Limited Asset Markets Participation, Monetary Policy and (Inverted) Keynesian Logic.

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ABSTRACT. This paper incorporates limited asset markets participation in dynamic general equilibrium and develops a simple analytical framework for monetary policy analysis. Aggregate dynamics and stability properties of an otherwise standard business cycle model depend nonlinearly on the degree of asset market participation. While 'moderate' participation rates strengthen the role of monetary policy, low enough participation causes an inversion of results dictated by ('Keynesian') conventional wisdom. The slope of the 'IS' curve changes sign, the 'Taylor principle' is inverted, optimal welfare-maximizing discretionary monetary policy requires a passive policy rule and the effects and propagation of shocks are changed. However, a targeting rule implementing optimal policy under commitment delivers equilibrium determinacy regardless of the degree of asset market participation. Our results may justify Fed's behavior during the 'Great Inflation' period.

Keywords: *limited asset markets participation; dynamic general equilibrium; aggregate demand; Taylor Principle; optimal monetary policy; real (in)determinacy.*

JEL codes: *E32; G11; E44; E31; E52; E58.*

1. Introduction

At the heart of modern macroeconomic literature dealing with monetary policy issues lies some form of 'aggregate Euler equation', or 'IS' curve: an inverse relationship between aggregate consumption today and the expected real interest rate. This relationship is derived from the households' individual Euler equations assuming that all households substitute consumption intertemporally - for example using assets. Normative prescriptions are then derived by using this equation as a building block, together with an inflation dynamics equation ('Phillips curve') derived under the assumption of imperfect price adjustment[1].

This paper introduces *limited asset markets participation* ($LAMP$) into an otherwise standard dynamic general equilibrium model and studies the implications of this for monetary policy. We model LAMP in a way that has become standard in the macroeconomic literature reviewed below. Namely, we assume that a fraction of agents have zero asset holdings, and hence do not smooth consumption but merely consume their current disposable income, while the rest of the agents hold all assets and smooth consumption[2]. This modelling choice is motivated both by direct data on asset holdings and by an extensive empirical literature studying consumption behavior. The latter seems to suggest that, regardless of whether aggregate time series or micro data are used, consumption tracks current income for a large fraction of the US population. To give just some prominent examples, Campbell and Mankiw (1989) used aggregate time series data to find that a fraction of 0.4 to 0.5 of the US population merely consumed their current income. More recent studies using micro data also find that a significant fraction of the US population fails to behave as prescribed by the permanent income hypothesis (e.g. Hurst (2004), Johnson, Parker and Souleles (2004))[3]. Finally, direct data on asset holdings shows that a low fraction of US population holds assets in various forms[4]. Models incorporating this insight have been recently used in the macroeconomic literature. First, some version of this assumption -whereby a fraction of agents does not hold physical capital- has been proposed by Mankiw (2000) and extended by Galí, Lopez-Salido and Valles (2003) for fiscal policy issues[5]. Second, it is the norm in the monetary policy literature trying to capture the 'liquidity effect', where it is assumed that

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[1]See Woodford (2003) for a state-of-the art review of this literature. Earlier overviews comprise, amongst others, Clarida, Gali and Gertler (1999) and Goodfriend and King (1997).

[2]In an appendix of an earlier working paper version, Bilbiie (2005), we outline a simple model in which high enough proportional transaction costs can rationalize limited participation. We also review some evidence concerning the magnitude of these costs necessary to generate observed non-participation levels.

[3]Johnson et al. show that a large part of the US population consumed the unexpected increase in transitory income generated by the 2001 tax rebate and find that the response was higher for households with low wealth. Relatedly, Wolff and Caner (2004) use 1999 PSID data to find that 41.7 percent of the US population can be classified as asset-poor when home equity is excluded from net worth, whereas 25.9 percent are asset-poor based on net worth data.

[4]Vissing-Jorgensen (2002) reports based on the PSID data that of US population 21.75 percent hold stock and 31.40 percent hold bonds. Data from the 1989 Survey of Consumer Finances (see e.g. Mulligan and Sala-i-Martin (2000)) shows that 59 percent of US population had no interest-bearing financial assets, while 25 percent had no checking account either.

[5]The latter paper argues that this modelling assumption can help explaining the effects of government spending shocks. See also Bilbiie and Straub (2003).

asset markets are 'segmented' (e.g. Alvarez, Lucas and Weber (2001)). This modelling choice has only recently been incorporated into the sticky-price monetary policy research in a paper that we review in detail below.

We show how the general equilibrium model with LAMP can be reduced to a familiar 2-equations system, consisting of a Phillips- and an IS- curve, which nests the standard New Keynesian model; since the resulting system is very simple, it might be of independent interest to some researchers. Notably, we capture the influence of LAMP on aggregate dynamics through an unique parameter, the elasticity of aggregate demand to real interest rates, which depends non-linearly on the degree of asset market participation and is at the core of the intuition for all our results. In a nutshell, we show that limited asset market participation has a non-linear effect on most predictions of the standard full-participation model.

Interest rate changes modify the intertemporal consumption and labor supply profile of *asset holders*, agents who smooth consumption by trading in asset markets. This affects the real wage, and the demand thereby of agents who have no asset holdings, are oversensitive to real wage changes, and insensitive directly to interest rate changes. Variations in the real wage (marginal cost) lead to variations in profits and hence in the dividend income of asset holders. These variations can either reinforce (if participation is not 'too' limited) or *overturn* the initial impact of interest rates on aggregate demand. The latter case occurs if the share of non-asset holders is high enough and/or and the elasticity of labor supply is low enough, for the potential variations in profit income offset the interest rate effects on the demand of asset holders. This is the main mechanism identified by this paper to change dramatically the effects of monetary policy as compared to a standard full-participation case whereby aggregate demand is completely driven by asset holders.

If participation is restricted below a certain threshold, the predictions are **strengthened**: as more (but not 'too many') people do not hold assets, the link between interest rates and aggregate demand becomes stronger, and monetary policy is more effective. However, when participation is restricted beyond a given threshold (i.e. enough agents do not participate in asset markets), standard theoretical prescriptions or predictions are **reversed**. First, the 'IS curve' has a positive slope: current aggregate output is positively related to real interest rates. Secondly, the 'Taylor Principle' (Woodford (2001)) is *inverted:* the central bank needs to adopt a *passive* policy rule whereby it increases the nominal interest rate by *less* than inflation (i.e. *decreases* the real interest rate), for policy to be consistent with a unique rational expectations equilibrium[6]. Relatedly, an interest rate peg can also lead to a determinate equilibrium. Thirdly, the welfare-maximizing optimal policy problem can still be cast in a linear-quadratic framework; but an expectations-based instrument rule implementing the *discretionary* optimum ought to be passive. Importantly, however, a targeting rule that implements *optimal policy under commitment* (or *timeless-optimal policy* in the sense of Woodford (2003)) leads to

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[6]This finding is closely related to other studies that have explored robustness of this policy prescription to changes in various assumptions. To mention but a few, some studies found that the Taylor principle is either not necessary or not sufficient if: taxes do not respond enough to the level of outstanding debt (Leeper (1991), Woodford (1996)), if the model features physical capital accumulation (Dupor (2000), Sveen and Weinke (2005)), or if consumers are not infinitely-lived (Benassy (2005)). Others have argued that the Taylor principle is sufficient under milder conditions, e.g. in a multi-sector economy even if applied to only one sector's prices (Carlstrom, Fuerst and Ghironi (2006)).

equilibrium determinacy regardless of the degree of asset markets participation. And finally, the effects of some shocks are overturned (for example, unanticipated positive shocks to interest rates are expansionary). In the limit (when nobody holds assets), aggregate demand ceases to be linked to interest rates and monetary policy becomes ineffective. The required share of non-asset holders for these results to hold can be compared to empirical estimates or to direct data on asset holding.

The paper closest related to ours is Galí, Lopez-Salido and Valles (2004, henceforth GLV); that paper studies determinacy properties of interest rate rules in a sticky-price model in which a fraction of agents does not hold physical capital and follows a 'rule-of-thumb'. The general message of that paper is that the Taylor principle is not a good guide for policy under some parametrizations. Namely, GLV argue that if the central bank responds to current inflation via a simple Taylor rule, when the share of 'rule-of-thumb' agents is high enough the Taylor principle is *strengthened*: the response to inflation needs to be higher than in the benchmark model. On the contrary, for a rule responding either to past or future expected inflation, GLV suggest, based on numerical simulations, that for a high share of non-asset holders the policy rule needs to violate the Taylor principle to ensure equilibrium uniqueness.

The aspects that differentiate our paper from GLV pertain to three issues: assumptions, girth and, where the focus of the paper does overlap, message. Concerning assumptions, we model the asset market explicitly and emphasize its interaction with the labor market, which is at the core of the intuition for all our results; any discussion of this is absent from GLV. We also abstract from physical capital accumulation and non-separability in the utility function, since these features can by themselves dramatically change determinacy properties of interest rate rules[7]. This simplification allows us to focus on the role of LAMP exclusively, derive all results analytically, and hence provide clear economic intuition for them. Concerning girth, determinacy properties of interest rate rules are only a subset of our paper's focus, which regards these issues together with others (most notably: welfare-based optimal monetary policy) as part of a more general theme having to do with LAMP's influence on the aggregate demand side. Finally, within the issue of determinacy properties, our conclusions are different from GLV's as follows. We show analytically that an *Inverted Taylor principle* holds *in general* when asset market participation is restricted enough. This result depends only to a small extent on whether the rule is specified in terms of current or expected future inflation. As discussed in text in more detail, this is in contrast to GLV who, while having noted the possibility to violate the Taylor principle for a forward-looking rule, also argue that a strengthening of the Taylor principle is required for a contemporaneous rule to result in equilibrium uniqueness. A very strong response to current inflation would also insure determinacy in our model, but we find the implied coefficient is higher than any plausible estimates and makes policy non-credible. We also show how the Taylor principle can be restored by either an appropriate response to output or via distortionary redistributive taxation of dividend income.

Our results can be perhaps most relevant for analyzing (i) developing economies, in which participation in asset markets is notoriously limited; (ii) historical episodes during which even developed economies experienced exceptionally low asset market

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[7]E.g. capital accumulation in itself may overturn or at least weaken the Taylor principle (see Dupor (2000) and Sveen and Weinke (2005), respectively).

participation. Regarding the latter, many authors have argued that policy before Volcker was 'badly' conducted along one or several dimensions, which led to worse macroeconomic performance as compared to the Volcker-Greenspan era. One such argument relies upon the estimated pre-Volcker policy rule non fulfilling the 'Taylor principle', hence containing the seeds of macroeconomic instability driven by non-fundamental uncertainty (CGG (2000), Lubik and Schorfheide (2004)). In a companion paper (Bilbiie and Straub (2006)), we take a *positive* standpoint and argue that Fed policy was better managed than conventional wisdom dictates, if financial market imperfections were pervasive during the 'Great Inflation' period. We split the data in two conventional sub-samples (pre-1979 and post-1982, eliminating the 'Volcker disinflation) and estimate a richer version of this paper's model using Bayesian methods; we find that the degree of asset market participation changed, inducing a change in the sign of the slope of the aggregate IS curve. The tremendous financial innovation and deregulation process in the 1979-1982 period and the abnormally high degree of regulation in the 1970's provide some support to this view. Moreover, we confirm previous literature's finding that monetary policy switched from passive in the former sample to active in the latter. However, in contrast to previous studies, we argue that a passive policy rule implied a determinate equilibrium, was close to optimal policy and allows for the effects of fundamental shocks to be studied. Other parameter estimates and the results of some stochastic simulation exercises using the estimated shock processes are also in line with the stylized facts. Therefore, the change in financial imperfections might help explain both the change in macroeconomic performance and the change in the policy response; the abrupt change in the policy rule might not be a mere coincidence, but a response to the structural change.

The rest of the paper is organized as follows. Sections 2 and 3 introduce the LAMP general equilibrium model and its reduced log-linear form, and discuss our core results intuitively. A discussion of the labor market equilibrium useful for further intuition is also presented. Section 4 outlines the 'Inverted Taylor Principle' and discusses ways to restore Keynesian logic, and the Taylor principle. Section 5 analyzes welfare-maximizing optimal monetary policy, Section 6 outlines some positive implications of our model and Section 7 concludes. Most technical details are contained in the Appendices.

2. A General Equilibrium Model with LAMP

The model we use is a standard cashless dynamic general equilibrium model, augmented for limited asset markets participation. We assume that some households are excluded from asset markets, while others trade in complete markets for state-contingent securities (including a market for shares in firms). The failure to trade in asset markets could come from a variety of sources, e.g. asset market frictions, transaction costs, etc.[8]. We assume the fraction of non-asset holders to be exogenous, as in most papers on market segmentation and limited participation, e.g. Alvarez, Lucas and Weber (2001). Our baseline model is similar to GLV (2004), but there are important differences in assumptions, focus and conclusions that have been outlined in the Introduction and will be further emphasized when

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[8]In Appendix A of the working paper version Bilbiie (2005) we outline a simple asset pricing model with proportional transaction costs and show how a distribution of proportional transaction costs can be found that rationalizes the exclusion of a given share of households from asset markets.

discussing our results. A role for monetary policy is introduced by assuming that prices are slow to adjust. There is a continuum of households, a single perfectly competitive final-good producer and a continuum of monopolistically competitive intermediate-goods producers setting prices on a staggered basis. There is also a monetary authority setting its policy instrument, the nominal interest rate.

