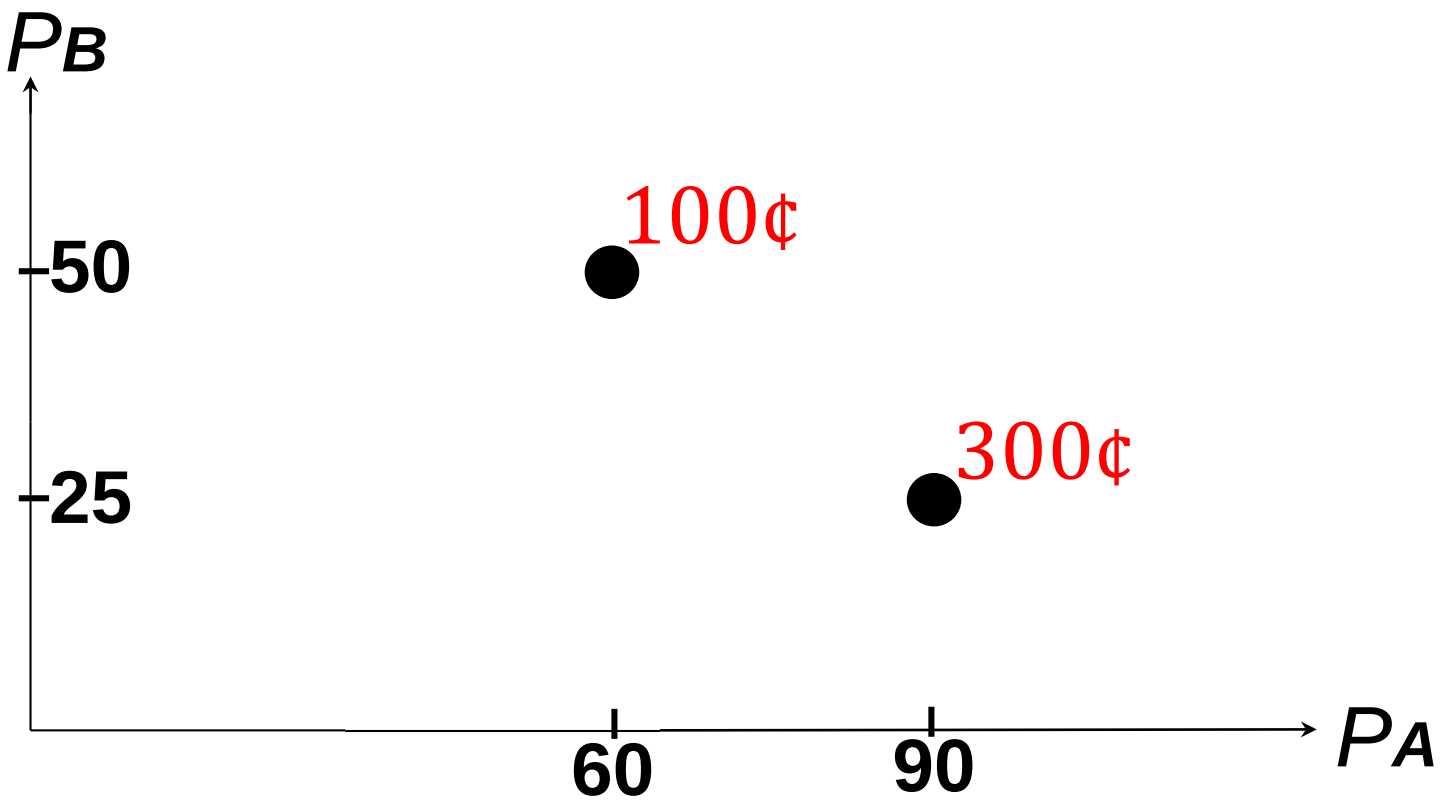
Generalised Bank of England bidding language

The *generalised* bids (60,50; **2,1**), (90,25; **2,3**), together result in these purchases (as a function of the prices (P_A, P_B) set by the auction).



Fixed Budgets (as in Iceland's implementation)

If bidders have fixed budgets, a bid is a list (of any length) of price vectors, and an associated budget for each price vector, e.g., (60,50; 100¢), (90,25; 300¢), ….

