Abstract:
Using firm-level data supplemented with wages estimated from household survey data, this study estimates translog cost and production functions to calculate elasticities between five occupations and capital in South Africa. It concludes that, to improve income inequality and increase output, it is essential that training or immigration policy concentrate on skilled technicians or artisans and not semi-skilled workers. Own-price elasticities for each occupation are between –0.56 and –0.8. In arriving at these results, the study extends Uzawa’s (1962) duality result beyond the constant returns to scale case, reveals analytically and empirically how not accounting for firm-size effects on wages can misleadingly reject profit-maximizing and technology restrictions, adjusts for imperfectly elastic product demand, provides innovative inference measures, and demonstrates how essential disaggregated studies are for arriving at the correct policy conclusions.
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Alberto Behar
# Elasticity Concepts

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<th>Name</th>
<th>Effect measured</th>
<th>Symbol(s)</th>
<th>General Expression</th>
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<tr>
<td>Allen Elasticity of Substitution (AES)</td>
<td>% change in factor quantity ratio</td>
<td>$\sigma_{AES}$ or $\sigma_{ij}$</td>
<td>$\frac{CC_{ij}}{C_iC_j}$ where C is the cost function and subscripts are first and second cross partial derivatives</td>
<td>$b_{ij} + 1$; where $b_{ij}$ comes from the cost function regression and $s_i$ is the factor cost share</td>
</tr>
<tr>
<td>Own Allen Elasticity of Substitution</td>
<td>Responsiveness of factor demand elasticity to change in factor price</td>
<td>$\sigma_{AES}$ or $\sigma_{ii}$</td>
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<td>Elasticity of Factor demand</td>
<td>% change in factor i quantity</td>
<td>$\lambda_j$ - constant output</td>
<td>$s_j \sigma_{ij}$</td>
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</tr>
<tr>
<td></td>
<td>% change in factor j price</td>
<td>$\lambda_j$ - variable output</td>
<td>$s_j (\sigma_{ij} + \eta)$ Where $s_j$ is the factor share of costs and $\eta$ is product demand elasticity ($&lt;0$)</td>
<td></td>
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<td>% change in factor i price</td>
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**Wage series (for each occupation):**
- Wage\textsubscript{ind}: classified by industry only
- Wage\textsubscript{all}: classified by industry and location and classified according to whether firm/worker is unionised
- Wage\textsubscript{some}: possible classifications based on those used for wage\textsubscript{all}, except some groups are combined
- Wage\textsubscript{size}: wage\textsubscript{some} adjusted for firm-size effects on wages

**Wage series (for each occupation):**
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- Wage\textsubscript{size}: wage\textsubscript{some} adjusted for firm-size effects on wages

**Occupations:**
- Managerial/Professional$^*$
- Sales/Clerical$^*$
- Skilled/Artisan$^+$
- Semiskilled$^+$
- Unskilled$^+$

$^*$ = non-production; $^+$ = production
<table>
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<tr>
<th>Hicks Elasticity of Complementarity (HEC) $(i \neq j)$</th>
<th>% change in factor i quantity</th>
<th>$R_{ij}$</th>
<th>$qq_{ij}$ where $q$ is the production function and $q_i, q_j$ and subscripts are first and second cross partial derivatives</th>
<th>$\frac{\beta_{ij}}{\zeta_i \zeta_j} + 1$; where $\beta_{ij}$ comes from the production function regression and $\zeta_i$ is the factor’s share of revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Own Hicks Elasticity of Complementarity (HEC)</td>
<td>Responsiveness of factor price elasticity to change in quantity</td>
<td>$R_{iij}$</td>
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<td>Elasticity of Factor price $(i \neq j)$</td>
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<td>$e_{ij}$ - perfectly elastic product demand $E_{ij}$ - imperfectly elastic product demand</td>
<td>$\frac{\zeta_j R_{ij}}{q_i} + \frac{\zeta_j}{\eta}$</td>
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</tr>
<tr>
<td>Elasticity of factor quantity for factor $u$ with rigid wages</td>
<td>% change in factor u quantity</td>
<td>$\Phi_{uij}$</td>
<td>$-\frac{\zeta_j R_{uij}}{\zeta_i}$</td>
<td>n/a</td>
</tr>
<tr>
<td>Elasticity of Factor price $(i \neq j)$ when factor $u$ has rigid wages</td>
<td>% change in factor i price</td>
<td>$\Phi_{ij}$</td>
<td>$\frac{\zeta_j (-R_{uui} R_{uij} + R_{uj})}{R_{uu}}$</td>
<td>n/a</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

Background and motivation

“Government’s ambition to grow SA’s manufacturing base risks being stillborn unless the country addresses a worsening skills crisis.” - Paton (2003:18)

The importance of skills to manufacturing is a topical issue in South Africa, as evidenced by the above quote from a lead article in the Financial Mail. Paton refers to artisans, for example welders and tool-makers. By stating that artisans are “essential to every aspect of manufacturing [...] production” and that their shortage will “severely hinder SA’s ability to deliver on [...] capital investment projects” (pg 18), Paton implies that artisans and other occupations are complements in production; that there are limited opportunities for substitution by other occupations and that the main effect of shortages is to lower output and demand for all factors.

Over one third of the workforce in South Africa is unemployed (Nattrass, 2004). Much of this unemployment appears to be structural in that an oversupply of unskilled labour exists alongside as many as 500 000 vacancies for skilled workers (The Economist, 2004). These unfilled vacancies are evidence of skills-shortages constraining output: filling them would allow production and employment to rise for all occupations.

Both these observed features of the economy assume or imply that skilled and unskilled labour are, loosely, complements and not substitutes in production. If they are complements, a rise in the supply of skilled workers (in the face of excess demand for skilled labour) has benefits for all occupations, including the unskilled. On the other hand, if skilled and unskilled labour are substitutes, then unskilled labour will be worse off if there is a rise in the supply of skilled labour. In particular, if vacancies for skilled workers are being partially filled by less skilled workers, then improved availability of the first-choice factor will result in these suboptimal substitutes losing out.

A major determinant of wage inequality in South Africa is explained by skill endowments. Increasing the skills of the workforce is seen as a key requirement for reducing wage inequality (Bhorat, Leibbrandt, Maziya, van der Berg & Woolard, 2001). People who acquire such skills or training are likely to earn productivity linked wage increases (Fallon & Lucas, 1998), but what will happen to those who remain unskilled? If skilled and unskilled labour are substitutes, training aimed at a small subsection of the labour force may actually worsen wage inequality.
In order to encourage firms to train their workers, the South African Skills Development Act of 1998 introduced a system where firms incur a tax on payroll, which is reduced if they equip workers with skills in co-operation with and recognised by Sector Education Training Authorities (SETAs) (Paton, 2003). 45 898 people were enrolled in such programmes at the start of 2004 and this number is rising rapidly (Mdladlana, 2004). However, too few of these are artisans and there are fears that the artisan shortage has become and will get worse before it gets better (Paton, 2003). It is important to gauge whether the types of skills being produced are those most conducive to growth and most beneficial to the unskilled.

The South African government intends relaxing immigration requirements and removing unnecessary obstacles. It is only the incompetence of the legislators that has turned this intention into what is so far a five-year saga (Ellis, 2004). The question is especially pertinent in this case: will skilled immigration harm or benefit the unskilled?

Elasticity concepts and occupation definitions

Estimates of various elasticity parameters are appropriate for answering these questions. The most common measure of substitution is the Allen (1938) Partial Elasticity of Substitution (AES), which measures the percentage change in the ratio of two factor quantities in response to a percentage change in their relative prices, holding output constant. Closely related are cross-elasticities of factor demand, which measure the percentage change in a factor’s quantity in response to a percentage change in another factor’s price.

The less famous Hicks (1970) elasticity of complementarity (HEC) measures the percentage change in the ratio of endogenous factor prices to an exogenous change in their relative quantities. Similarly, the cross-elasticities of factor price measure the percentage change in a factor price in response to an exogenous change in another factor’s quantity.\(^1\)

Such elasticities are measured between various factor inputs, including labour inputs disaggregated according to skill. While skill can be defined by education level, this study divides the workforce into five occupations. Non-production workers are divided into managers/professionals and sales/clerical workers. Production workers are divided into skilled

---

\(^1\)Except in the two-good case, the HEC is not the inverse of the AES and the elasticity of factor price is not the inverse of the elasticity of factor demand.
workers / artisans (like welders or toolmakers), semi-skilled workers (like machine operators) and unskilled workers. While elasticities between all five occupations and with capital are estimated, the relationships of interest are within the production occupations.

How this thesis adds to existing empirical work

For South Africa, no documented empirical measures exist at this level of disaggregation, using firm-level data, and/or using an appropriate technological representation. There are, however, studies of somewhat disaggregated labour elasticities. Moolman (2003) attempts industry-level demand estimations for skilled and unskilled labour, but the equations are rudimentary and the fact that the wage variables are aggregated across skill/occupation types severely limits the usefulness of her study. Du Toit and Koekemoer (2003) estimate macroeconomic models for skilled and unskilled labour demand and supply based on a Cobb Douglas technology. Edwards (2002, 2003) uses firm-level manufacturing data from South Africa’s Gauteng Province to estimate relative demand functions for two occupations using Constant Elasticity of Substitution (CES) production technologies. Studies using Cobb Douglas technologies are inappropriate because they assume the AES is unity, while CES functions are not easily conducive to multiple factors and impose various technological restrictions on the technology and elasticities (Heathfield & Wibe, 1987). Translog functions, introduced by Christensen, Jorgenson & Lau (1973), overcome these disadvantages.

This thesis therefore estimates a translog production function to derive the Hicks elasticity of complementarity and related factor price elasticities and also estimates a translog cost function to derive the AES and related factor demand elasticities. Besides estimating a superior function, this study employs detailed national firm-level data, capturing in particular the interactions between occupations at the low end of the skill spectrum.

However, the dataset does not contain any wage data, which are essential for cost function estimates. Therefore, household data are used to predict wages for each firm according to characteristics that are common to both the firm and household surveys, after which the wages are adjusted for firm-size effects.
**Summary of results**

The HEC results reveal that at least 64% of firms employ skilled/ artisans and unskilled workers as complements, while at least 96% of firms have semi-skilled and unskilled labour as substitutes. The elasticity of factor price results suggest a 10% rise in the supply of skilled/artisans will increase unskilled wages by 4.2%. Accounting for rigid unskilled wages preserves the relationship and predicts a rise in unskilled employment, so the key consequence is a rightward shift in the demand curve for unskilled workers. In contrast, a 10% rise in the supply of semi-skilled workers will lead to a 4.2% fall in unskilled wages, or at least a leftward shift in the demand curve for unskilled workers. These conclusions apply to at least 75% of firms in the sample and are robust to inference procedures. The findings are consistent with the view that a shortage of artisans is holding back production and that relieving the shortage will raise wages/employment for unskilled labour. The policy recommendation is that, to maximise output growth and improve wage inequality, training should focus on skilled workers / artisans and not semi-skilled workers.

The elasticity of factor demand suggests that a 10% rise in skilled/artisan wages will lead to a 1.2% rise in demand for unskilled labour, holding output constant. Should semi-skilled workers get a 10% wage rise, unskilled worker demand will fall by 3.4%. The AES and HEC concepts are applicable to different questions and hence based on different assumptions, which limits comparability. It is possible for the HEC to signify two factors are complements and for the AES to signify they are substitutes (and *vice versa*) (Hamermesh, 1993), which demonstrates how important it is to use the appropriate elasticity measure. Nonetheless, both estimation procedures conclude capital and the occupations are generally substitutes while most occupation groups are complements. Own-price elasticities of factor demand are between 0 and –1, with the elasticity for unskilled labour being –0.65.