2.1. Households. There is a continuum of households $[0, 1]$, all having the same utility function $U(.)$. A $1 - \lambda$ share is represented by households who are forward looking and smooth consumption, being able to trade in all markets for state-contingent securities: *'asset holders'* or savers. Each asset holder (subscript S denotes the representative asset holder) chooses consumption, asset holdings and leisure solving the standard intertemporal problem: $\max E_t \sum_{i=0}^{\infty} \beta^i U(C_{S,t+i}, N_{S,t+i})$, subject to the sequence of constraints:

$$B_{S,t} + \Omega_{S,t+1} V_t \leq Z_{S,t} + \Omega_{S,t} (V_t + P_t D_t) + W_t N_{S,t} - P_t C_{S,t}.$$

Asset holder's momentary felicity function takes the additively separable log-CRRA form $U(C_{S,t}, N_{S,t}) = \ln C_{S,t} - \omega N_{S,t}^{1+\varphi}/(1+\varphi)$, which has been used in many DSGE studies[9]. $\beta \in (0, 1)$ is the discount factor, $\omega > 0$ indicates how leisure is valued relative to consumption, and $\varphi > 0$ is the inverse of the labor supply elasticity. $C_{S,t}, N_{S,t}$ are consumption and hours worked by saver (time endowment is normalized to unity), $B_{S,t}$ is the nominal value at end of period t of a portfolio of all state-contingent assets held, except for shares in firms. We distinguish shares from the other assets explicitly since their distribution plays a crucial role in the rest of the analysis. $Z_{S,t}$ is beginning of period wealth, not including the payoff of shares. V_t is average market value at time t shares in intermediate good firms, D_t are real dividend payoffs of these shares and $\Omega_{S,t}$ are share holdings.

Absence of arbitrage implies that there exists a stochastic discount factor $\Lambda_{t,t+1}$ such that the price at t of a portfolio with uncertain payoff at $t + 1$ is (for state-contingent assets and shares respectively):

$$(2.1) \qquad B_{S,t} = E_t [\Lambda_{t,t+1} Z_{S,t+1}] \text{ and } V_t = E_t [\Lambda_{t,t+1} (V_{t+1} + P_{t+1} D_{t+1})]$$

Note that the Euler equation for shares iterated forward gives the fundamental pricing equation: $V_t = E_t \sum_{i=t+1}^{\infty} \Lambda_{t,i} P_i D_i$. The riskless gross short-term nominal interest rate R_t is a solution to:

$$(2.2) \qquad \frac{1}{R_t} = E_t \Lambda_{t,t+1}$$

Substituting the no-arbitrage conditions (2.1) into the wealth dynamics equation gives the flow budget constraint. Together with the usual 'natural' no-borrowing limit for *each* state, this will then imply the usual intertemporal budget constraint:

$$(2.3) \qquad E_t \sum_{i=t}^{\infty} \Lambda_{t,i} P_i C_{S,i} \leq Z_{S,t} + V_t + E_t \sum_{i=t}^{\infty} \Lambda_{t,i} W_i N_{S,i}$$

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[9]This function is in the King-Plosser-Rebelo class and leads to constant steady-state hours. In Appendix A we conduct some robustness checks for an utility function that does not.

Maximizing utility subject to this constraint gives the first-order necessary and sufficient conditions at each date and in each state:

$$\beta \frac{U_C\left(C_{S,t+1}\right)}{U_C\left(C_{S,t}\right)} = \Lambda_{t,t+1} \frac{P_{t+1}}{P_t}$$

$$\omega N_{S,t}^{\varphi} = \frac{1}{C_{S,t}} \frac{W_t}{P_t}$$

along with (2.3) holding with equality (or alternatively flow budget constraint hold with equality and transversality conditions ruling out over-acummmulation of assets and Ponzi games be satisfied: $\lim\limits_{i\to\infty} E_t\left[\Lambda_{t,t+i} Z_{S,t+i}\right] = \lim\limits_{i\to\infty} E_t\left[\Lambda_{t,t+i} V_{t+i}\right] = 0$). Using (2.3) and the functional form of the utility function, the short-term nominal interest rate must obey:

$$\frac{1}{R_t} = \beta E_t\left[\frac{C_{S,t}}{C_{S,t+1}} \frac{P_t}{P_{t+1}}\right].$$

The rest of the households on the $[0,\lambda]$ interval have no assets[10] (*'non-asset holders',* indexed by H) and solve:

(2.4) $$\max_{C_{H,t}, N_{H,t}} \ln C_{H,t} - \omega \frac{N_{H,t}^{1+\varphi}}{1+\varphi} \text{ s.t. } C_{H,t} = \frac{W_t}{P_t} N_{H,t}.$$

The first order condition is:

(2.5) $$\omega N_{H,t}^{\varphi} = \frac{1}{C_{H,t}} \frac{W_t}{P_t},$$

which further allows *reduced-form solutions* for $C_{H,t}$ and $N_{H,t}$ (functions only of W_t/P_t and exogenous processes). Due to the very form of the utility function, hours are constant for these agents: the utility function is chosen to obtain constant hours in steady state, and these agents are 'as if' she were in the steady state always. In this case labour supply of non-asset holders is fixed, no matter φ, as income and substitution effects cancel out. While this facilitates algebra, it is in no way necessary for our results (elastic labor supply will be discussed below-see Appendix A for preferences that do not lead to constant steady state hours). Hours are given by: $N_{H,t} = \omega^{-\frac{1}{1+\varphi}}$ and consumption will track the real wage to exhaust the budget constraint.

2.2. Firms. The firms' problem is completely standard and can be skipped by some readers without loss of continuity. The **final good** is produced by a representative firm using a CES production function (with elasticity of substitution ε) to aggregate a continuum of intermediate goods indexed by i: $Y_t = \left(\int_0^1 Y_t\left(i\right)^{(\varepsilon-1)/\varepsilon} di\right)^{\varepsilon/(\varepsilon-1)}$. Final good producers behave competitively, maximizing profit $P_t Y_t - \int_0^1 P_t\left(i\right) Y_t(i) di$ each period, where P_t is the overall price index of the final good and $P_t\left(i\right)$ is the price of intermediate good i. The demand for each intermediate input is $Y_t(i) = \left(P_t\left(i\right)/P_t\right)^{-\varepsilon} Y_t$ and the price index is $P_t^{1-\varepsilon} = \int_0^1 P_t\left(i\right)^{1-\varepsilon} di$. Each intermediate good is produced by a monopolist indexed by i using a technology given by: $Y_t(i) = A_t N_t(i) - F$, if $N_t(i) > F$ and 0 otherwise. F is a fixed cost assumed to be common to all firms: this can be treated

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[10]These households are labeled 'non-traders' by Alvarez, Lucas and Weber, 'rule-of-thumb' or 'non-Ricardian' by GLV, and 'spenders' by Mankiw 2000.

as a free parameter governing the degree of returns to scale. Cost minimization taking the wage as given, implies that nominal marginal cost is $MC_t = W_t/A_t$. The

profit function in real terms is given by: $D_t(i) = [P_t(i)/P_t] Y_t(i) - (W_t/P_t) N_t(i)$, which aggregated over firms gives total profits $D_t = [1 - (MC_t/P_t) \Delta_t] Y_t$. Total profits are rebated to the asset holders as dividends. The term Δ_t is relative price dispersion defined following Woodford (2003) as $\Delta_t \equiv \int_0^1 (P_t(i)/P_t)^{-\varepsilon} di$ and will play a major role in the welfare analysis.

To introduce a role for monetary policy in affecting the real allocation in this simple cashless model we follow Calvo (1983) and Yun (1996) and introduce sticky prices. Intermediate good firms adjust their prices infrequently, θ being both the history-independent probability of keeping the price constant and the fraction of firms that keep their prices unchanged. Asset holders (who in equilibrium will hold all the shares in firms) maximize the value of the firm, i.e. the discounted sum of future nominal profits, choosing the price $P_t(i)$ and using $\Lambda_{t,t+i}$, the relevant stochastic discount factor (pricing kernel) for nominal payoffs: $\max E_t \sum_{s=0}^{\infty} (\theta^s \Lambda_{t,t+s} [P_t(i)Y_{t,t+s}(i) - MC_{t+i}Y_{t,t+s}(i)]$, subject to the demand

equation. The optimal price of the firm obeys:

$$P_t^o(i) = E_t \sum_{s=0}^{\infty} \frac{\theta^s \Lambda_{t,t+s} P_{t+s}^{\varepsilon-1} Y_{t+s}}{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} P_{t+k}^{\varepsilon-1} Y_{t+k}} MC_{t+s}$$

In equilibrium each producer that chooses a new price $P_t(i)$ in period t will choose the same price and the same level output, hence the price index is: $P_t^{1-\varepsilon} = (1 - \theta)(P_t^o)^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon}$. The combination of these two conditions leads in the log-linearized equilibrium to the well known New Keynesian Phillips curve given below.

2.3. Monetary policy. We consider two policy frameworks prominent in the literature. First, we study *instrument rules* in the sense of a feedback rule for the instrument (short-term nominal interest rate) as a function of macro variables, mainly inflation. We focus on rules within the family (where variables with a star denote variables calculated under flexible prices, defined below):

$$(2.6) \qquad\qquad R_t = (R_t^*)^{\phi^*} R \left(E_t \frac{P_{t+1}}{P_t} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*} \right)^{\phi_x} e^{\varepsilon_t}.$$

We shall also consider *targeting rules* resulting from policymaking under either discretion or commitment, whereby the path of the nominal rates is found by optimization by the central bank - this is described in detail in Section 6 below. Such a framework will also imply a behavioral relationship for the instrument rule, but this is only an *implicit instrument rule* (see Svensson (1999)).

2.4. Market clearing, aggregation and accounting. All agents take as given prices (with the exception of monopolists who reset their good's price in a given period), as well as the evolution of exogenous processes. A rational expectations equilibrium is then as usually a sequence of processes for all prices and quantities introduced above such that the optimality conditions hold for all agents and all markets clear at any given time t. Specifically, labor market clearing requires that labor demand equal total labor supply, $N_t = \lambda N_{H,t} + (1-\lambda) N_{S,t}$. State-contingent assets are in zero net supply (markets are complete and agents

trading in them are identical), whereas equity market clearing implies that share holdings of each asset holder are:

$$\Omega_{S,t+1} = \Omega_{S,t} = \Omega = \frac{1}{1-\lambda}.$$

Finally, by Walras' Law the goods market also clears (this is also the social resource constraint) $Y_t = C_t$, where $C_t \equiv \lambda C_{H,t} + (1-\lambda) C_{S,t}$ is aggregate consumption.

2.5. Steady state and linearized equilibrium. We study the dynamics of the above model by taking its (log-)linear approximation around the unique non-stochastic steady state. The latter is found by evaluating the optimality conditions in the absence of shocks and assuming that all variables are constant. From the Euler equation of asset holders, the steady-state riskless interest rate is $R = \beta^{-1}$, where $R \equiv 1 + r$. Defining the steady-state net mark-up as $\mu \equiv (\varepsilon - 1)^{-1}$ and the share of the fixed cost in steady-state output $F_Y \equiv F/Y$, the share of real wage in total output can easily be shown to be: $WN/PY = (1 + F_Y)/(1 + \mu)$; profits' share in total output is: $D_Y = D/Y = (\mu - F_Y)/(1 + \mu)$. We assume that hours are the same for the two groups in steady state, $N_H = N_S = N$. Then, using the budget constraint for each group, consumption shares in total output are:

$$\frac{C_S}{Y} = \frac{1 + F_Y}{1 + \mu} + \frac{1}{1-\lambda}\frac{\mu - F_Y}{1 + \mu}; \quad \frac{C_H}{Y} = \frac{1 + F_Y}{1 + \mu}$$

Since preferences are homogenous, steady-state consumption shares are also equal across groups, since intratemporal optimality conditions evaluated at steady-state imply:

$$C_H = C_S = \frac{1}{\omega N^\varphi}\frac{W}{P}.$$

This instead requires either restrictions on technology making the share of asset income zero in steady state, or an appropriate taxation scheme (something we have abstracted from until now, but shall return to when analyzing optimal policy). For example, if the share of the fixed cost is equal to net markup $\mu = F_Y$ the share of profits in steady-state D_Y is zero, consistent with evidence and arguments in i.a. Rotemberg and Woodford (1995), and with the very idea that the number of firms is fixed in the long run[11]. Consumption shares are then:

$$\frac{C_H}{Y} = \frac{C_S}{Y} = C_Y = 1.$$

Alternatively, we could assume that either (i) hours worked are not equal across groups or (ii) preferences are not homogenous across groups. Any of these assumptions would allow for consumption shares to be different across groups in steady-state, complicating the subsequent analysis. We studied such generalizations in an earlier working paper version (Bilbiie (2005)) and have found that all results of the simplest case carry through largely unaltered. Therefore, we have decided to adopt the simplest setup in order to facilitate exposition.

We take a (log-)linear approximation of the equilibrium conditions around this steady state. We let a small-case letter denote the log-deviation of a variable from its steady state value, e.g. $y_t = \log(Y_t/Y) \simeq (Y_t - Y)/Y$, with the exception of

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[11]However, profits will vary across firms and over time around this steady-state. Indeed, as it shall soon become clear, profit -and asset income- variations are at the heart of this paper's intuition.

profits, which are defined as a fraction of steady-state output (since their steady-state value D is zero), i.e. $d_t \simeq D_t/Y$. The linearized equilibrium conditions are conveniently summarized in Table 1, where we have already imposed asset market clearing, including $\Omega = (1-\lambda)^{-1}$, and substituted the steady-state ratios found above. We use these equations to express dynamics in terms of aggregate variables only; this makes our model readily comparable with the standard full-participation framework (see CGG (1999), Woodford (2003)) and amenable to policy exercises.

   Table 1 here.

