

Endogenous Entry, Product Variety, and Business Cycles*

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Abstract

This paper builds a framework for the analysis of macroeconomic fluctuations that incorporates the endogenous determination of the number of producers over the business cycle. Economic expansions induce higher entry rates by prospective entrants subject to irreversible investment costs. The sluggish response of the number of producers (due to the sunk entry costs) generates a new and potentially important endogenous propagation mechanism for real business cycle models. The stock-market price of investment in the creation of new productive units determines household saving decisions, producer entry, and the allocation of labor across sectors. The model performs at least as well as the traditional setup with respect to the implied second-moment properties of key macroeconomic aggregates. In addition, our framework jointly predicts a procyclical number of producers and procyclical profits even for preference specifications that imply countercyclical markups. When we include physical capital, the model can reproduce the variance and autocorrelation of GDP found in the data.

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1 Introduction

The number of firms in the economy varies over the business cycle. Figure 1 shows the quarterly growth rates of real GDP, profits, and net entry in the U.S. economy (measured as the difference between new incorporations and failures) for the period 1947-1998.¹ Net entry is strongly procyclical and comoves with real profits, which are also procyclical. Figure 2 shows cross correlations between real GDP, profits, and net entry (Hodrick-Prescott filtered data in logs) at various leads and lags, with 95 percent confidence bands. The strong procyclicality of net entry and profits is evident, with net entry strongly correlated to profits. Importantly, Figure 2 shows that net entry tends to lead GDP and profit expansions, suggesting that entry in the expectation of future profits may play an important role in GDP expansions.²

This paper studies the role of producer entry and product creation in propagating business cycle fluctuations in a dynamic, stochastic, general equilibrium (DSGE) model with monopolistic competition and sunk entry costs. We seek to understand the contributions of the intensive and extensive margins to the response of the economy to changes in aggregate productivity and market regulation (which affects the size of sunk entry costs). Our theoretical model will equate a producer with a production line for an individual variety/good. We naturally want to account for the empirical reality that new products are not only introduced by new firms, but also by existing firms (most often at their existing production facilities).³ We therefore take a broad view of producer entry and exit as also incorporating product creation and destruction by existing firms (although our model does not address the determinants of product variety within firms). Although new firms account for a small share of overall production (for U.S. manufacturing, new firms account for 2-3% of both overall production and employment), the contribution of new products (including those produced at existing firms) is substantially larger – important enough to be a major source of aggregate output fluctuations.

This important contribution of product creation and destruction to aggregate output dynamics is convincingly documented in a new paper by Bernard, Redding, and Schott (2006). They are the first to measure product creation and destruction within firms, across an entire subset of an

¹Authors' computations from the following sources: GDP, GDP implicit price deflator, profits before tax, and new business incorporations from Basic Economics Database, Global Insight, Inc.; Business failures from NBER Macroeconomic History Database (1947-1965), The Economic Report of the President (1965-1982), and Basic Economics Database, Global Insight, Inc. (1982-1998).

²The procyclical pattern of net entry is the result of a strongly procyclical pattern of new incorporations and a countercyclical pattern of failures, which correlate negatively with GDP and profits.

³Bernard, Redding, and Schott (2006) report that 94% of product additions by U.S. manufacturing firms occur within their pre-existing production facilities (as opposed to new plants or via mergers and acquisitions).

economy (all of U.S. manufacturing). For every firm, they record production levels (dollar values) across 5-digit U.S. SIC categories (which still represent a very coarse definition of products).⁴ Bernard, Redding, and Schott (2006) document that product creation and destruction within firms is prevalent: 68% of firms change their product mix within a 5-year census period (representing 93% of firms weighted by output). Of these firms, 66% both add and drop products (representing 87% of firms weighted by output). Thus, product creation over time is not just a secular trend at the firm-level (whereby firms steadily increase the range of products they produce over time). Most importantly, Bernard, Redding, and Schott (2006) show that product creation and destruction accounts for an important share of overall production: Over a 5-year period, the value of new products (produced at existing firms) is 33.6% of overall output during that period (-30.4% of output for the lost value from product destruction at existing firms). These numbers are almost twice (1.8 times) as large as those accounted for by changes at the intensive margin – production increases and decreases for the same product at existing firms. The overall contribution of the extensive margin (product creation and destruction) would be even higher if a finer level of product disaggregation (beyond the 5-digit level) were available.⁵

Put together, product creation (both by existing firms and new firms) accounts for 46.6% of output in a 5-year period while the lost value from product destruction (by existing and exiting firms) accounts for 44% of output. This represents a minimal annual contribution of 9.3% (for product creation) and 8.8% (for product destruction).⁶ As previously mentioned, annual contributions using a finer level of product disaggregation are probably substantially higher.

We assume symmetric, homothetic preferences over a continuum of goods that nest analytically tractable specifications as special cases. When preferences are specified in the familiar C.E.S. form of Dixit and Stiglitz (1977), frictionless price adjustment results in constant markups. If preferences take the translog form proposed by Feenstra (2003), demand-side pricing complementarities arise that result in time-varying markups. To keep the setup simple, we do not model multi-product firms. In our model presentation below, and in the discussion of results, there is a one-to-one identi-

⁴As an example, the 5-digit SIC codes within the 4-digit SIC category 3949–Sporting and Athletics Goods– are: 39491–Fishing tackle and equipment, 39492–Golf equipment, 39493–Playground equipment, 39494–Gymnasium and exercise equipment, and 39495–Other sporting and athletic goods. For all of U.S. manufacturing, there are 1848 5-digit products.

⁵Returning to the example of 5-digit SIC 39494 (Gymnasium and exercise equipment) from the previous footnote: Any production of a new equipment product, whether a treadmill, an elliptical machine, a stationary bike, or any weight machine, would be recorded as production of the same product and hence be counted towards the intensive margin of production.

⁶The true annual contributions are higher as additions and reductions to output across years within the same 5-year interval (for a given firm-product combination) are not recorded.

fication between a producer, product, and firm. This is consistent with much of the macroeconomic literature with monopolistic competition, which similarly uses ‘firm’ to refer to the producer of an individual good. However, each productive unit in our setup is best interpreted as a production line within a multi-product firm whose boundaries we can leave unspecified thanks to the assumption of a continuum of goods. The producer is then the profit maximizing manager of this production line. In this interpretation, producer entry and exit in our model capture the product-switching dynamics within firms documented by Bernard, Redding, and Schott (2006).⁷

In our baseline setup, each individual producer/firm produces output using only labor. However, the number of firms that produce in each period can be interpreted as the capital stock of the economy, and the decision of households to finance entry of new firms is akin to the decision to accumulate physical capital in the standard real business cycle (RBC) model.⁸ Methodologically, just as the RBC model is a discrete-time, stochastic, general equilibrium version of the exogenous growth model that abstracts from growth to focus on business cycles, our model can be viewed as a discrete-time, stochastic, general equilibrium version of variety-based, endogenous growth models (see e.g. Romer, 1990, and Grossman and Helpman, 1991) that abstracts from endogenous growth.

This difference in methodology generates significant differences in results relative to RBC theory. First and foremost, the investment in new productive units that we emphasize is financed by households through the accumulation of shares in the portfolio of firms that operate in the economy. The stock-market price of this investment fluctuates endogenously in response to shocks and is at the core of our propagation mechanism. It determines household saving decisions, producer entry, and the allocation of labor across sectors of the economy. This is in contrast with the price of physical capital in standard RBC models, which is constant absent capital adjustment costs. Our approach to investment, capital, and their price provides an alternative to assuming adjustment costs in order to obtain a time-varying price of capital and introduces a direct link between investment and (the expectation of) economic profits.⁹ Moreover, we show that entry plays an important role in the propagation of responses to shocks. If aggregate productivity increases permanently, the expansion of aggregate GDP initially takes place at the intensive margin, with an increase in output of existing firms. Higher productivity makes entry more attractive and labor is reallocated

⁷The ability to leave the boundaries of firms unspecified afforded by continuity differentiates our approach from Jaimovich’s (2004), who assumes a discrete set of producers within each sector. In that case, the boundaries of firms crucially determine the strategic interaction between individual competitors.

⁸In fact, we show that our model relates quite transparently to the traditional RBC model pioneered by Kydland and Prescott (1982).

⁹We explore the implications of our framework for asset prices and investment in Bilbiie, Ghironi, and Melitz (in progress, a).

to creation of new firms. Over time, the number of firms in the economy increases, and output per firm decreases. Further aggregate GDP expansion is the result of an increasing number of producers. In the long run, when preferences are of C.E.S. form, output per firm returns to the initial steady-state level and permanent GDP expansion is entirely driven by the extensive margin. These labor reallocation dynamics and intensive-extensive margin effects are absent in the standard RBC framework. Importantly, even if total labor supply is fixed, and hence net job creation is absent, our model predicts sizeable gross job flows, precisely due to intersectoral reallocations.¹⁰ With translog preferences (for which the elasticity of substitution is increasing in the number of goods produced), our model is further able to simultaneously generate countercyclical markups and procyclical profits. This is a well-known challenge for models of countercyclical markups based on sticky prices (see Rotemberg and Woodford, 1999, for a discussion).

Our model’s performance in matching key second moments of the U.S. business cycle is virtually indistinguishable from that of a traditional RBC model. Importantly, however, our model can additionally account for stylized facts pertaining to entry, profits, and markups. To the best of our knowledge, our framework is the first to address and explain these issues simultaneously.¹¹ Since our model performs comparably to the RBC benchmark relative to key aggregate business cycle statistics, and it explains and reproduces features of evidence on which RBC theory is silent, we view the balance of results as favorable to the mechanisms we highlight. Moreover, inclusion of physical capital both in production of existing goods and creation of new ones – combining our propagation mechanism with the traditional capital accumulation of RBC theory – dramatically improves the performance of the model along several important dimensions. Most notably, our model can then closely match the volatility and persistence of GDP, strengthening the notoriously weak endogenous propagation mechanism of basic RBC models (see Cogley and Nason, 1995).¹²

Chatterjee and Cooper (1993) and Devereux, Head, and Lapham (1996a,b) had already documented the procyclical nature of entry and developed general equilibrium models with monopolistic competition to study the effect of entry and exit on the dynamics of the business cycle. However, entry is frictionless in their models: There is no sunk entry cost, and firms enter instantaneously in

¹⁰This is consistent with evidence of small net job flows, but large gross flows in Davis, Haltiwanger, and Schuh (1996).

¹¹Perfect-competition models, such as the standard RBC, address none of these facts. Imperfect-competition versions (with or without sticky prices) generate fluctuations in profits (and, for sticky prices, in markups) but no entry. Free-entry models reviewed below generate fluctuations in entry (and, in some versions – such as Cook (2001), Jaimovich (2004), or Comin and Gertler (2006)–, also markups) but with zero profits.

¹²The extended model also predicts more volatile consumption and hours than both our benchmark and the RBC case. However, it also counterfactually predicts less volatile investment.

each period until all profit opportunities are exploited. A fixed period-by-period cost then merely ensures that the number of operating firms is finite; the free-entry condition sets profits to zero in all periods, and the number of firms that produce in each period is not a state variable. This is clearly inconsistent with two pieces of evidence presented above, namely the cyclical variation of profits (Figure 1) and the fact that net entry leads both output and profits (Figure 2). Moreover, this is also inconsistent with observed barriers to entry in most industries. In contrast, entry in our model is subject to a sunk entry cost and a time-to-build lag, and the free entry condition equates the expected present discounted value of profits to the sunk cost. Thus, profits are allowed to vary and the number of firms is a state variable in our model, consistent with the evidence presented above and the widespread view that the number of producing firms is fixed in the short run.¹³ Furthermore, Devereux, Head, and Lapham interpret the number of firms as an endogenous productivity shifter, whereas it is best interpreted as the capital stock of the economy in our model.¹⁴ Finally, our model exhibits a steady state in which: (i) the share of profits in capital is constant and (ii) the share of investment is positively correlated with the share of profits. These are among the ‘Kaldorian’ growth facts outlined in Cooley and Prescott (1995), which neither the standard RBC model nor the frictionless entry model can account for (the former because it is based on perfect competition, the latter because the share of profits is zero).¹⁵

Entry subject to sunk costs, with the implications that we stressed above, also distinguishes our model from more recent contributions such as Comin and Gertler (2006) and Jaimovich (2004), who also assume a period-by-period, zero-profit condition.¹⁶ Our model further differs from Comin and Gertler’s along three dimensions: (i) we focus on a standard definition of the business cycle, whereas they focus on the innovative notion of ‘medium term’ cycles; (ii) our model generates countercyclical markups due to demand-side pricing complementarities, whereas Comin and Gertler, like Galí (1995), postulate a function for markups which is decreasing in the number of firms; and finally (iii) our model features exogenous, RBC-type technology shocks, whereas Comin and Gertler consider

¹³In fact, our model features a fixed number of producing firms within each period and a fully flexible number of firms in the long run.

¹⁴Benassy (1996) analyzes the persistence properties of the model developed by Devereux, Head, and Lapham. See also Kim (2004). The dynamics of firm entry and exit have recently received attention in open economy studies. See, for instance, Corsetti, Martin, and Pesenti (2005) and Ghironi and Melitz (2005).

¹⁵Lewis (2006) builds on the framework of this paper and estimates VAR responses (including those of profits and entry) to macroeconomic shocks, finding support for the sunk-cost driven dynamics predicted by our model.

¹⁶Sunk entry costs are a feature of Hopenhayn and Rogerson’s (1993) model, which is designed to analyze the employment consequences of firm entry and exit, and thus directly addresses the evidence in Davis, Haltiwanger, and Schuh (1996). However, Hopenhayn and Rogerson assume perfect competition in goods markets (as in Hopenhayn’s, 1992, seminal model) and abstract from aggregate dynamics by focusing on stationary equilibria in which prices, employment, output, and the number of firms are all constant.

endogenous technology and use shocks to the marginal rate of substitution between consumption and leisure as a driving force of business cycles.¹⁷ The source of cyclical movements in markups further differentiates our work from Jaimovich’s (and Cook, 2001), where countercyclical markups occur due to supply-side considerations – i.e., increased competition leading to lower markups. We prefer a demand-, preference-based explanation for countercyclical markups since data suggest that most of the entering and exiting firms are small, and much of the change in the product space is due to product switching within existing firms rather than entry of entirely new firms, pointing to a limited role for supply-driven competitive pressures in explaining markup dynamics over the business cycle.¹⁸

The structure of the paper is as follows. Section 2 presents the benchmark model. Section 3 discusses some key properties of the model and solves for its steady state. Section 4 illustrates the dynamic properties of the model for transmission of economic fluctuations by means of a numerical example, computing impulse responses and second moments of the artificial economy. Section 5 extends the model to include physical capital. Section 6 concludes.

2 The Benchmark Model

Household Preferences and the Intra-temporal Consumption Choice

The economy is populated by a unit mass of atomistic households. All contracts and prices are written in nominal terms. Prices are flexible. Thus, we only solve for the real variables in the model. However, as the composition of the consumption basket changes over time due to firm entry (affecting the definition of the consumption-based price index), we introduce money as a convenient unit of account for contracts. Money plays no other role in the economy. For this reason, we do not model the demand for cash currency, and resort to a cashless economy as in Woodford (2003).

The representative household supplies L_t hours of work each period t in a competitive labor market for the nominal wage rate W_t and maximizes expected intertemporal utility $E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} U(C_s, L_s) \right]$, where C is consumption and $\beta \in (0, 1)$ the subjective discount factor. The period utility function takes the form $U(C_t, L_t) = \ln C_t - \chi (L_t)^{1+1/\varphi} / (1 + 1/\varphi)$, $\chi > 0$, where $\varphi \geq 0$ is the Frisch elastic-

¹⁷Consistent with standard RBC theory, aggregate productivity shocks affect all firms uniformly in our model. We abstract from the more complex technology diffusion processes across firms of different vintages studied by Caballero and Hammour (1994) and Campbell (1998). We also do not address the growth effects of changes in product variety. Bils and Klenow (2001) document that these effects are empirically relevant for the U.S.