**Outline and innovations**

In arriving at these results, this thesis innovates by:

- Extending the Uzawa (1962) result that Allen Elasticities of Substitution can be derived using a cost function without recourse to linear homogeneity assumptions. Chapter 2 also explains the historical evolution of the various elasticity concepts, in particular clarifying confusion over when they are equivalent and when they are not, and showing that the
lesser-known HEC and elasticity of factor price concepts are the appropriate measures for analysing the effects of an exogenous rise in skill supply.

- Deriving the translog expressions for elasticity calculations in chapter 3, as derivations in the production setting are not readily available in the literature. The chapter motivates why translog functions are the most appropriate for estimating substitution elasticities, and discusses when estimating cost functions is more appropriate than estimating production functions and *vice versa*.

- Using detailed disaggregated firm-level data, which is essential for uncovering interactions between labour types at the low end of the skill spectrum. This requires substantial time using household data to estimate firm-level wages. Details of the firm level data and wage-collection procedure are discussed at length in chapter 4.

- Showing analytically, also in chapter 4, how not accounting for firm-size effects on wages can overstate returns to scale as well as falsely reject price homogeneity and homotheticity assumptions, before proposing a way to adjust wages for firm-size. Chapter 5 covers econometric issues, tests for separability, and discusses analytically and empirically why two-stage least squares estimates are not appropriate to this study.

- Constructing valid confidence intervals for translog elasticity parameters, exploiting the advantages of translog functions by showing how elasticities vary across firms in the sample, and allowing for the effects of imperfect product demand elasticity on the elasticity of factor price. This is done in chapter 6, where the key results are presented.

- Using the results from chapters 3 and 4 to verify the effects of omitting firm-size effects on wages and derive highly disaggregated AES and factor quantity elasticities in chapter 7.

Most importantly, in the concluding chapter, this thesis provides clear policy recommendations regarding skills-formation and immigration. To maximise output growth and improve wage inequality, training should focus on the skilled/artisan occupations and not semi-skilled workers. The study also supplies the necessary empirical parameters for future analysis, being the first to provide disaggregated values on a national level for South Africa.
2. ELASTICITIES OF SUBSTITUTION, COMPLEMENTARITY, FACTOR DEMAND AND FACTOR PRICE

“What now emerges is that [Joan Robinson] ought to have the sole right to the Elasticity of Substitution. Mine should have been defined by its reciprocal, which should have been given another name – Elasticity of Complementarity? It should then have been proved that in the two-factor case (alone) one was the reciprocal of the other. There would have been perfect duality between the two concepts. But it is much too late for that.” – John Hicks (1970:296)

2.1 INTRODUCTION

This chapter explains the evolution of four elasticity terms, showing how concepts thought equivalent at various stages of the 20th century are actually quite different. It clarifies the conditions under which the measures are appropriate and confirms that certain methods can be used to calculate some of them under more general technological settings than constant returns to scale. Although all four measures are estimated, the chapter concludes by explaining why the Hicks Elasticity of Complementarity and the elasticity of factor price are appropriate for analysing the effects of an exogenous change in skill supply.

Although this chapter will introduce a variety of elasticity concepts, four are estimated:

1a) The Allen Elasticity of Substitution (AES) measures the percentage change in the ratio of factor quantities after a percentage change in the ratio factor prices, holding other factor prices constant (\( \sigma_{AES} \)).

1b) The elasticity of factor demand measures the percentage change in factor quantity for a given percentage change in that or another factor’s price, holding other factor prices constant (\( \lambda \)).

2a) The Hicks Elasticity of Complementarity (HEC) measures the percentage change in the ratio of factor prices after a percentage change in the ratio of factor quantities, holding other factor quantities and usually product price constant (\( \theta \)).

2b) The elasticity of factor price measures the percentage change in factor price after a given percentage change in that or another factor’s quantity (\( \varepsilon \)).
2.2 MARSHALL’S RULES AND FOUR ELASTICITY CONCEPTS

It is necessary to revisit a framework nesting all four concepts, so we refer to the rules developed by Marshall (1920) (pg 383) and formalised by Hicks (1963) (pg 244):

\[ \lambda = \frac{\sigma (|\eta| + e) + se(|\eta| - \sigma)}{|\eta| + e - s(|\eta| - \sigma)} \]  

(2.1)

The demand for a factor in an industry is more elastic (high \(|\lambda|\)) if:

I. It can be easily substituted by another factor (high \(\sigma\)).

II. Its share of costs is higher (high \(s\)); the term factor share of output (\(\zeta\)) will be more appropriate in some contexts, as explained from chapter 3 onwards; of course, under perfect competition, \(s = \zeta\).

III. The supply of other factors is more elastic (high \(e\)).

IV. Product demand is more elastic (high \(|\eta|\)).

Point I requires a measure of the elasticity of substitution. The confusion started in the 1930s, with the simultaneous development of the concept of an elasticity of substitution by Hicks in his Theory of Wages and Robinson (1933) in her Economics of Imperfect Competition. His measure was, positing two factors in the production function:

\[ \sigma_h = \frac{q_i q_j}{q q_{ij}} \]

(2.2)

where \(q\) is output, \(q_i\) or \(q_j\) is the derivative of the production function for \(q\) with respect to factors \(i\) or \(j\) and \(q_{ij}\) is the second cross partial derivative. Robinson (1933) (pp 255-6) assumes no “economies of large scale industry” and that the supply of other factors is perfectly elastic to propose the more common:

\[ \sigma_r = \frac{\partial \log \frac{x_i}{x_j}}{\partial \log \frac{q_i}{q_j}} \]

(2.3)

\(x_i\) and \(x_j\) are factors \(i\) and \(j\) while the denominator is a ratio of marginal products. The equivalence or more general relationship between the two was unclear until, in the second edition of the Theory of Wages, Hicks (1963) finally confirmed they are the same in a two good case with constant returns to scale.
The equivalence leads many authors, including Heathfield & Wibe (1987), to state that the elasticity of substitution can be expressed either in terms of the production function ($\sigma_h$) or in terms of the curvature of the isoquant ($\sigma_r$). However, generalising to three goods ends the equivalence and the subsequent terminology sometimes confuses the ancestry of the $n$-factor concepts.

Allen (1938) proposes the (partial) elasticity of substitution (AES) between 2 factors, holding output and other factor prices constant\(^2\). Using a production function and the system of first order conditions for the cost-minimizing factor demands, define the AES between factors $i$ and $j$ as:

$$
\sigma_{AES,ij} = \frac{q_{ij} \sum q_r x_r}{|q| x_i x_j}
$$

(2.4)

$|q|$ is the determinant of the bordered Hessian of equilibrium conditions and $q_{ij}$ is the cofactor of $q_{ij}$ in $q$. By Euler’s theorem, the summation term equals $q$ under constant returns to scale. Allen labels this an extension of the 2-factor case, but is not clear whether he is referring to $\sigma_h$ or $\sigma_r$. It may “look” familiar to (2.2), but Blackorby & Russel (1989) agree that Allen makes no attempt to show his is an intuitive or natural extension of the two-factor version and assert it exhibits none of the salient Hicksian 2-factor characteristics.

Hicks (1970) revisits the 40-year issue with a three factor example. Starting with equation (2.1), he distinguishes between two constant returns to scale cases. In the first, he holds the price of the third factor fixed and derives the cross price elasticity of the quantity of factor $i$ with respect to the price of factor $j$ to be:

$$
\lambda_{ij} = \frac{\partial \ln x_i}{\partial \ln w_j} = s_j (\sigma_{r,ij} + \eta)
$$

(2.5)

Hicks states this equation refers to Robinson’s definition of elasticity. Indeed, Allen (1938) produces the same result after defining $\sigma_{AES}$. Given what Hicks writes in 1970, and given the assumption that other factor prices are constant, Allen’s definition, if anything, is Robinson’s, hence the $r$ subscript. The fact that this is sometimes referred to as a Hicks-Allen elasticity, analogous to the consumption treatment, is counterproductive.

In the second case, Hicks keeps the quantity of the third factor fixed, yielding the expression for what Hamermesh (1993) calls the (partial) elasticity of factor price:

\(^2\) This is equivalent to assuming perfectly elastic supply of other factors.
The elasticity of substitution is unchanged from (2.2), except the subscripts are now explicit. Derivation of equations very similar to (2.5) and (2.6) follow in the next two sections, which, now that we have introduced the four elasticity concepts, examine them closely.

2.3 COST/PRODUCTION DUALITY, THE ALLEN ELASTICITY OF SUBSTITUTION AND ELASTICITIES OF FACTOR DEMAND

A cost-minimizing firm chooses its conditional factor demands as a function of (constant) output and factor prices. The constant output elasticity of factor demand is the change in the demand for one factor in response to the change in price for another factor, holding all other factor prices and output constant, so the assumptions regarding what is held constant are consistent with those of cost-minimization.

The AES as expressed in (2.4) imposes a cumbersome calculation, especially when the number of factors is large. Uzawa (1962) uses the duality between production and cost functions to show that (2.4) can be replaced by:

$$\sigma_{AES} \equiv \sigma_{AU} = \frac{C_iC_{ij}}{C_jC_j}$$  (2.7)

Uzawa’s proof uses a unit cost function, which only uniquely represents the underlying production function under constant returns to scale (Varian, 1992) and his result thus appears strictly applicable to constant returns to scale only. However, countless studies use this result in more general settings. For example, of the twelve listed in Chung (1994), only five have a linearly homogenous production technology. While the validity of (2.7) under more general technological settings may be “folk knowledge”, it is instructive to confirm and document this\(^3\).

The conditional factor demands are derived from the cost minimization problem:

$$\min \sum_i w_i x_i \text{ subject to } q(x_1, \ldots, x_n) = y$$  (2.8)

The first order conditions are, where \(\mu\) is the Lagrange multiplier:

---

\(^3\) I am particularly grateful to Dr Margaret Stevens for her role in establishing this result.
\[ w_i = \mu \frac{\partial q}{\partial x_i} \quad (i = 1, \ldots, n) \]  
\[ q() = y \]  

The cost function is:
\[ C(w_1, \ldots, w_n, y) = \sum_i w_i x_i(w_1, \ldots, w_n, y) \]  

Following Allen (1938), but without assuming constant returns to scale, differentiate the first-order conditions with respect to \( w_i \), divide each equation by \( \mu \) and define
\[ q_i = \frac{\partial q}{\partial x_i} \quad \text{and} \quad q_{ij} = \frac{\partial^2 q}{\partial x_i \partial x_j} : \]
\[
\begin{align*}
0 & + q_1 \frac{\partial x_1}{\partial w_i} + q_2 \frac{\partial x_2}{\partial w_i} + \ldots + q_n \frac{\partial x_n}{\partial w_i} = 0 \\
\frac{1}{\mu} q_1 \frac{\partial \mu}{\partial w_i} & + q_{11} \frac{\partial x_1}{\partial w_i} + q_{12} \frac{\partial x_2}{\partial w_i} + \ldots + q_{1n} \frac{\partial x_n}{\partial w_i} = \frac{1}{\mu} \\
\frac{1}{\mu} q_2 \frac{\partial \mu}{\partial w_i} & + q_{21} \frac{\partial x_1}{\partial w_i} + q_{22} \frac{\partial x_2}{\partial w_i} + \ldots + q_{2n} \frac{\partial x_n}{\partial w_i} = 0 \\
& \quad \ldots \quad \ldots \quad \ldots \\
\frac{1}{\mu} q_n \frac{\partial \mu}{\partial w_i} & + q_{n1} \frac{\partial x_1}{\partial w_i} + q_{n2} \frac{\partial x_2}{\partial w_i} + \ldots + q_{nn} \frac{\partial x_n}{\partial w_i} = 0
\end{align*}
\]

By Cramer’s rule:
\[
\frac{\partial x_i}{\partial w_i} = \frac{1}{|q|} \begin{vmatrix} 
0 & q_1 & \cdots & q_n \\
q_1 & q_{11} & \cdots & q_{1n} \\
q_2 & q_{12} & 0 & \cdots & q_{2n} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
q_n & q_{1n} & \cdots & q_{nn} \\
\end{vmatrix}
\]

Therefore:
\[
\frac{\partial x_i}{\partial w_i} = \frac{1}{\mu} \frac{q_{i2}}{|q|} \quad (2.13)
\]

where, as in (2.4), \( |q| \) is the determinant of the bordered Hessian of equilibrium conditions and \( q_{ij} \) is the cofactor of \( q_{ij} \) in \( q \). Using (2.4):
\[
\sigma_{AES,12} = \sum_k q_k x_k \frac{x_k}{\partial x_k} \frac{\partial x_k}{\partial x_2} \]

But:
\[
\mu \sum_k x_k q_k = \sum_k w_k x_k = C \quad (2.15)
\]
(by the first order conditions) and
\[ x_k = \frac{\partial C}{\partial w_k} \tag{2.16} \]
(by Shephard’s Lemma), so:
\[ \sigma_{AES} = C \frac{\partial^2 C}{\partial w_1 \partial w_2} \tag{2.17} \]
This is the same as (2.7) and shows the duality result holds without assuming constant returns to scale.