3. Aggregate dynamics

   First, as noted above hours of non-asset holders are constant $n_{H,t} = 0$; there-fore, their consumption tracks real wage, $c_{H,t} = w_t$ and total labor supply (from the labor market clearing condition) is $n_t = [1-\lambda]\, n_{S,t}$ (see Appendix A for the case whereby labor supply of non-asset holders is also elastic). Using these two expressions and asset holders' labor supply equation into the definition of total consumption we find: $c_t = \frac{\lambda}{1-\lambda}\varphi n_t + c_{S,t}$. Finally, using the production function, we obtain the equivalent of the 'planned expenditure line' familiar from standard Keynesian models (see for example David Romer's textbook):

$$(3.1) \qquad c_t = c\left(\underset{+}{y_t}, \underset{-}{rr_t}, a_t\right) = \frac{\lambda}{1-\lambda}\frac{\varphi}{1+\mu}y_t + c_{S,t} - \frac{\lambda}{1-\lambda}\varphi a_t$$

   This equation emphasizes that aggregate expenditure (on the consumption good) depends on two endogenous objects: positively on current total income (out-put) y and negatively on consumption of asset holders c_S (and, using the Euler equa-tion of asset-holders in Table 1, on the ex-ante real interest rate $rr_t \equiv r_t - E_t\pi_{t+1}$). Equation (3.1) is not a reduced-form relationship since c, y and c_S are all endoge-nous variables, which will be determined in general equilibrium. However, we can think of (3.1) as a schedule in the (y, c) space, for a *given* level of $c_{S,t}$ (or, equiva-lently, keeping the real interest rate fixed). While this reasoning may seem contrived at first sight, it is completely standard; for example, it is the equivalent of thinking of the constant-consumption labor supply schedule, which relates hours worked to wages for a given level of (endogenous) consumption. We can take this analogy further and define ξ, the *income elasticity of consumption* (for constant interest rates, or constant-c_S), as the partial derivative of log consumption with respect to log income:

$$\xi \equiv \partial c_t/\partial y_t = \frac{\lambda}{1-\lambda}\frac{\varphi}{1+\mu} > 0,$$

just as one defines the (constant-consumption) elasticity of labor supply.

   The income elasticity of consumption ξ is increasing in (i) the share of non-asset holders λ, for this means that a higher fraction of total population simply consumes the real wage and is insensitive to interest rate movements and (ii) the extent to which labor supply is inelastic φ, for this implies that small variations in hours (and output) are associated to large variations in real wage and hence in the consumption of non-asset holders. Therefore, ξ depends finally on income distribution, which changes as aggregate income and the wage rate change; this gives the model a distinctly Keynesian flavor. In fact, together with the condition that consumption equal output $c_t = y_t$, equation (3.1) leads to a 'Keynesian cross' in case $\xi < 1$: this is pictured in Figure 1 by the black thick line labelled 'K'. A

positive but low enough λ makes the economy 'more Keynesian' since the income elasticity of consumption becomes larger than zero, its value under full participation (whereby the PEL line is the horizontal axis).

   More interestingly perhaps, ξ can become *greater than one.* This case, which we label **'non-Keynesian'**, occurs when enough agents consume their wage income w_t (λ high) and/or wage is sensitive enough to real income y_t (φ high), more precisely when:

$$(3.2) \qquad \lambda > \lambda^* = \frac{1}{1 + \varphi/\left(1 + \mu\right)}.$$

This is illustrated in Figure 1 by the red thin line labelled NK plotting (3.1) in this case along with the $c_t = y_t$ schedule, resulting in a 'Non-Keynesian cross'. An *increase* in the real interest rate moves the (3.1) schedule rightward (by intertemporal substitution) leading to *higher* consumption and output. An increase in technology also moves the (3.1) schedule rightward and boosts consumption and output, since the resulting increase in real wage boosts consumption of non-asset holders.
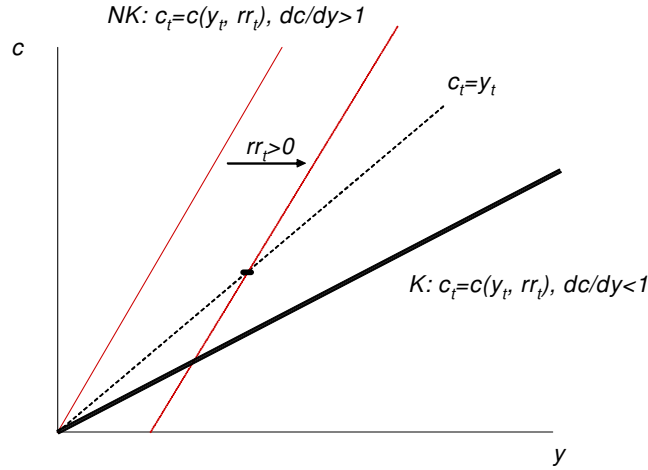

Fig.1: The Keynesian and Non-Keynesian Crosses.

   An immediate implication of the above is that the aggregate IS curve, relating total output growth with the ex-ante real interest rate, swivels (its slope *changes sign*). Consumption of asset holders is related to total output, combining (3.1) with $c_t = y_t$, by:

$$(3.3) \qquad c_{S,t} = \delta y_t + \left(1 + \mu\right)\left(1 - \delta\right) a_t\,, \text{ where } \delta \equiv 1 - \varphi \frac{\lambda}{1 - \lambda} \frac{1}{1 + \mu} = 1 - \xi.$$

Note that δ becomes negative when $\xi > 1$, i.e. precisely when (3.2) holds (see Section 3.2 below for further intuition as to how asset-holders' consumption can be systematically negatively related to output). Substituting (3.3) back into the Euler equation we find the *aggregate Euler equation,* or 'IS curve':

$$(3.4) \qquad y_t = E_t y_{t+1} - \delta^{-1}\left[r_t - E_t \pi_{t+1}\right] + \left(1 + \mu\right)\left(1 - \delta^{-1}\right)\left[a_t - E_t a_{t+1}\right]$$

   Direct inspection of (3.4) suggests the impact that LAMP has on the dynamics of a standard business cycle model through modifying the elasticity of aggregate

demand to real interest rates $-\delta^{-1}$ in a non-linear way[12]. For high enough partic-
ipation rates $\lambda < \lambda^*$, δ is positive and we are in a 'Keynesian' region, whereby
real interest rates restrain aggregate demand. As λ increases towards λ^*, the sen-
sitivity to interest rates increases in absolute value, making policy more effective
in containing demand. However, once λ is above the threshold λ^* we move to the
'non-Keynesian' region where increases in real interest rates become expansionary
(see also Figure 1). As λ tends to its upper bound of 1, $-\delta^{-1}$ decreases towards
zero - policy is ineffective when nobody holds assets. We will call 'non-Keynesian'
an economy in which participation in asset markets is limited enough such that
$\delta < 0$. In Section 3.2. below we turn to an intuitive explanation of this result.

We now have a first glance at the magnitude of λ required for the non-Keynesian
case to occur. To that end, in Figure 2 we plot the threshold λ^* as a function of φ,
assuming a conventional value for steady-state markup of $\mu = 0.2$ corresponding to
an elasticity of substitution of intermediate goods ϵ of 6. Values under the curve give
the Keynesian ($\delta > 0$) case, whereas above the curve we have the non-Keynesian
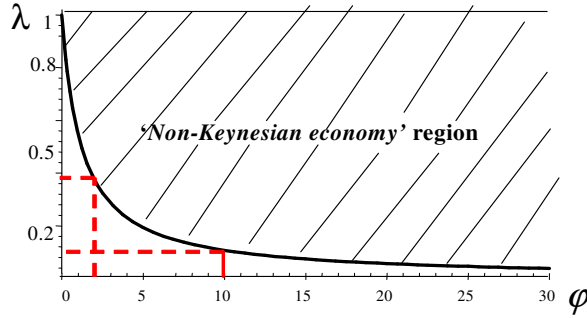economy with $\delta < 0$.

Fig. 2: Threshold share of non-asset holders as a function of inverse labor supply
elasticity.

For Keynesian logic to work, the Frisch elasticity of labor supply (and of intertem-
poral substitution in labor supply), should be high, and the higher, the higher the
share of non-asset holders λ. For a range of φ between 1 (unit elasticity) and 10
(0.1 elasticity) the threshold share of non-asset holders should be lower than 0.5 to
as low as around 0.1 respectively. Compared to some empirical evidence by Camp-
bell and Mankiw (1989) that place this at around 0.4-0.5 for the US economy for
data running up to the mid eighties, or to data on asset holdings reviewed in the
Introduction, this shows that the required share of non-asset holders to end up in
the non-Keynesian case is not empirically implausible.

3.1. Aggregate supply and gap dynamics. In the foregoing we have fo-
cused on the aggregate demand side and hence ignored aggregate supply, the dy-
namics of inflation and its interaction with the above insights, to which we now turn.
By substituting (3.3) and the production function in the labor supply equation of
asset holders we obtain an expression relating real wage to total output:

$$w_t = \chi y_t - \varphi a_t, \text{ where } \chi \equiv 1 + \varphi/(1+\mu) \geq 1 \geq \delta.$$

[12]Note that the only way for δ to be independent of λ is for φ to be zero, i.e. labor supply of
asset holders be infinitely elastic. In this case, consumption of all agents is independent of wealth,
making the heterogeneity introduced in this paper irrelevant.

Substituting this in the expression for real marginal cost in Table 1 and further in the New Phillips curve we obtain:

$$(3.5) \qquad \pi_t = \beta E_t \pi_{t+1} + \kappa y_t - \psi \left(1 + \varphi\right) a_t, \text{ where } \kappa \equiv \psi \chi.$$

This is the Phillips curve written over the level of output, where ψ is defined in Table 1[13].

We now seek to write both the IS and Phillips curve in terms of the *output gap* $x_t \equiv y_t - y_t^*$, defined as the difference between actual output and *natural output* y_t^*. *Natural* levels of all variables are defined as values occurring in the notional equilibrium in which all prices are flexible ($\theta = 0$), and hence inflation is nil and real marginal cost and markup are constant. We see directly from (3.5) that natural output is[14]: $y_t^* = (1 + \varphi) \chi^{-1} a_t$, so marginal cost is related to the output gap by: $mc_t = \chi x_t + \psi^{-1} u_t$ where, following e.g. Clarida, Galí and Gertler (1999) or Galí (2002) we also introduce cost-push shocks $\tilde{u}_t = \psi^{-1} u_t$, i.e. variations in marginal cost not due to variations in excess demand[15]. Therefore, the Phillips curve written in terms of the output gap is:

$$(3.6) \qquad \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t.$$

Finally, returning to the aggregate demand side and evaluating the IS curve under flexible prices we can define the natural interest rate[16]: $r_t^* = \left[1 + \mu \left(1 - \delta/\chi\right)\right] \left[E_t a_{t+1} - a_t\right]$. Using this, the IS curve can be rewritten in terms of the output gap:

$$(3.7) \qquad x_t = E_t x_{t+1} - \delta^{-1} \left[r_t - E_t \pi_{t+1} - r_t^*\right],$$

The Phillips and IS curves (3.6) and (3.7), together with the feedback interest rate rule fully determine dynamics of endogenous variables as a function of exogenous shocks. Note that while we have not specified a process for technology, the model can be solved even for non-stationary technology a_t since r_t^* (which is the relevant shock for determining dynamics) is stationary. For instance, one can assume that technology growth $(\Delta a_t \equiv a_t - a_{t-1})$ is given by an AR(1) process $\Delta a_t = \rho^a \Delta a_{t-1} + \varepsilon_t^a$, which implies shocks to technology have permanent effects on the level of output (see Galí (1999)).

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[13]Note that the Phillips curve is invariant to the share of non-asset holders λ and hence identical to that obtained in the standard framework; this is due to the assumption that stead state consumption shares are equal across groups. In the more general case studied in the working paper (Bilbiie (2005)) the presence of non-asset holders modifies χ (the elasticity of marginal cost to movements in the output gap) and hence the response of inflation to aggregate demand variations. However, the induced modifications are quantitatively minor.

[14]Following the discussion above, note that under flexible prices consumption of asset holders will always increase in response to technology shocks (despite its partial elasticity to total output δ being negative) since $c_{S,t}^* = \left[1 + \mu \left(1 - \delta/\chi\right)\right] a_t$ and hence is procyclical. Real profits under flexible prices are given by $d_t^* = \left[\mu/\left(1 + \mu\right)\right] y_t^*$, and are evidently procyclical.

[15]These could come from the existence of sticky wages creating a time-varying wage markup, time-varying elasticity of substitution among intermediate goods or other sources creating this inefficiency wedge although we do not model this explicitly here (see Woodford, 2003).

[16]Note that r_t^* is stationary even when a_t is not. Moreover, the *sign* of the response of r_t^* to a_t is the same as under full participation: permanent technology shocks have permanent effects on natural output and positive temporary effects on the r_t^* (since $\delta < \chi$), whereas temporary technology shocks cause a fall in r_t^*.