¹⁸Bergin and Corsetti (2005) have used the same type of translog preferences in a similar model, looking at issues of stabilization policy.

ity of labor supply to wages, and the intertemporal elasticity of substitution in labor supply. As in Campbell (1994), our choice of functional form for the utility function is guided by results in King, Plosser, and Rebelo (1988): Given separable preferences, log utility from consumption ensures that income and substitution effects of real wage variation on effort cancel out in steady state; this is necessary to have constant steady-state effort and balanced growth if there is productivity growth.

At time t , the household consumes the basket of goods C_t , defined over a continuum of goods Ω . At any given time t , only a subset of goods $\Omega_t \subset \Omega$ is available. Let $p_t(\omega)$ denote the nominal price of a good $\omega \in \Omega_t$. Our model can be solved for any parametrization of symmetric homothetic preferences. For any such preferences, there exists a well defined consumption index C_t and an associated welfare-based price index P_t . The demand for an individual variety, $c_t(\omega)$, is then obtained as $c_t(\omega)d\omega = C_t \partial P_t / \partial p_t(\omega)$, where we use the conventional notation for quantities with a continuum of goods as flow values.¹⁹

Given the demand for an individual variety, the symmetric price elasticity of demand ζ is in general a function of the number N_t of goods/producers (where N_t is the mass of Ω_t):

$$\zeta(N_t) \equiv \frac{\partial c_t(\omega)}{\partial p_t(\omega)} \frac{p_t(\omega)}{c_t(\omega)}, \quad \text{for any symmetric variety } \omega.$$

The benefit of additional product variety is described by the relative price ρ :

$$\rho_t(\omega) = \rho(N_t) \equiv \frac{p_t(\omega)}{P_t}, \quad \text{for any symmetric variety } \omega,$$

or, in elasticity form:

$$\epsilon(N_t) \equiv \frac{\rho'(N_t)}{\rho(N_t)} N_t.$$

Together, $\zeta(N_t)$ and $\rho(N_t)$ completely characterize the effects of preferences in our model; explicit expressions can be obtained for these objects upon specifying functional forms for preferences, as will become clear in the discussion below.

Firms

There is a continuum of monopolistically competitive firms, each producing a different variety $\omega \in \Omega$. Production requires only one factor, labor. Aggregate labor productivity is indexed by Z_t , which represents the effectiveness of one unit of labor. Z_t is exogenous and follows an $AR(1)$

¹⁹See the appendix for more details.

process (in logarithms). Output supplied by firm ω is $y_t(\omega) = Z_t l_t(\omega)$, where $l_t(\omega)$ is the firm's labor demand for productive purposes. The unit cost of production, in units of the consumption good C_t , is w_t/Z_t , where $w_t \equiv W_t/P_t$ is the real wage.

Prior to entry, firms face a sunk entry cost of $f_{E,t}$ effective labor units, equal to $w_t f_{E,t}/Z_t$ units of the consumption good. The sunk entry cost $f_{E,t}$ is exogenous and subject to shocks. (We interpret a permanent decrease as deregulation that lowers the size of entry barriers below.) There are no fixed production costs. Hence, all firms that enter the economy produce in every period, until they are hit with a “death” shock, which occurs with probability $\delta \in (0, 1)$ in every period.²⁰

Given our modeling assumption relating each firm to an individual variety, we think of a firm as a production line for that variety, and the entry cost as the development and setup cost associated with the latter (potentially influenced by market regulation). The exogenous “death” shock also takes place at the individual variety level. Empirically, a firm may comprise more than one of these production lines. Our model does not address the determination of product variety within firms, but our main results would be unaffected by the introduction of multi-product firms.

Given the demand function for each good (where the elasticity of demand can depend on the number of goods), firms set prices in a flexible fashion as markups over marginal costs. In units of consumption, firm ω 's price is $\rho_t(\omega) \equiv p_t(\omega)/P_t = \mu_t w_t/Z_t$, where the markup is a function of the number of producers: $\mu_t = \mu(N_t) \equiv \zeta(N_t)/(\zeta(N_t) + 1)$. The firm's profit in units of consumption, returned to households as dividend, is $d_t(\omega) = d_t = \left(1 - \mu(N_t)^{-1}\right) Y_t^C/N_t$, where Y_t^C is total output of the consumption sector and will in equilibrium be equal to total consumption demand C_t .

Preference Specifications and Markups

In our quantitative exercises, we consider two alternative preference specifications. The first features constant elasticity of substitution between goods as in Dixit and Stiglitz (1977). For these C.E.S. preferences, the consumption aggregator is $C_t = \left(\int_{\omega \in \Omega} c_t(\omega)^{\theta-1/\theta} d\omega\right)^{\theta/(\theta-1)}$, where $\theta > 1$ is the symmetric elasticity of substitution across goods. The consumption-based price index is then $P_t = \left(\int_{\omega \in \Omega_t} p_t(\omega)^{1-\theta} d\omega\right)^{1/(1-\theta)}$, and the household's demand for each individual good ω is $c_t(\omega) = (p_t(\omega)/P_t)^{-\theta} C_t$. It follows that the markup and the benefit of variety are independent of

²⁰For simplicity, we do not consider endogenous exit. Appropriate calibration of δ makes it possible for our model to match several important features of the data.

the number of goods ($\epsilon(N_t) = \epsilon, \mu(N_t) = \mu$) and related by $\epsilon = \mu - 1 = 1/(\theta - 1)$.²¹ The second specification uses the translog expenditure function proposed by Feenstra (2003), which introduces demand-side pricing complementarities. For this preference specification, the symmetric price elasticity of demand is $-(1 + \sigma N_t)$, $\sigma > 0$: As N_t increases, goods become closer substitutes, and the elasticity of substitution $1 + \sigma N_t$ increases. If goods are closer substitutes, then the markup $\mu(N_t)$ and the benefit of additional varieties in elasticity form ($\epsilon(N_t)$) must decrease.²² The change in $\epsilon(N_t)$ is only half the change in net markup generated by an increase in the number of producers. Table 1 contains the expressions for markup, relative price, and the benefit of variety (the elasticity of ρ to the number of firms), for each preference specification.²³

Table 1. Two frameworks	
C.E.S.	Translog
$\mu(N_t) = \mu = \frac{\theta}{\theta-1}$	$\mu(N_t) = \mu_t = 1 + \frac{1}{\sigma N_t}$
$\rho(N_t) = N_t^{\mu-1} \left(= N_t^{\frac{1}{\theta-1}} \right)$	$\rho(N_t) = e^{-\frac{1}{2} \frac{\tilde{N}-N_t}{\sigma \tilde{N} N_t}}, \quad \tilde{N} \equiv Mass(\Omega)$
$\epsilon(N_t) = \mu - 1$	$\epsilon(N_t) = \frac{1}{2\sigma N_t} = \frac{1}{2} [\mu(N_t) - 1]$

Firm Entry and Exit

In every period, there is a mass N_t of firms producing in the economy and an unbounded mass of prospective entrants. These entrants are forward looking, and correctly anticipate their expected future profits $d_s(\omega)$ in every period $s \geq t + 1$ as well as the probability δ (in every period) of incurring the exit-inducing shock. Entrants at time t only start producing at time $t + 1$, which introduces a one-period time-to-build lag in the model. The exogenous exit shock occurs at the very end of the time period (after production and entry). A proportion δ of new entrants will therefore never produce. Prospective entrants in period t compute their expected post-entry value

²¹An alternative setup would have the household consume a homogeneous good produced by a competitive sector that bundles intermediate goods using a production function that has the form of our consumption basket. All our results would hold also in that setup, though the interpretation would be different. In our setup, consumers derive welfare directly from availability of more varieties. In the alternative setup, an increased range of intermediate goods shows up as increasing returns to specialization. Empirical problems associated with increasing returns to specialization and a C.E.S. production function induce us to adopt the specification without intermediate varieties.

²²This property for the markup occurs whenever the price elasticity of residual demand decreases with quantity consumed along the residual demand curve.

²³Note that while none of the two preference specifications is nested in the other, they are both nested in the general class of (homothetic) preferences we consider. Moreover, insofar as the log-linear version of the model is concerned, the translog case will nest the C.E.S. one, as we spell out below.

$(v_t(\omega))$ given by the present discounted value of their expected stream of profits $\{d_s(\omega)\}_{s=t+1}^{\infty}$:

$$v_t(\omega) = E_t \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left(\frac{C_s}{C_t}\right)^{-1} d_s(\omega). \quad (1)$$

This also represents the value of incumbent firms *after* production has occurred (since both new entrants and incumbents then face the same probability $1 - \delta$ of survival and production in the subsequent period). Entry occurs until firm value is equalized with the entry cost, leading to the free entry condition $v_t(\omega) = w_t f_{E,t}/Z_t$. This condition holds so long as the mass $N_{E,t}$ of entrants is positive. We assume that macroeconomic shocks are small enough for this condition to hold in every period. Finally, the timing of entry and production we have assumed implies that the number of producing firms during period t is given by $N_t = (1 - \delta)(N_{t-1} + N_{E,t-1})$. The number of producing firms represents the stock of capital of the economy. It is an endogenous state variable that behaves much like physical capital in the benchmark RBC model, but in contrast to the latter has an endogenously fluctuating price given by (1).

Symmetric Firm Equilibrium

All firms face the same marginal cost. Hence, equilibrium prices, quantities, and firm values are identical across firms: $p_t(\omega) = p_t$, $\rho_t(\omega) = \rho_t$, $l_t(\omega) = l_t$, $y_t(\omega) = y_t$, $d_t(\omega) = d_t$, $v_t(\omega) = v_t$. In turn, equality of prices across firms implies that the consumption-based price index P_t and the firm-level price p_t are such that $p_t/P_t \equiv \rho_t = \rho(N_t)$. An increase in the number of firms implies necessarily that the relative price of each individual good increases $\rho'(N_t) > 0$. When there are more firms, households derive more welfare from spending a given nominal amount, *i.e.*, *ceteris paribus*, the price index decreases. It follows that the relative price of each individual good must rise.²⁴ The aggregate consumption output of the economy is $Y_t^C = N_t \rho_t y_t = C_t$.

Importantly, in the symmetric firm equilibrium, the value of waiting to enter is zero, despite the entry decision being subject to sunk cost and exit risk; *i.e.*, there are no option-value considerations pertaining to the entry decision.²⁵ This happens because all uncertainty in our model (including the “death” shock) is aggregate. Let the option value of waiting to enter for firm ω be $\Lambda_t(\omega) \geq 0$. In all periods t , $\Lambda_t(\omega) = \max[v_t(\omega) - w_t f_{E,t}/Z_t, \beta \Lambda_{t+1}(\omega)]$, where the first term is the payoff of

²⁴In the alternative setup with homogeneous consumption produced by aggregating intermediate goods, an increase in the number of intermediates available implies that the competitive sector producing consumption becomes more efficient, and the relative price of each individual input relative to consumption rises accordingly.

²⁵This is in contrast with models such as Caballero and Hammour (1995) and Campbell (1998). See also Jovanovic (2006) for a more recent contribution in that vein.

undertaking the investment and the second term is the discounted payoff of waiting. If firms are identical (there is no idiosyncratic uncertainty) and exit is exogenous (uncertainty related to firm death is also aggregate), this becomes: $\Lambda_t = \max[v_t - w_t f_{E,t}/Z_t, \beta\Lambda_{t+1}]$. Because of free entry, the first term is always zero, so the option value obeys: $\Lambda_t = \beta\Lambda_{t+1}$. This is a contraction mapping because of discounting, and by forward iteration, under the assumption $\lim_{T \rightarrow \infty} \beta^T \Lambda_{t+T} = 0$ (*i.e.*, there is a zero value of waiting when reaching the terminal period), the only stable solution for the option value is $\Lambda_t = 0$.

Household Budget Constraint and Intertemporal Decisions

Households hold two types of assets: shares in a mutual fund of firms and risk-free bonds. (We assume that bonds pay risk-free, consumption-based real returns.) Let x_t be the share in the mutual fund of firms held by the representative household *entering* period t . The mutual fund pays a total profit in each period (in units of currency) equal to the total profit of all firms that produce in that period, $P_t N_t d_t$. During period t , the representative household buys x_{t+1} shares in a mutual fund of $N_{H,t} \equiv N_t + N_{E,t}$ firms (those already operating at time t and the new entrants). Only $N_{t+1} = (1 - \delta) N_{H,t}$ firms will produce and pay dividends at time $t + 1$. Since the household does not know which firms will be hit by the exogenous exit shock δ at the *very end* of period t , it finances the continuing operation of all pre-existing firms and all new entrants during period t . The date t price (in units of currency) of a claim to the future profit stream of the mutual fund of $N_{H,t}$ firms is equal to the nominal price of claims to future firm profits, $P_t v_t$.²⁶

The household enters period t with bond holdings B_t in units of consumption and mutual fund share holdings x_t . It receives gross interest income on bond holdings, dividend income on mutual fund share holdings and the value of selling its initial share position, and labor income. The household allocates these resources between purchases of bonds and shares to be carried into next period, and consumption. The period budget constraint (in units of consumption) is:

$$B_{t+1} + v_t N_{H,t} x_{t+1} + C_t = (1 + r_t) B_t + (d_t + v_t) N_t x_t + w_t L_t, \quad (2)$$

where r_t is the consumption-based interest rate on holdings of bonds between $t - 1$ and t (known

²⁶New entrants finance entry on the stock market in our model. This is consistent with observed behavior of existing firms, raising capital on the stock market to finance new projects – new production lines – as in our favored model interpretation. On the other hand, the empirical evidence on new firms is that they mostly borrow from banks to cover entry costs. Cetorelli and Strahan (2006) find that monopoly power in banking constitutes a barrier to firm entry in the U.S. economy. Stebunovs (in progress) extends the model of this paper to study the consequences of finance as a barrier to entry.

with certainty as of $t - 1$). The household maximizes its expected intertemporal utility subject to (2).

The Euler equations for bond and share holdings are:

$$(C_t)^{-1} = \beta (1 + r_{t+1}) E_t \left[(C_{t+1})^{-1} \right], \quad \text{and} \quad v_t = \beta (1 - \delta) E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} (v_{t+1} + d_{t+1}) \right].$$

As expected, forward iteration of the equation for share holdings and absence of speculative bubbles yield the asset price solution in equation (1).²⁷

Finally, the allocation of labor effort obeys the standard intratemporal first-order condition:

$$\chi (L_t)^{\frac{1}{\varphi}} = \frac{w_t}{C_t}. \quad (3)$$

Aggregate Accounting, Equilibrium, and the Labor Market

Aggregating the budget constraint (2) across households and imposing the equilibrium conditions $B_{t+1} = B_t = 0$ and $x_{t+1} = x_t = 1 \forall t$ yields the aggregate accounting identity $C_t + N_{E,t}v_t = w_tL_t + N_t d_t$: Total consumption plus investment (in new firms) must be equal to total income (labor income plus dividend income).

Different from the benchmark, one-sector, RBC model of Campbell (1994) and many other studies, our model economy is a two-sector economy in which one sector employs part of the labor endowment to produce consumption and the other sector employs the rest of the labor endowment to produce new firms. The economy's GDP, Y_t , is equal to total income, $w_tL_t + N_t d_t$. In turn, Y_t is also the total output of the economy, given by consumption output, $Y_t^C (= C_t)$, plus investment output, $N_{E,t}v_t$. With this in mind, v_t is the relative price of the investment "good" in terms of consumption.

Labor market equilibrium requires that the total amount of labor used in production and to set up the new entrants' plants must equal aggregate labor supply: $L_t^C + L_t^E = L_t$, where $L_t^C = N_t l_t$ is the total amount of labor used in production of consumption, and $L_t^E = N_{E,t} f_{E,t} / Z_t$ is labor used to build new firms.²⁸ In the benchmark RBC model, physical capital is accumulated by using as investment part of the output of the same good used for consumption. In other words, all labor is

²⁷We omit the transversality conditions for bonds and shares that must be satisfied to ensure optimality. Note that the interest rate is determined residually in our economy (it appears only in the Euler equation for bonds and is fully determined once consumption is determined). This is due to the absence of physical capital. Indeed, what is crucial in our economy for the allocation of intertemporal consumption is the return on shares.