The relationship between the AES and the constant output elasticity of factor demand is:
\[ \lambda_{ij} = s_j \sigma_{AES,ij} \tag{2.18} \]
\( \lambda_{ij} \) is the partial elasticity of the quantity of factor \( i \) with respect to the price of factor \( j \) and \( s_j \) is the cost share of factor \( j \). Heathfield & Wibe (1987) assert the relationship between \( \lambda_{ij} \) and \( \sigma_{ij} \) holds only under conditions of constant returns to scale (pg 61). Indeed, they refer to Allen (1938), who in his exposition uses linear homogeneity. However, constant returns to scale is not a requirement for this result to hold. Using (2.14) to (2.16),
\[ \sigma_{AES,12} = C \frac{\partial x_1}{x_1 x_2 \partial w_2} = C \frac{\partial \log x_1}{w_2 x_2 \partial \log w_2} \tag{2.19} \]
But:
\[ s_i = \frac{w_i x_i}{C} \tag{2.20} \]
Therefore:
\[ \sigma_{AES,12} = \frac{\lambda_{12}}{s_2} \tag{2.21} \]
so the equivalence holds under more general conditions than constant returns to scale.

(2.18) holds output constant, and, true to the spirit of Robinson’s concept, refers to a movement along an isoquant. However, once one endogenises output to optimization by profit-maximizing firms, the analysis changes. The optimum choice of output is a function of the industry product price and the industry input prices. In a competitive industry, a unit fall in the price of one factor will lower average and marginal cost by that factor’s share if the technology is homothetic (Estrin & Laidler, 1995). Profit-maximising industry output will rise and so will demand for all factors, by
their share. However, as industry output rises, the product price falls, which lowers the value marginal product of each factor and mitigates the increase in demand for all factors.

Under constant returns to scale, average cost for the industry stays constant with respect to output, so output rises until product price equals the new lower average cost. The percentage increase in output with respect to a 1% fall in price is of course the elasticity of product demand. Therefore, the output effect for the industry is \( \frac{s}{\eta} \). Under constant returns to scale, the elasticity of factor demand with respect to output is 1 (Fallon & Verry, 1988), so the expression translates to factor demand.

Provided the technology is homothetic, the output effect can be adjusted for non-constant returns to scale (and hence non-constant average cost). This is mentioned by Robinson (1933) in her discussion of Marshall’s Rules and is shown in Fallon & Verry (1988) and Mosak (1938), but is not shown here because this study needs constant returns to scale for another reason. The reason arises because this study uses firm-level data to infer effects on firms that are necessarily driven by industry level effects (otherwise, for a perfectly competitive firm, product demand and hence derived demand would be infinitely elastic; more importantly, input prices realistically fall for the whole industry, not just 1 firm).

In their treatments, Hicks (1963) and Marshall (1920) speak of industry-level phenomena despite referring to firm-level technologies in their elasticity of substitution measures. Hamermesh (1993) also freely moves between the industry and firm level. They can do this because they assume constant returns to scale and can therefore think of industry outcomes independently of the number of firms in the industry. If, on the other hand, there are decreasing returns to scale, it clearly matters for average costs and hence price how many firms are in the industry. Average costs now become a function of industry output and the number of firms in the industry, which itself is a function of the underlying cost structures in each firm. The assumption of constant returns to scale allows one to think of the effects on the equilibrium product price and each firm’s optimal percentage change in output independently of firm-entry/exit. Therefore, under the assumption of constant returns to scale, the output-compensated elasticity of factor demand for each firm is:

\[
\bar{\lambda}_j = s_j (\sigma_{AES,j} + \eta) \quad (2.22)
\]

It is however more common for constant output to be assumed. When \( \lambda_j > 0 \), a rise in the price of factor \( j \) will lead to a rise in the quantity of factor \( i \) demanded. Such factors are said to be \( p-\)
substitutes. On the other hand, when \( \lambda_{ij} < 0 \), factors are said to be \( p \)-complements (Hicks, 1970). These terms can loosely be applied to the signs on \( \sigma_{AES,ij} \).

### 2.4 THE HICKS ELASTICITY OF COMPLEMENTARITY AND ELASTICIES OF FACTOR PRICE

The AES is more common, but \( \sigma_h \) does manifest itself in some studies. We adopt the term “aleph” \( \aleph = 1/\sigma_h \) and keep the name Hicks Elasticity of Complementarity (HEC). The reason for the inversion will be apparent shortly.

For a profit-maximising firm, the HEC is the percentage change in ratio of two factors’ marginal products relative to the exogenous percentage change in the ratio of the same two factor quantities, holding other factor quantities constant. In this setting, factor prices are determined by their value marginal products, so the HEC can be interpreted as the percentage change in relative factor prices after an exogenous change in the ratio of the two factor quantities. The interpretation can only readily be interpreted after an industry-level change in factor supply, which affects all firms, for if each firm genuinely had its own fixed labour supply, it wouldn’t pay labour its marginal product. The HEC between factors \( x_i \) and \( x_j \) is given by:

\[
\aleph_{ij} = \frac{q_i q_{ij}}{q_j q_{ij}} \tag{2.23}
\]

Sato & Koizumi (1973) develop an expression for \( \aleph_{ij} \) analogous to (2.4), but it is much easier to estimate the underlying production function and use (2.23). The symmetry between (2.23) and (2.7) is beauty with a purpose: chapter 3 will show that they are calculated using exactly the same regression coefficients, but the coefficients are for input quantities in one case and input prices in another. This convenience is the main reason for the inversion of \( \sigma_h \).

The relationship between the partial elasticity of factor price and the partial elasticity of complementarity is (Sato & Koizumi, 1973):

\[
\epsilon_{ij} = \zeta_j \aleph_{ij} \tag{2.24}
\]

An increase in the supply of \( j \) will be partially accommodated by decreases in demand for some of the other factors and, \textit{ceteris paribus}, a fall in their price and may also lead to a rise in demand for other factors. However, for all perfectly inelastically supplied factors to be employed, output must necessarily increase, which in turn raises demand for all the factors. The overall demand
effect will determine the overall factor price effect: to what extent must the wage of factor $i$ rise or fall for demand to equal the (unchanged) supply.

(2.24) holds product price constant, which is unrealistic because, given the fall in average cost, this means firms are making abnormal profits\(^4\). This is analogous to perfectly elastic product demand, as is possibly the case for firms in an industry that is so small relative to the world market that the industry as a whole is a price taker. (2.24) omits any effect of a fall in product price on each factor's value of marginal product, overstates the increase in demand for each factor and overstates the output induced increase in wages. (2.25) accounts for lower prices:

$$\bar{\epsilon}_q = \zeta_j \phi_j + \frac{\zeta_j}{\eta}$$

(2.25)

No documented empirical studies actually use (2.25) instead of (2.24), which means they are over inclined towards finding two inputs are complements. To be more precise, two factors are \textit{q-complements} if $\epsilon_{ij} > 0$ and they are \textit{q-substitutes} if $\epsilon_{ij} < 0$ (Hicks, 1970). If $\epsilon_{ij} > 0$, an exogenous rise in the supply of $j$ will lead to a rise in the wage of factor $i$.

Constant returns to scale are necessary to justify interacting firm-level data and behaviour with industry features. Otherwise, one must consider how the increase in factor supply is distributed across firms in the industry, and whether new firms would enter and employ the factors instead. This would have an impact on which firms, including entrants, expand, and by how much, and therefore produce a different industry equilibrium. Under constant returns to scale, the industry equilibrium output and price is independent of how the supply increase is distributed and how many firms there are in the industry, as there is no difference between one firm being the only one to increase industry output and all the firms increasing output equally.

\section*{2.5 THE APPROPRIATE USE OF THE FOUR ELASTICITY MEASURES}

One reason the AES may be more generally accepted is the nature of the exogeneity assumptions. The view that factor prices are exogenous to a firm and factor quantities are endogenously chosen is favoured to the other way round, certainly at a firm level. However, neither phenomenon is entirely correct (Hamermesh, 1993; Fallon & Verry, 1988).

\(^4\) Were it not for the fixed factor supply, under constant returns to scale, the increase in output and hence factor demand would be infinite.
At a micro-level, there is evidence for firm-effects on wages, especially firm size effects (Troske, 1991). Whether the cause is that (i) workers are more productive because of their abilities or the higher capital:labour ratio, (ii) compensating differentials for a less-pleasant environment or (iii) any other effect, wages are at least partly endogenous to the firms (Oi & Idson, 1999). The cost of capital a firm gets tends to fall as it gets bigger, certainly up to a point, because small and/or young firms incur risk premia, and the nature of their activities affects the cost of capital. Firms that are highly leveraged also have risk premia.

As one moves to an industry or macro-level, the exogeneity of factor prices becomes less accurate (Hamermesh, 1993). A big rise in demand for a factor at even relatively low levels of aggregation will start affecting factor prices, especially when specific skills are scarce as in South Africa.

Exogenous quantities and endogenous prices are consistent with the context of perfectly inelastic labour supply. Some may find this hard to reconcile with an unemployment rate of over 30%, but the view that these people are structurally unemployed and to a large extent non-participants means they are effectively not part of the supply. Furthermore, the whole premise on which this thesis is based is insufficient supply of skills. This is not because of a lack of demand for skills provision (Department of Labour, 1997), which is possibly endogenous to wages, but because of a lack of supply of skills provision. A fixed skill supply, certainly with respect to the wage, is a credible point of departure.

Chapter 4 will show there is collective bargaining in South Africa. The wages are at least partially set by those firms in the collective bargaining process, making wages at least partially endogenous for those firms, but there is also extension by the Minister of Labour to other firms, making wages exogenous for the others (Nattrass, 2000). Such wage agreements hamper the responsiveness of wages to quantities, but this is more of an argument about the smooth operation of market forces than about endogeneity or exogeneity. Finally, the existence of collective bargaining is equivocal in this argument, as there is evidence of collective bargaining taking place over quantity as well as price. There is one compelling recent example, where the National Union of Mineworkers literally “…stopped wage talks to focus on the Harmony [goldmine] job losses” (Bridge, 2004).
To the extent that wages and quantities are neither completely endogenous nor exogenous, it becomes important to see which elasticity concepts are most suitable. To this end, it is clearly the case that the relevant exogenous move is the supply of skilled labour, through immigration or more successful training, and it is certainly to be expected that the relief of the shortages will lead to changes in wages in the economy, even if distorted by labour market institutions. Whether a rise in the supply of one factor leads to a smooth fall in the wage of another or to a trade union being forced to accept a smaller wage increase, the estimated elasticities provide a useful insight into the gains and losses other factors are likely to see. Furthermore, an adjustment can be made to cope with rigid wages, as discussed next.