3.2. Further intuition and the labor market. Consumption of asset hold-
ers can be systematically negatively related to total output, as in (3.3) for $\lambda > \lambda^*$,
since an increase in demand can only be satisfied by movements of (as opposed to
movements along) the labor supply schedule when enough people hold no assets
and labor supply is inelastic enough. But the necessary rightward shift of labor
supply can only come from a *negative income effect* on consumption of asset hold-
ers. This negative income effect is ensured in general equilibrium by a *potential*
fall in dividend income. Note that asset holders have in their portfolio $(1 - \lambda)^{-1}$
shares: if total profits fell by one unit, dividend income of one asset holder would
fall by $(1 - \lambda)^{-1} > 1$ units[17]. The *potential* decrease in profits is a natural re-
sult of inelastic labor supply, since the increase in marginal cost (real wage) would
more than outweigh the increase in sales (hours). Therefore, this mechanism relies
on *the consumption of the wealth-owning agents being sensitive to changes in the
money-value of their wealth.*

The key to understanding the results obtained here is the labor market equi-
librium - and its interaction with the asset market. In system (3.8) we outline the
labor supply and the equilibrium wage-hours locus. The labor supply schedule LS
represents the locus of wages and hours for a given level of consumption of asset
holders (all the intertemporal substitution in labor supply comes naturally from as-
set holders). The equilibrium wage-hours locus labeled WN is derived taking into
account the endogenous relationship between asset holders' consumption and total
output (3.3) and the production function. This schedule is invariant to endogenous
forces in equilibrium (in fact, it will be shifted by technology shocks only).

$$(3.8) \qquad LS \quad : \quad w_t = \frac{\varphi}{1 - \lambda} n_t + c_{S,t}$$

$$WN \quad : \quad w_t = \left[(1 + \mu) \delta + \frac{\varphi}{1 - \lambda} \right] n_t + (1 + \mu) a_t$$

A *non-Keynesian economy* ($\delta < 0$) has an intuitive interpretation in labor market
terms, for it implies that the equilibrium wage-hours locus is less upward sloping
than (and hence cuts from above) the labor supply curve. Intuitively, the presence
of non-asset holders generates overall a 'negative income effect', which cannot be
obtained ceteris paribus when $\lambda = 0$ and $\delta = 1$. In the latter, standard case, the
wage-hours locus is more upward sloping than LS. The difference between the two
is the intertemporal elasticity of substitution in consumption, normalized to 1 in
our case (multiplied by returns to scale $1 + \mu$). Ceteris paribus, if the labor demand
shifts out, labor supply shifts leftward due to the usual income effect, since agents
anticipate higher income and higher consumption. If labor supply shifts up due
to a positive income effect, same effect makes labor demand shift out. This gives
a WN locus more upward sloping than the labor supply curve LS. The threshold
value for λ for this insight to change is λ^*, the same as that making $\delta < 0$. When
the share of non-asset holders is higher than this threshold (or equivalently for a
given share, labor supply of asset holders is inelastic enough), the wage-hours locus
becomes less upward sloping than the labor supply. An intuition for that follows, as

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[17]In the standard model all agents hold assets, so this mechanism is completely irrelevant.
Any increase in wage exactly compensates the decrease in dividends, since all output is consumed
by asset holders.

illustrated in Figure 3 where we assume that the real interest rate is kept constant for simplicity[18].

Take first an exogenous outward *shift in labor demand.* Keeping supply fixed, there would be an increase in real wage and an increase in hours. The increase in the real wage would boost consumption of non-asset holders, amplifying the initial demand effect. When labor supply is relatively inelastic, this increase in wage is large and the increase in hours is small compared to that necessary to generate the extra output demanded; note that the effect induced on demand is larger, the higher the share of non-asset holders. The only way for supply to meet demand is for labor supply to shift right. This is insured in equilibrium by the potential fall in profits resulting from: (i) increasing marginal cost (since wage increases) and (ii) the weak increase in hours and hence in output and sales. This is like and indirect negative income effect induced on asset holders by the presence of non-asset holders. Next consider a *shift in labor supply,* for example leftward as would be the case if consumption of asset holders increased. Keeping demand fixed, wage increases and hours fall. The increase in wage (and the increase in consumption of asset holders itself) has a demand effect due to sticky prices. As labor demand shifts right, the real wage would increase by even more; hours would increase, but by little due to the relatively inelastic labor supply (the overall effect would again depend on the relative slopes of the two curves). The increase in the real wage means extra demand through non-asset holders' consumption[19]. The only way to increase output in order to meet this demand is an increase in labor supply. This obtains only if labor supply shifts right, consistently with the potential fall in profits. This explains why in a 'non-Keynesian economy' the wage-hours locus cuts the labor supply curve from above, which will help our intuition in explaining the further results[20].

Note that such a wage-hours locus implies that the model generates a higher partial elasticity of hours to the real wage, and more so more negative δ is. Importantly, despite the potential decrease, in general equilibrium *actual* profits may not fall, precisely due to the negative income effect making asset holders willing to work more; for as a result of this effect hours will increase by more and marginal cost by less, preventing actual profits from falling. In fact, for certain combinations of parameters, shocks or policies our model would *not imply countercyclical profits* in equilibrium (or at least implies more procyclical profits than a standard full-participation model with countercyclical markups). This is an important point, since it is widely believed that profits are procyclical[21]. It is also important to note that the negative income effect does not mean that for a given increase in output, the consumption of asset holders will necessarily *decrease.* In fact, if the

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[18]How the nominal interest rate reacts to inflation, generated here by variations in demand, will be crucial in the further analysis.

[19]The assumptions on preferences ensuring constant steady-state hours are less crucial than it might seem - see Appendix A.

[20]Note that the intuition for real indeterminacy to obtain in standard models (see e.g. Benhabib and Farmer 1994) requires the wage hours locus be upward sloping but cut the labor supply curve from *below.* This is also the case in standard sticky-price models, and gives rise to a certain requirement for the monetary policy rule to result into real determinacy - see below. Our intuition will be that having the wage-hours locus cut the labor suply from *above,* changes determinacy properties in a certain way.

[21]See Section 7 of the working paper version Bilbiie (2005) for a detailed discussion.

increase in output is due to technology, $c_{S,t}$ will *increase* in most cases (i.e. when the equilibrium elasticity of output to technology is less than $(1 + \mu) \left(1 - \delta^{-1}\right)$).
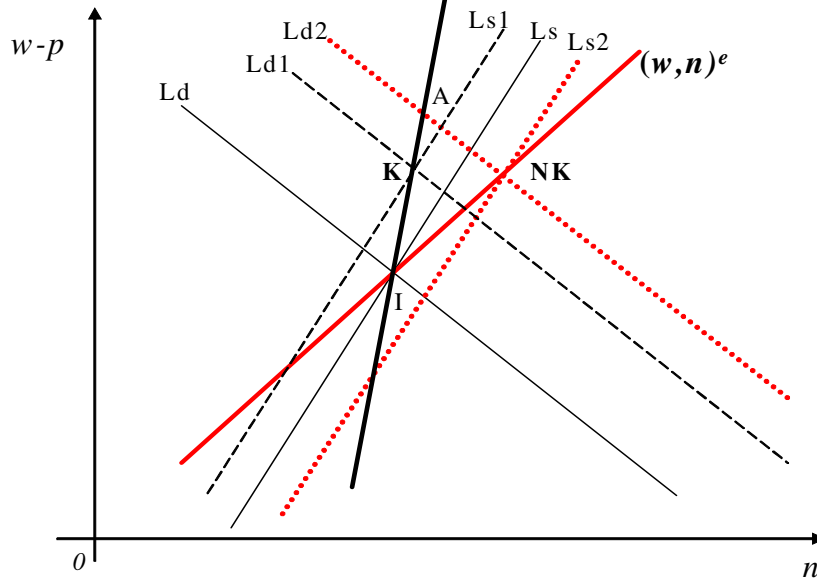

Fig. 3: The equilibrium wage-hours locus and labor supply curve with LAMP.

Having derived the equilibrium wage-hours locus gives us a simple way of thinking intuitively about the effects of shocks and of monetary policy in general; monetary policy, by changing nominal interest rates, modifies real interest rates and hence shifts the labor supply curve (by changing the intertemporal consumption profile of asset holders). But this has no effect on the wage-hours locus by construction, since this describes a relationship that holds in equilibrium always and is shifted only by technology shocks.

4. The Inverted Taylor Principle: Determinacy properties of interest rate rules

The implications of LAMP and the insights outlined above for determinacy properties of simple interest rate rules are dramatic. Assume first that nominal interest rates are set as a function of expected inflation as e.g. in CGG (2000)[22]. This specification provides simpler and sharper determinacy conditions (for reasons discussed below), and captures the idea that the central bank responds to a larger set of information than merely the current inflation rate:

(4.1) $$r_t = \phi_\pi E_t \pi_{t+1} + \varepsilon_t$$

where ε_t is the non-systematic part of policy-induced variations in the nominal rate. The dynamic system for the $z_t \equiv (x_t, \pi_t)'$ vector of endogenous variables and the $\nu_t \equiv (\varepsilon_t - r_t^*, u_t)'$ vector of disturbances is obtained by replacing (4.1) into (3.7) and (3.6) as:

$$E_t z_{t+1} = \mathbf{\Gamma} z_t + \Psi \nu_t,$$

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[22]For analytical simplicity we abstract from inertia (interest rate smoothing) but this extension should be straightforward.

where coefficient matrices are given by:

$$(4.2) \qquad \mathbf{\Gamma} = \begin{bmatrix} 1 - \beta^{-1}\delta^{-1}\kappa\left(\phi_\pi - 1\right) & \delta^{-1}\beta^{-1}\left(\phi_\pi - 1\right) \\ -\beta^{-1}\kappa & \beta^{-1} \end{bmatrix},$$

$$\mathbf{\Psi} = \begin{bmatrix} \delta^{-1} & -\beta^{-1}\delta^{-1}\left(\phi_\pi - 1\right) \\ 0 & -\beta^{-1} \end{bmatrix}.$$

Since both inflation and output gap are forward-looking variables, determinacy requires that both eigenvalues of $\mathbf{\Gamma}$ be outside the unit circle. The determinacy properties of rule (4.1) are emphasized in Proposition 2 (the proof is in Appendix B).

PROPOSITION 1. **The Inverted Taylor Principle**: *Under policy rule (4.1) there exists a locally unique rational expectations equilibrium (i.e. the equilibrium is determinate) if and only if:*

Case I: When $\delta > 0$: $\phi_\pi \in \left(1, 1 + \delta\frac{2(1+\beta)}{\kappa}\right)$;

Case II: When $\delta < 0$: $\phi_\pi \in \left(1 + \delta\frac{2(1+\beta)}{\kappa}, 1\right) \cap [0, \infty)$.

Case I corresponds to the standard 'Keynesian' case and is a generalization of the Taylor principle (Woodford (2001)): as in the full-participation case, the central bank should respond more than one-to-one to increases in inflation[23]. Case II is the 'non-Keynesian economy'. In this case, the Central Bank should follow an *Inverted Taylor Principle*: only passive policy is consistent with a unique rational expectations equilibrium. Obviously, the condition for the *Inverted Taylor Principle* to hold is the same as the one causing a change in the sign of δ, as in (3.2). In other words, the point $\delta = 0$ (or, equivalently, $\lambda > \lambda^*$) constitutes a (local) *bifurcation point*: given the other parameter values (notably, given a value of ϕ_π), the system changes determinacy properties when crossing this threshold[24]. It is worth noting that only the product of δ^{-1} and $(\phi_\pi - 1)$ appears in the matrix $\mathbf{\Gamma}$, and hence in its roots. This is a technical explanation as to why determinacy conditions are clearer (i.e. the inversion of the Taylor principle precisely coincides with the non-Keynesian case $\delta < 0$) for a forward looking-rule than for a contemporaneous rule as that studied in Appendix B, where this is no longer the case. A more intuitive explanation of these issues and of Proposition 1 more generally follows below.

4.1. Intuition: sunspot equilibria with LAMP. Why do sunspot shocks have no real effect in the cases covered by Proposition 1, whereas in the opposite cases they lead to self-fulfilling expectations?[25] Start by substituting the rule (4.1) into (3.7) to obtain aggregate demand as a function of expected inflation:

$$(4.3) \qquad x_t - E_t x_{t+1} = -\delta^{-1}\left(\phi_\pi - 1\right)E_t\pi_{t+1}.$$

Suppose for simplicity and without losing generality that a sunspot shock hits inflationary expectations. In a *Non-Keynesian economy* ($\delta < 0$) a non-fundamental

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[23]It should also not respond 'too much', which is a well-established result for froward-looking rules first noted by Bernake and Woodford (1997). Note that this upper bound is decreasing in the share of non-asset holders.

[24]This case ($\delta = 0$) is observationally equivalent to having linear utility of consumption in the standard, full-participation model. In this case, equilibrium is indeterminate regardless of the value of ϕ_π.

[25]In Section 7 of the working paper version Bilbiie (2005) we compute sunspot equilibria formally.

increase in expected inflation generates an increase in the output gap today if the policy rule is active ($\phi_\pi > 1$) as can be seen from (4.3). By the Phillips curve inflation today increases, validating the initial non-fundamental expectation. This is not the case in the Keynesian economy ($\delta > 0$), since an active rule generates a fall in output gap and (by Phillips curve logic) actual inflation, contradicting initial expectations.

How does a passive policy rule ensure equilibrium determinacy when $\delta < 0$? A non-fundamental increase in expected inflation causes a fall in the real interest rate, a fall in the output gap today by (4.3) and deflation, contradicting the initial expectation that are hence not self-fulfilling. At a more micro level, the transmission is as follows. The fall in the real rate leads to an increase in consumption of asset holders, and an increase in the demand for goods; but note that these are now partial effects. To work out the overall effects one needs to look at the component of aggregate demand coming from non-asset holders and hence at the labor market. The partial effects identified above would cause an increase in the real wage (and a further boost to consumption of non-asset holders) and a fall in hours. Increased demand, however, means that (i) some firms adjust prices upwards, bringing about a further fall in the real rate (as policy is passive); (ii) the rest of firms increase labor demand, due to sticky prices. Note that the real rate will be falling along the entire adjustment path, amplifying these effects. But since this would translate into a high increase in the real wage (and marginal cost) and a low increase in hours, it would lead to a fall in profits, and hence a negative income effect on labor supply. The latter will then not move, and no inflation will result, ruling out the effects of sunspots. This happens when asset markets participation is limited 'enough' in a way made explicit by (3.2).

If the policy rule is instead active ($\phi_\pi > 1$) sunspot equilibria can be constructed. The shock to inflationary expectations leads to an increase in the real rate and in aggregate demand by (4.3). This generates inflation and makes the initial expectations self-fulfilling. At a micro level, transmission is as follows: consumption of asset holders increases due to the real rate increase, which implies a rightward shift of labor supply, and hence a fall in wage and increase in hours. Consumption of non-asset holders also falls one-to-one with the wage, and hence aggregate demand falls by more than it would in a full-participation economy. Firms who can adjust prices will adjust them downwards, causing deflation, and a further fall in the real rate. Firms who cannot adjust prices will cut demand, causing a further fall in the real wage and a small fall in hours (since labor supply is inelastic). But this will mean higher profits (since marginal cost is falling), and eventually a positive income effect on labor supply of asset holders. As labor supply starts moving leftward, demand starts increasing, its increase being amplified by the sensitivity of non-asset holders to wage increases. The economy will establish at a point on the wage-hours locus consistent with the overall negative income effect on labor supply of asset holders, i.e. with higher inflation and real activity. Hence, the initial inflationary expectations become self-fulfilling.