²⁸We used the equilibrium condition $y_t = Z_t l_t = c_t = (\rho_t)^{-\theta} C_t$ in the expression for L_t^C .

allocated to the only productive sector of the economy. When labor supply is fixed, there are no labor market dynamics in the model, other than the determination of the equilibrium wage along a vertical supply curve. In our model, even when labor supply is fixed (the case $\varphi = 0$), labor market dynamics arise in the allocation of labor between production of consumption and creation of new plants. The allocation is determined jointly by the entry decision of prospective entrants and the portfolio decision of households who finance that entry. The value of firms, or the relative price of investment in terms of consumption v_t , plays a crucial role in determining this allocation, and is at the center of our model's propagation mechanism. Based on this price, the household decides how much to invest in the financing of entry, and prospective entrants decide whether to enter or not. In turn, entry determines the amount of labor that is allocated to setting up new production lines (rather than producing consumption goods).²⁹

Aggregate Production and the Role of Variety

Rearranging the labor market clearing condition yields the following expression for aggregate production of the consumption good:

$$Y_t^C = C_t = Z_t \rho_t \left(L_t - \frac{f_{E,t}}{Z_t} N_{E,t} \right).$$

In turn, substituting this into $C_t + N_{E,t} v_t = Y_t$ and using firm pricing and free entry gives:

$$Y_t = Z_t \rho_t \left(L_t - \frac{\mu_t - 1}{\mu_t} \frac{f_{E,t}}{Z_t} N_{E,t} \right).$$

These expressions resemble analogous expressions in Chatterjee and Cooper (1993) and Devereux, Head, and Lapham (1996*a,b*). An increase in the number of entrants $N_{E,t}$ absorbs productive resources in the form of effective labor and acts like an overhead cost. This cost is accounted for differently in GDP, since this recognizes that firm entry is productive. *Ceteris paribus*, when $\mu_t \rightarrow 1$ (perfect competition), $\rho_t \rightarrow 1$, and the expression for Y_t reduces to a familiar Cobb-Douglas production function with a zero share of capital: $Y_t = Z_t L_t$.³⁰

Chatterjee and Cooper (1993) and Devereux, Head, and Lapham (1996*a,b*) interpret the effect of changes in the number of active firms N_t in the expressions above as an endogenous aggregate

²⁹With elastic labor supply, labor market dynamics operate along two margins as the interaction of household and entry decisions determines jointly the total amount of labor and its allocation to the two sectors of the economy.

³⁰In the C.E.S. case, $\rho_t = (N_t)^{\frac{1}{\theta-1}} \rightarrow 1$ when $\theta \rightarrow \infty$. In the translog case, perfect competition similarly corresponds to infinite substitutability across goods, with $\rho_t = \rho(N_t) \rightarrow 1$ (see Table 1).

productivity shifter. Since $\rho'(N_t) > 0$, an increase in the number of active firms N_t has a similar effect to that of an endogenous increase in productivity. As there is no sunk entry cost nor time-to-build lag in those studies, such endogenous productivity changes do not impart endogenous persistence in those models, in contrast to ours. If we think of the effect of entry as endogenously affecting productivity in our model, entry at time t affects labor demand at $t + 1$ because it affects productivity of labor at $t + 1$ by causing N_{t+1} to rise.

We prefer to interpret N_t as the stock of capital (production lines) of the economy during period t , treating aggregate productivity Z_t as exogenous in production of the consumption good, $Y_t^C = Z_t \rho_t N_t l_t$, and in production of new plants. There are three reasons for this. First, it is transparent to treat labor per firm and the number of firms as the factors of production in the aggregate function $Y_t^C = Z_t \rho_t N_t l_t$, in which Z_t is exogenous productivity, and the relative price ρ_t converts units of individual goods into units of consumption. In this interpretation, which does not hinge on the properties of homothetic-preferences-based demand to endogenize productivity relative to the number of firms, entry at t affects labor demand at $t + 1$ because it increases the number of producing firms at $t + 1$. This is akin to the benchmark RBC model, where investment at t affects labor demand at $t + 1$ by increasing the capital stock used in production at $t + 1$. Second, capital accumulation in the RBC model can be viewed as an extreme case in which all observed investment goes toward the production of existing goods. Our baseline framework studies another possible extreme case, in which all investment is accounted for by the creation of new products. In a final section, we study a setup in which investment is split endogenously between the creation of new products and augmenting the physical capital stock of the economy. Finally, as argued in Ghironi and Melitz (2005), empirically relevant variables – as opposed to welfare-consistent concepts – net out the effect of changes in the range of available varieties. The reason is that construction of CPI data by statistical agencies does not adjust for availability of new varieties as in the welfare-consistent price index.³¹ CPI data are closer to p_t than P_t . For this reason, when investigating the properties of the model in relation to the data (for instance, when computing second moments or impulse responses for comparison with empirical evidence), one wants to focus on real variables deflated by a data-consistent price index. For any variable X_t in units of the consumption basket, such data-consistent counterpart is obtained as $X_{R,t} \equiv P_t X_t / p_t = X_t / \rho_t = X_t / \rho(N_t)$. Therefore, data-consistent measures of consumption output and GDP in our model are $Y_{R,t}^C = Z_t L_t - f_{E,t} N_{E,t}$

³¹ Adjustment for variety, when it happens, certainly does not happen at the frequency represented by periods in our model.

and $Y_{R,t} = Z_t L_t - \frac{\mu_t - 1}{\mu_t} f_{E,t} N_{E,t}$, which remove the role of variety as an endogenous productivity shifter.³²

Model Summary

Table 2 summarizes the main equilibrium conditions of the model.³³ The equations in the table constitute a system of ten equations in ten endogenous variables: ρ_t , μ_t , d_t , w_t , L_t , $N_{E,t}$, N_t , r_t , v_t , C_t . Of these endogenous variables, two are predetermined as of time t : the total number of firms, N_t , and the risk-free interest rate, r_t . Additionally, the model features two exogenous variables: aggregate productivity, Z_t , and the sunk entry cost, $f_{E,t}$. The latter may be interpreted in at least two ways. Part of the sunk entry cost $f_{E,t}$ originates in the economy's technology for creation of new plants, which is exogenous and outside the control of policymakers. But another part of the entry cost is motivated by regulation and entry barriers induced by policy. Holding the technology component of $f_{E,t}$ given, we interpret changes in $f_{E,t}$ below as changes in market regulation facing firms.³⁴

Table 2. Benchmark Model, Summary

Pricing	$\rho_t = \mu_t \frac{w_t}{Z_t}$
Markup	$\mu_t = \mu(N_t)$
Variety effect	$\rho_t = \rho(N_t)$
Profits	$d_t = \left(1 - \frac{1}{\mu_t}\right) \frac{C_t}{N_t}$
Free entry	$v_t = w_t \frac{f_{E,t}}{Z_t}$
Number of firms	$N_t = (1 - \delta)(N_{t-1} + N_{E,t-1})$
Intratemporal optimality	$\chi(L_t)^{\frac{1}{\varphi}} = \frac{w_t}{C_t}$
Euler equation (bonds)	$(C_t)^{-1} = \beta(1 + r_{t+1}) E_t \left[(C_{t+1})^{-1} \right]$
Euler equation (shares)	$v_t = \beta(1 - \delta) E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} (v_{t+1} + d_{t+1}) \right]$
Aggregate accounting	$C_t + N_{E,t} v_t = w_t L_t + N_t d_t$

³²Treating aggregate productivity as exogenous in the absence of firm heterogeneity and endogenous exit is also consistent with Melitz (2003).

³³The labor market equilibrium condition is redundant once the variety effect equation is included in the system in Table 2.

³⁴We focus on random shocks to Z_t as the source of business cycle fluctuations in our model, assuming that changes in market regulation do not take place at business cycle frequency. Results on the consequences of government spending shocks in our model are available on request.

3 Benchmark Model Properties and Solution

We can reduce the system in Table 2 to a system of two equations in two variables, N_t and C_t . To see this, write firm value as a function of the endogenous state N_t and the exogenous state $f_{E,t}$ by combining free entry, the pricing equation, and the two equations emphasizing the dependence of markup and variety effect on the number of firms:

$$v_t = f_{E,t} \frac{\rho(N_t)}{\mu(N_t)}. \quad (4)$$

Equation (4) and the free entry condition in Table 2 yield a first, important set of results: Since the number of producing firms is predetermined and does not react to exogenous shocks on impact, firm value and the real wage are also predetermined with respect to some exogenous shocks. Namely, firm value is predetermined with respect to all shocks but deregulation, while the real wage ($w_t = Z_t \rho(N_t) / \mu(N_t)$) is predetermined with respect to all shocks but productivity. A fall in the sunk entry cost encourages entry and decreases firm value since more firms start producing at $t+1$, which implies an expected decrease in demand for each individual firm. An increase in productivity results in a proportional increase in the real wage on impact through its effect on labor demand. Since the entry cost is paid in effective labor units, this does not affect firm value. An implication of the wage schedule $w_t = Z_t \rho(N_t) / \mu(N_t)$ is also that marginal cost, w_t / Z_t , is predetermined with respect to all shocks.

The number of new entrants as a function of consumption and number of firms is $N_{E,t} = Z_t L_t / f_{E,t} - C_t / (f_{E,t} \rho(N_t))$. Substituting this, equations (4) and (3), and the expression for profits in the law of motion for N_t (scrolled one period forward) and the Euler equation for shares yields:

$$N_{t+1} = (1 - \delta) \left[N_t + \frac{Z_t}{f_{E,t}} \left(\frac{1}{C_t} \frac{\rho(N_t)}{\mu(N_t)} \frac{Z_t}{\chi} \right)^\varphi - \frac{C_t}{f_{E,t} \rho(N_t)} \right], \quad (5)$$

$$f_{E,t} \frac{\rho(N_t)}{\mu(N_t)} = \beta (1 - \delta) E_t \left\{ \frac{C_t}{C_{t+1}} \left[f_{E,t+1} \frac{\rho(N_{t+1})}{\mu(N_{t+1})} + \left(1 - \frac{1}{\mu(N_{t+1})} \right) \frac{C_{t+1}}{N_{t+1}} \right] \right\}. \quad (6)$$

Steady State and Calibration

We assume that exogenous variables are constant in steady state and denote steady-state levels of variables by dropping the time subscript: $Z_t = Z$, and $f_{E,t} = f_E$. We conjecture that all endogenous variables are constant in steady state and show that this is indeed the case.³⁵ The

³⁵Our model would exhibit endogenous growth if the cost of entry were a decreasing function of the number of producers, as in Grossman and Helpman (1991). We abstract from by now well understood growth considerations in

steady-state interest rate is pinned down as usual by the rate of time preference, $1 + r = \beta^{-1}$; the gross return on shares is $1 + d/v = (1 + r)/(1 - \delta)$, which captures a premium for expected firm destruction. The number of new entrants makes up for the exogenous destruction of existing firms: $N_E = \delta N / (1 - \delta)$. We follow Campbell (1994) below and exploit $1 + r = \beta^{-1}$ to treat r as a parameter in the solution.

Calculating the shares of profit income and investment in consumption output and GDP allows us to draw another transparent comparison between our model and the standard RBC setup. The steady-state profit equation gives the share of profit income in consumption output: $dN/Y^C = (\mu - 1)/\mu$. Using this result in conjunction with those obtained above, we have the share of investment in consumption output:

$$\frac{vN_E}{Y^C} = \frac{\delta}{r + \delta} \frac{\mu - 1}{\mu}.$$

This expression is similar to its RBC counterpart. There, the share of investment in output is given by $s_K \delta / (r + \delta)$, where δ is the depreciation rate of capital and s_K is the share of capital income in total income. In our framework, $(\mu - 1)/\mu$ can be regarded as governing the share of “capital” since it dictates the degree of monopoly power and hence the share of profits that firms generate from producing consumption output (dN/Y^C). Noting that $Y = Y^C + vN_E$, the shares of investment and profit income in GDP are $vN_E/Y = \delta / \left[\delta + \frac{\mu}{\mu-1} (r + \delta) \right]$ and $dN/Y = (r + \delta) / \left[\delta + \frac{\mu}{\mu-1} (r + \delta) \right]$, respectively. It follows that the share of private consumption in GDP is $C/Y = \frac{\mu}{\mu-1} (r + \delta) / \left[\delta + \frac{\mu}{\mu-1} (r + \delta) \right]$. The share of labor income in total income is $wL/Y = 1 - (r + \delta) / \left[\delta + \frac{\mu}{\mu-1} (r + \delta) \right]$.³⁶ Importantly, all these ratios are constant, consistent with the Kaldorian growth facts. In fact, our model’s long-run properties are consistent with two stylized facts originally found by Kaldor (1957): a constant share of profits in total capital, $dN/vN = (r + \delta) / (1 - \delta)$, and, relatedly, a high correlation between the profit share in GDP and the investment share in GDP.³⁷ These facts are absent from both the standard RBC model and the frictionless entry models reviewed in the Introduction.

order to focus on the business cycle implications of entry.

³⁶Note that all these ratios are identical if we compute them in terms of empirically relevant variables deflated by the average price p .

³⁷These properties would hold also along the balanced growth path of the model with C.E.S. preferences if we incorporated either exogenous productivity growth or endogenous growth stemming from an entry cost (in units of effective labor) that decreases with the number of products. Furthermore, these properties would hold even if the model did not exhibit balanced growth (such as when the markup is a function of the number of products). Balanced growth would be restored in the translog case by assuming that σ decreases at the same rate as N_t increases in the long run.

Equations (5)-(6) allow us to solve for the steady-state number of firms and consumption (and therefore all other variables) by solving the equations:

$$N = [\chi(r + \delta)]^{-\varphi} \left[(1 - \delta) \frac{Z}{f_E} \right]^{1+\varphi} \frac{\left(\frac{\mu(N)-1}{N\mu(N)} \right)^\varphi}{\delta + \frac{r+\delta}{\mu(N)-1}}, \quad (7)$$

$$C = \frac{(r + \delta) \rho(N)}{(1 - \delta)(\mu(N) - 1)} N f_E. \quad (8)$$

To obtain a closed-form solution for the steady state, we distinguish according to the two functional forms for preferences (and therefore the markup and variety functions) considered above. In the C.E.S. case, the markup is always equal to a constant: $\mu(N) = \theta / (\theta - 1)$, and the variety effect is governed by $\rho(N) = N^{\frac{1}{\theta-1}}$. The solution is:

$$N^{CES} = \frac{(1 - \delta)}{\chi\theta(r + \delta)} \left[\frac{\chi\theta(r + \delta)}{\theta(r + \delta) - r} \right]^{\frac{1}{1+\varphi}} \frac{Z}{f_E}, \quad (9)$$

$$C^{CES} = \frac{(r + \delta)(\theta - 1)}{1 - \delta} f_E (N^{CES})^{\frac{\theta}{\theta-1}}. \quad (10)$$

Intuitively, an increase in long-run productivity results in a larger number of firms (and hence higher firm value, $v = [(\theta - 1) / \theta] f_E (N^{CES})^{\frac{1}{\theta-1}}$, and consumption). Deregulation (a lower sunk entry cost) also generates an increase in the long-run number of firms and consumption, and it increases firm value as a proportion of the sunk cost itself (v/f_E).³⁸ The effect of deregulation on v^{CES} depends on whether θ is larger or smaller than two. Empirically plausible values of θ , which satisfy $\theta > 2$, imply that deregulation has a negative effect on firm value. Importantly, C^{CES} and N^{CES} tend to zero if θ tends to infinite. For firms to find it profitable to enter, the expected present discounted value of the future profit stream must be positive, so as to offset the sunk entry cost. But profits tend to zero in all periods if firms have no monopoly power. This implies that no firm will enter the economy, driving N^{CES} and C^{CES} to zero.