2.6 THE ELASTICITY OF FACTOR PRICE WITH RIGID WAGES

Given that wages are influenced by collective bargaining institutions, one can adjust for this using the methods employed in Grant & Hamermesh (1981) and Johnson (1980). In the simple case, where that factor’s wage is completely rigid, the effect of an exogenous change in the quantity of one factor on the quantity of that factor can be calculated. This means the HEC can be used to infer effects on employment rather than wages for a particular factor. Following Grant & Hamermesh (1981), assume the prices of all factors are flexible except for unskilled labour, which has wage \( w_u \). The profit-maximising firms’ marginal productivity conditions are

\[
\begin{align*}
    w_u^* &= q_u\left(x_u, x_1^*, \ldots, x_n^*ight), \\
    w_k &= q_k\left(x_u, x_1^*, \ldots, x_n^*\right),
\end{align*}
\]

\[k = 2, \ldots, n\]  

(2.26)

where \( w_k \) are the flexible wages, \( x_k \) and \( x_u \) are factor quantities and \( q_u \) and \( q_k \) are marginal products. Differentiating the first order equations yields:

\[
\frac{\partial x_j}{\partial x_i} = -\frac{q_{ij}}{q_{jj}}, \quad j = 2, \ldots, n
\]

(2.27)

\[
\frac{\partial w_i}{\partial x_j} = -\frac{q_{ui}q_{ij} + q_{uj}q_{ju}}{q_{uu}}
\]

(2.28)

A factor’s share of output is:

\[
\zeta_k = \frac{w_k x_k}{q}, \quad \forall k
\]

(2.29)

By (2.29), \( x_k = \frac{q_i x_i}{q_k} \). By (2.23) \( q_{ij} = \frac{w_i w_j x_i x_j}{q} \). Using this and multiplying (2.27) by \( \frac{x_j}{q} \) yields:

\[
\frac{\partial \ln x_j}{\partial \ln x_i} = \phi_{ij} = -\frac{\zeta_k}{\zeta_u x_i}, \quad j = 2, \ldots, n
\]

(2.30)

Similarly, (2.28) results in:
\[
\frac{\partial \ln w_i}{\partial \ln x_j} = \Phi \frac{\zeta_j (-K_{w}K_{wj} + K_{j})}{K_{ww}}, j = 2, \ldots, n
\]  

(2.31)

So, the HEC, which is most appropriate for examining exogenous changes in factor supply, can be used to adjust for rigid wages in a factor.

The AES and related factor demand elasticity are nonetheless more generally accepted concepts and more commonly used as inputs into other research. This is the main reason for estimates of these models. The two sets of measures are not directly comparable, and may even have different signs (Hamermesh, 1993). See Sato & Koizumi (1973) for a discussion of the complex dualities.

2.7 SUMMARY

This study estimates four elasticities. On the presumption of endogenous factor prices and exogenous factor quantities, the Hicks Elasticity of Complementarity (HEC or \( \aleph \) “aleph”), measures the change in the ratio of factor prices in response to a change in the ratio of factor quantities. Related to this is the elasticity of factor price (\( \varepsilon \)), which measures the change in one factor’s price in response to a change in that or another factor’s quantity. These measures hold the quantities of other factors constant, may or may not adjust for imperfect product demand elasticity, and can be transformed into effects on employment if wages are rigid. These concepts are best suited to analysing the effects of an exogenous rise in skill supply.

The HEC is used in Grant & Hamermesh (1981) to analyse the effects of greater female labour force participation on youths, while Johnson (1980) puts the concept to a variety of uses, including the effects of training programmes for unskilled youths. Mak (2000) estimates the HEC and elasticity of factor price for a number of different skill levels defined by education.

On the presumption of exogenous factor prices and endogenous factor quantities, the Allen Elasticity of Substitution (AES or \( \sigma \)), measures the change in the ratio of factor quantities after a change in the ratio of factor prices. Related to this is the familiar elasticity of factor demand (\( \lambda \)), which measures the change in one factor’s quantity after a change in that or another factor’s price. These measures may or may not allow for changes in output and the estimates hold the prices of other factors constant.
3. ESTIMATING ELASTICITIES USING TRANSLOG FUNCTIONS

3.1 INTRODUCTION

This chapter compares cost and production function estimation, because the thesis estimates the HEC and elasticity of factor price using a production function and estimates the AES and elasticity of factor demand using a cost function and cost share equations. The chapter explains why translog functions are superior to other functional forms for elasticity estimates, showing that they impose no technology or elasticity assumptions and allow for elasticities to vary across the sample. The chapter derives factor share equations from the production and cost function and shows how the elasticities are calculated from the regression estimates. It concludes with a discussion of separability, which is necessary for the use of the value-added specification in a production function. But first, it may be useful to recall some technological concepts.

3.2 TECHNOLOGICAL CONCEPTS

A production function is homogeneous of degree $k$ if $q(tx_1,\ldots,tx_n) = t^k q(x_1,\ldots,x_n)$ (Varian, 1992), so we can say it has returns to scale of value $k$ independent of output. As a special case, $k=1$ means there are constant returns to scale. Homogeneous functions fall in the more general class of homothetic functions. Functions are homothetic if they are a positive monotonic transformation of homogeneous functions. Their key feature is that (ibid.):

$$\frac{\partial q(tx_1,\ldots,tx_n)}{\partial x_i} = \frac{\partial q(x_1,\ldots,x_n)}{\partial x_i}$$

(3.1)

The marginal rates of technical substitution are independent of the scale of output and the shape of the isoquants is preserved along the expansion path (Heathfield & Wibe, 1987), so the factor shares are independent of output (Chung, 1994). Furthermore, we can express the returns to scale as a function of output only and not the factor input values or factor prices.

Loosely speaking, a technology is separable if the elasticity of substitution between two inputs is independent of the quantity of the other inputs. This allows for the exclusion of some inputs that are not of direct interest and for value added to be used instead of gross output (Chung, 1994).
3.3 COST FUNCTIONS AND PRODUCTION FUNCTIONS

There are many suggestions why cost functions are more popular than production functions for estimation purposes (Binswanger, 1974a). First, as a consequence of optimising behaviour, cost functions exhibit homogeneity of degree one in prices, which can be imposed to improve estimates without recourse to technological assumptions. Second, cost functions are more consistent with the view that wages are exogenous. In the production function, estimated on the basis of exogenous input quantities, the input quantities may possibly be endogenous to wages, the production technology and market shocks. Not accounting for this can result in biased estimates. The main reason, however, for using a cost function in this study is that, as shown in chapter 2, the AES and elasticity of factor demand can be far more tractably arrived at than by using production functions. This is of particular importance when one attempts to conduct inference on the elasticities. Furthermore, the exogeneity assumptions consistent with cost function estimates are also consistent with those underlying the AES.

Similarly, those assumptions underlying the HEC are consistent with production functions, which are more tractable for estimating the HEC and related elasticities of factor price. The inputs can in practice be endogenous to market or technology shocks, possibly biasing the coefficients, so econometric methods for mitigating the bias are presented in chapter 5. Production functions also offer the advantage of not relying on wage data. This is an advantage because wage data are not available, so firm-level wages must themselves be estimated before use in the cost function. The process is explained in chapter 4.3.

3.3 CHOICE OF TECHNOLOGICAL REPRESENTATION

In a macroeconomic model of skilled and unskilled labour demand and supply, Du Toit & Koekemoer (2003) use a Cobb Douglas production function. Although they claim it was “estimated and validated as representative of the South African production structure” (pg 7), the homogeneity and separability assumptions it carries are too restrictive to go untested in a new study. More importantly, the implication that the elasticity of substitution is unity completely circumvents one of the aims of this thesis (Chung, 1994).

Constant Elasticity of Substitution functions allow the elasticity of substitution to differ from one, but are still homogeneous (Heathfield & Wibe, 1987). More importantly, the elasticity of substitution is the same between all input pairs, although they may differ across observations.
(Chung, 1994). For example, different industries can have different elasticities, but the elasticity within each industry is the same for the substitution between all factors. This is still a major restriction, but the resulting factor demand equations yield easily estimable elasticities between two factors. For example, Edwards (2003) estimates an equation for the demand for skilled relative to unskilled labour as a function of relative wages, import penetration variables (M), export orientation (X), and technology variables (Φ) in South Africa.

\[
\ln \left( \frac{S}{U} \right) = \theta_0 + \theta_1 \Phi + \theta_2 M + \theta_3 X - \sigma \ln \left( \frac{w_i}{w_u} \right) + \epsilon,
\]

(3.2)

If σ – the AES – is the only parameter of interest, the assumptions of the function preclude the need for other factors to be included. In other words, if there are many factors, one need only estimate σ between two of them and apply that value to the other pairs, provided there are no biased estimates. Edwards (2003) estimates σ to be −0.47 between skilled and unskilled labour and −0.41 between less-skilled and unskilled labour. The values are quite close, suggesting the CES restriction may not be seriously inaccurate.

However, if other parameters are being researched, adding factors requires more complex non-linear techniques or step-wise regression to estimate the production function directly (Fallon & Verry, 1988; Hamermesh, 1993). Fallon & Lucas (1998) include capital in their CES function to estimate, with non-linear 3 stage least squares and calibration techniques, demand for black and white labour as proxies for unskilled and skilled labour. They produce long run elasticities of demand for unskilled labour of about −0.7.

More flexible functional forms do not impose a priori technological assumptions like separability of factor inputs or homotheticity. Besides allowing for a more accurate representation of the underlying technology, elasticities can vary across the sample, allowing for more policy-relevant insights. To pre-empt a discussion in later chapters, it can be far more useful to know that 75% of firms have a positive HEC than that the mean value is 0.75.

Two functions falling in this “flexible” class are the Generalised Leontief function due to Diewert (1971, in Berndt, 1991) and the transcendental logarithmic (translog) function developed by Christensen, Jorgenson & Lau (1973). There appears to be no relevant application of either of these to heterogeneous labour in the South African literature. This study uses translog functions. While the gains from using a flexible functional form are clear, the reasons for adopting the translog instead of Generalised Leontief are a moderate gain in tractability, especially if constant returns to scale are not assumed, at no apparent cost, and the fact that the majority of
international studies appear to use translogs for this application. However, a study wishing to focus on the effects of non-wage or non-input variables – for example the effects of trade liberalisation – might benefit from the easy way in which such variables can be included (Chung, 1994). In a rare comparison of both technologies, Humphrey & Wolkowitz (1976) get somewhat different elasticity estimates, so there certainly is merit in comparing this study’s results with those using Generalised Leontief technology.

3.4 THE TRANSLOG PRODUCTION FUNCTION AND ASSOCIATED ELASTICITIES

The translog production function can be viewed as a second order Taylor approximation to an unknown technology. The main advantage is that separability and homotheticity are not assumed, but that they can be tested for and, if accepted, imposed on the system. The function is:

\[
\ln y = \ln y + \sum \alpha_i \ln x_i + \frac{1}{2} \sum \sum \beta_{ij} \ln x_i \ln x_j, \quad (i, j = 1, ..., 6)
\]  

(3.3)

\(y\) is output and \(x_i\) are the six factor inputs in this study. A twice continuously differentiable production function implies the underlying marginal productivity conditions satisfy the symmetry conditions \(B_{ij} = B_{ji}\) (Conrad & Jorgenson, 2000). The translog production function is homogeneous of degree \(k\) if:

\[
\sum j \beta_{ij} = \sum i \beta_{ij} = 0 \\
\sum i \alpha_i = k
\]

(3.4)

If \(k=1\), the translog is linearly homogeneous. If all \(\beta_{ij}=0\), it is also separable and reduces to a Cobb Douglas function (Kim, 1992).