4.2. Output stabilization may restore the Taylor Principle. We now study whether a policy rule incorporating an output stabilization motive can make the Taylor principle a good policy prescription even in a 'non-Keynesian economy'

where $\delta < 0$. Consider a rule of the form:

$$(4.4) \qquad\qquad r_t = \phi_\pi E_t \pi_{t+1} + \phi_x x_t,$$

where ϕ_x is the response to output gap. Intuitively, we may expect this to restore Keynesian logic since it effectively reduces the slope of the planned expenditure line, by introducing a negative dependence of asset holders' consumption on current income (through the Euler equation). However, there are non-trivial interactions among parameters due to the nature of the price-adjustment equation as emphasized by the following Proposition. Replacing (4.4) into (3.7) and (3.6), the $\boldsymbol{\Gamma}$ matrix becomes:

$$\boldsymbol{\Gamma} = \left[\begin{array}{cc} 1 - \delta^{-1} \left[\beta^{-1} \kappa \left(\phi_\pi - 1 \right) - \phi_x \right] & \delta^{-1} \beta^{-1} \left(\phi_\pi - 1 \right) \\ -\beta^{-1} \kappa & \beta^{-1} \end{array} \right]$$

Applying exactly the same method as in the proof of Proposition 1 it can be shown that the determinacy conditions are as follows.

PROPOSITION 2. *(a) Under (4.4), there exists a locally unique rational expectations equilibrium if and only if:*
Case I: When $\delta > 0$: $\phi_\pi + \frac{1-\beta}{\kappa}\phi_x > 1$ and $\phi_\pi < 1 + \frac{1+\beta}{\kappa}\left(\phi_x + 2\delta\right)$ (the 'Taylor Principle')
Case II: When $\delta < 0$: EITHER II.A: $\phi_x < -\delta\left(1-\beta\right)$ and $\phi_\pi + \frac{1-\beta}{\kappa}\phi_x < 1$ and $\phi_\pi > 1 + \frac{1+\beta}{\kappa}\left(\phi_x + 2\delta\right)$ OR II.B: $\phi_x > -\delta\left(1+\beta\right)$ and $\phi_\pi + \frac{1-\beta}{\kappa}\phi_x > 1$ and $\phi_\pi < 1 + \frac{1+\beta}{\kappa}\left(\phi_x + 2\delta\right)$
(b) The equilibrium is indeterminate regardless of ϕ_π if $\delta < 0$ and $\phi_x \in \left(-\delta\left(1-\beta\right); -\delta\left(1+\beta\right)\right)$.

Part (a) studies equilibrium uniqueness. Case I is the standard Taylor principle for an economy where $\delta > 0$. In contrast to Proposition 1, in Case II the inversion of the Taylor Principle is now *not* granted. If either δ is very large in absolute value (a high degree of limited participation λ) or the response to output is low, we end up in case II.A and an instance of the Inverted Taylor Principle is observed. However, for moderate values of λ and/or a high enough response to the output gap, the Taylor Principle is restored. Another way to put this is that for a given share of non-asset holders, the Taylor Principle is a good guide for policy only insofar as the response to output is high enough. The response to output, however, can generate perverse effects if it is not high enough and participation in asset markets is very limited. As part (b) of the Proposition shows, the equilibrium is indeterminate if ϕ_x is in a certain range, regardless of the magnitude of the inflation response. This region is increasing with the share of non-asset holders.

To assess the magnitude of the policy coefficients needed for restoring the Taylor principle, consider a standard quarterly parameterization with $r = 0.01$ and an average price duration of one year, implying $\theta = 0.75$, for a 'non-Keynesian economy' with $\lambda = 0.4$ and $\varphi = 2$ (giving $\delta = -0.11$ and $\kappa = 0.228$).The conditions for Case II.B are $\phi_x > 0.21$; $\phi_\pi > 1 - 0.043\phi_x$; $\phi_\pi < 8.728\,\phi_x - 0.920\,16$.The figure below shows that as soon as the Central Bank responds to output, the Taylor principle is restored under the baseline parameterization for a large parameter region. However, this result should be taken with care, for the very dangers associated with responding to output might outweigh potential benefits. As soon as the share of non-asset holders increases or labor supply becomes more inelastic, equilibrium is more likely to become indeterminate for any inflation response.
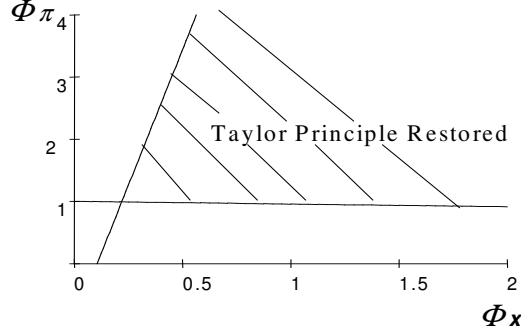
Fig. 4 : Policy parameter region whereby Taylor Principle is restored in a 'LAMP
economy' by responding to output under baseline parameterization.

Finally, in appendix B we show that a version of the *Inverted Taylor Principle*
holds for a contemporary rule also. This is done to further illustrate the differences
of our determinacy results from GLV (2003*b*), where there is a dramatic distinction
between forward-looking and contemporaneous rules. GLV do note (relying upon
numerical simulations and not as a general result) a result similar to our Proposition
1: namely, the Taylor principle may need to be violated for a forward-looking
rule if the share of 'rule-of-thumb' consumers is high. But for a contemporaneous
rule to be compatible with a unique equilibrium, they argue that the central bank
should respond to inflation more strongly than in the full-participation economy
(and indeed very strongly under some parameter constellations). The message of
our paper in what regards determinacy properties of policy rules is different: we
provide analytical conditions for an inverted Taylor principle to hold *generically*,
independently on the policy rule followed, and as part of a general theme having to
do with LAMP's influence on the aggregate demand elasticity to interest rates. Our
results for a simple Taylor rule have the same flavor as for a forward-looking rule:
in the 'non-Keynesian economy' the inverted Taylor principle holds 'generically'
(i.e. if we exclude some extreme values for some of the parameters) for a somewhat
larger share of non-asset holders than was the case under a forward-looking rule. It
is also the case, as in GLV, that a policy rule responding to current inflation very
strongly would insure equilibrium uniqueness[26]. But the implied response ($\phi_\pi = 35$
under the baseline parameterization) is much larger than any plausible empirical
estimate and would have little credibility. This is in contrast with GLV, who do not
consider a possible inversion of the Taylor principle in their numerical analysis of
such rules, but instead argue that for a large share of non-asset holders making the
required policy response too strong under a Taylor rule, the central bank should
switch to a passive forward-looking rule.

4.3. Redistribution restores Keynesian logic. The mechanism of all the
previous results relies on the interaction between labor and asset markets, namely
income effects on labor supply of asset holders from the return on shares. This
hints to an obvious way to restore Keynesian logic relying on a specific fiscal policy

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[26]This is not the case under a forward looking rule, since there, even in a standard full-
participation economy too strong a response leads to indeterminacy - see Bernake and Wodford
1997.

rule that shuts off this channel: tax dividend income and redistribute proceeds as transfers to non-asset holders. We focus on the non-Keynesian case whereby in the absence of fiscal policy $\delta < 0$. To make this point, consider the following simplified fiscal rule: profits are taxed at rate τ_t^D and the budget is balanced period-by-period, with total tax income $\tau_t^D D_t$ being distributed lump-sum to all non-asset holders. We focus on the case where profits are zero in steady-state. The balanced-budget rule then is $\tau_t^D D_t = \lambda L_{H,t}$ which around the steady state (both profits and transfers are shares of steady-state GDP, $l_{H,t} \equiv L_{H,t}/Y$) is approximately: $\lambda l_{H,t} = \tau^D d_t$. Replacing this into the new budget constraint of non-asset holders we get $c_{H,t} = w_t + \frac{\tau^D}{\lambda} d_t$. Asset holders' consumption will then be given by (substituting the expression for profits): $c_{S,t} = (1 - \lambda)^{-1} \left[1 - \tau^D \mu / (1 + \mu) \right] y_t + \left[(\tau^D - \lambda) / (1 - \lambda) \right] w_t$. The wage-hours locus is then obtained following the same method as before: $w_t = (1 - \tau^D)^{-1} \left[1 + \varphi + \mu (1 - \tau^D) \right] n_t$. Finally, consumption of asset holders as a function of total output is:

$$c_{S,t} = \delta_\tau y_t, \text{ where } \delta_\tau = \frac{1}{1 - \tau^D} \left[1 - \tau^D \frac{\mu}{1 + \mu} + \frac{\tau^D - \lambda}{1 - \lambda} \frac{\varphi}{1 + \mu} \right].$$

For a given λ there exists a minimum threshold for the tax rate such that $\delta_\tau > 0$ when in the absence of such fiscal policy $\delta < 0$. This threshold is (note that $\varphi / (1 - \lambda) - \mu > 0$ where $\delta < 0$):

$$1 > \tau^D > 1 - \frac{1 + \varphi}{(1 - \lambda)^{-1} \varphi - \mu} .$$

The necessary tax rate is higher, the more inelastic is labor supply and the higher the share of agents with no assets. The intuition for this result is straightforward: a higher tax rate on asset income eliminates some of the income effect of dividend variation on asset holders' labor supply.

5. Optimal monetary policy.

Our next task is to characterize optimal policy in the presence of non-asset holders. We are particularly interested in whether optimal policy arrangements exist that insure equilibrium determinacy regardless of the degree of asset markets participation[27]. The objective function is calculated as follows. Following Woodford (2003) we use a second-order approximation to a convex combination of households' utilities, described in detail in appendix C, where we use the more general CRRA functional form introduced in Appendix A, without restricting the coefficient of risk aversion γ to equal unity. We make a series of assumptions that allow us to use these second-order approximation techniques. Firstly, we assume that *efficiency* of the steady state is obtained by appropriate fiscal instruments inducing marginal cost pricing in steady state (subsidies for sales at a rate equal to the stead-state net mark-up financed by lump-sum taxes on firms). Since this policy makes steady-state profit income zero, the steady-state is also *equitable*: steady-state consumption

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shares of the two agents are equal, making aggregation much simpler. This ensures consistency with the model outlined above[28].

Secondly, we assume that the social planner maximizes the (present discounted value of a) convex combination of the utilities of the two types, weighted by the mass of agents of each type: $U_t(.) \equiv \lambda U_H(C_{H,t}, N_{H,t}) + [1 - \lambda] U_S(C_{S,t}, N_{S,t})$. This is consistent with our view that limited participation in asset markets comes from *constraints* and not preferences, since in the latter case maximizing intertemporally the utility of non-asset holders would be hard to justify on welfare grounds. The following Proposition shows how the objective function can be represented (up to second order) as a discounted sum of squared output gap and inflation (the proof is in appendix C).

PROPOSITION 3. *If the steady state of the model in Section 3 is* **efficient** *the aggregate welfare function can be approximated by (ignoring terms independent of policy and terms of order higher than 2):*

$$(5.1) \qquad \mathbf{U}_t = -\frac{U_C C}{2} \frac{\varepsilon}{\psi} E_t \sum_{i=t}^{\infty} \left\{ \alpha x_{t+i}^2 + \pi_{t+i}^2 \right\},$$

$$\alpha \equiv \frac{\varphi + \gamma}{1 - \lambda} \left[1 - \lambda (1 - \gamma)(1 + \varphi) \right] \frac{\psi}{\varepsilon}.$$

Note that when $\lambda = 0$ we are back to the standard case $\alpha = (\varphi + \gamma)\psi/\varepsilon$. In the case studied extensively in the rest of this paper ($\gamma = 1$), the relative weight on output gap is $\alpha = (1 + \varphi)\psi/[(1 - \lambda)\varepsilon]$ and is increasing in the share of non-asset holders. In general, an increase in the share of non-asset holders leads to an increase in the relative weight on output (if $\gamma \geq \frac{\varphi}{1+\varphi}$, which is empirically plausible)[29]. When λ tends to one, the implicit relative weight on output stabilization tends to infinity (for $\varphi > 0$). Hence, the presence of non-asset holders modifies the trade-off faced by the monetary authority. The intuition for this result is simple. Since aggregate real profits can be written as $D_t = [1 - (MC_t/P_t)\Delta_t] Y_t$, relative price dispersion Δ_t (related here linearly to squared inflation) erodes aggregate profit income for given levels of output and marginal cost. Given that only a fraction of $(1 - \lambda)$ receives profit income, when this fraction falls the welfare-based relative weight on inflation (price dispersion) also falls. Inflation becomes completely irrelevant for welfare purposes when $\lambda \to 1$: since nobody holds assets, asset income need not be stabilized.

We study optimal policy, consisting of choosing a path $\{r_t^o\}_0^{\infty}$ for the nominal interest rate to minimize $-U_t$, under two alternative regimes. The first is the *discretionary, time-consistent* equilibrium in which the central bank does not internalize the effect of its actions on private sector expectations (i.e. treats these expectations parametrically). The second is the *commitment equilibrium*, in which the central bank takes into account that policy influences expectations by attaching a Lagrange multiplier to the dynamic constraint (3.6). (In the *timeless-optimal policy* studied by Woodford (2003) the central bank additionally commits to a policy to which

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[28]Note, however, that since steady-state consumption shares are equal we do not need to assume increasing returns. Under these assumptions, the reduced-form coefficients simply modifiy as follows: $\chi^o = 1 + \varphi$ and $\delta^o = 1 - \varphi\lambda/(1 - \lambda)$.