Of particular interest is the behavior of the real wage, given by:

$$w^{CES} = \frac{\theta - 1}{\theta} Z (N^{CES})^{\frac{1}{\theta-1}} = \frac{\theta - 1}{\theta} Z \left[\frac{Z}{f_E} \frac{(1 - \delta)}{\chi\theta(r + \delta)} \right]^{\frac{1}{\theta-1}} \left[\frac{\chi\theta(r + \delta)}{\theta(r + \delta) - r} \right]^{\frac{1}{(1+\varphi)(\theta-1)}}.$$

Both higher productivity and deregulation result in a higher wage, as a larger number of firms

³⁸See Alesina, Ardagna, Nicoletti, and Schiantarelli (2005) for empirical evidence supporting the view that deregulation generates entry and therefore stimulates investment.

puts pressure on labor demand. Most importantly, deregulation and higher productivity cause steady-state marginal cost w/Z to increase (the long-run elasticity being $1/(\theta - 1)$). This is in sharp contrast to models with a constant number of firms, where marginal cost would be constant relative to long-run changes in productivity. (To see this, set $N = 1$ for convenience and note that $w/Z = (\theta - 1)/\theta$ in this case.) Changes in productivity would be reflected in equal percentage changes in the real wage, so that marginal cost remains constant.³⁹ In a model with endogenous number of firms, higher productivity results in a more attractive business environment, which leads to more entry and a larger number of firms. This puts pressure on labor demand that causes w to increase by more than Z , so that the new long-run marginal cost is higher than the original one.⁴⁰ Given solutions for v , C , N , and w/Z , it is easy to recover solutions for all other variables in Table 2, which we omit. To complete the information on the steady-state properties of the model with CES preferences, Table 3 reports the long-run elasticities of endogenous variables to permanent changes in Z and f_E .

Table 3. Benchmark Model, C.E.S., Long-Run Elasticities

Elasticity of \Downarrow w.r.t. \Rightarrow	Z	f_E
N, N_E	1	-1
w	$\frac{\theta}{\theta-1}$	$-\frac{1}{\theta-1}$
v, d	$\frac{1}{\theta-1}$	$\frac{\theta-2}{\theta-1}$
C, Y	$\frac{\theta}{\theta-1}$	$-\frac{1}{\theta-1}$

In the translog case, the steady-state markup function is $\mu(N) = 1 + \frac{1}{\sigma N}$. Substituting this in (7), we obtain:

$$N = \left((1 - \delta) \frac{Z}{f_E} \right)^{1+\varphi} \left(\frac{1}{\chi(r + \delta)} \right)^\varphi \frac{[N(1 + \sigma N)]^{-\varphi}}{(\delta + \sigma N(r + \delta))} \equiv H(N), \quad (11)$$

which shows that N^{Trans} is a fixed point of the function $H(N)$. Since $H(N)$ is continuous and $\lim_{N \rightarrow 0} H(N) = \infty$ and $\lim_{N \rightarrow \infty} H(N) = 0$, $H(N)$ has a unique fixed point if and only if $H'(N) \leq$

³⁹In fact, marginal cost ($w_t/Z_t = \rho_t/\mu_t$) would be constant in all periods, in and out of the steady state, if the number of firms were constant – and $N_t = 1$ would imply $\rho_t = 1$ and $\mu_t = \mu$, as in standard models without entry. (To see this in the translog case in Table 2, set $N_t = \bar{N} = 1$.) In our model, even the data-consistent measure of marginal cost $w_{R,t}/Z_t = (w_t/Z_t)/\rho_t = 1/\mu_t$ is not constant (except for C.E.S. preferences): Indeed, it is procyclical whenever markups are countercyclical.

⁴⁰This mechanism is central for Ghironi and Melitz's (2005) result that a permanent increase in productivity results in higher average prices and an appreciated real exchange rate in the country that experiences such higher productivity relative to its trading partners.

0. Straightforward differentiation of $H(N)$ shows that this is indeed the case, and hence there exists a unique N^{Trans} that solves the nonlinear equation (11). In the special case of inelastic labor ($\varphi = 0$), a closed-form solution can be obtained as:

$$N_{\varphi=0}^{Trans} = \frac{-\delta + \sqrt{\delta^2 + 4\sigma \frac{Z}{f_E} (r + \delta) (1 - \delta)}}{2\sigma (r + \delta)}. \quad (12)$$

An intuitive explanation of the effects of long-run increases in technology and deregulation is possible. Suppose that $Z = f_E = 1$. An increase in Z , or a decrease in f_E , from the initial value of 1 shifts the $H(N)$ schedule upward, and hence leads to an increase in N^{Trans} . This increase is larger, the larger the elasticity of labor supply. Since $\rho'(N) > 0$ and $\mu'(N) < 0$, the increase in technology also translates into a permanent increase in consumption, the value of the firm, wage and marginal cost (recall that for $f_E = 1$, $v = w/Z = \rho(N)/\mu(N)$).

In the quantitative exercises below, we use a specific calibration scheme, which ensures that steady-state number of firms and markup under translog preferences are the same as under C.E.S. (We make this assumption since we only observe one set of data, and hence only one value for N and μ .) We can achieve this for translog preferences by an appropriate choice of the parameter σ (denoted with σ^* below), which we describe in detail in the appendix.

In our baseline calibration, we interpret periods as quarters and set $\beta = .99$ – a standard choice for quarterly business cycle models. We set the size of the exogenous firm exit shock $\delta = .025$ to match the U.S. empirical level of 10 percent job destruction per year.⁴¹ Under C.E.S. preferences, we use the value of θ from Bernard, Eaton, Jensen, and Kortum (2003) and set $\theta = 3.8$, which was calibrated to fit U.S. plant and macro trade data.⁴² We set initial productivity to $Z = 1$. The initial steady-state entry cost f_E does not affect any of the impulse responses under C.E.S. preferences and under translog for the σ^* calibration;⁴³ we therefore set $f_E = 1$ without loss of

⁴¹Empirically, job destruction is induced by both firm exit and contraction. In our model, the “death” shock δ takes place at the product level. In a multi-product firm, the disappearance of a product generates job destruction without firm exit. Since we abstract from the explicit modeling of multi-product firms, we include this portion of job destruction in δ . As a higher δ implies less persistent dynamics, our choice of δ is also consistent with not overstating the ability of the model to generate persistence.

⁴²It may be argued that the value of θ results in a steady-state markup that is too high relative to the evidence. However, it is important to observe that, in models without any fixed cost, $\theta/(\theta - 1)$ is a measure of both markup over marginal cost and average cost. In our model with entry costs, free entry ensures that firms earn zero profits *net* of the entry cost. This means that firms price at average cost (inclusive of the entry cost). Thus, although $\theta = 3.8$ implies a fairly high markup over marginal cost, our parametrization delivers reasonable results with respect to pricing and average costs. The main qualitative features of the impulse responses below are not affected if we set $\theta = 6$, resulting in a 20 percent markup of price over marginal cost as in Rotemberg and Woodford (1992) and several other studies.

⁴³The total number of firms in steady state is inversely proportional to f_E – and the size and value of all firms

generality. The value of σ that ensures equality of steady-state markup and number of firms across preference specifications for the baseline parameterization is $\sigma^* = .35323$. Steady-state labor effort under both preference scenarios is:

$$L = \left\{ \frac{1}{\chi} \left[1 - \frac{r}{\theta(r + \delta)} \right] \right\}^{\frac{\varphi}{1+\varphi}}. \quad (13)$$

(Note that hours are indeed constant relative to variation in productivity and regulation.) We consider different values for the elasticity of labor supply (though Figure 3 above is plotted for $\varphi = 0$), and we set the weight of the disutility of labor in the period utility function, χ , so that the steady-state level of labor effort in (13) is 1 – and steady-state levels of all variables are the same – regardless of φ .⁴⁴

Dynamics

We solve for the dynamics in response to exogenous shocks by log-linearizing the model around the steady state obtained above. For simplicity, we assume initial steady-state levels of productivity and sunk cost $Z = f_E = 1$. Using sans-serif fonts to denote percent deviations from steady-state levels, log-linearization under assumptions of log-normality and homoskedasticity yields:

$$\begin{aligned} \mathbf{N}_{t+1} = & \left[1 - \delta + \frac{r + \delta}{\mu - 1} \epsilon + \left(\frac{r + \delta}{\mu - 1} + \delta \right) \varphi (\epsilon - \eta) \right] \mathbf{N}_t - \left[\varphi \left(\frac{r + \delta}{\mu - 1} + \delta \right) + \frac{r + \delta}{\mu - 1} \right] \mathbf{C}_t \\ & + (1 + \varphi) \left(\frac{r + \delta}{\mu - 1} + \delta \right) \mathbf{Z}_t - \delta \mathbf{f}_{E,t}, \end{aligned} \quad (14)$$

$$\begin{aligned} \mathbf{C}_t = & \frac{1 - \delta}{1 + r} E_t \mathbf{C}_{t+1} - \left[\frac{1 - \delta}{1 + r} (\epsilon - \eta) - \frac{r + \delta}{1 + r} \left(1 - \frac{\eta}{\mu - 1} \right) \right] \mathbf{N}_{t+1} + (\epsilon - \eta) \mathbf{N}_t \\ & - \frac{1 - \delta}{1 + r} E_t \mathbf{f}_{E,t+1} + \mathbf{f}_{E,t}, \end{aligned} \quad (15)$$

where $\eta \equiv \mu'(N) N / \mu(N) \leq 0$ is the elasticity of the markup function with respect to N , which takes the value of 0 under C.E.S and $-(1 + \sigma N)^{-1}$ under translog preferences. Equation (14) states that the number of firms producing at $t + 1$ increases if consumption at time t is lower (households save more in the form of new firms), if the sunk entry cost is below the initial level, or if productivity is higher. Equation (15) states that consumption at time t is higher the higher expected future consumption and the larger the number of firms producing at time t . Current deregulation lowers

are similarly proportional to f_E . Basically, changing f_E amounts to changing the unit of measure for output and number of firms. As in the C.E.S. case, the ‘long-run scale’ of the economy Z/f_E does not matter for dynamics under translog preferences under the σ^* calibration.

⁴⁴This requires $\chi = .924271$.

current consumption, because households save more to finance faster firm entry. However, expected future deregulation boosts current consumption as households anticipate the availability of more varieties in the future. The effect of N_{t+1} depends on parameter values. For realistic parameter values, we have $\epsilon - \eta > (r + \delta) / (1 - \delta)$: An increase in the number of firms producing at $t + 1$ is associated with lower consumption at t . (Higher productivity at time t lowers contemporaneous consumption through this channel, as households save to finance faster entry in a more attractive economy. However, we shall see below that the general equilibrium effect of higher productivity will be that consumption rises.)

In the appendix, we find conditions under which the system (14)-(15) has a unique, non-explosive solution. To solve the system, we assume $Z_t = \phi_Z Z_{t-1} + \varepsilon_{Z,t}$, where $\varepsilon_{Z,t}$ is an i.i.d., Normal innovation with zero mean and variance $\sigma_{\varepsilon_Z}^2$. Differently from productivity, we do not treat $f_{E,t}$ as a stochastic process subject to random innovations at business cycle frequency. We think of changes in sunk entry costs as changes in market regulation, and we assume that market regulation is controlled by a policymaker, who can change it in more or less persistent fashion, so that $f_{E,t} = \phi_{f_E} f_{E,t-1}$ in all periods after an initial change.

4 Business Cycles: Propagation and Second Moments

In this section we explore the properties of our benchmark model by means of a numerical example. We compute impulse responses to productivity and deregulation shocks. The responses substantiate the results and intuitions in the previous section. Then, we compute second moments of our artificial economy and compare them to second moments in the data and those produced by a standard RBC model.

Impulse responses

Productivity

Figure 3 shows the responses (percent deviations from steady state) to a permanent 1 percent increase in productivity for the inelastic labor case, comparing the two alternative preference structures, C.E.S. and translog. Periods are interpreted as quarters, and the number of years after the shock is on the horizontal axis. Consider first the long-run effects in the new steady state for C.E.S. preferences (round markers). As was previously described, the business environment becomes more attractive, drawing a permanently higher number of entrants (N_E), which translates into a perma-

nently higher number of producers (N). This induces marginal cost (w/Z) and the relative price of each product ρ to be higher. GDP Y and consumption (C) also rise permanently, and they do so by more than the increase in productivity due to the expansion in the range of available varieties. Individual firm output (y) is not affected as the increase in the relative price offsets the larger demand resulting from higher consumption. Profits per firm (d) and firm value (v) are also permanently higher.⁴⁵

The long-run effects in the case of translog preferences (cross markers) are clearly different from the C.E.S. ones, despite the initial steady state being the same by construction. The main difference comes from the dampening effect of the increase in the number of producers on the markup (μ), due to the demand-side pricing complementarities generated by these preferences. The increased profitability drawing new producers into the market and the benefit of additional variety are lower than under C.E.S. preferences precisely because of this negative effect on the markup. Hence, the new steady-state number of producers is lower, and firm value, relative price, real wage, and marginal cost are all lower than under C.E.S. Since less resources are used for the creation of new firms, relatively more resources are used for producing existing goods. Thus, individual firm output is permanently above its initial steady state – in contrast to the C.E.S. case, and consistent with the evidence in Bernard, Redding, and Schott (2006) that adjustments along both the intensive and extensive margins coexist in the medium to long run.

Transition dynamics highlight the role of the number of firms as the key endogenous state variable, and of firm value v_t as the key price for household finance and firm entry decisions in our model. Absent sunk entry costs, and the associated time-to-build lag before production starts, the number of producing firms N_t would immediately adjust to its new steady-state level. Sunk costs and time-to-build imply that N_t is a state variable that behaves very much like the capital stock in the standard RBC model: The number of entrants (new production lines) $N_{E,t}$ represents the consumers' investment, which translates into increases in the stock of production lines N_t over time. The number of entrants $N_{E,t}$ overshoots on impact because the price of shares v_t (which agents forecast in a rational expectations equilibrium) is expected to increase permanently in the future, making it profitable to over-invest today relative to the new long-run level of firm creation. Marginal cost and the relative price ρ_t react to the shock with a lag and start increasing only in the period after the shock as a larger number of producing firms puts pressure on labor demand.

⁴⁵Permanently higher firm value and number of entrants result in permanently higher investment (in units of consumption), $I^E \equiv vN_E$.

The responses of firm-level output and GDP highlight the different roles of intensive and extensive margins during economic expansions in response to permanent productivity improvements. In both cases, firm-level output booms on impact in response to larger consumption. In the C.E.S. case, the increase in ρ_t pushes firm-level output back to the initial steady state. Over time, the expansion along the intensive margin is reabsorbed as the increase in the number of firms puts pressure on labor costs. Since output per firm returns to the initial steady state in the long run, the increase in productivity is offset by a matching decrease in firm-level employment as the cost of labor increases during the transition. Eventually, the expansion operates only through the extensive margin. Thus, our model predicts that the expansionary effect of higher productivity is initially transmitted through the intensive margin as output per firm rises, but it is the extensive margin that delivers GDP expansion in the long run with C.E.S. preferences. In the translog case, the intensive margin does not fade out in the long run, since the increase in the number of firms is not enough to prevent output per firm from increasing; therefore, while it is still true that the intensive margin accounts for all of the initial GDP expansion, the intensive and extensive margins coexist in the long run.⁴⁶

Importantly, during the transition in both cases, there is a reallocation of the fixed labor supply from production of consumption to production of new firms, as implied by the increase in L_t^E and the decrease in L_t^C . As the increase in productivity boosts entry, labor shifts to the construction of new production lines. Over time, the rising cost of effective labor – and thus the rising burden of the entry cost – redistributes this labor back to production of consumption. The gradual increase in the cost of effective labor is the labor market counterpart to the dynamics of firm value in explaining why the number of new entrants overshoots its new long-run equilibrium in the short run: The steady increase in the relative price of the investment good makes it optimal to reallocate resources (in this case, labor) from this good to producing consumption. This effect is weaker with translog preferences because of the negative effect on profit opportunities of an increased number of producers.