The factor share of output equations are linear in the parameters (Varian, 1992). Differentiating the production function with respect to a factor input yields that factor’s share of output \(\zeta_i\) (Mak, 2000). The production function derivations don’t appear to be documented, so we make space to do so:

\[
\frac{d \ln y}{d \ln x_i} = \alpha_i + \sum_j \beta_{ij} \ln x_j
\]

(3.5)

But\(^5\), using \(\frac{\partial y}{\partial x_i} = w_j\),

\[
\frac{\partial \ln y}{\partial \ln x_i} = \frac{x_i \partial y}{y \partial x_i} = \frac{w_i x_i}{y} = \zeta_i
\]

(3.6)

Hence:

\(^5\) Mak (2000) claims to use Shephard’s Lemma here, although this is only applicable to the cost case shown later.
\[
\zeta_i = \alpha_i + \sum_j \beta_{ij} \ln x_j \tag{3.7}
\]
If all \(\beta_{ij} = 0\), the Cobb-Douglas constant factor share results, but in (3.7), the factor share is a function of the inputs and the technological parameters. It is common for the system of factor share equations to be estimated to improve the precision of the estimates (Greene, 2003), as explained in chapter 5.

I am not aware of any full derivations of the expressions for elasticities, especially in the production context, so it is informative to do so here for the perfect product demand elasticity case. To derive the elasticity of factor price, which is the change in the wage of factor \(i\) in response to a change in the quantity of factor \(j\), observe that:

\[
w_i = \frac{y}{x_i} \zeta_i \tag{3.8}
\]
Therefore, recalling that \(\frac{\partial}{\partial x_i} = w_j\) and holding product price is constant:

\[
\epsilon_{ij} = \frac{\partial \log w_i}{\partial \log x_j} = \frac{x_j}{w_i} \frac{\partial}{\partial x_j} \left( \frac{y}{x_i} \zeta_i \right)
\]
\[
= \frac{x_j}{w_i} \left( \frac{y\beta_{ij}}{x_i} + \frac{w_j \zeta_i}{x_i} \right)
\]
\[
= \frac{\beta_{ij}}{\zeta_i} + \zeta_i \left( \frac{w_j x_j}{y} \right) \left( \frac{y}{w_j x_i} \right)
\]
Therefore:

\[
\epsilon_{ij} = \frac{\partial \log w_i}{\partial \log x_j} = \frac{\beta_{ij}}{\zeta_i} + \zeta_j \tag{3.9}
\]
Using the fact that \(\epsilon_{ij} = \zeta_i \kappa_{ij}\) for perfectly elastic product demand, it follows that:

\[
\kappa_{ij} = \frac{\beta_{ij}}{\zeta_i} + 1 \tag{3.10}
\]
It is immediately apparent that \(\beta_{ij} = 0\) yields a Hicks elasticity of complementarity (HEC) of unity.

The expression for the own elasticity of factor price is (Binswanger, 1974a):

\[
\epsilon_{ii} = \frac{\beta_{ii}}{\zeta_i} + \zeta_i - 1 \tag{3.11}
\]
while the HEC is:

\[
\kappa_{ii} = \frac{\beta_{ii}}{\zeta_i} + 1 - \zeta_i \tag{3.12}
\]
The own elasticity of factor price should be interpreted as the amount a factor’s price would have to go down to absorb an increase of its quantity into production (and vice versa). The meaning of (3.13) is not quite clear, nor is its usefulness. It could be a measure of the responsiveness of a
factor price elasticity to a change in a factor’s quantity. Grant & Hamermesh (1981) and Mak (2000) estimate HECs and elasticities of factor demand using translog production functions.

3.5 THE TRANSLOG COST FUNCTION AND ASSOCIATED ELASTICITIES

Christensen et al (1973) also develop translog cost functions. Some insist the cost function is not the strict dual of the production function (Heathfield & Wibe, 1987), but the symmetry in treatment will become apparent shortly. Roman letters will be used to denote the coefficients on the cost function:

\[
\ln C = \ln a_0 + \sum_i a_i \ln w_i + a_y \ln y + \sum_i \sum_j b_{ij} \ln w_i \ln w_j + b_y \ln^2 y + \sum_j b_{iy} \ln w_i \ln y; \quad (i; j = 1, \ldots, 6)
\]

\[C\] is cost and \(w_i\) are the factor prices. The cost share equation for factor \(i\) is derived by differentiating the cost function with respect to \(\ln w_i\). Following Chung (1994):

\[
\frac{d \ln C}{d \ln w_i} = a_i + \sum_j b_{ij} \ln w_j + b_{iy} \ln y \quad (3.15)
\]

But, using Shephard’s Lemma for the penultimate equality:

\[
\frac{\partial \ln C}{\partial \ln w_i} = w_i \frac{\partial C}{\partial w_i} = \frac{w_i x_i}{C} = s_i \quad (3.16)
\]

Therefore:

\[
s_i = a_i + \sum_j b_{ij} \ln w_j + b_{iy} \ln y \quad (3.17)
\]

Consistent with cost minimizing behaviour (Berndt & Khaled, 1979):

\[b_j = b_j, \text{(Slutsky symmetry)}\]

\[
\frac{\partial \ln C}{\partial \ln W} = 1 \text{ (price homogeneity) iff } \sum_j b_{ij} = \sum_i b_{ij} = 0; \sum_i a_i = 1; \sum_i b_{yi} = 0 \quad (3.18)
\]

where \(\partial \ln W = \partial \ln w_i \forall i\)

In addition, restrictions can be imposed on the technology. This is easily seen by observing that returns to scale are calculated as the inverse of:

\[
\frac{d \ln C}{d \ln y} = h = \frac{1}{r} = a_y + b_y \ln y + \frac{1}{2} \sum_i b_{iy} \ln w_i \quad (3.19)
\]

To get a measure of returns to scale that is independent of the factor prices, as implied by homotheticity, requires \(b_{yi} = 0 \forall i\). This is also seen in(3.17), where the factor share is no longer a function of output. The cost function is homogeneous of degree \(r\) if \(b_y = 0\), with \(r = \frac{1}{a_y}\).

To derive the elasticity of factor demand, which is the change in the quantity of factor \(i\) in response to a change in the price of factor \(j\), observe that:
\[ x_i = \frac{C}{w_i} s_i \]

\[ \lambda_{ij} = \frac{\partial \log x_i}{\partial \log w_j} = \frac{w_j}{x_i} \frac{\partial}{\partial w_j} \left( \frac{C}{w_i} s_i \right) \]

\[ = \frac{w_j}{x_i} \left( \frac{C b_{ij} x_j^i}{w_i w_j} + \frac{x_j s_i}{w_i} \right) \quad \text{(using Shephard's Lemma)} \]

\[ = \frac{b_{ij}}{s_i} + s_i \left( \frac{w_j x_j}{C} \right) \left( \frac{C}{w_i x_i} \right) \]

Therefore:

\[ \lambda_{ij} = \frac{\partial \log x_i}{\partial \log w_j} = \frac{b_{ij}}{s_i} + s_i \quad (3.21) \]

The AES is:

\[ \sigma_{ij} = \frac{b_{ij}}{s_i s_j} + 1 \quad (3.22) \]

\( b_{ij} = 0 \) yields an AES of unity. The expression for the own elasticity of factor demand is (Binswanger, 1974a):

\[ \lambda_{ii} = \frac{b_{ii}}{s_i} + s_i - 1 \quad (3.23) \]

while the AES is:

\[ \sigma_{ii} = \frac{b_{ii}}{s_i s_i} + 1 - s_i \quad (3.24) \]

Humphrey & Wolkowitz (1976) suggest the own AES can be interpreted as a change in a factor’s demand responsiveness to a change in its own price.

The symmetry between equations (3.10) to (3.13) and (3.21) to (3.24) is striking. The coefficients in the calculation are exactly the same, except the first set of elasticities uses the coefficients on the inputs of a production function while the second set uses the coefficients on the factor prices in a cost function.

The typical procedure is to estimate the cost or production function or the relevant factor share equations in order to get the coefficients. Some studies use the actual factor shares to calculate elasticities (Chung, 1994), but it is correct to use the regression’s predicted shares (Berndt, 1991). It is typical for studies to predict the shares using mean factor prices or factor quantities and calculate a single elasticity based on this point (Greene, 2003). However, this study predicts shares for each firm in the sample and calculates the elasticity for each firm in the sample for a richer analysis.
Early cost function studies were interested in the relationships between capital, labour, materials and energy prices. Famous examples include Berndt & Wood (1975) and Griffin & Gregory (1976). Taking place in the early seventies, it is likely that the attention to energy was motivated by the oil crises, which provide an excellent example of an exogenous price shock. Another popular application is a test of the capital skill complementarity (CSC) hypothesis of Griliches (1969). This hypothesis is that, in strong form, capital and skilled labour are complements ($AES<0$) while capital and unskilled labour are substitutes ($AES>0$). In weak form, the hypothesis is that unskilled labour and capital are bigger substitutes than skilled labour and capital (Bergström & Panas, 1992). Teal (2000) estimates substitution possibilities between skilled and unskilled labour using Ghanaian firm-level data.

3.6 SEPARABILITY AND THE VALUE-ADDED SPECIFICATION

Production functions are either estimated with value added as the dependent variable or with gross sales/output as the dependent variable and with raw materials as an additional input. Some do so by assumption while others test for separability. If the separability conditions are met, then excluding an input will not affect elasticity estimates of the inputs of interest (Chung, 1994). The discussion, by referring to an input, is distinct from the notion of global separability of all inputs (Berndt & Christensen, 1973a), which is assumed by a Cobb Douglas technology. The term “factors” refers to labour and capital while “inputs” include raw materials or other intermediate inputs.

This issue applies to the exclusion of any input, often capital, or perhaps the aggregation of labour inputs. It also applies to the omission of raw materials inputs, usually with value added being used instead of gross output. The validity of the value-added specification in the production function, which is tested in chapter 5, is the emphasis in this discussion. These tests also have a bearing on the cost function, as estimates will have to exclude raw materials simply because of a lack of raw materials prices, and because value added or output can be used in (3.14) and/or (3.17).

The core of the separability issue, exposed by Sato (1975)\(^6\), is the conditions under which the production function expressing gross output in terms of skilled labour ($x_s$), unskilled labour ($x_u$) and raw materials ($x_r$):

$$q = q(x_s, x_u, x_r)$$

\(^{(3.25)}\)

can be expressed as a function of the labour inputs alone:

\(^6\) His exposition refers to capital, labour and raw materials
\[ v = g(x, x_u) \]  

where \( v = q - \gamma x, x = q - w, x_u \) is value added. Sato (1975) shows (3.26) is valid if and only if:

\[ q = Q[g(x, x_u), x_u] \]  

Although the literature refers to additivity and separability in both weak and strong forms, weak separability is the concept of interest for our purposes. Raw materials are weakly separable from the factors if (3.27) holds. Adjusting the notation in Berndt & Christensen (1973a), the essential characteristic of (3.27) is that changes in raw materials do not affect the marginal rate of technical substitution between the factors of production.

\[ \frac{\partial}{\partial x_r} \left( \frac{F_r}{F_u} \right) = 0 \]  

(3.28)

\( F \) is the production function its subscripts refer to derivatives with respect to inputs \( s, u \) and \( r \). This implies (Berndt & Christensen, 1973a):

\[ F_r F_u - F_u F_{ru} = 0 \]  

(3.29)

and, using the parameters of a translog production function, Berndt & Christensen (1973b) show this means:

\[ \zeta_r \beta_{ur} - \zeta_u \beta_r = 0 \]  

(3.30)

\( \beta_{ru} \) refers to the relevant coefficient estimate and \( \zeta_i \) to the factor share. Monotinicity of the production function implies each \( \zeta_i \) is positive so a sufficient condition is the additive separability condition:

\[ \beta_{ur} = \beta_{ru} = 0 \]  

(3.31)

(3.31) can be tested empirically. If it holds for the particular variables whose elasticities are of interest, then the function is additively separable and excluding raw materials is justified. If this does not hold for some \( \beta_{ur} \), one can substitute the expression for \( \zeta_i \) given by (3.7) into (3.30) and re-arrange to find, for those \( \beta_{ur} \) of interest:

\[ \alpha_u \beta_{ru} - \alpha_i \beta_{ur} = 0 \]
\[ \beta_r \beta_{ru} - \beta_u \beta_{ru} = 0 \]
\[ \beta_u \beta_{ru} - \beta_{ur} \beta_{ru} = 0 \]

(Chung, 1994; Berndt & Christensen, 1973b). Using the symmetry conditions \( \beta_{ji} = \beta_{ij} \), this means the following parameter restriction can be tested:

\[ \frac{\alpha_u}{\alpha_i} = \frac{\beta_{ru}}{\beta_{ir}} = \frac{\beta_{ru}}{\beta_{ir}} = \frac{\beta_r}{\beta_u} \]  

(3.33)

If (3.33) holds, the input is said to be non-additively separable from the inputs of interest. A production function with six factors in addition to raw materials requires performing six tests
like (3.31) and, if necessary, fifteen tests like (3.33) for all marginal technical rates of substitution.