[29]When this condition is not fulfilled, so $\gamma < \varphi/(1 + \varphi)$, the relative weight on output gap is decreasing in λ and can even become negative when $\lambda > [(1 - \gamma)(1 + \varphi)]^{-1}$. We exclude this parameter region on grounds of its being empirically irrelevant.

it wished it had committed in a remote past[30]. For our purposes, the distinction between these two regimes is largely immaterial as explained below).

Since the problems for both discretion and commitment are entirely isomorphic to the standard problems (see e.g. CGG (1999) and Woodford (2003)), we will skip solution details for the sake of brevity. The optimal discretionary rule $\left\{r_t^d\right\}_0^\infty$ is found by minimizing $-\mathbf{U}_t$ taking as a constraint the system given by (3.6) and (3.7) and re-optimizing every period. Solving this standard problem, we find that the central bank adjusts its instrument in order to ensure that the following *targeting rule under discretion* hold at all times:

$$(5.2) \qquad\qquad x_t = -\frac{\kappa}{\alpha}\pi_t.$$

When inflation increases (decreases) the central bank has to act in order to contract (expand) demand. Assuming an AR(1) process for the cost-push shock $E_t u_{t+1} = \rho_u u_t$ for simplicity, we obtain the following reduced forms for inflation and output from the aggregate supply curve:

$$(5.3) \qquad\qquad \pi_t = \alpha\Upsilon u_t; \quad x_t = -\kappa\Upsilon u_t,$$

where $\Upsilon \equiv \left[\kappa^2 + \alpha\left(1 - \beta\rho_u\right)\right]^{-1}$. Since α is generally increasing in λ, in an economy with limited asset market participation optimal policy results in greater inflation volatility and lower output gap volatility than in a full participation economy ($\lambda > 0$). Optimal policy in this case requires more output stabilization at the cost of accommodating inflationary pressures. It is insightful to derive the implicit instrument rule that implements (5.2), or more precisely one of the possible instrument rules (see below); for consistency with the foregoing determinacy discussion, we choose the expectations-based instrument rule that has the same functional form as (4.1)[31]:

$$(5.4) \qquad\qquad
\begin{aligned}
r_t^d &= r_t^* + \phi_\pi^o E_t\pi_{t+1}, \\
\phi_\pi^o &= \left[1 + \frac{\delta\kappa}{\alpha}\frac{1 - \rho_u}{\rho_u}\right].
\end{aligned}$$

Some of the results obtained in a full-participation economy carry over: from the existence of a trade-off between inflation and output stabilization, to convergence of inflation to its target under the optimal policy (e.g. CGG (1999)). Also, real disturbances affect nominal rates only insofar as they affect the Wicksellian interest rate, as discussed for example by Woodford (2003 p.250). There is one important exception however, emphasized in the following Proposition.

PROPOSITION 4. *In a non-Keynesian economy ($\delta < 0$) the implied instrument rule for optimal policy is passive $\phi_\pi^o < 1$. The optimal response to inflation is decreasing in the share of non-asset holders $\partial\phi_\pi^o/\partial\lambda < 0$ and changes from passive to active as δ changes sign.*

This Proposition shows the precise way in which the central bank has to change its instrument in order to meet the targeting rule (5.2): contract demand when

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[30]This is the way proposed by Woodford to circumvent the time inconsistency of optimal policy in a forward-looking model, inconsistency which arises because optimal policy today is not a continuation of optimal policy yesterday.

[31]Equation (5.4) is obtained by solving for the interest rate from the IS curve, and using (5.2) to express expected output gap in terms of expected inflation and (5.3) to express the current output gap in terms of expected inflation.

inflation increases, but move nominal rates such that the real rate *decreases* when δ is negative. This happens because, as explained previously, real interest rate cuts are associated with a fall in current aggregate demand when the slope of the IS curve is positive ($\delta < 0$). However, as it is well understood in the benchmark model (see e.g. Woodford (2003) or Jensen, (2002)) the rule (5.4) is only one out of an infinity of possible ways to implement the targeting rule. Among these representations, many would lead to indeterminacy regardless of the values of deep parameters. Moreover, in a discretionary policy regime, the necessary commitment to a rule that would ensure equilibrium determinacy (be it the targeting rule or the instrument rule (5.4)) is not granted. Therefore, we next study policymaking under commitment.

In our model, as in the standard model (see Proposition 7.15 in Woodford (2003)) *optimal policy from a timeless perspective* is implemented if the central bank commits to follow the targeting rule:

$$(5.5) \qquad\qquad \pi_t = -\frac{\alpha}{\kappa}\left(x_t - x_{t-1}\right)$$

for any date starting from (and including) the date at which policy is chosen. Under commitment policy that is *not* timeless-optimal, the same targeting rule would at all times except for the initial date, at which (5.2) would instead hold - this distinction is immaterial for determinacy issues. The determinacy properties of this targeting rule are worth emphasizing in the following Proposition.

PROPOSITION 5. *Commitment to a policy that ensures that the targeting rule (5.5) is fulfilled at every date is 'robustly optimal', in the sense of delivering equilibrium determinacy regardless of the value of λ.*

The proof of this proposition is simple and mirrors the standard case. We combine (5.5) with the Phillips curve (3.6) to obtain the second-order difference equation:

$$E_t x_{t+1} = \left[1 + \beta^{-1}\left(1 + \frac{\kappa^2}{\alpha}\right)\right] x_t - \beta^{-1} x_{t-1} + \frac{\beta^{-1}\kappa}{\alpha} u_t,$$

whose roots are on opposite sides of the unit circle as required by determinacy, regardless of the value of λ. We view this as complementing existing support for such rules provided e.g. by Giannoni and Woodford (2004). In a sense, our results imply that targeting rules are *robustly optimal*, where the qualifier 'robust' pertains in our case to a specific form of model uncertainty, namely uncertainty related to the degree of asset markets participation (whereas in Giannoni and Woodford robustness pertains to the stochastic properties of the shocks). Therefore, among all the policy strategies studied within the 'optimal policy' class, we conclude that commitment to a targeting rule that implements timeless-optimal policy (5.5) is preferable insofar as it delivers equilibrium determinacy regardless of the degree of asset markets participation[32].

To complete the discussion of optimal policy we finally mention that when cost-push shocks are absent (and there is no inflation-output stabilization trade-off),

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[32]As in the case of discretionary policymaking, many possible implicit instrument rules implementing the targeting rule (5.5) would result in an indeterminate equilibrium. One exception is provided by the 'expectations-based reaction function' studied by Evans and Honkapohja (2003), whereby the central bank conditions its policy on observed expectations $E_t x_{t+1}$ and $E_t \pi_{t+1}$, much as in the case of the corresponding rule derived in the discretionary case, (5.4).

the flexible-price allocation can be achieved by having the nominal rate equal the Wicksellian rate at all times $r_t^o = r_t^*$, as in the standard model (e.g. Woodford, Ch. 4). However, there is an important difference with respect to the baseline model: when $\delta < 0$ this policy can also be *consistent with a unique rational expectations equilibrium*[33]. To see this, we observe that such this policy is equivalent from an equilibrium-determinacy standpoint to an interest rate peg, i.e. an interest rate rule with $\phi_\pi = 0$. From Proposition 1, Case II, one can easily see that $\phi_\pi = 0$ leads to equilibrium determinacy if and only if $1 + \delta \frac{2(1+\beta)}{\kappa} \leq 0$.However, the ability of the central bank to achieve full price stability as the *unique* equilibrium applies to the simple model assumed here and relies upon the ability/willingness of the bank to monitor the natural rate of interest and match its movement one-to-one by movements in the nominal rate. Moreover, the natural interest rate can sometimes be negative. All these caveats suggest that this result is unlikely to have much practical relevance.

6. Some positive implications

While we have drawn on empirical evidence to justify our assumption of limited asset markets participation, we now wish to argue that some of the implications of our theoretical model are in line with existing empirical results[34]. For the sake of brevity, we have chosen one such result and the corresponding experiment, namely the effects of cost-push shocks - and the potential of our model to explain the Great Inflation and the change in macroeconomic performance thereafter. Namely, we show that such a shock (even of the same magnitude) generates relatively much higher and more volatile inflation in the pre-Volcker scenario as compared to the Volcker-Greenspan one, consistently with stylized facts pertaining to the Great Inflation. We focus on this experiment for its potential added value in explaining these stylized facts, with respect to the benchmark model of CGG (2000) and Lubik and Schorfheide (2004). In those models, under a passive policy rule, fundamental shocks cannot even be studied, since in an indeterminate equilibrium their effects are indexed by a parameter (matrix) that can take any value. Relying on sunspot shocks instead has also major shortcomings: the size, location and stochastic properties of the sunspot are arbitrary, and the effect of a sunspot is to induce positive co-movement between inflation and output; this is in contrast with the stylized facts pertaining to the 1970s, where inflation coexisted with recessions. In our model, instead, the pre-Volcker equilibrium is determinate and fundamental shocks can be studied despite a passive policy rule, if participation in asset markets was limited enough to make the economy 'non-Keynesian'. In Bilbiie and Straub (2006), we pursue this exercise in great detail estimating a richer version of this model using Bayesian methods, and present evidence that this was indeed the case in the 1970s[35].

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[33]In the baseline model instead, the bank needs to commit to respond to inflation by fulfilling the Taylor principle $r_t^o = r_t^* + \phi_\pi \pi_t, \phi_\pi > 1$ in order to pin down a unique equilibrium.

[34]The earlier working paper version contains an analytic calculation of the responses to all shocks (including sunspots), under either determinacy or inteterminacy.

[35]We find that asset market participation changed generating a switch in the sign of the slope of the IS curve, and equilibrium was determinate in the pre-Volcker sample characterized by a passive policy rule. We show that fundamental shocks can explain stylized facts pertaining to the Great Inflation, but the change in economy's structure is paramount in accounting for the change in mamcroeconomic outcomes.

We study the response of the economy to a unit cost-push shock[36] under two scenarios labelled pre-Volcker and Volcker-Greenspan (notably, we keep the stochastic structure of the shock process unchanged across the two periods).We consider the case whereby there are two parameters changing across the two periods: the response of interest rates to inflation, and the degree of asset markets participation. It is fairly well established (see e.g. CGG (2000), Taylor (1999) or Lubik and Schorfheide (2004)) that the response of monetary policy in the pre-Volcker period implied a (long-run) response to inflation of less than one. We follow those studies and parameterize the policy rule using $\phi_\pi = 0.8$ for the pre-Volcker sample and 2 for the Volcker-Greenspan sample. The benchmark share of agents with no assets in the pre-Volcker period is taken to be the lower bound of the estimates of Campbell and Mankiw (1989), i.e. $\lambda = 0.4$, while for the Volcker-Greenspan period we consider a low value chosen arbitrarily, $\lambda = 0.05$ (the main point is not affected qualitatively by considering different value for this parameter, as long as we are on different sides of the threshold implied by the other parameter values, namely $\lambda^* = 0.285$). This parameterization, together with the assumption that the inverse labor elasticity φ is unchanged across samples at a value of 3, is also consistent with the estimation results of the richer version of this model in Bilbiie and Straub (2006).

The impulse responses of output (gap), inflation and nominal and interest rates to a unit cost shock under the two scenarios are plotted in Figure 5 (circles for 'pre-Volcker' and triangles otherwise). Indeed, the responses conform to both conventional wisdom and what we view as a good test for a theory purported to explain dynamics in that period: higher inflation, low real rates, and negative co-movement of inflation and the output gap. Moreover, responses of output and inflation have the same sign under both scenarios, as shown analytically in Bilbiie (2005). But the response of inflation is much larger and more persistent in the pre-Volcker scenario. The response of output is not much different, and the real rate is negative as expected, since the policy rule is passive. The responses to the same shock under the assumption of discretionary optimization (not pictured) look very similar (in fact, they have been calculated analytically in (5.3) and we already noted that in the LAMP economy there would be too little inflation stabilization)[37].

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[36]Arthur Burns emphasized the cost-push nature of inflation in the 1970s time and again in various speeches and statements as documented e.g. in Mayer (1999). Variance decompositions from estimated 'new synthesis' models seem to support cost shocks' being the main cause of fluctuations in the pre-Volcker era (see e.g. Ireland (2004) and Lubik and Schorfheide (2004)).

[37]The responses of y and π to the same shock under timeless-optimal commitment (not pictured, avilable upon request) are virtually indistinguishable across the two scenarios, despite the markedly different value of λ. While this feature further illustrates the desirability of commiting to timeless-optimal policy, it also casts doubt on the accuracy of this scenario as a good description of policymaking in the pre-Volcker era, at least under the assumptions that cost-push shocks were the main source of fluctuations and drawn from the same distribution across the two periods. While the former assumption finds empirical support as detailed above, the latter does not - indeed, there is overwhelming evidence that shocks have been drawn from different distributions. We refer the interested reader to the companion paper Bilbiie and Straub (2006) and references therein for a study of the relative merits of the changes in structure and shocks in explaining the changes in macroeconomic performance.
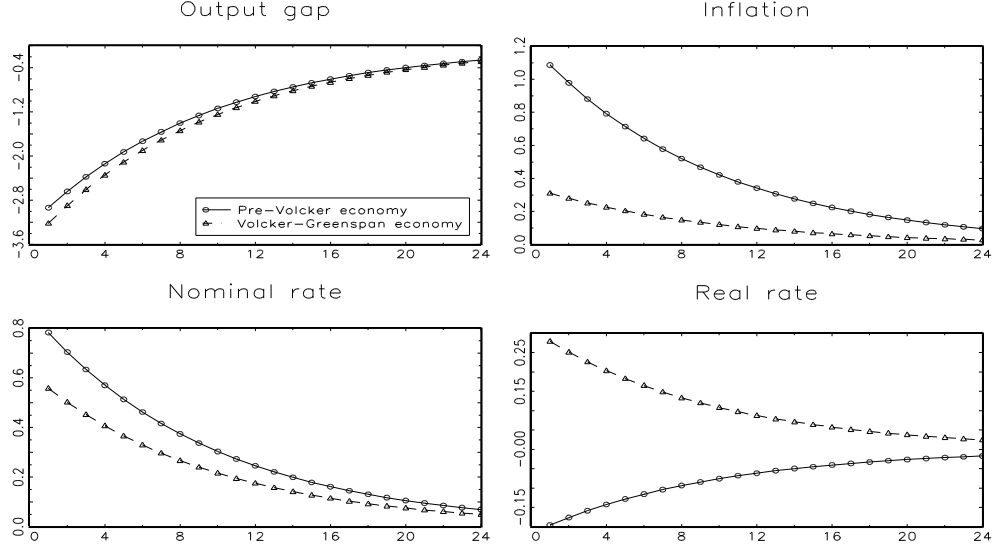
Fig. 5: Impulse responses to unit cost push-shock. Line with circles has $\lambda = 0.4$ and $\phi_\pi = 0.8$; line with triangles has $\lambda = 0.05$ and $\phi_\pi = 2$.