An important implication of our model is that, in the translog case, it generates a countercyclical markup without necessarily implying countercyclical profits. Both firm-level and aggregate profits ($D_t \equiv N_t d_t$) increase in response to the shock. Generating this result is a notorious difficulty of other models of countercyclical markups with a constant number of producers (for instance, based

⁴⁶ A muted increase in the number of firms with translog preferences is consistent with the fact that the benefit of variety is now smaller than the markup, reducing the household's incentive to invest in new firms.

on sticky prices). These models imply that profits become countercyclical, in stark contrast with the data (see Rotemberg and Woodford, 1999). We return to this issue when computing the second moments of our artificial economy.

The responses of several key macroeconomic variables deflated by average prices rather than with the consumption based price index are qualitatively similar to those in Figure 3.⁴⁷ Two key results are worth mentioning: Aggregate real profits ($D_{R,t} \equiv N_t d_t / \rho_t$) still increase in procyclical fashion under both preference specifications, consistent with the evidence in Figure 1. The data-consistent firm value $v_{R,t} = f_{E,t} / \mu(N_t)$ is not affected at all by the shock in the C.E.S. case (because the markup is constant and the real wage $w_{R,t}$ increases exactly as much as the shock); but, in the translog case, it increases in response to technology since markups are countercyclical.

To further illustrate the properties of our model, Figure 4 shows the responses to a transitory but persistent 1 percent productivity shock (with persistence .9). The direction of movement of endogenous variables on impact is the same as in Figure 3, though all variables return to the steady state in the long run. Interestingly, for C.E.S. preferences, firm-level output is below the steady state during most of the transition, except for a short-lived initial expansion. Different from the permanent shock case, the effect of a higher relative price prevails on the expansion in consumption demand to push individual firm output below the steady state for most of the transition. For translog preferences, however, the initial expansion is more persistent for reasons described above (it is relatively more profitable to keep producing old goods since investing in new ones would erode profit margins). As in the permanent-shock case, despite markups being countercyclical, total profits are still procyclical. Notably, the dynamics of firm entry result in responses that persist beyond the duration of the exogenous shock in both cases (but relatively more so in the C.E.S. case), and, for some key variables, display a hump-shaped pattern.

The responses of most variables with elastic labor are qualitatively similar to the inelastic-labor case.⁴⁸ However, elastic labor implies that the household has an additional margin of adjustment in the face of shocks. This enhances the model's propagation mechanism and amplifies the impact responses of most endogenous variables. (The long-run responses are identical – independent of

⁴⁷For instance, this is the case for $C_{R,t}$ and $Y_{R,t}$ for C.E.S. preferences, even if the increase after the initial impact is muted and removal of the variety effect implies that these empirically-consistent variables do not increase by more than the size of the shock in the new steady state. For translog preferences, however, $C_{R,t}$ and $Y_{R,t}$ still increase by more than the size of the shock. This is a consequence of markups being countercyclical, best illustrated by the dynamics of the data-consistent firm value $v_{R,t}$ mentioned below.

⁴⁸We computed impulse responses for the cases $\varphi = 2$ and $\varphi = 10$. Figures are available on request. The case in which $\varphi \rightarrow \infty$ corresponds to linear disutility of effort and is often studied in the business cycle literature. See, for instance, Christiano and Eichenbaum (1992). With $\varphi = 10$, disutility of labor is essentially linear in a neighborhood of $L = 1$ (the steady-state level of effort).

labor supply elasticity – as explained in Section 2.) Consider the case of a transitory shock with persistence .9 and C.E.S. preferences. Faced with an increase in the real wage, the household optimally decides to work more hours in order to attain a higher consumption level, the more so the higher φ . The impact responses of labor in the investment sector and investment in new production lines are correspondingly larger as labor supply becomes more elastic. This adds to the capital stock of the economy (the number of firms) and makes both GDP and consumption increase by relatively more as φ increases. Except in the initial quarters, where the increase is amplified, firm-level profits decrease by more than in the inelastic-labor case since profit margins are eroded by the increased entry of new firms. But total profits increase more than in the inelastic-labor case, since the increase in the number of producers is more than enough to compensate for the fall of profits per firm. In the translog case with elastic labor, the markup becomes relatively more countercyclical. This amplifies the fall in firm-level profits that follows the initial expansion, but total profits remain more procyclical than under inelastic labor – as in the C.E.S. scenario – because of the larger increase in the number of producers.

Deregulation

Figure 5 shows the responses to a 1 percent permanent deregulation shock with inelastic labor supply. In the C.E.S. case (round markers), deregulation attracts new entrants and firm value decreases (the relative price of the investment good falls). Since investment is relatively more attractive than consumption, there is intersectoral labor reallocation from the latter to the former. Consumption falls initially as households postpone consumption to invest more in firms whose productivity has not increased. The number of firms starts increasing, but GDP initially falls as the decline in consumption dominates the increase in investment. All variables then move monotonically toward their new steady-state levels.⁴⁹ As for productivity shocks, the number of producers increases by less with translog preferences (cross markers) than in the C.E.S. case due to the demand complementarity leading to falling markups; the long-run expansion of GDP is smaller, but output per firm falls by less, precisely because there is less entry into the market. A key difference between C.E.S. and translog preferences in this scenario is that deregulation eventually results in higher aggregate profits with C.E.S. preferences, whereas aggregate profits fall in the

⁴⁹In the C.E.S. case with $\varphi = 2$, consumers decide optimally to supply more labor and accommodate the extra labor demand generated by the increasing number of firms. The initial decrease in labor used in production of consumption goods is thus muted. The number of entrants increases relatively more, and, as for a permanent productivity shock, the total number of firms, firm value, and the real wage all converge faster to their new steady-state levels. GDP now increases due to a larger increase in investment and a smaller decrease in consumption. Figure available on request.

translog case, because muted entry is not enough to offset the effect of lower markups.⁵⁰

Second Moments

To further evaluate the properties of our baseline model, we compute the implied second moments of our artificial economy for some key macroeconomic variables and compare them to those of the data and those produced by the benchmark RBC model. To start with, we compute standard moments of GDP, consumption, investment, and hours worked. Table 4 presents the results for our C.E.S. model with elastic labor. We use the same productivity process as King and Rebelo (1999), with persistence .979 and a standard deviation of innovations equal to .0072, to facilitate comparison of results with the baseline RBC setup. As in King and Rebelo’s benchmark calibration, we set $\varphi = 2$.⁵¹ In each column of Table 4, the first number (bold fonts) is the empirical moment implied by the U.S. data reported in King and Rebelo (1999), the second number (normal fonts) is the moment implied by our model, and the third number (italics) is the moment generated by King and Rebelo’s baseline RBC model. We compute model-implied second moments for HP-filtered variables for consistency with data and standard RBC practice, and we measure investment in our model with the real value of household investment in new firms ($v_R N_E$). Remarkably, the performance of the simplest model with entry subject to sunk costs and constant markups is very similar to that of the baseline RBC model in reproducing some key features of U.S. business cycles. On the other hand, our baseline framework faces the same well-known difficulties of the standard RBC model: Consumption and hours are too smooth relative to output; there is not enough endogenous persistence (as indicated by the first-order autocorrelations); and all variables are too procyclical relative to the data.

⁵⁰The differences between C.E.S. and translog cases are similar with $\varphi = 2$.

⁵¹The period utility function is defined over leisure ($1 - L_t$) in King and Rebelo (1999), where the endowment of time in each period is normalized to 1. The elasticity of labor supply is then the risk aversion to variations in leisure (set to 1 in their benchmark calibration) multiplied by $(1 - L)/L$, where L is steady-state effort, calibrated to 1/3. This yields $\varphi = 2$ in our specification.

Table 4. Moments for: Data, C.E.S. Model, and Baseline RBC

Variable X	σ_X			σ_X/σ_{Y_R}			$E[X_t X_{t-1}]$			$corr(X, Y_R)$		
Y_R	1.81	1.34	1.39	1.00			0.84	0.70	0.72	1.00		
C_R	1.35	0.65	0.61	0.74	0.48	0.44	0.80	0.75	0.79	0.88	0.97	0.94
Investment, $v_R N_E$	5.30	5.23	4.09	2.93	3.90	2.95	0.87	0.69	0.71	0.80	0.99	0.99
L	1.79	0.63	0.67	0.99	0.47	0.48	0.88	0.69	0.71	0.88	0.98	0.97

Source for **data** and *RBC* moments: King and Rebelo (1999)

Additionally however, our model can jointly reproduce important business cycle facts: procyclical entry (product creation), procyclical profits and, in the version with translog preferences, countercyclical markups. To substantiate this point, Figure 6 plots model-generated cross-correlations of entry, aggregate real profits, and the markup with GDP at various lags and leads for C.E.S preferences and translog preferences. In both cases, entry and profits are strongly procyclical, and the contemporaneous correlation of profits and entry (not reported) is positive, as in the data reviewed in the Introduction.⁵² With translog preferences, our model can additionally reproduce the contemporaneous countercyclical of markups; furthermore, the time profile of the markup's correlation with the business cycle is very similar to that documented by Rotemberg and Woodford (1999).⁵³

We view the performance of our benchmark model as a relative success. First, the model, although based on a different propagation mechanism from which traditional physical capital is absent, has second moment properties that are virtually indistinguishable from the RBC model's concerning macroeconomic variables of which that model speaks. Second, our model can explain (at least qualitatively) stylized facts about which the benchmark RBC model is silent. Third, to the best of our knowledge, our model is the first that can account for all these additional facts simultaneously; as reviewed in the Introduction, previous models that address entry fail to account for the cyclicity of profits (since they assume frictionless entry), and models that generate procyclical profits (due

⁵²Quantitative differences emerge, with procyclicity of entry and profits excessively strong relative to the data. Moreover, the model-generated correlations involving entry peak at the contemporaneous horizon rather than one period later. However, the latter problem could be addressed by modifying the time-to-build requirement appropriately.

⁵³Of the various labor share-based empirical measures of the markup considered by Rotemberg and Woodford, the one that is closest to our model's is the version with overhead labor (reported in column 2 of their Table 2, page 1066), which we reproduce in Figure 6. That is because markups in our model can be written as the inverse of the share (in consumption) of labor beyond the 'overhead' quantity used to set up new product lines, $\mu_t = C_t / [w_t(L_t - L_{E,t})]$. There is of course an additional issue: this measure is specified as a share of consumption, not output as in Rotemberg and Woodford. For issues pertaining to cyclicity, however, this makes little difference, since the share of consumption in output is relatively acyclical.

to monopolistic competition) abstract from changes in product space. Finally, we view the ability to generate procyclical profits with a countercyclical markup in the simplest version of our model as a major plus relative to other (e.g., sticky-price-based) theories of cyclical markup variation.

5 The Role of Physical Capital

To complete the exploration of our model as a description of business cycles, we extend our setup to incorporate physical capital as usually featured in RBC models. There are two reasons for this. First, our benchmark model studies an extreme case in which all investment goes toward the creation of new production lines and the associated new products. While this is useful to emphasize the new transmission mechanism provided by firm entry, it is certainly unrealistic: Part of observed investment is accounted for by the need to augment the capital stock used in production of existing goods. Second, the introduction of physical capital may improve the model's performance in explaining observed macroeconomic fluctuations. Since inclusion of capital in the model does not represent a major modeling innovation, we relegate the presentation of the augmented setup to an appendix, and limit ourselves to mentioning the main assumptions here.

We assume that households accumulate the stock of capital (K_t), and rent it to firms producing at time t in a competitive capital market. Investment in the physical capital stock (I_t) requires the use of the same composite of all available varieties as the consumption basket. Physical capital obeys a standard law of motion with rate of depreciation δ^K . In the version we present here, physical capital is used to produce both existing goods and new product lines as follows.⁵⁴ Producing firms use capital and labor to produce goods according to the Cobb-Douglas production function $y_t(\omega) = Z_t l_t(\omega)^\alpha k_t(\omega)^{1-\alpha}$, and we assume that the same Cobb-Douglas production function holds for new product creation, with the same labor share α as for production of goods. Thus, the technology for setting up the mass $N_{E,t}$ of new entrants is: $f_{E,t} N_{E,t} = Z_t (L_t^E)^\alpha (K_t^E)^{1-\alpha}$, where L_t^E and K_t^E are labor and capital used in setting up new firms, respectively. The free entry condition requires $v_t = f_{E,t} \lambda_t$, where λ_t is marginal cost. We outline this model in an appendix.

This model poses problems for determinacy and non-explosiveness of the solution, which are no longer guaranteed for standard parameter values (as there are now increasing returns to an

⁵⁴We have also studied a setup in which the creation of new firms does not require physical capital, following Grossman and Helpman (1991). That model's behavior and performance in terms of second moments were almost identical to those of the benchmark model without physical capital. Incidentally, note that Grossman and Helpman showed that extending their variety-based endogenous growth model to incorporate physical capital in production of goods did not alter the main conclusions of the model pertaining to growth. Our findings for business cycles echo that finding and are available upon request.

accumulated factor, physical capital). A unique, non-explosive solution is now guaranteed only if we assume very fast physical capital depreciation – specifically, $\delta^K \geq .32$, holding the other parameter values unchanged. In fact, values of δ^K close to the threshold .32 result in empirically implausible (converging) oscillations in impulse responses. For these reasons, we set $\delta^K = .5$ and limit ourselves to repeating the exercise of Table 4 and studying how this version of the model with capital affects the second moment properties of the model. The results are reported in Table 5.⁵⁵ Bold fonts denote data moments, normal fonts the C.E.S. case, and italics now denote the translog case (as before, we set $\varphi = 2$ and use the same productivity process as in King and Rebelo, 1999).

Table 5. Moments for: **Data**, and Model with Physical Capital: C.E.S and Translog

Variable X	σ_X			σ_X/σ_{Y_R}			$E[X_t X_{t-1}]$			$corr(X, Y_R)$		
Y_R	1.81	1.82	<i>1.69</i>	1.00			0.84	0.82	<i>0.82</i>	1.00		
C_R	1.35	0.85	<i>0.97</i>	0.74	0.47	<i>0.57</i>	0.80	0.76	<i>0.77</i>	0.88	0.96	<i>0.95</i>
Investment, TI_R	5.30	3.30	<i>2.83</i>	2.93	1.81	<i>1.67</i>	0.87	0.81	<i>0.80</i>	0.80	0.99	<i>0.99</i>
L	1.79	0.85	<i>0.68</i>	0.99	0.47	<i>0.40</i>	0.88	0.69	<i>0.59</i>	0.88	0.93	<i>0.89</i>

Source for **data** moments: King and Rebelo (1999)

Inclusion of physical capital improves the performance of the model remarkably (subject to the caveat of having to assume fast depreciation of capital). Most notably: (1) the volatility of GDP essentially matches the data (the ‘Prescott ratio’, i.e. the ratio of model-generated GDP variance to data variance, is close to one); and (2) the autocorrelation of GDP is also very close to the data value. The latter result suggests that this setup features a strong endogenous propagation mechanism that is not subject to the Cogley and Nason (1995) critique. In addition, this version of the model fares somewhat better (with respect to both our benchmark setup and the RBC model) in what concerns the volatility of consumption and the cyclicity of hours, especially for translog preferences. Excessive procyclicality of consumption and investment remain features of our results, and so does too little volatility of consumption and labor. Also, the issue remains open of how to introduce capital in creation of new firms while preserving determinacy and non-explosiveness for standard values of physical capital depreciation. But the results in Table 5 support the view that combining the traditional propagation mechanism of RBC theory with our extensive margin mechanism is a promising avenue for further research.

⁵⁵Investment is now measured as total investment, $TI_R \equiv v_R N_E + I_R$, where I_R is real investment in physical capital accumulation.