We restrict the discussion to the specialist cases of all six factors either being additively separable or non-additively separable from raw materials. Blackorby & Russel (1977) discuss separability in quadratic flexible functional forms, allowing for combinations of additive separability and non-additive separability. Adapting their general theorem to (3.31), the translog representation of (3.27) is:

$$\ln y = \ln \alpha_0 + \sum_{i=1}^{6} \alpha_i \ln x_i + \frac{1}{2} \sum_{i=1}^{6} \sum_{j=1}^{6} \beta_{ij} \ln x_i \ln x_j + \left[ \alpha_r \ln x_r + \frac{1}{2} \beta_{ir} \ln^2 x_r \right]$$

(3.34)

This is a Cobb-Douglas aggregation of two translog functions. Differentiating to get share equations will not include a term in raw materials, hence the zero $\beta_{ir}$ coefficient. If non-additive separability holds, then using (3.33) results in a quadratic in linear logarithmic aggregators:

$$\ln y = k_i \sum_{i=1}^{6} \gamma_i \ln x_i + \frac{1}{2} \sum_{j=1}^{6} \sum_{k=1}^{6} \beta_{jk} \ln x_j \ln x_k + k_r \alpha_r \ln x_r + \frac{1}{2} \beta_{rr} \ln^2 x_r$$

$$+ \delta_{11} \left( \sum_{i=1}^{6} \gamma_i \ln x_i \right) \left( T^1 (x_1; \ldots; x_6) \right) + \delta_{12} \left( \sum_{i=1}^{6} \gamma_i \ln x_i \right) \left( T^2 (x_i) \right)$$

$$+ \delta_{21} \left( \sum_{i=1}^{6} \gamma_i \ln x_i \right) \left( T^1 (x_1; \ldots; x_6) \right) + \delta_{22} \left( \sum_{i=1}^{6} \gamma_i \ln x_i \right) \left( T^2 (x_i) \right)$$

(3.35)

Neither form provides an obvious way for raw materials inputs to be subtracted from both sides of the equation to result in a value-added specification, which is why the result of Sato (1975) is essential. It means that expressions of the form of (3.32) or (3.33) can be used to test whether the translog function in seven inputs can be expressed as either (3.34) or (3.35). Chapter 5 discusses the empirical implementation of the tests in this study and others.

### 3.7 SUMMARY

This chapter has shown that HEC and elasticity of factor price parameters will be estimated on the basis of a production function and that the AES and elasticity of factor quantity parameters will be estimated on the basis of a cost function. It has shown that the flexibility of the translog function enriches a study of elasticities, and that technological restrictions can be imposed if they are consistent with the data. In showing how the elasticity parameters are derived from the higher order coefficients, the symmetry between the HEC and AES estimates was revealed. The chapter also showed that the separability can be tested for; which is appropriate before using a value-added specification.

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7 Sincere thanks to Eric Budish for clarifying some of the set terminology in this paper, which allowed me to derive these translog examples.
4. DATA DESCRIPTION AND CONSTRUCTION

4.1 INTRODUCTION

The chapter begins with a description of the manufacturing firm dataset, which forms the basis of this study. The more important constructions based on the dataset, notably of value added, are disclosed here.

There are two potential drawbacks to the firm-level dataset, the first being that it is a cross section. The chapter will propose there are enough candidates to act as controls or instruments for firm-specific effects and hence mitigate omitted variable bias.

The second potential drawback is that there are no factor prices, so these must be constructed. After justifying such the procedure, this chapter dedicates substantial space to explaining how wage data are estimated for each firm using household survey data. In summary, this is done by predicting wages according to characteristics that are common to both the firm and household surveys. The chapter further explains the likely effects on the translog estimates of not accounting for firm-size effects on wages, before suggesting how the imported wages can be adjusted for firm-size. The way the cost of capital is constructed is also presented and the chapter continues by showing how these data are used to calculate total costs and factor cost shares.

4.2 DATA FROM THE FIRM-LEVEL MANUFACTURING SURVEY

The dataset used is from the National Enterprise Manufacturing Survey (NE survey) covering the period of 1998. After adjusting for non-response and outliers, there are about 300 firms with the appropriate variables. Unlike the Greater Johannesburg Metropolitan Council Survey (GJMC survey), the NE survey is national in coverage. For a thorough analysis of the data, see Bhorat & Lundall (2002). Edwards (2003) uses it and includes further descriptions.

The dataset is a single cross section, so variables are required to control for firm-specific effects and avoid omitted variable bias. Fortunately, the NE dataset has a rich set of variables for the purpose. There are nine industries and nine provinces. There is information on whether the firm is a member of a bargaining council or otherwise subject to a bargaining council agreement. Consistent with the view that trade unions are more likely to survive in some industries than others (Booth, 1995) and that they may have a non-price effect on factor quantity, this is a useful
variable. There are also ordinal variables for how much difficulty firms have recruiting each occupation, which may capture something about the nature of the firm’s activity and may also have a non-price effect on factor choice. Other factors range from the percentage of sales that a firm exports, the age of the firm’s equipment, the manager’s satisfaction with productivity, the percentage of raw materials the firm imports and the percentage of assets invested in computers. This is not an extensive list; such a list, with details of construction if necessary, is provided in appendix 4.1. Many of these variables might be better candidates for instruments, an issue discussed in chapter five in the context of controlling for endogeneity in the production function.

The key variables for the production function are the capital stock and employment numbers by occupation group. The five groups are:

- Managerial/Professional
- Sales/Clerical
- Skilled/Artisan (technicians, welders)
- Semi-skilled (machinery operators)
- Unskilled (labourers, security guards)

In the absence of better information, part-time workers are given a weighting of a half in computing a weighted measure of the total workforce. Capital stock is available in currency (Rand) values. The correct procedure is to adjust this for capacity utilization, but, while this percentage is available for large firms, it is not for small firms. Using data on the actual and maximum average shift length and the number of shifts per week, it is possible to construct a shift capacity utilization variable as a reasonable proxy. The capital input is therefore the capital stock adjusted for shift capacity utilization.

There is information on what percentage of total costs is comprised of raw materials costs, but there is no data on total costs or on raw materials costs. To derive a measure of raw materials costs, it is necessary to assume that turnover equals total costs. Instead of multiplying raw materials as a percentage of costs by costs to get raw materials costs, raw materials are multiplied by turnover to get a measure of raw materials costs. Value added is constructed as sales minus the constructed raw materials. The implications of any measurement error are discussed in chapter 5.

All variables except some controls are converted to logs as appropriate. The translog function uses higher order terms. For example, there will be an interaction term for half the log of capital

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8 The dataset is a combination of two questionnaires, so not all questions are exactly the same.
multiplied by the log of unskilled labour, say \( \beta_{16} \), and one for half the log of unskilled labour multiplied by the log of capital, say \( \beta_{61} \). By imposing one of the conditions for profit maximization \( \beta_{ii} = \beta_{ji} \) directly, it is far more convenient to construct only one interaction term and double it; i.e. to have a variable for the log of capital multiplied by the log of unskilled labour. All other restrictions are tested before being imposed.

Chapter 3 suggested the estimation of factor output share equations instead of the production function. There are no data on factor shares. For the cost function and cost share estimation, wages are collected and used in constructing factor shares, but it is deemed better to perform the production estimations using data from the firm survey only instead of importing them from external sources. Therefore, the single translog production function is estimated. The factor shares necessary for constructing the elasticities will be predicted using the coefficients from the production function (see chapter 3.4).

4.3 WAGE CONSTRUCTION

The National Enterprise dataset does not have wage data. Edwards (2003) instruments for wages and other industry specific factors by including industry dummies in his labour demand equations, which is inappropriate in a study where wages form an integral part. Therefore, appropriate wage information is transplanted from household survey data, where characteristics common to both the NE survey and the household data are used to predict wages by occupation for each firm. This section justifies the procedure and describes it in more detail. Thereafter, it demonstrates the need to adjust wages for firm-size effects and demonstrates a way to do this.

4.3.1 Why firm-level wages can realistically be represented by supra-firm data

Average wages by industry and occupation are a good approximation to those faced by firms in South Africa. Nattrass (2000) reports that the main wage setting institutions are industrial level bargaining councils (BC), noting that 65% of manufacturing workers are covered by a BC. Furthermore, the Minister of Labour is obliged to extend BC agreements to non-members. Nattrass concludes that extension is at the core of wage setting in an industry. Also, Moll (1996) shows how extensions of bargaining council agreements make some firms become more capital intensive and other firms, which tend to be small and labour intensive, leave the industry. This leads to convergence in technologies and wages in the industry.
The NE survey provides data on whether the firm is subject to collective bargaining and/or a BC agreement. On average, over 70% of firms are subject to a BC agreement. Small firms (< 50 employees) are almost 100% covered while large firms vary from 32% to 61% by industry in coverage. There is therefore support for convergence of wages in industries and justification for wages being calculated at a supra-firm level. However, the NE survey does not reveal which occupations within a firm are subject to BC wages, so it is not clear whether only unskilled workers are covered or perhaps all production workers. One could argue that more skilled wages are less likely to be influenced by collective bargaining, but the household data show that the proportion of trade union membership does not vary much across occupation. Even if not influenced by bargaining, more skilled people tend to be mobile, which standardises wages across firms through ordinary market processes.

Chapter 2 introduced arguments for why wages may vary because of firm-specific characteristics, which means they are not fully exogenous and therefore not perfectly suited to cost equations. However, constructing the wages using these exogenous factors succeeds in removing the endogenous component from wages, so the process of predicting the wages using other criteria could crudely be described as a two-stage procedure with instrumented wages. Like all such procedures, one still has to worry about the quality of the “instruments”.

Predicting wage data has precedence. Teal (2000) generates predicted values from earnings functions using a matching panel: at the same time as firm-level data was collected, employees within the firms were asked questions on earnings and related variables. Classifying workers as skilled or unskilled, he generates firm-level wages using the human capital characteristics observed in those workers sampled for each firm, controlling for other factors. Unlike his work, this study unfortunately neither uses matching data nor is able to draw on individuals’ characteristics, because neither feature is available in the NE survey.