7. Conclusions

The above analysis has shown how limited asset markets participation (LAMP), by changing aggregate demand's sensitivity to interest rates nonlinearly, changes monetary policy prescriptions likewise. Despite their insensitivity to real interest rates, non-asset holders affect the sensitivity of aggregate demand to interest rates since these agents are oversensitive to real wage variations. Real wages are related to interest rates through the labor supply decision of asset holders and the way this interacts with their asset holdings (through income effects and intertemporal substitution). Their asset income, in turn, consists of dividend income and is also related to real wages which are equal to marginal costs. Therefore, non-asset holders and asset holders interact through the interdependent functioning of labor and asset markets. These interactions can either strengthen (if participation is not 'too' limited) or *overturn* the systematic link between interest rates on aggregate demand. The latter case occurs if the share of non-asset holders is high enough and/or and the elasticity of labor supply is low enough. This is the main mechanism identified by this paper to make monetary policy analysis dramatically different when compared to a standard full-participation case whereby aggregate demand is completely driven by asset holders. This paper develops an analytical framework incorporating the foregoing insight and uses it to study in detail the dynamics of a simple general equilibrium model, the determinacy properties of interest rate rules and optimal, welfare-based, monetary policy. Its aim is to make a contribution to the literature emphasizing the role of LAMP in shaping macroeconomic policy and helping towards a better understanding of the economy. In that respect, we just seek to add to a new developing literature analyzing the role of non-asset holders in

macroeconomic dynamic general equilibrium models (see Mankiw (2000), Alvarez, Lucas and Weber (2001), or Galí, Lopez-Salido and Valles (2003)).

Our results have clear normative implications. In a nutshell, central bank policy should be pursued with an eye to the aggregate demand side of the economy. Empirical results on consumption behavior and asset market participation, on the one hand, and labor supply elasticity and the degree of monopoly power in goods markets, on the other, would become an important part of the policy input. While the degree of development of financial markets may well make this not a concern in present times in the developed economies, central banks in developing countries with low participation in financial markets might find this of practical interest. The theoretical results hinting to such policy prescriptions are related to limited participation beyond a certain threshold making the economy behave in a 'non-Keynesian' way. Namely, the IS curve changes sign, and an *Inverted Taylor Principle* applies generally: the central bank needs to adopt a passive policy rule to ensure equilibrium uniqueness and rule out self-fulfilling, sunspot-driven fluctuations. Moreover, optimal time-consistent monetary policy also requires that the central bank move nominal rates such that real rates decline (thereby containing aggregate demand). The effects and transmission of shocks are also radically modified. Importantly, however, we have shown that commitment to a targeting rule that implements timeless-optimal policy delivers equilibrium determinacy regardless of the degree of asset markets participation. We view this as an additional argument in favor of adopting targeting rules purported to implement commitment policy.

In this paper we modeled limited asset market participation in a very simple way, and were as a result able to isolate and study its implications for monetary policy analytically in the same type of framework used in standard, full-participation analyses. This simplicity (shared with the rest of the literature), while justified on tractability grounds, also implies that many realistic features have been left out. For instance, one could try to break the link between asset markets participation and consumption smoothing behavior, which this paper assumed. Another important extension would endogenize the decision to participate in asset markets. Lastly, an empirical assessment of limited participation, its dynamics and implications at the aggregate level, is in our view a necessary step for understanding business cycles. We pursue such extensions in current work.

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Table 1. Model Summary

Euler equation, S	$E_t c_{S,t+1} - c_{S,t} = r_t - E_t \pi_{t+1}$
Labor supply, S	$\varphi n_{S,t} = w_t - c_{S,t}$
Budget constraint S	$c_{S,t} = (w_t + n_{S,t}) + \frac{1}{1-\lambda} d_t.$
Labor supply, H	$\varphi n_{H,t} = w_t - c_{H,t}$
Budget constraint, H	$c_{H,t} = w_t + n_{H,t}$
Production function	$y_t = (1 + \mu) n_t + (1 + \mu) a_t$
Real marginal cost	$mc_t = w_t - a_t,$
Real profits	$d_t = -mc_t + \frac{\mu}{1+\mu} y_t,$
Phillips curve	$\pi_t = \beta E_t \pi_{t+1} + \psi mc_t, \ \psi \equiv (1-\theta)(1-\theta\beta)/\theta$
Labor mkt. clearing	$n_t = \lambda n_{H,t} + (1-\lambda) n_{S,t}$
Aggregate cons.	$c_t = \lambda c_{H,t} + (1-\lambda) c_{S,t}.$
Monetary policy	$r_t = \phi_\pi E_t \pi_{t+1} + \phi_x x_t + \varepsilon_t.$
Note: By Walras' law the resource constraint $y_t = c_t$ also holds	

Appendix A. Robustness

One might rightly wonder whether the mere theoretical possibility of a change in the sign of δ is entirely dependent upon the specification of preferences. It turns out this possibility is robust to two obvious candidates: an elastic labor supply of non-asset holders, and a non-unitary elasticity of intertemporal substitution in consumption; we briefly study these extensions jointly. Consider preferences given by a general CRRA utility function for both agents j (γ is relative risk aversion for both agents and also inverse of intertemporal elasticity of substitution in consumption

for asset holders):

$$(A.1) \qquad\qquad U_j\left(.,.\right) = \frac{C_{j,t}^{1-\gamma}}{1-\gamma} - \omega\frac{N_{j,t}^{1+\varphi}}{1+\varphi}$$

Following the same method as before one can show that the solution to non-asset holders' problem will be (in log-linearized terms, where elasticity of hours to wage $\eta \equiv (1-\gamma)/(\gamma + \varphi)$ is positive iff $\gamma < 1$): $n_{H,t} = \eta w_t$ and $c_{H,t} = (1+\eta)\,w_t$. For asset holders, the new Euler equation and intratemporal optimality in log-linearized form are, respectively: $E_t c_{S,t+1} - c_{S,t} = \gamma^{-1}\left(r_t - E_t\pi_{t+1}\right)$ and $\varphi n_{S,t} = w_t - \gamma c_{S,t}$. Using the same method as previously, one finds the new condition to be fulfilled in order for δ to become negative and hence end up in a 'non-Keynesian economy':

$$(A.2) \qquad\qquad \lambda > \frac{1}{1 + \varphi\left(1 - \eta\mu\right)/\left(1+\mu\right)}$$

By comparing (A.2) with (3.2) one immediately notices that under the more general preferences (A.1) the threshold value of λ is lower (higher) than under log utility if $\eta > 0 \; (< 0)$. The intuition is that while making aggregate labor supply more elastic, a positive η also makes *equilibrium* hours more elastic to wage changes since it makes consumption of non-asset holders more responsive to the wage. In general however, the difference induced by having $\gamma \neq 1$ on the threshold value of λ is quantitatively negligible[38]. In view of the relative innocuousness of these assumptions, we shall continue using the log-CRRA utility function in the remainder, since it preserves constant steady-state hours and hence allows analyzing permanent technology shocks.

Appendix B. Proof of Proposition 1

Necessary and sufficient conditions for determinacy are as follows (given in Woodford (2003), Appendix to Chapter 4). Either **Case A**: $(Aa) \det \mathbf{\Gamma} > 1$; $(Ab) \det \mathbf{\Gamma} - tr\mathbf{\Gamma} > -1$ and $(Ac) \det \mathbf{\Gamma} + tr\mathbf{\Gamma} > -1$ or **Case B**: $(Ba) \det \mathbf{\Gamma} - tr\mathbf{\Gamma} < -1$ and $(Bb) \det \mathbf{\Gamma} + tr\mathbf{\Gamma} < -1$. For our forward -looking rule case, the determinant and trace are:

$$(B.1) \qquad\qquad \begin{aligned} \det \mathbf{\Gamma} &= \beta^{-1} > 1 \\ tr\mathbf{\Gamma} &= 1 + \beta^{-1} - \beta^{-1}\delta^{-1}\kappa\left(\phi_\pi - 1\right) \end{aligned}$$

Imposing the determinacy conditions in Case A above (where Case B can be ruled out due to sign restrictions), we obtain the requirement for equilibrium uniqueness:

$$\delta^{-1}\left(\phi_\pi - 1\right) \in \left(0, \frac{2\left(1+\beta\right)}{\kappa}\right)$$

This implies the two cases in Proposition 1: Case I: $\delta > 0, \phi_\pi \in \left(1, 1 + \delta\frac{2(1+\beta)}{\kappa}\right)$, which is a non-empty interval; Case II: $\delta < 0, \phi_\pi \in \left(1 + \delta\frac{2(1+\beta)}{\kappa}, 1\right)$. Notice that (i) $1 + 2\delta\left(1+\beta\right)/\kappa < 1$ so the interval is non-empty; (ii) $1 + 2\delta\left(1+\beta\right)/\kappa > 0$ implies instead that we can rule out an interest rate peg, whereas a peg is consistent with

---

[38]Even evaluating the difference between the threshold values corresponding to $\gamma = 0$ and 100 respectively, one obtains under the baseline parameterization (for values of $\varphi = 0.5; 1; 5; 10$ respectively): $0.13; 0.09; 0.03; 0.01$.

a unique REE for $1 + \delta\frac{2(1+\beta)}{\kappa} < 0$. The last condition instead holds if and only if

$\lambda \geq \left(1 + \frac{1}{1+\mu}\varphi\frac{(1-\theta)(1-\beta\theta)}{(1+\theta)(1+\beta\theta)}\right) / \left(1 + \frac{1}{1+\mu}\varphi\right) \geq \left(1 + \frac{1}{1+\mu}\varphi\right)^{-1}$.

When this condition is not fulfilled, we have $0 < 1 + \delta\frac{2(1+\beta)}{k} < 1$, so there still exist policy rules $\phi_\pi \in \left(1 + \delta\frac{2(1+\beta)}{k}, 1\right)$ bringing about a unique rational expectations equilibrium. But in this case an interest rate peg, and any policy rule with too weak a response $\phi_\pi \in \left[0, 1 + \delta\frac{2(1+\beta)}{k}\right)$ is not compatible with a unique equilibrium.

B.1. Determinacy properties of a simple Taylor rule. We consider rules of the form:

(B.2) $$r_t = \phi_\pi \pi_t + \varepsilon_t$$

Replacing this in the IS equation and using the same method as previously we obtain the following Proposition.

PROPOSITION 6. *An interest rate rule such as* (B.2) *delivers a unique rational expectations equilibrium if and only if:*

Case I: If $\delta > 0, \phi_\pi > 1$ *(the 'Taylor Principle')*
Case II: If $\delta < 0$,

$$\phi_\pi \in \left[0, \min\left\{1, \delta\frac{\beta-1}{\kappa}, \delta\frac{-2(1+\beta)}{\kappa} - 1\right\}\right) \cup \left(\max\left\{1, \delta\frac{-2(1+\beta)}{\kappa} - 1\right\}, \infty\right)$$

It turns out that the 'inverted Taylor principle' holds in Case II for a somewhat larger share of non-asset holders than was the case under a forward-looking rule.

PROOF. Substituting the Taylor rule in the IS equation and writing the dynamic system in the usual way for the $z_t \equiv (y_t, \pi_t)'$ vector of endogenous variables and the $\nu_t \equiv (\varepsilon_t - r_t^*, u_t)'$ vector of disturbances :

$$E_t z_{t+1} = \mathbf{\Gamma} z_t + \Psi\nu_t$$

The coefficient matrices are given by:

$$\mathbf{\Gamma} = \begin{bmatrix} 1 + \beta^{-1}\delta^{-1}\kappa & \delta^{-1}\left(\phi_\pi - \beta^{-1}\right) \\ -\beta^{-1}\kappa & \beta^{-1} \end{bmatrix} \text{ and } \Psi = \begin{bmatrix} \delta^{-1} & 0 \\ 0 & -\beta^{-1} \end{bmatrix}$$

Determinacy requires that both eigenvalues of $\mathbf{\Gamma}$ be outside the unit circle. Note that:

$$\det\mathbf{\Gamma} = \beta^{-1}\left(1 + \delta^{-1}\kappa\phi_\pi\right) \text{ and } tr\mathbf{\Gamma} = 1 + \beta^{-1}\left(1 + \delta^{-1}\kappa\right)$$

For Case A we have: (Aa) implies:

$$\delta^{-1}\phi_\pi > \frac{\beta-1}{\kappa}$$

(Ab) implies

$$\delta^{-1}\left(\phi_\pi - 1\right) > 0$$

(Ac) implies

$$\delta^{-1}\left(1 + \phi_\pi\right) > \frac{-2(1+\beta)}{\kappa}$$

The determinacy requirements are as follows. First, note that (Ab) merely requires that δ^{-1} and $(\phi_\pi - 1)$ have the same sign. Hence, we can distinguish two cases:

Case I: $\delta^{-1} > 0, \phi_\pi > 1$. The standard case is encompassed here and the Taylor principle is at work as one would expect. The other conditions are automatically satisfied, since both $\delta^{-1}\phi_\pi$ and $\delta^{-1}(1+\phi_\pi)$ are positive, and $\frac{\beta-1}{\kappa}, \frac{-2(1+\beta)}{\kappa} < 0$.