6 Conclusions

We developed a model of business cycle transmission with an endogenous number of producers subject to sunk entry costs, a time-to-build lag, and exogenous risk of firm destruction. The assumption of a general structure of homothetic preferences allows the model to nest the familiar C.E.S. specification with constant markups and a translog setup with time-varying markups as special cases. The model shows that variation in the number of producers and products over the horizon generally associated to the length of a business cycle can be an important propagation mechanism for fluctuations, consistent with the evidence documented by Bernard, Redding, and Schott (2006). Our setup explains stylized facts such as the procyclical behavior of entry and profits. Assuming translog preferences, it results in countercyclical markups with procyclical profits, resolving a puzzle for models that motivate cyclical markup variation with nominal rigidity. Consistent with evidence of slow adjustment in the number of producers, the model predicts that output expansion by existing producers – *i.e.*, expansion at the intensive margin – is the main channel through which a GDP expansion takes place in the short run, but a large portion of the expansion takes place at the extensive margin in the long run. Finally, when it comes to the second moment properties of variables that are the focus of traditional RBC models, our setup does at least as well as the latter for a benchmark productivity process. The endogenous stock-market price of investment in new product creation is at the center of our propagation mechanism. A version of the model that features also investment in traditional physical capital can reproduce the volatility and persistence of output found in the data, albeit subject to caveats on parameter values required for stability and determinacy.

There are certainly other avenues for improvement of the model’s performance: First, the ability of the benchmark model without physical capital to replicate business cycle moments improves along most dimensions if we include government spending shocks as an additional source of fluctuations. Assuming wasteful government consumption (with the same composition as private consumption) that follows the same process as in Christiano and Eichenbaum (1992), in addition to the King-Rebelo (1999) productivity process, results in standard deviations and contemporaneous correlations with GDP that are much closer to the data.⁵⁶ The only moment along which the performance of the model worsens is the correlation between consumption and GDP, which becomes negative in most scenarios due to the familiar crowding-out effect of government consumption on

⁵⁶Details are available on request.

private consumption.⁵⁷

A second dimension along which there is much room for improvement is measurement. To facilitate comparison with results from RBC models, we have used an off-the-shelf productivity process in our second-moment calculations. However, the structure of our model would require corrections to the standard formula for calculation of the Solow residual in order to estimate a model-consistent productivity process, as it is well-understood at least since Hall (1988).⁵⁸ A measurement problem arises also with reference to a government spending shock process: One should pay attention to the fact that, while the spending shocks may be specified in units of the consumption basket, estimation will in fact yield a process for government spending deflated of the variety effect for reasons discussed above. Computation of model-consistent Solow residuals and use of a data-consistent government spending process in a model in which non-Ricardian elements solve the consumption-crowding-out problem are two directions along which this work will evolve in the future.

Finally, we study the consequences of risky investment in product creation for asset prices in Bilbiie, Ghironi, and Melitz (in progress, a), and we take on the implications of a monetary version of our model for inflation dynamics and the conduct of monetary policy in Bilbiie, Ghironi, and Melitz (in progress, b).⁵⁹

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⁵⁷While the empirical evidence is that positive government spending shocks cause consumption to increase (for instance, Blanchard and Perotti, 2002), the negative wealth effect of higher expected taxes causes government spending shocks to crowd out real private consumption $C_{R,t}$ in our model and in standard RBC models, as explained by Baxter and King (1993). A solution to this problem pursued in the literature is to introduce non-Ricardian features in the model in the form of a fraction of households who simply consume their disposable income in each period (e.g., Galí, López-Salido, and Vallés, 2004).

⁵⁸Devereux, Head, and Lapham (1996a) seek to address the issue of measurement of Solow residuals in their model with varying product space. However, in contrast to this paper, they attribute an increase in the variety effect to endogenous productivity rather than removing the variety effect from welfare-consistent variables (see Section 2).

⁵⁹Bergin and Corsetti (2005) use a model with entry and sticky prices to study issues of stabilization policy. See also Mancini-Griffoli and Elkhoury (2006) for a model in which the entry cost is sticky.

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Appendix

A Homothetic Consumption Preferences

Consider an arbitrary set of homothetic preferences over a continuum of goods Ω . Let $p(\omega)$ and $c(\omega)$ denote the prices and consumption level (quantity) of an individual good $\omega \in \Omega$. These preferences are uniquely represented by a price index function $P \equiv h(\mathbf{p})$, $\mathbf{p} \equiv [p(\omega)]_{\omega \in \Omega}$, such that the optimal expenditure function is given by PC , where C is the consumption index (the utility level attained for a monotonic transformation of the utility function that is homogeneous of degree 1). Any function $h(\mathbf{p})$ that is non-negative, non-decreasing, homogeneous of degree 1, and concave, uniquely represents a set of homothetic preferences. Using the conventional notation for quantities with a continuum of goods as flow values, the derived Marshallian demand for any variety ω is then given by:

$$c(\omega)d\omega = C \frac{\partial P}{\partial p(\omega)}.$$

B Equating Steady States under C.E.S. and Translog Preferences

The choice of σ that ensures equalization of steady states across C.E.S. and translog preferences can be explained intuitively for the case $\varphi = 0$ with reference to Figure B.1. In the C.E.S. case, the relevant $H(N)$ function is a constant, and the equilibrium is given by $H^{CES} \equiv \frac{1-\delta}{\theta(r+\delta)-r} \frac{Z}{f_E} = N$, represented by the dotted horizontal line. The intersection of this with the 45 degree line determines the number of firms in steady state. Choosing the value of σ that equates the steady-state number of firms across C.E.S. and translog cases (denoted σ^*) amounts to choosing the $H(N)$ function for the translog case whose fixed point is precisely the same (*i.e.*, which crosses the 45 degree line at the same point); this is given by the solid curve in the figure.

Algebraically, this can be achieved as follows in the general case $\varphi \geq 0$. For any preference specification, the steady-state number of firms solves equation (7), which can be rewritten as:

$$N = [\chi(r+\delta)]^{-\frac{\varphi}{1+\varphi}} (1-\delta) \frac{Z}{f_E} \frac{\left(\frac{\mu(N)-1}{\mu(N)}\right)^{\frac{\varphi}{1+\varphi}}}{\left[\delta + \frac{r+\delta}{\mu(N)-1}\right]^{\frac{1}{1+\varphi}}}.$$

Since the terms up to Z/f_E in the right-hand side of this equation are independent of N , equalization of N for translog and C.E.S. preferences reduces to ensuring that the last fraction is invariant to the preference specifications. That is, we need to find the value of σ that ensures that $N^{Trans} = N^{CES}$,

which holds as long as

$$\frac{\left(\frac{1}{1+\sigma N^{CES}}\right)^{\frac{\varphi}{1+\varphi}}}{[\delta + (r + \delta) \sigma N^{CES}]^{\frac{1}{1+\varphi}}} = \frac{\left(\frac{1}{\theta}\right)^{\frac{\varphi}{1+\varphi}}}{[\delta + (r + \delta) (\theta - 1)]^{\frac{1}{1+\varphi}}},$$

where we used the expression for N^{CES} in (9). It is easily verified that $\sigma^* = (\theta - 1)/N^{CES}$ is a solution, and is unique (exploiting monotonicity of the markup function). Substituting the expression for N^{CES} , the value of σ^* can then be written as a function of structural parameters:

$$\sigma^* = \frac{\theta - 1}{1 - \delta} \frac{[\chi \theta (r + \delta)]^{\frac{\varphi}{1+\varphi}} f_E}{[\theta (r + \delta) - r]^{-\frac{1}{1+\varphi}} Z}.$$

C Local Equilibrium Determinacy and Non-Explosiveness

To analyze local determinacy and non-explosiveness of the rational expectation equilibrium, we can focus on the perfect foresight version of the system (14)-(15) and restrict attention to endogenous variables. Rearranging yields:

$$\begin{bmatrix} C_{t+1} \\ N_{t+1} \end{bmatrix} = M \begin{bmatrix} C_t \\ N_t \end{bmatrix}, \quad M \equiv \begin{bmatrix} \frac{1+r}{1-\delta} - A \frac{r+\delta}{\mu-1} & AB - \frac{1+r}{1-\delta} (\epsilon - \eta) \\ -\frac{r+\delta}{\mu-1} & B \end{bmatrix}.$$

where $A \equiv \epsilon - \eta - \frac{r+\delta}{1-\delta} \left(1 - \frac{\eta}{\mu-1}\right)$ and $B \equiv 1 - \delta + \frac{r+\delta}{\mu-1} \epsilon$. Existence of a unique, non-explosive, rational expectations equilibrium requires that one eigenvalue of M be inside and one outside the unit circle. The characteristic polynomial of M takes the form $J(\lambda) = \lambda^2 - (\text{trace}(M)) \lambda + \det(M)$, where the trace is

$$\text{trace}(M) = 1 - \delta + \frac{1+r}{1-\delta} + \eta \frac{r+\delta}{\mu-1} \left(1 - \frac{r+\delta}{(1-\delta)(\mu-1)}\right) + \frac{r+\delta}{1-\delta} \frac{r+\delta}{\mu-1},$$

and the determinant

$$\det(M) = 1 + r + \frac{r+\delta}{\mu-1} \frac{1+r}{1-\delta} \eta.$$

The condition for existence of a unique, non-explosive rational expectations equilibrium is $J(-1) J(1) < 0$, where

$$J(1) = -\frac{r+\delta}{1-\delta} \left(\delta + \frac{r+\delta}{\mu-1}\right) + \eta \frac{(r+\delta)^2}{1-\delta} \frac{\mu}{(\mu-1)^2} < 0 \text{ if and only if } \eta < \frac{\mu-1}{\mu} \frac{r+\delta}{r+\delta}.$$

Since $\eta \leq 0$ and the right-hand side of the latter inequality is always positive, this condition is always satisfied. Moreover, $J(-1) = 4 + 2r - J(1) > 0$ whenever $J(1) < 0$, so there exists a unique, stable, rational expectations equilibrium for any possible parametrization. The elasticity of the number of firms producing in period $t + 1$ to its past level is the stable root of $J(\lambda) = 0$, *i.e.*, $\left[\text{trace}(M) - \sqrt{(\text{trace}(M))^2 - 4 \det(M)} \right] / 2$.

D The Model with Physical Capital

On the household side, we now have the capital accumulation equation:

$$K_{t+1} = (1 - \delta^K)K_t + I_t, \quad (16)$$

where I_t is investment and $\delta^K \in (0, 1)$ is the rate of depreciation. The budget constraint becomes:

$$B_{t+1} + v_t N_{H,t} x_{t+1} + C_t + I_t + T_t = (1 + r_t)B_t + (d_t + v_t) N_t x_t + w_t L_t + r_t^K K_t,$$

where r_t^K is the rental rate of capital. Euler equations for bonds and share holdings, and the labor supply equation, are unchanged. The Euler equation for capital accumulation requires:

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} (r_{t+1}^K + 1 - \delta^K) \right]. \quad (17)$$

On the firm side, the production function is now Cobb-Douglas in labor and capital: $y_t(\omega) = Z_t l_t(\omega)^\alpha k_t(\omega)^{1-\alpha}$. When $\alpha = 1$, this model reduces to our previous model without physical capital. Imposing symmetry of the equilibrium, cost minimization taking factor prices w_t, r_t^K as given implies:

$$r_t^K = (1 - \alpha) \frac{y_t}{k_t} \lambda_t; \quad w_t = \alpha \frac{y_t}{l_t} \lambda_t,$$

where λ_t is marginal cost. The profit function becomes $d_t = \rho_t y_t - w_t l_t - r_t^K k_t$, where optimal pricing yields $\rho_t = \mu_t \lambda_t$.

As for entry, we now assume that the production technology is uniform across sectors, *i.e.*, the production function for setting up new firms is Cobb-Douglas with the same parameters as the consumption sector. Creating a new firm/product requires an aggregate of labor l_t^E and capital k_t^E in amount $f_{E,t} = Z_t (l_t^E)^\alpha (k_t^E)^{1-\alpha}$. Therefore, the technology for setting up all new entrants, $N_{E,t}$, is: $f_{E,t} N_{E,t} = Z_t (L_t^E)^\alpha (K_t^E)^{1-\alpha}$. The free entry condition requires $v_t = f_{E,t} \lambda_t$. Cost minimization

in this sector implies $r_t^K = (1 - \alpha) \lambda_t f_{E,t} N_{E,t} / K_t^E$ and $w_t = \alpha \lambda_t f_{E,t} N_{E,t} / L_t^E$. Since both capital and labor are perfectly mobile across sectors, the relative capital/labor shares are equalized across sectors (by comparing the expressions for factor prices): $(k_t^E / l_t^E =) K_t^E / L_t^E = K_t^C / L_t^C (= k_t^C / l_t^C)$.

Finally, market clearing for physical capital requires: $K_{t+1} = N_{t+1} k_{t+1}^C + N_{E,t+1} k_{t+1}^E = K_{t+1}^C + K_{t+1}^E$, *i.e.*, capital is rent both to firms that are producing at time $t + 1$ and for setting up new firms at $t + 1$. At the end of the period (when the capital market clears) there is a ‘reshuffling’ of capital among firms such that there is no scrap value for the capital of disappearing firms. The other equations remain unchanged.

We can write the equations as in the version without capital, using only aggregate variables. The complete model can then be summarized by adding the equations in the following table to the equations in Table 2 that remain unchanged (markup, variety effect, number of firms, intratemporal optimality, Euler equations for bonds and shares).

Table C.1. Model with Physical Capital in Production of Consumption and Firm Creation, Summary

Pricing	$\rho_t = \mu(N_t) \lambda_t$
Profits	$d_t = \left(1 - \frac{1}{\mu(N_t)}\right) \frac{Y_t^C}{N_t}$
Free entry	$v_t = f_{E,t} \lambda_t$
Capital accumulation	$K_{t+1} = (1 - \delta^K) K_t + I_t$
Euler equation (capital)	$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-1} (r_{t+1}^K + 1 - \delta^K) \right]$
Aggregate accounting	$Y_t^C + v_t N_{E,t} = w_t L_t + N_t d_t + r_t^K K_t$
Total manufacturing output	$Y_t^C = C_t + I_t$
Real wage	$w_t = \alpha Z_t \left(\frac{L_t}{K_t}\right)^{\alpha-1} \lambda_t$
Rental rate	$r_t^K = (1 - \alpha) Z_t \left(\frac{L_t}{K_t}\right)^\alpha \lambda_t$
Capital market clearing	$K_{t+1} = K_{t+1}^C + K_{t+1}^E$
Labor market clearing	$L_t = L_t^C + L_t^E$
Factor shares equalization	$K_t^E / L_t^E = K_t^C / L_t^C$
New firm creation	$f_{E,t} N_{E,t} = Z_t (L_t^E)^\alpha (K_t^E)^{1-\alpha}$

The Steady State

In steady state, the Euler equation for shares, combined with expressions for firm value, pricing and profits, gives:

$$f_E \rho(N) = \frac{1 - \delta}{r + \delta} \frac{Y_C}{N} (\mu(N) - 1).$$

From the aggregate accounting identity, combined with factor prices, the free entry condition, profit and pricing equations, and the steady-state number of entrants, manufacturing output is:

$$Y^C = \rho(N) \left(Z L^\alpha K^{1-\alpha} - f_E \frac{\delta}{1 - \delta} N \right).$$

Combining these two results, we have:

$$N = \frac{(1 - \delta) Z L^\alpha K^{1-\alpha}}{\left(\frac{r + \delta}{\mu(N) - 1} + \delta \right) f_E}.$$

From the rental rate expression:

$$K = \left(\frac{1 - \alpha}{r + \delta^K} \frac{\rho(N)}{\mu(N)} Z \right)^{\frac{1}{\alpha}} L$$

Thus, when labor supply is inelastic ($L = 1$), N solves:

$$N = \frac{(1 - \delta) Z^{\frac{1}{\alpha}} \left[\frac{1 - \alpha}{r + \delta^K} \right]^{\frac{1}{\alpha} - 1} \left[\frac{\rho(N)}{\mu(N)} \right]^{\frac{1}{\alpha} - 1}}{f_E \frac{r + \delta}{\mu(N) - 1} + \delta}$$