4.3.2 Data and procedure

The 1997 October Household Survey sample was reduced to include those 3 500 people working for somebody else in formal manufacturing industries. Definitional correspondence to the NE survey in terms of industry, province and occupation is good, but, as explained in 4.3.1, the

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9 The 1998 survey was much smaller due to funding problems. This and an allowance for adjustment lags make the 1997 survey the preferred edition. Inflationary increases are easily dealt with.
correspondence regarding union membership / collective bargaining is not. Details of the dataset and survey methodology are available in Statistics South Africa (1998).

Survey design effects are taken into account, because not accounting for the probability weights can bias estimates. People in the 10 households interviewed per geographical cluster don’t have independent characteristics, being more likely to have similar features, so the survey sample variance of the wage would be lower than would be the case in a random sample. Failing to account for clustering often results in standard error estimates that are half what they should be. In contrast, stratification guarantees that this similarity will not happen across strata. It mitigates the chances of there being a non-representative sample and therefore standard error estimates should (correctly) be lower than in the absence of stratification Deaton (1997).

This study accounts for probability weights and clustering but only partially adjusts for stratification. The reason for this is that many magisterial districts (strata) have only one cluster – many have only one observation – and at least two are needed for variance estimates. A standard procedure for dealing with this is to collapse or merge strata (Statacorp, 2003a), but the number of cases to collapse is high in this study. Therefore, compromise stratification by province, which sometimes has close to 100 magisterial districts, is carried out. An aggregate estimate of monthly salary has a Deff statistic, which is the ratio of the estimated variance accounting for survey design to the unadjusted variance, of 2.44. This indicates the importance of dealing with the survey design effects.

For each occupation, the characteristics available in both data sources are:

- economic activity (broken down into nine industries)
- province group (the nine provinces were ex post broken down into two groups with similar wages)
- individual trade union membership (household data); collective bargaining and bargaining council membership (firm data)

Construction entails calculating the survey-adjusted means for groupings of people for each occupation. A total of four different wage series are constructed for each of the five occupations. The next few pages describe each of four wage series, namely wage_{ind}, wage_{all}, wage_{some} and wage_{size}.

1) Wage_{ind} only classifies wages by industry. This measure is used mainly as a check against other series and for consistency in preliminary analysis. It provides too course a measure of wages to be used in regressions. Because such data are not available elsewhere, a table of
average wages by occupation and industry, using only the household data, is provided in appendix 4.2\(^\text{10}\). The manufacturing industry data are consistent with the overall manufacturing wage calculated by Statistics South Africa (2000). Using the same survey, they calculate a mean unskilled hourly wage of R7.86 for example, which corresponds roughly to R 1 400 (£120) per month.

2) It is appropriate to classify wages further by location and trade-union membership to generate \( \text{Wage}_{\text{all}} \). There are nine industries and nine provinces, meaning that, together with a trade-union membership dummy, there are potentially 162 different wages. However, while some means are calculated using a comfortable number of observations, others are based on few data points, sometimes only 1. This means the standard errors on the wage estimates can be high (or non-existent). To mitigate this, the nine provinces are divided into two groups, as variation within each of the two groups is low.

3) However, a third measure (\( \text{Wage}_{\text{some}} \)) is also generated. It achieves more precise estimates by combining some locations and industries and/or not distinguishing by trade-union membership in cases where wages do not differ substantially. Before discussing the process, it is helpful to look at one example of classifications, so table 4.1 presents six of the fifteen groups the skilled/artisan wages are divided into and the associated estimates.

<table>
<thead>
<tr>
<th>Mean Monthly Salary: Skilled/Artisan</th>
<th>Estimate</th>
<th>Std Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; Beverages</td>
<td>1562</td>
<td>161</td>
</tr>
<tr>
<td>Wood, Pulp &amp; Paper - Prov0</td>
<td>1116</td>
<td>229</td>
</tr>
<tr>
<td>Wood, Pulp &amp; Paper - Prov1</td>
<td>1993</td>
<td>169</td>
</tr>
<tr>
<td>Chemicals, Rubber &amp; Plastic - Prov0, not unionised</td>
<td>786</td>
<td>152</td>
</tr>
<tr>
<td>Chemicals, Rubber &amp; Plastic - Prov0, unionised</td>
<td>2316</td>
<td>264</td>
</tr>
<tr>
<td>Chemicals, Rubber &amp; Plastic - Prov1</td>
<td>2067</td>
<td>284</td>
</tr>
</tbody>
</table>

Table 4.1: Some of the groups into which \( \text{Wage}_{\text{some}} \) is classified.

The first row contains wages for all skilled/artisans in the Food & Beverages industry, regardless of location or union membership. The Wood Pulp & Paper industry is subdivided by province group but not union membership (rows 2 and 3). Wages in the Chemicals, Rubber and Plastic industries are subdivided by province group. One group of provinces is further divided into unionised and non-unionised workers (rows 4 and 5) while the other group is not (row 6). In some cases, industries are combined, with the possibility of disaggregation by other criteria.

\(^{10}\) The data are also available at http://www.nuff.ox.ac.uk/users/Behar/data/wage1data.xls
This selection could of course be seen as a prediction from a regression. However, all the explanatory variables are dummies or interactions thereof, so calculating means for each group is an equivalent and more convenient procedure, especially when taking account of multiple interactions.

Classifying the wages for \( \text{Wage}_{\text{some}} \) involves a number of tradeoffs. While averages across two or more groups are different, the standard errors may be large, resulting in imprecise estimates. This is often because of a small sample size for that group. One way to proceed is to separate all groups with statistically significant differences in means. However, this is imperfect. An extreme but not infrequent occurrence is that of one observation per group, generating no standard error and occurring outside the confidence interval of another group. Similarly, inference based on very few observations is not reliable. On the other hand, some estimates, even if based on few observations, are so radically different that the groups should be classified separately. The aim is to produce a set of estimates per group with better precision characteristics but sufficient variation to represent the firm-level data. To do this, various combinations are carefully inspected. Factors considered are differences in log wages, the number of observations, and comparisons of the standard errors and confidence intervals of the separate and combined groups. Of course, all the criteria are related.

Comparing the confidence intervals of two groups is naturally akin to performing a two-sample t-test. However, visual inspection is quicker for all the combinations and allows for analysis in conjunction with the other criteria. The choice of confidence interval is a matter of taste in this application, so 85% bands are used. As a control against this judgement-based procedure, standard t-tests, regressions and non-parametric procedures are performed on certain groups.

Going through the above procedure on a case-by-case basis therefore produces a set of wages, for each occupation, which partially disaggregates each industry by location and/or trade union membership in a way that optimises the trade-off between achieving representative wage estimates and having precise estimates. Appendix 4.3 shows the number of categories for \( \text{wage}_{\text{some}} \) ranges from 7 to 15, with the number of observations per category ranging from 16.9 to 43.7. The average number of observations for \( \text{wage}_{\text{all}} \) ranges from 3.9 to 14.6. It is important to stress that most groups of wages do not differ substantially from those in \( \text{wage}_{\text{all}} \), the exceptions

\footnote{Tests of median equality are performed, but they do not factor in survey design. The results do not imply material differences in classification. Another useful way to compare specific groups is to use Anova and Scheffe’s method of comparing the means of each group to those of all the others (Van den Honert, 1997). This method is used but there is also no readily available way to adjusting for survey design.}
being those in wage_{all} based on very few observations and in which one can have very little confidence anyway.

4) The fourth measure of wages is wage_{size}. It adjusts wage_{some} for firm-size, as discussed next.

### 4.3.3 Adjusting wages for firm-size

The start of the chapter suggested there were no firm-specific variables by which the household wages could be classified and that this preserves their exogeneity. However, there is a way to account for firm size. This section (and chapter seven) will show why failing to account for firm size in wages explains some initially poor results. Any exogeneity issues are minor in that context, so a way to adjust wages for firm-size using existing estimates is proposed.

Chapter 2 suggested firm-size may affect wages. The following paragraphs explain what impact ignoring this effect may have on translog estimates, concluding that the estimations are more likely to (falsely) reject homotheticity and linear price homogeneity and overstate returns to scale. Abstracting from individuals’ characteristics, wages for occupation $i$ can be seen as a simple function of firm size measured according to sales ($y$) and those variables available from the household survey ($x$).

$$
\ln w_i = \beta \ln x_i + \gamma_i \ln y; \quad \gamma_i > 0
$$

(4.1)

Algebra shows a translog cost function without accounting for firm size is the same as:

$$
\ln C = \sum_i a_i \ln w_i + \Gamma \ln y + \sum_j \sum_i \frac{1}{2} b_{ij} \ln w_i \ln w_j + \Phi \ln^2 y + \Omega \ln w_i \ln y
$$

where

$$
\Gamma = \sum_i a_i \gamma_i + a_y; \quad \Phi = \sum_j \sum_i \frac{1}{2} b_{ij} \gamma_i \gamma_j + \sum_i b_y \gamma_i + b_{yy}; \quad \Omega = \sum_j \sum_i b_{ij} \gamma_j + \sum_i b_y
$$

(4.2)

The coefficients containing value added may be vastly different to what they are supposed to be. To gauge the likely nature of the bias in a simple setting, we ignore the econometric reasons for the estimates on the $a$ and $b$ coefficients being biased and assume they are correctly estimated. Furthermore, on the assumption that linear price homogeneity and constant returns to scale:

$$
\begin{aligned}
\sum_i a_i = 1; \sum_i b_y = \sum_j b_{ij} = 0; b_{yy} = b_{yy} = 0; a_y = 1
\end{aligned}
$$

are valid for the true cost function:

$$
\Gamma' = \sum_i a_i \gamma_i + 1; \Phi' = \sum_j \sum_i \frac{1}{2} b_{ij} \gamma_i \gamma_j; \Omega' = \sum_j \sum_i b_{ij} \gamma_j
$$

(4.3)
We can’t be sure $\Gamma > 1$, as it is not necessarily the case that all $a_i > 0$, neither in theory (Varian, 1992) nor in the results in chapter 7. However, linearly homogeneous prices imply that, if all the values of $\gamma_i$ for each occupation are close enough to the average across occupations, the result will tend to be an upward bias on the value added coefficient. If the firm size effect is equal for all occupations, the bias is $\gamma$.

It is not possible to tell what direction the bias will be for $\Phi'$. However, if there is an equal firm size effect, price homogeneity implies this will be zero and in fact not biased. If the firm-size effect is not equal for each occupation, there is the possibility of $\Phi'$ being found significant when it actually is not. This would falsely reject a homogeneous technology. A similar analysis concludes the coefficient on $\Omega$ may be found significant and therefore falsely reject homotheticity or that linear price homogeneity is rejected by distorted coefficient values.

To understand the likely effects on returns to scale, assume for simplicity a common firm-size effect across all occupations. The assumption of a homogeneous technology is relaxed but homotheticity and price homogeneity are maintained. Returns to scale are given by:

$$\left[ \frac{\partial C}{\partial y} \right]^{-1} = \left[ \gamma + a_y + b_y \ln y \right]^{-1}$$  \hspace{1cm} (4.4)

Using these assumptions, one can gauge that omitting the firm size variable will underestimate the denominator by $\gamma$ on average, so returns to scale will be overestimated. This is intuitive: if wages rise for bigger firms, the returns to scale are less than otherwise. Therefore, including a measure of $\gamma$ will reduce the estimated returns to scale.