Case II: $\delta^{-1} < 0, \phi_\pi < 1$. Condition (Aa) implies (note that since $\delta < 0$ the right-hand quantity will be positive): $\phi_\pi < \delta\frac{\beta-1}{\kappa}$. The third requirement for uniqueness (Ac) implies: $\phi_\pi < \delta\frac{-2(1+\beta)}{\kappa} - 1$. Since $\phi_\pi \geq 0$, this last requirement implies a further condition on the parameter space, namely $\delta\frac{-2(1+\beta)}{\kappa} - 1 \geq 0$. Overall, the requirement for determinacy when $\delta^{-1} < 0$ is hence:

(B.3)
$$0 \leq \phi_\pi < \min\left\{1, \delta\frac{\beta-1}{\kappa}, \delta\frac{-2(1+\beta)}{\kappa} - 1\right\}$$

Case B, instead, involves fulfilment of the following conditions: (Ba) implies $\delta^{-1}(\phi_\pi - 1) < 0$ and (Bb) implies $\delta^{-1}(1+\phi_\pi) < \frac{-2(1+\beta)}{\kappa}$. In Case I, whereby $\delta^{-1} > 0$, these conditions cannot be fulfilled due to sign restrictions (this is the case in a standard economy as in Woodford (2003), e.g.). In Case II however, the two conditions imply:

(B.4)
$$\phi_\pi > \max\left\{1, \delta\frac{-2(1+\beta)}{\kappa} - 1\right\}$$

$(B.3)$ and $(B.4)$ together imply the following overall determinacy condition for the policy parameter:

$$\phi_\pi \in \left[0, \min\left\{1, \delta\frac{\beta-1}{\kappa}, \delta\frac{-2(1+\beta)}{\kappa} - 1\right\}\right) \cup \left(\max\left\{1, \delta\frac{-2(1+\beta)}{\kappa} - 1\right\}, \infty\right)$$

$\square$

Note that the determinacy conditions are considerably more complicated than for a forward-looking rule, since the coefficients in matrix $\mathbf{\Gamma}$ (and hence its eigenvalues) depend on δ^{-1} and ϕ_π separately, and not only on the ratio $\delta^{-1}(\phi_\pi - 1)$ as in Proposition 1. To assess the magnitude of policy responses needed for determinacy as a function of deep parameters, we can distinguish a few cases for different parameter regions (note that we are always looking at the subspace whereby $\delta^{-1} < 0$):

Share of non-asset holders	Determinacy condition
$\lambda < \bar{\lambda}_1$	$\phi_\pi > 1$
$\lambda \in [\bar{\lambda}_1, \bar{\lambda}_2)$	$\phi_\pi \in \left[0, \delta\frac{-2(1+\beta)}{\kappa} - 1\right) \cup (1, \infty)$
$\lambda \in [\bar{\lambda}_2, \bar{\lambda}_3)$	$\phi_\pi \in \left[0, \delta\frac{\beta-1}{\kappa}\right) \cup (1, \infty)$
$\lambda \in [\bar{\lambda}_3, \bar{\lambda}_4)$	$\phi_\pi \in \left[0, \delta\frac{\beta-1}{\kappa}\right) \cup \left(\delta\frac{-2(1+\beta)}{\kappa} - 1, \infty\right)$
$\lambda \in [\bar{\lambda}_4, 1)$	$\phi_\pi \in [0, 1) \cup \left(\delta\frac{-2(1+\beta)}{\kappa} - 1, \infty\right)$

where

$$\bar{\lambda}_i = \left(1 + \frac{1}{1+\mu}\varphi\frac{(1-\theta)(1-\beta\theta)}{h_i(\theta)}\right) \Big/ \left(1 + \frac{1}{1+\mu}\varphi\right)$$

$h_1(\theta) = (1+\theta)(1+\beta\theta); h_2(\theta) = 1 + \beta\theta^2 + 2\beta\theta; h_3(\theta) = 1 + \beta\theta^2; h_4(\theta) = 1 - \beta\theta^2$
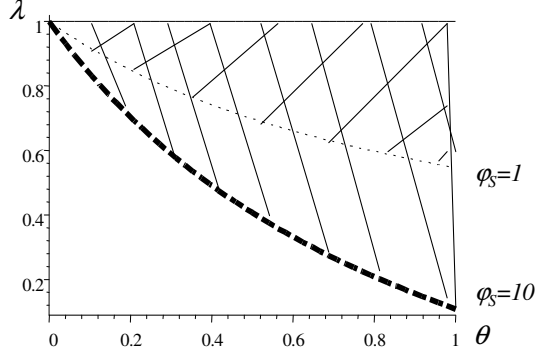
Fig.6: Threshold value for share of non-asset holders making determinacy conditions closest to Inverted Taylor Principle.

We plot the last case $\lambda \in \left[\bar{\lambda}_4, 1 \right); \phi_\pi \in [0,1) \cup \left(\delta \frac{-2(1+\beta)}{\kappa} - 1, \infty \right)$ in Figure 6 above, where the region above the curve and below the horizontal line gives parameter combinations compatible with the above condition. The different curves correspond to different labor supply elasticities ($\varphi = 1$ dotted line and $\varphi = 10$ thick line). In view of usual estimates of λ in the literature (e.g. 0.4-0.5 Campbell and Mankiw (1989)) we shall consider this case as the most plausible. Whenever these parameter restrictions are met, determinacy is insured by either a violation of the Taylor principle, or for a strong response to inflation. However, note that the lower bound on the inflation coefficient then becomes very large (35.433 under the baseline), which is far from any empirical estimates. Indeed, the threshold inflation coefficient is sharply increasing in the share of non-asset holders and inverse elasticity of labor supply, as can be seen by merely differentiating $\delta \frac{-2(1+\beta)}{\kappa} - 1$ with respect to these parameters.

Appendix C. Proof of Proposition 3: derivation of aggregate welfare function

Assume the steady-state is efficient and equitable, in the sense that consumption shares across agents are equalized. This is ensured by having a fiscal authority tax/subsidize sales at the constant rate τ and redistribute the proceedings in a lump-sum fashion T such that in steady-state there is marginal cost pricing, and profits are zero. The profit function becomes $D_t(i) = (1 - \tau) \left[P_t(i)/P_t \right] Y_t(i) - (MC_t/P_t) N_t(i) + T_t$, where by balanced budget $T_t = \tau P_t(i) Y_t(i)$. Efficiency requires $\tau = -\mu$, such that under flexible prices $P_t^*(i) = MC_t^*$ and hence profits are $D_t^*(i) = 0$. Note that under sticky prices profits will not be zero since the mark-up is not constant. Under this assumption we have that in steady-state:

$$\frac{V'(N_H)}{U'(C_H)} = \frac{V'(N_S)}{U'(C_S)} = \frac{W}{P} = 1 = \frac{Y}{N},$$

where $N_H = N_S = N = Y$ and $C_H = C_S = C = Y$.

Suppose further that the social planner maximizes a convex combination of the utilities of the two types, weighted by the mass of agents of each type: $U_t(.) \equiv \lambda U_H(C_{H,t}, N_{H,t}) + [1 - \lambda] U_S(C_{S,t}, N_{S,t})$. A second-order approximation to type

$j's$ utility around the efficient flex-price equilibrium delivers:

$$
\begin{aligned}
\hat{U}_{j,t} &\equiv U_j\left(C_{j,t}, N_{j,t}\right) - U_j\left(C_{j,t}^*, N_{j,t}^*\right) = \\
&= U_C C_j \left[\hat{C}_{j,t} + \frac{1-\gamma}{2}\hat{C}_{j,t}^2 + (1-\gamma)\, c_{j,t}^* \hat{C}_{j,t}\right] - \\
&\quad - V_N N_j \left[\hat{N}_{j,t} + \frac{1+\varphi}{2}\hat{N}_{j,t}^2 + (1+\varphi)\, n_{j,t}^* \hat{N}_{j,t}\right] + t.i.p + O\left(\|\zeta\|^3\right),
\end{aligned}
$$

where variables with a hat denote log-deviations from the flex-price level (or 'gaps') $\hat{C}_t \equiv \log\frac{C_t}{C_t^*} = c_t - c_t^*$, and variables with a star flex-price values as above $c_t^* \equiv \log\frac{C_t^*}{C}$. Note that since $U_C C_H = U_C C_S$ and $V_N N_H = V_N N_S$ and using $c_{H,t}^* = c_{S,t}^* = c_t^*$ (which holds since asset income in the flex-price equilibrium is zero, as profits are zero) we can aggregate the above into:

$$
\begin{aligned}
\hat{U}_t &= U_C C\left[\hat{C}_t + (1-\gamma)\, c_t^* \hat{C}_t + \frac{1-\gamma}{2}\left(\lambda\hat{C}_{H,t}^2 + (1-\lambda)\hat{C}_{S,t}^2\right)\right] - \\
&\quad - V_N N\left[\hat{N}_t + (1+\varphi)\, n_t^* \hat{N}_t + \frac{1+\varphi}{2}\left(\lambda\hat{N}_{H,t}^2 + (1-\lambda)\hat{N}_{S,t}^2\right)\right] + t.i.p + O\left(\|\zeta\|^3\right)
\end{aligned}
$$

Note that $\hat{C}_t = \hat{Y}_t$ and $\hat{N}_t = \hat{Y}_t + \Delta_t$, where Δ_t is price dispersion as in Woodford (2003), $\Delta_t = \log\int_0^1\left(P_t\left(i\right)/P_t\right)^{-\varepsilon}di$. Since $U_C C = V_N N$ we can show that the linear term boils down to:

$$
\begin{aligned}
& U_C C\left[\hat{C}_t + (1-\gamma)\, c_t^* \hat{C}_t\right] - V_N N\left[\hat{N}_t + (1+\varphi)\, n_t^* \hat{N}_t\right] \\
&= -U_C C\left[-\hat{Y}_t + (\gamma-1)\, c_t^* \hat{Y}_t + \hat{Y}_t + \Delta_t + (1+\varphi)\, n_t^* \hat{Y}_t\right] + O\left(\|\zeta\|^3\right) \\
&= -U_C C\left[\Delta_t\right] + O\left(\|\zeta\|^3\right)
\end{aligned}
$$

Quadratic terms can be expressed as a function of aggregate output. For that purpose, note that in evaluating quadratic terms we can use first-order approximations of the optimality conditions (higher order terms would imply terms of order higher than 2, irrelevant for a second-order approximation). Up to first order, we have that $\hat{C}_{H,t} = (1+\eta)\hat{W}_t$, $\hat{N}_{H,t} = \eta\hat{W}_t$ and $\hat{W}_t = \varphi\hat{N}_t + (1-\gamma)\hat{C}_t = (\varphi+\gamma)\hat{Y}_t + \varphi\Delta_t$ so:

$$
\begin{aligned}
\hat{C}_{H,t}^2 &= (1+\varphi)^2\hat{Y}_t^2 + O\left(\|\zeta\|^3\right) \\
\hat{N}_{H,t}^2 &= (1-\gamma)^2\hat{Y}_t^2 + O\left(\|\zeta\|^3\right) \\
\hat{C}_{S,t}^2 &= \frac{1}{(1-\lambda)^2}\left[1 - \lambda(1+\varphi)\right]^2\hat{Y}_t^2 + O\left(\|\zeta\|^3\right) \\
\hat{N}_{S,t}^2 &= \frac{1}{(1-\lambda)^2}\left[1 - \lambda(1-\gamma)\right]^2\hat{Y}_t^2 + O\left(\|\zeta\|^3\right)
\end{aligned}
$$

The aggregate *per-period* welfare function is thence, up to second order (ignoring terms independent of policy and of order larger than 2):

$$
\begin{aligned}
\hat{U}_t &= -U_C C\left[\frac{\gamma-1}{2}\left(\lambda\hat{C}_{H,t}^2 + (1-\lambda)\hat{C}_{S,t}^2\right) + \frac{1+\varphi}{2}\left(\lambda\hat{N}_{H,t}^2 + (1-\lambda)\hat{N}_{S,t}^2\right) + \Delta_t\right] \\
&= -U_C C\left\{\frac{1}{2}\frac{\varphi+\gamma}{1-\lambda}\left[1 - \lambda(1-\gamma)(1+\varphi)\right]\hat{Y}_t^2 + \Delta_t\right\}
\end{aligned}
$$

The intertemporal objective function of the planner will hence be $\mathbf{U}_t = \sum_{i=t}^{\infty} U_{t+i}$. This is consistent with our view that limited participation in asset markets comes from constraints and not preferences. In the latter case, maximizing intertemporally the utility of non-asset holders would be hard to justify on welfare grounds. By usual arguments readily available elsewhere (see Woodford (2003), Galí and Monacelli (2004)) we can express price dispersion as a function of the cross-section variance of relative prices: $\Delta_t = (\varepsilon/2) \, Var_i \, (P_t(i)/P_t)$ and the (present discounted value of the) cross-variance of relative prices as a function of the inflation rate $\sum_{t=0}^{\infty} \beta^t Var_i \, (P_t(i)/P_t) = \psi^{-1} \sum_{t=0}^{\infty} \beta^t \pi_t^2$. Hence, the present discounted values of price dispersion and the inflation rate are related by $\sum_{t=0}^{\infty} \beta^t \Delta_t = (\varepsilon/2\psi) \sum_{t=0}^{\infty} \beta^t \pi_t^2$ and the intertemporal objective function becomes (5.1) in Proposition 3 (reintroducing the notation $\hat{Y}_t = x_t$).