This equation yields N as a function of structural parameters, including preference parameters. For example, under C.E.S. preferences, we have

$$N^{CES} = \left\{ \frac{(1 - \delta) Z^{\frac{1}{\alpha}} \left[\frac{1 - \alpha}{r + \delta^K} \frac{\theta}{\theta - 1} \right]^{\frac{1}{\alpha} - 1}}{f_E [(r + \delta)(\theta - 1) + \delta]} \right\}^{1 + \frac{1 - \alpha}{\alpha \theta - 1}}.$$

Otherwise, N is a function of the parameters governing the functional forms of $\rho(N)$ and $\mu(N)$, and the calibration scheme $\sigma^* N^{CES} = \theta - 1$ described above ensures that the steady-state markup and number of firms are the same under C.E.S. and translog, so that, again:

$$\eta(N) \equiv \frac{\mu'(N) N}{\mu(N)} = -\frac{1}{1 + \sigma N^{Trans}} = -\frac{1}{\theta} \quad \text{and} \quad \epsilon(N) \equiv \frac{\rho'(N) N}{\rho(N)} = \frac{1}{2\sigma N^{Trans}} = \frac{1}{2(\theta - 1)}.$$

The shares vN_E/Y^C and dN/Y^C are unchanged relative to the baseline model without capital. The shares of (physical) capital and labor income in consumption output Y^C are now:

$$\frac{r^K K}{Y^C} = (1 - \alpha) \left(\frac{1}{\mu} + \frac{vN_E}{Y^C} \right) \quad \text{and} \quad \frac{wL}{Y^C} = \alpha \left(\frac{1}{\mu} + \frac{vN_E}{Y^C} \right).$$

Each of these shares consists of two parts: the share in Y_C of the respective factor's portion used in the consumption and 'new products' sectors, respectively. The share of consumption output in GDP is: $Y^C/Y = (1 + vN_E/Y^C)^{-1}$. The share of total investment also consists of two components: investment in new products/firms vN_E (which includes the part of capital installed last period that is used to produce these new firms, $r^K K^E$) and investment in new capital I . The latter can be found from the expression for the rental rate, using also $I/K = \delta^K$ and $r^K = r + \delta^K$:

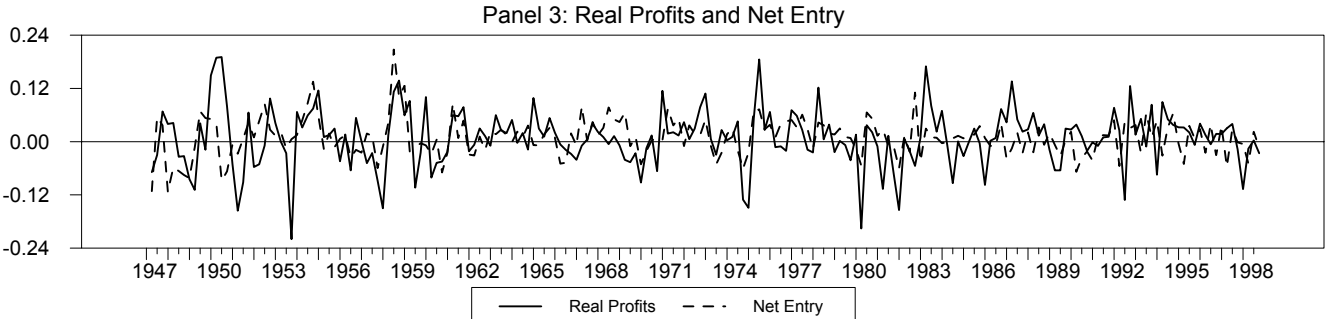
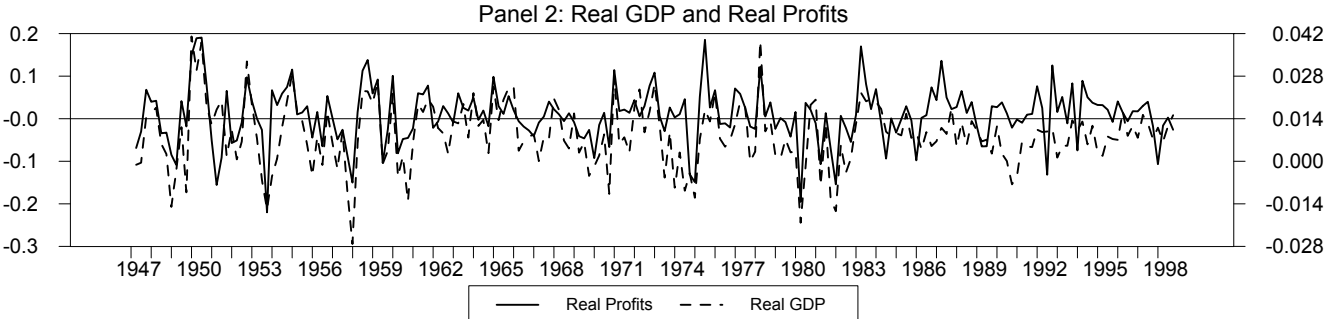
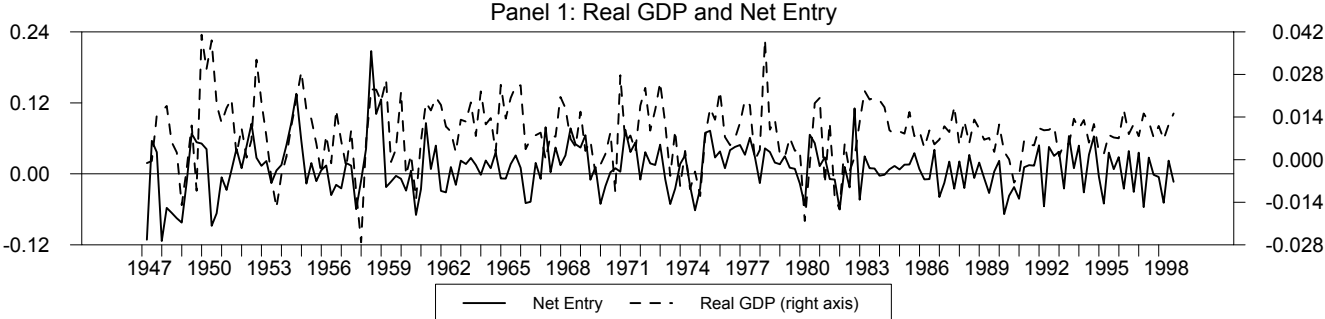
$$\begin{aligned} \frac{K}{Y^C} &= \frac{1 - \alpha}{r + \delta^K} \left(\frac{1}{\mu} + \frac{vN_E}{Y^C} \right) \Rightarrow \frac{K}{Y} = \frac{1 - \alpha}{r + \delta^K} \left(1 - \frac{\mu - 1}{\mu} \frac{1}{1 + \frac{vN_E}{Y^C}} \right), \\ \frac{I}{Y^C} &= (1 - \alpha) \frac{\delta^K}{r + \delta^K} \left(\frac{1}{\mu} + \frac{vN_E}{Y^C} \right) \Rightarrow \frac{I}{Y} = (1 - \alpha) \frac{\delta^K}{r + \delta^K} \left(1 - \frac{\mu - 1}{\mu} \frac{1}{1 + \frac{vN_E}{Y^C}} \right). \end{aligned}$$

Note that the share of physical capital investment in GDP is smaller than its RBC counterpart $((1 - \alpha) \delta^K / (r + \delta^K))$. But the share of total investment in total GDP can be higher since it includes investment in new firms. Specifically, the share of total investment in GDP is now:

$$\frac{TI}{Y} = \frac{vN_E}{Y} + \frac{I}{Y} = \frac{vN_E}{Y^C} \frac{1}{1 + \frac{vN_E}{Y^C}} + (1 - \alpha) \frac{\delta^K}{r + \delta^K} \left(1 - \frac{\mu - 1}{\mu} \frac{1}{1 + \frac{vN_E}{Y^C}} \right).$$

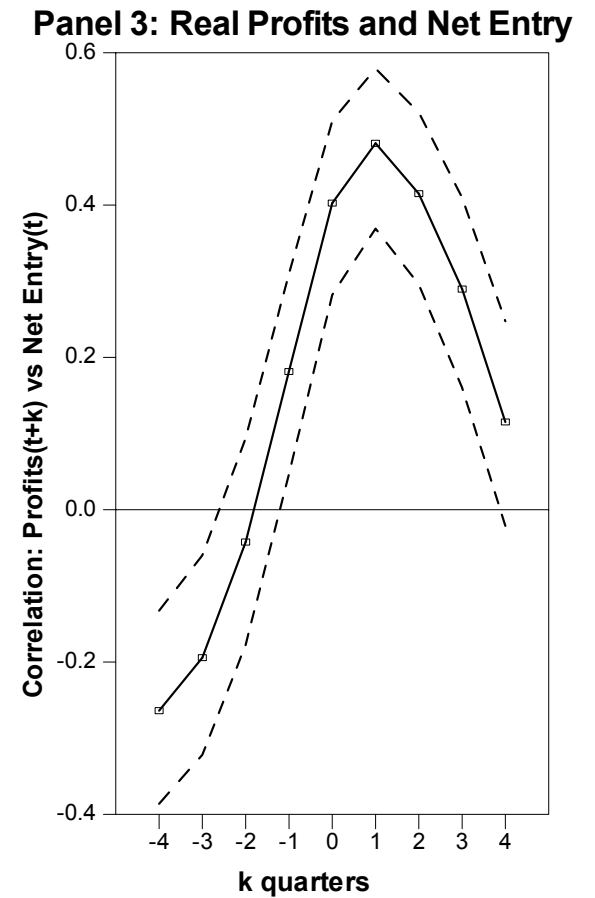
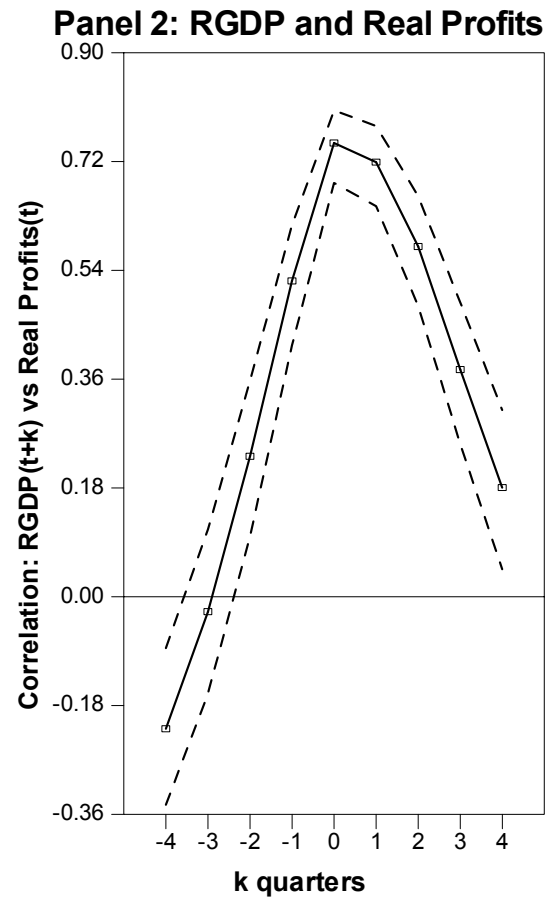
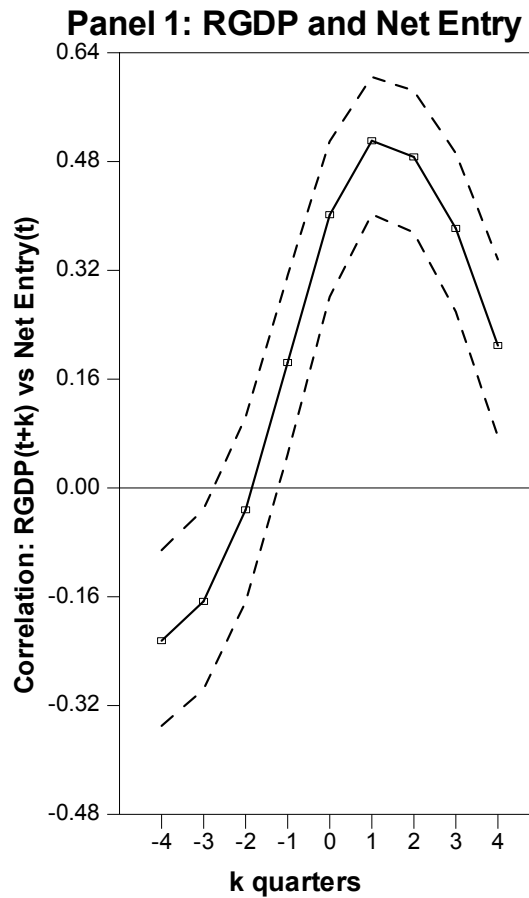
Growth Rates of Real GDP, Net Entry, and Real Profits

Sample period: 1947 - 1998



Cross Correlation: RGDP, Net Entry, and Real Profits

Hodrick-Prescott filtered data in logs, 95% confidence intervals, sample period: 1947 - 1998



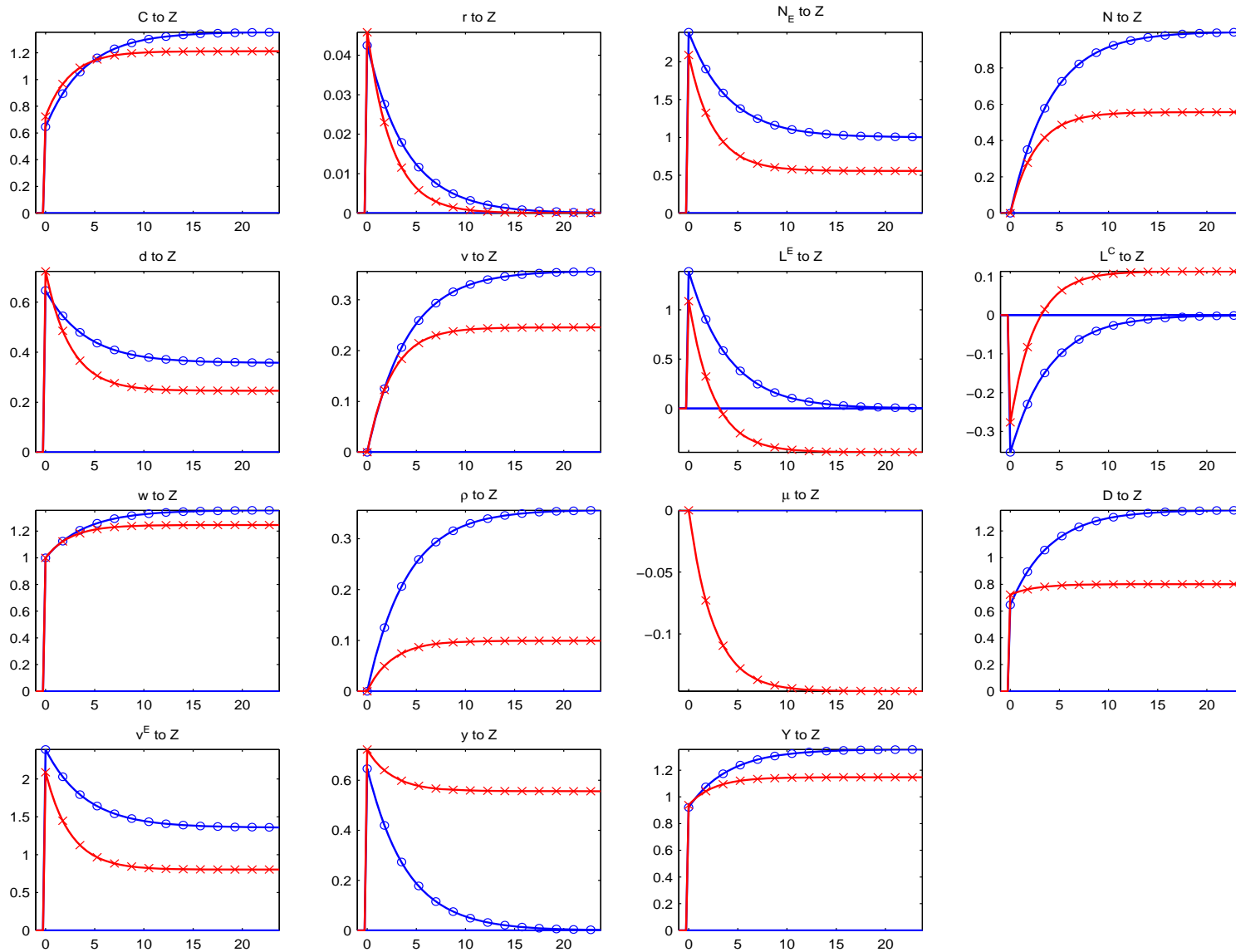


Figure 3. Responses to a Permanent Productivity Increase, C.E.S. vs. Translog, Inelastic Labor