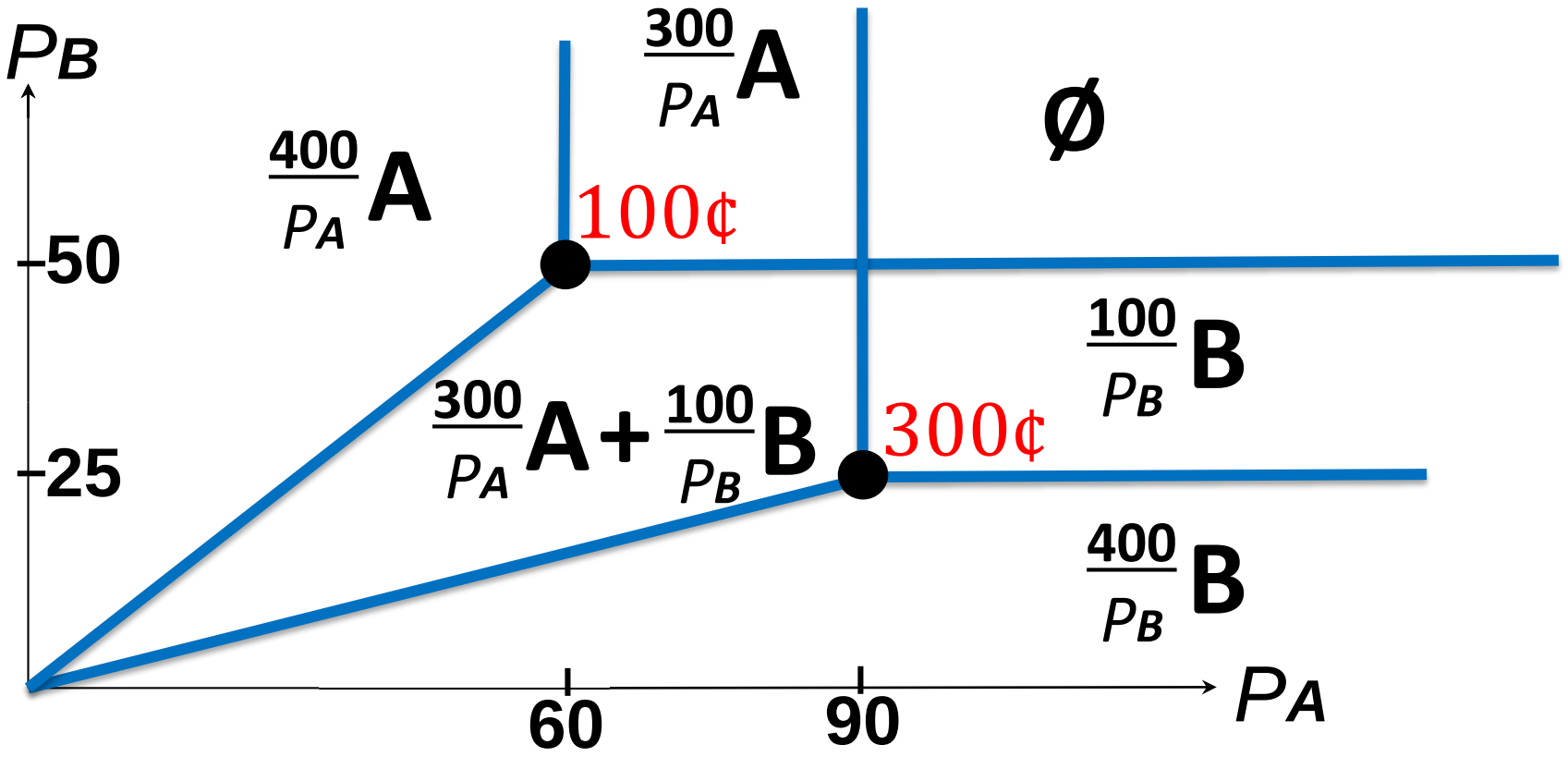


Fixed Budgets (as in Iceland's implementation)

If bidders have fixed budgets, the bids (60,50;100¢), (90,25;300¢), together result in these purchases.

*(recall, a bid (**V_A, V_B**; ·) wants A if **$V_A/P_A > V_B/P_B$** (& $P_A<V_A$)*

*vs., in Bank of England's (original) language, the bid wants A if **$V_A - P_A > V_B - P_B$** (& $P_A<V_A$))*



We have additional bidding languages:

A bidder with ***any*** substitutes preferences can *perfectly* represent its preferences using generalised positive ***and negative*** bids

Most-recent update of Bank of England's auction allows some other forms of preferences, permitting total supply to depend on bidding.

see paper "Product-Mix Auctions" (2008, updated in 2018) on my website

Product-Mix Auctions — Summary

1. Bidders simultaneously state their preferences (as bids); alternative ways to express these, depending on context.
Bids can be understood, explained, and analysed geometrically.

2. Implement Competitive Equilibrium allocation consistent with stated preferences ("bids")
(Basic version uses (lowest) competitive equilibrium prices, but pay-your-bid pricing is an option.)

Easy to start simple: e.g. allow only "single-variety" bids
-- i.e., each bid specifies a price and quantity for just one variety, but each bidder can make multiple bids

[More sophisticated: allow "OR" bids
-- i.e., each bid can specify a trade-off between varieties]

Our software can also be used to develop ideas, and to experiment with alternative allocations