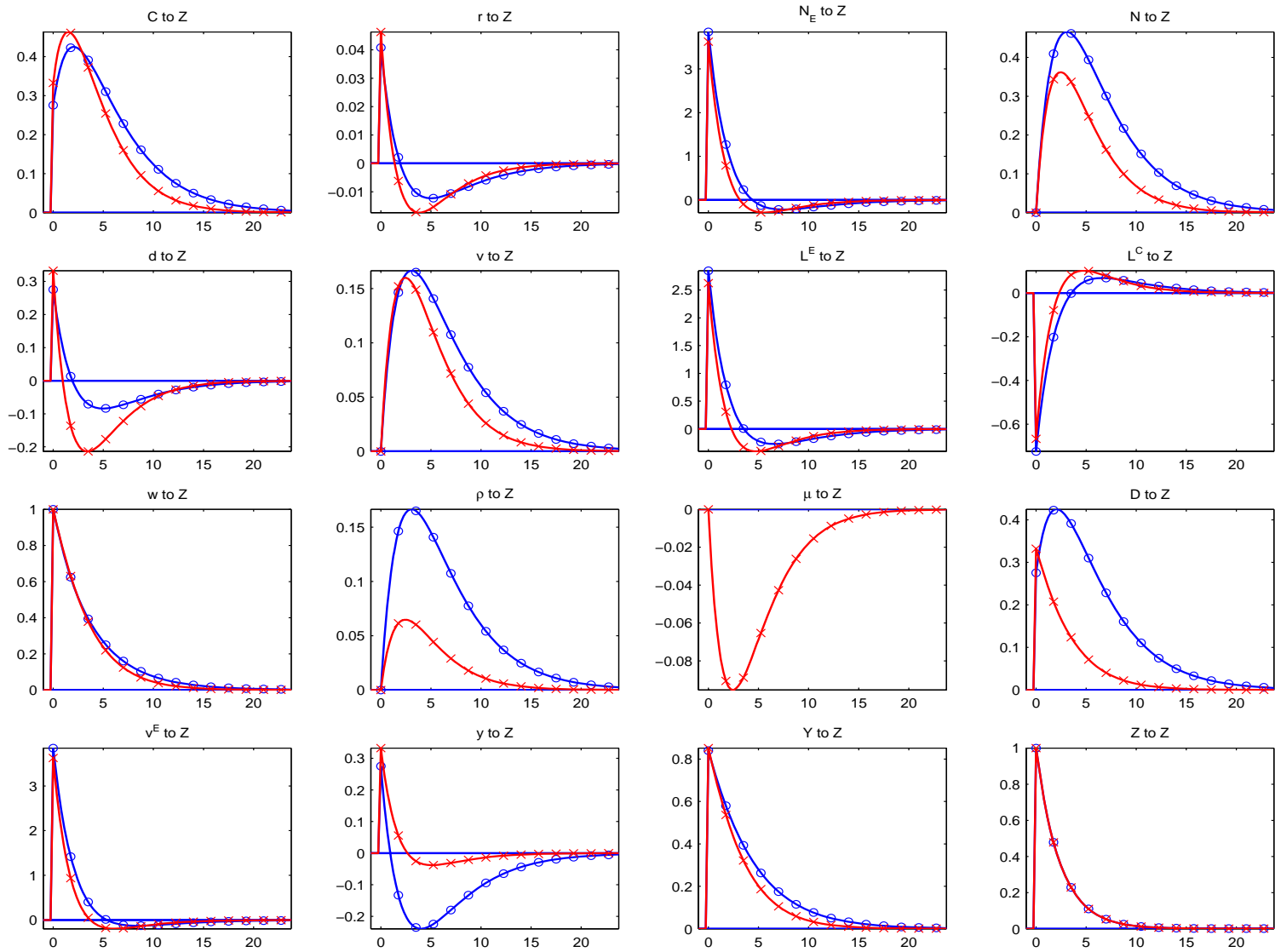


Figure 4. Responses to a Transitory Productivity Increase, Persistence .9, C.E.S. vs. Translog, Inelastic Labor

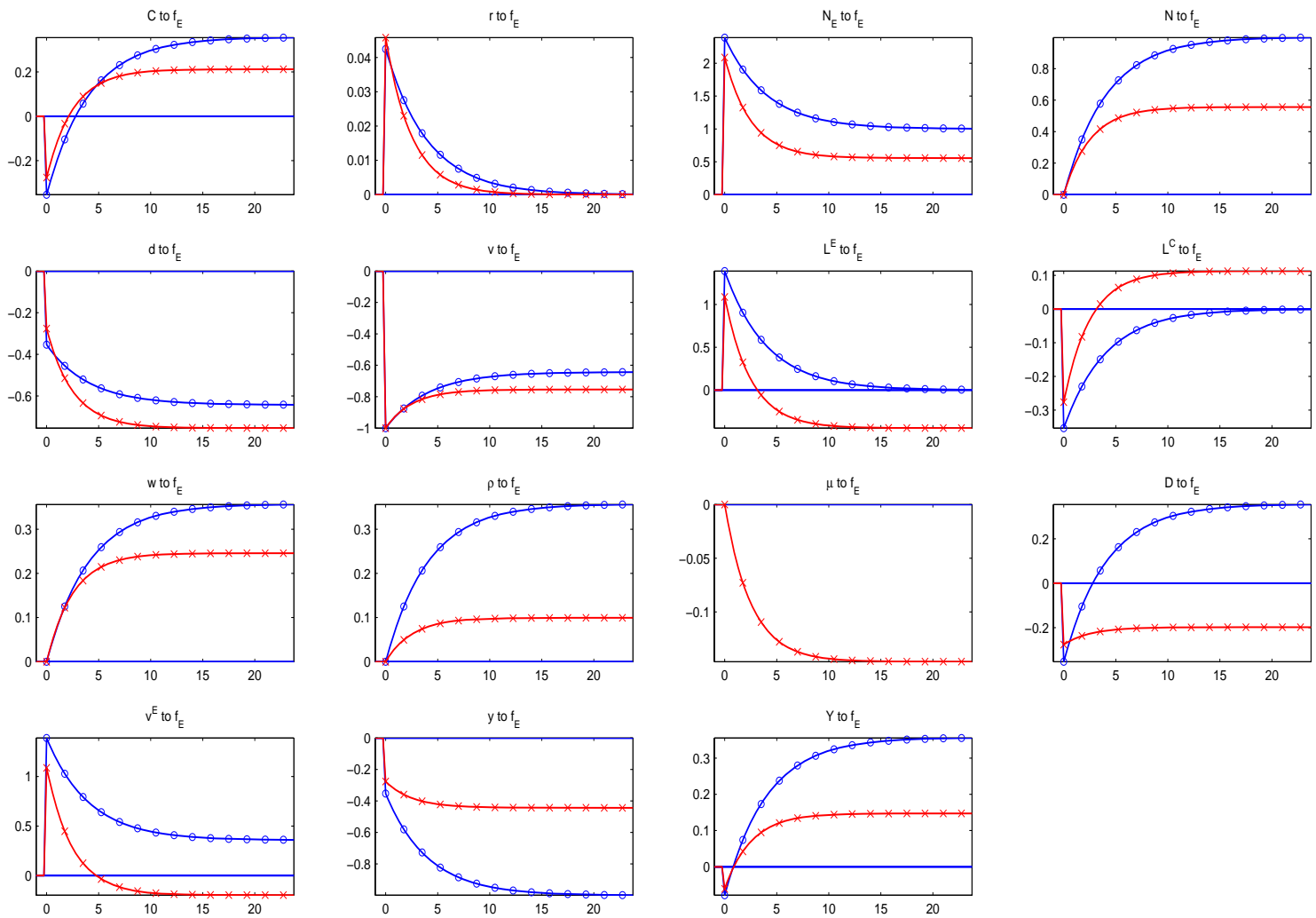


Figure 5. Responses to a Permanent Deregulation, C.E.S. vs. Translog, Inelastic Labor

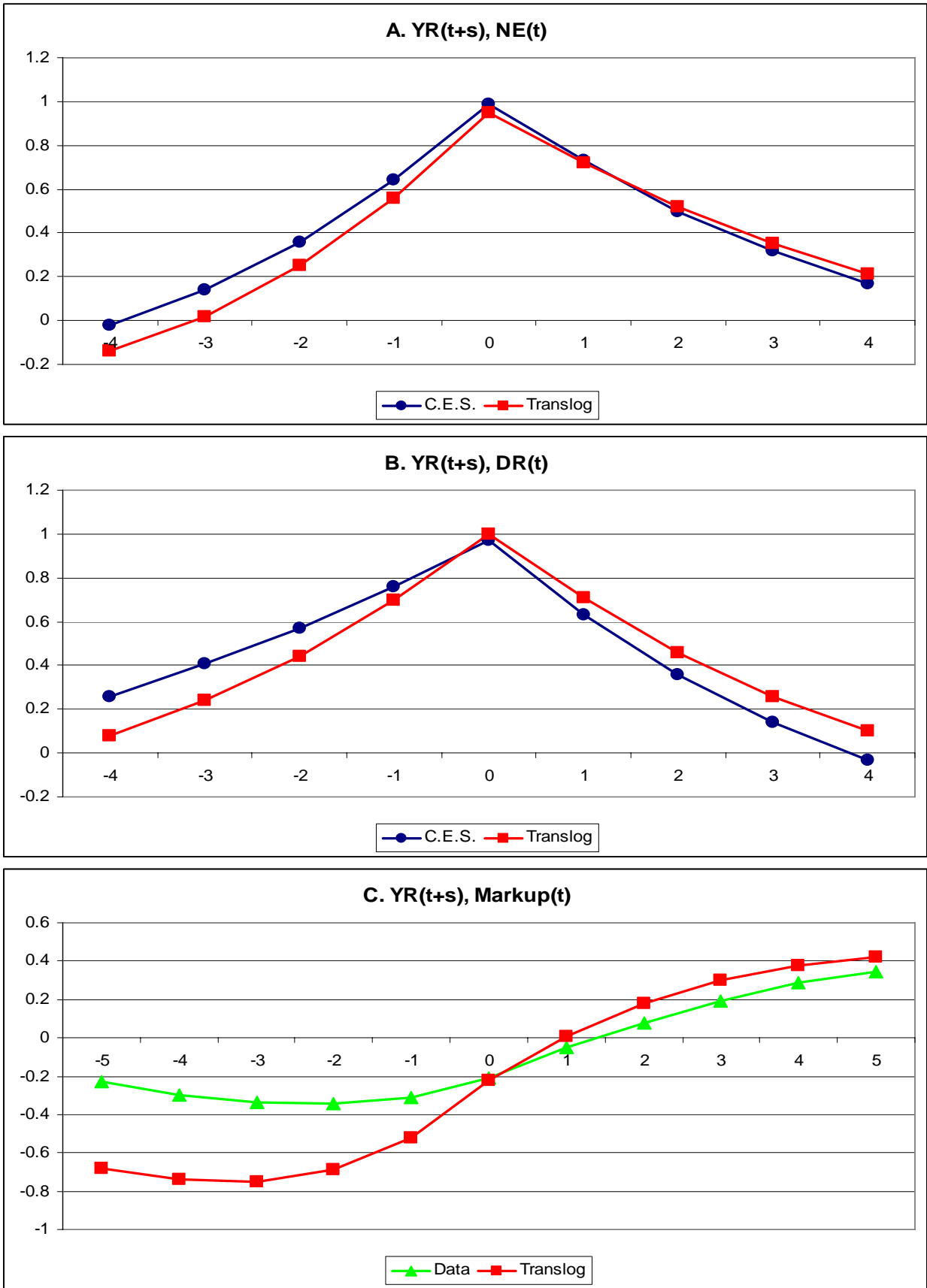


Figure 6. Model-Based Correlations: Entry, Real Profits, and the Markup*
 * Source for Data in Panel C: Rotemberg and Woodford (1999), page 1066, Table 2, column 2.

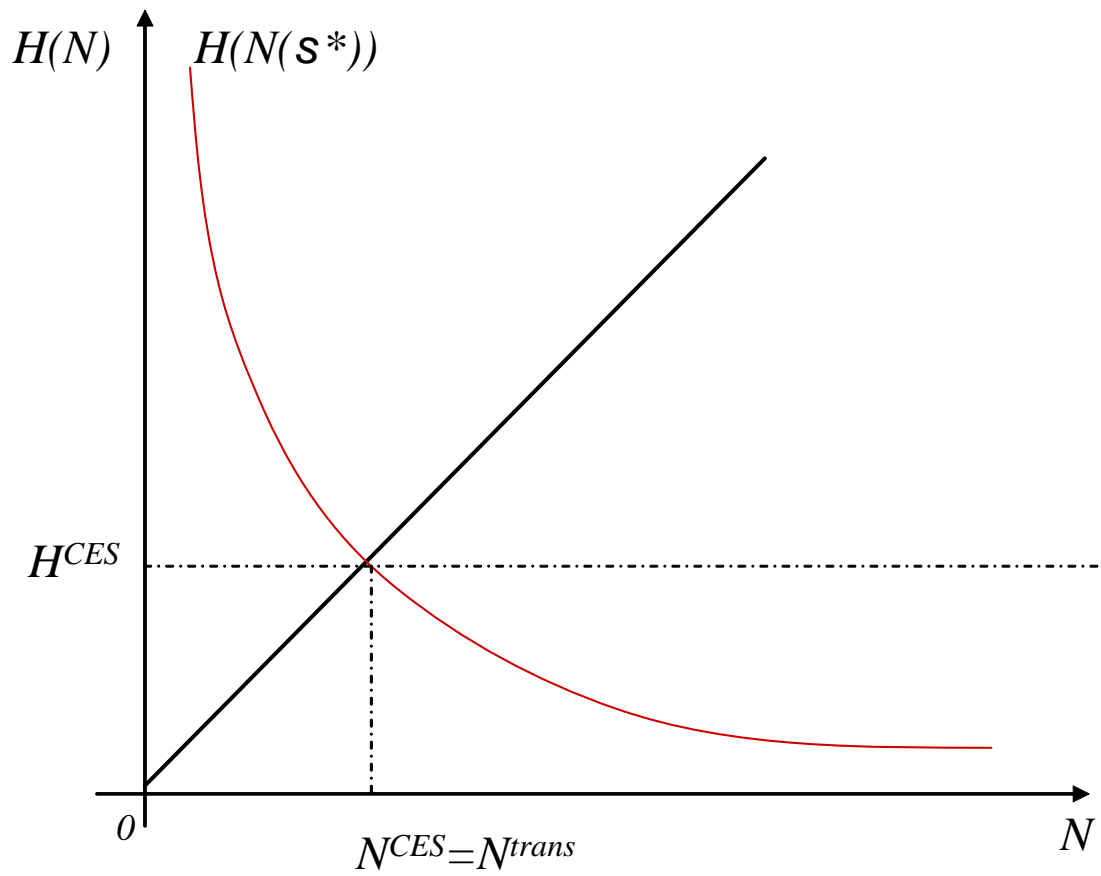


Figure B.1. The Steady-State Number of Firms, C.E.S. vs. Translog