Given the possible problems with ignoring firm-size effects, ways of capturing them must be found. There is no information on the size of firms individuals in the household survey work for. One way to proceed might be to assume constant returns to scale or some other degree of homogeneity, estimate the unconstrained model, and see the level of $\gamma$ necessary to produce the assumed value. Another is to attach values of $\gamma_i$ to the wage series. Bhorat & Lundall (2002) estimate the following manufacturing firm-size wage effects using GJMC data:

<table>
<thead>
<tr>
<th>Occupation</th>
<th>Managers &amp; Technical</th>
<th>Clerks</th>
<th>Sales &amp; Clerical</th>
<th>Craft</th>
<th>Operators</th>
<th>Labourers</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Managers</td>
<td>0.089</td>
<td>0.076</td>
<td>0.09</td>
<td>0.066</td>
<td>0.096</td>
<td>0.094</td>
<td>0.031</td>
</tr>
<tr>
<td>Clerical</td>
<td>0.066</td>
<td>0.096</td>
<td>0.094</td>
<td>0.096</td>
<td>0.094</td>
<td>0.031</td>
<td>0.065</td>
</tr>
<tr>
<td>Labourers</td>
<td>0.031</td>
<td>0.065</td>
<td>0.065</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2: Estimates used to infer firm-size effects; all except Labourers were significant

Their estimates are rudimentary, using only average firm wages and annual firm sales, but they claim similarity to the US study of Doms, Dunne & Troske (1997). The omission of other firm
effects like age and capital intensity and of individual characteristics means these numbers could be too high. This study does not adjust wages for other factors because no number is available, so the estimates in table 4.2 are taken, being adjusted using averages to match the occupations in the NE data. Assuming the unadjusted wages represent those for an average-sized firm, wage_{some} is inflated/deflated accordingly. This fourth wage is denoted by wage_{size}.

In summary, household survey data are used to generate four sets of wage estimates by occupation. One set uses the nine industries as groups (wage_{ind}). Another has a separate group for each combination of two locations, trade union membership and industry (wage_{all}). Given the trade-off between wage heterogeneity and the precision of estimates, a third set of wage estimates allows for some industries, locations and trade union membership to be combined and for others to be separated (wage_{some}). Wage_{size}, adjusts wage_{some} for firm-size effects. Finally, data from the Trade and Industrial Policy Secretariat shows wages rose by approximately 15% between the time of the household survey and the time of the NE survey. Wages are therefore raised by this percentage.

### 4.4 COST OF CAPITAL CONSTRUCTION

The Jorgenson (1963) “cost of capital” is:

\[
c = q \left( \frac{1 - uv}{1 - u} \delta + \frac{1 - uw}{1 - u} r \right)
\]

(4.5)

\(c\) is the cost of a unit of capital, \(q\) is the price of machines, \(\delta\) is the rate of depreciation, \(r\) is an interest rate, \(u\) is the corporate tax rate while \(v\) & \(w\) are the proportions of depreciation and interest expenditure chargeable against income for tax purposes. When capital is aggregated in a static setting, \(q\) is normalized to one and (4.5) is a percentage cost per unit of capital. Holding \(v=w=1\) yields:

\[
c = \delta + r
\]

(4.6)

Clague (1969) uses this version in his early study of developing countries. \(v = 1\) in South Africa and \(w\) is at least one because of accelerated depreciation allowances. Because \(w\) does not equal one and because firms pay tax even on normal profits, the dropped tax term needs to be reintroduced.

In an industry-level study of capital in South Africa, Fedderke et al (2001) use the following slightly ad hoc expression:

\[
c = (r - \pi) + \delta + \tau
\]

(4.7)
\( \Pi \) is the inflation rate and \( \tau \) is the corporate tax rate. Fedderke et al calculate industry-level data on \( \delta \) ranging from 11\% to 16\%.\textsuperscript{12} For the nominal interest rate (\( r \)), they use yields on 10-year government bonds and consumer price inflation, but we use the average prime lending rate and consumer price inflation for 1999.

Furthermore, we adjust the interest rate to account for risk. Adjustments range from –2\% for large firms older than 5 years to + 5\% for new small firms.\textsuperscript{13} This amendment can be accused of confusing Jorgenson’s notion of the cost of capital with the weighted average cost of capital developed by Modigliani and Miller in 1958 (Lau, 2000). Bergström & Panas (1992) use a weighted average cost of capital measure in their study, while Teal (2000) constructs predicted profit rates as a percentage of the capital stock with regressions containing firm- and industry-specific variables. In equilibrium, these two costs should be equal (Lau, 2000). To the extent that this equilibrium does not hold realistically, and because of the practical reality that smaller and newer firms have higher borrowing costs, accounting for these risk premia is necessary.

Fedderke et al (2001) use the nominal corporate tax rate for \( \tau \), which was 35\% for the fiscal year starting early in 1998 (RSA, 1998). They state it would be ideal to have the effective rates of taxation by industry as this is another source of divergence in costs of capital. Negash (1999) calculates effective tax rates to be about 15\% below nominal rates for the 1990s, so a 20\% average effective rate is applied to all firms. The resulting range of costs of capital in appendix 4.4 vastly improves on standard studies that simply assume a uniform interest rate as the cost of capital, like Maki & Meredith (1987) and Fallon & Layard (1975).

4.5 TOTAL COSTS AND FACTOR COST SHARES

Wages are also used in the determination of cost shares and total costs. The vast majority of studies, including but not restricted to Binswanger (1974a), Berndt (1973b), Teal (2000) and Bergström & Panas (1992), derive total cost and/or factor cost shares using factor price and quantity data. This means that variables on the right hand side of the share equation are used to construct the dependent variable, but there is no readily available alternative.

Labour costs are obtained by multiplying labour quantities by the constructed wage; because there are four wage variables, there are four cost variables. Capital costs are the cost of capital.

\textsuperscript{12} I thank Prof Fedderke for providing this data.
\textsuperscript{13} Adding 5\% is the standard rule of thumb premium added for new small ventures in South Africa.
percentage multiplied by the capacity-adjusted capital stock. Total factor cost \((C_f)\) is the sum of factor costs. Raw material input data is available only as a percentage of total costs \((p)\), so total input cost \((C_i)\), including raw materials, is calculated as \(C_i = \frac{C}{1-p}\). Raw materials costs \((rm)\) are easily derived and subtracted from output to get a second measure of value added.

Table 4.3 considers this value added measure \((V_2)\) and compares it with the value added measure calculated earlier by multiplying \(p\) by turnover \((V_1)\). While wage\(_{some}\) is used in this table, the results are consistent for all wage definitions.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>(V_1=py)</th>
<th>(V_2=y-rm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>8</td>
<td>8.71</td>
</tr>
<tr>
<td>p25</td>
<td>1.2</td>
<td>0.77</td>
</tr>
<tr>
<td>p50</td>
<td>2.8</td>
<td>3.21</td>
</tr>
<tr>
<td>p75</td>
<td>8.75</td>
<td>9.63</td>
</tr>
</tbody>
</table>

Table 4.3: Comparison of value added measures in R million. The first column uses data on raw material cost percentages and sales. The second uses data on raw material cost percentages, factor prices (wage\(_{some}\)) and quantities.

The measures of central tendency are close but there is moderate dispersion at the 25\(^{th}\) and 75\(^{th}\) percentiles. Correlations between the first measure and the wage-based measures vary from 0.89 to 0.91, depending on the wage definition. The similarities are considerable, in spite of the difference in calculation and the vulnerability of both calculations to error, so there are grounds from some confidence in the estimates.

In cost estimations, \(V_2\) would introduce very serious correlation with the dependent variable, which was constructed using the exact same factor prices and quantities. \(V_2\) would also be highly correlated with the other inputs. Therefore, while useful for comparison with \(V_1\), \(V_2\) is not used in regressions. \(V_1\) is used in the cost function (and in the production function).

Table 4.4 presents constructed factor shares of total factor cost, calculated from wage\(_{some}\).

<table>
<thead>
<tr>
<th>Statistic</th>
<th>ManProf</th>
<th>Salecle</th>
<th>Skilart</th>
<th>Semi</th>
<th>Un</th>
<th>Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>8.7%</td>
<td>6.9%</td>
<td>6.6%</td>
<td>15.3%</td>
<td>10.8%</td>
<td>50.9%</td>
</tr>
<tr>
<td>median</td>
<td>6.1%</td>
<td>4.4%</td>
<td>2.8%</td>
<td>9.8%</td>
<td>6.7%</td>
<td>52.2%</td>
</tr>
<tr>
<td>p5</td>
<td>0.9%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>12.8%</td>
</tr>
<tr>
<td>p25</td>
<td>3.2%</td>
<td>2.2%</td>
<td>0.1%</td>
<td>4.5%</td>
<td>2.4%</td>
<td>32.1%</td>
</tr>
<tr>
<td>p75</td>
<td>11.5%</td>
<td>8.6%</td>
<td>9.2%</td>
<td>22.5%</td>
<td>14.9%</td>
<td>69.2%</td>
</tr>
<tr>
<td>p95</td>
<td>24.8%</td>
<td>22.3%</td>
<td>25.1%</td>
<td>44.6%</td>
<td>36.0%</td>
<td>86.1%</td>
</tr>
</tbody>
</table>

Table 4.4: Share of each factor in total factor cost (wage\(_{some}\))
For comparison, the mean shares of skilled labour, unskilled labour and capital for Ghana in Teal (2000) are 11%, 29% and 60%. This corresponds roughly to 16% for the Managerial/Professional and Sales/Clerical categories, 33% for the three less-skilled categories, and 51% for capital.

Roughly 5% of labour shares are zero, simply because of zero reported labourers in that category. These firms are at corner solutions for their factor demand, signifying a sample selection problem that is common in demand studies. The phenomenon is especially pertinent in the translog context, because many calculations require division by one or more factor shares. Berndt (1991) asserts that, instead of actual shares, one should use predicted factor shares from the regression in such calculations (not necessarily because of this problem). This may avoid division by zero, but one condition for profit maximisation is strictly positive factor shares (Berndt & Christensen, 1973a). The post-estimation predicted shares should be tested for this feature, but clearly, under these circumstances, such a test is bound to fail as even relatively precise estimates will yield negative shares for some occupations in some firms. Nonetheless, these issues do not merit the additional complications of procedures designed to account for them.

4.6 SUMMARY

The variables needed for the production estimates are reasonably easily derived. In contrast, labour prices for each occupation are predicted by matching firm characteristics with those of a household survey for use in cost function estimates. The three different wage variables derived this way differ in the level of disaggregation, as summarised at the end of section 4.3.3. The chapter shows that ignoring firm-size effects on wages can falsely reject homotheticity, falsely reject price homogeneity and/or overestimate returns to scale before proposing a method to adjust wages for firm size. Factor shares of cost and total costs are derived using the wage data, and preliminary analysis of two constructed value added measures provides grounds for optimism regarding their quality.
5. FURTHER ISSUES IN ESTIMATING PRODUCTION AND COST FUNCTIONS

5.1 INTRODUCTION

This chapter addresses important practical issues relevant to estimating translog functions. The first is separability, which is necessary for the value-added specification to be appropriate. The chapter shows that the value added specification, which allows for raw materials to be dropped and value-added to be the regressand instead of gross output, is valid and preferable. The second issue is heteroskedasticity; a brief indication of how it is treated and used to improve specification is presented. The third issue is endogeneity bias: because the factor inputs may be endogenous to unobserved phenomena, coefficients may be biased. This chapter will assign substantial space to the possible problem, concluding that instruments are too weak and unreliable for two-stage least squares (2SLS) to improve on OLS with controls/proxies and that formal tests suggest there is no need for 2SLS in the first place.

The fourth issue is a review of the Iterated Zellner Efficient (IZEF) procedure used to estimate the system of factor cost shares. This is a standard approach so the discussion is brief. The fifth issue is inference. Unlike many other studies, this study conducts inference on the elasticities, which are functions of estimated coefficients. Besides constructing confidence intervals using the delta method, this chapter explains how the fact that elasticities vary across the sample can be used to infer whether summary estimates are truly representative of the firms in the sample.

As a prelude to some of the discussions, it is important to emphasise the relative importance of efficiency to this study. While not proposing to overturn the accepted preference for unbiased estimates over efficient ones, the nature of this study is that, if necessary, a moderate degree of bias should be incurred to gain efficiency. The main reason is policy-relevance. It is not too important whether an elasticity is 0.4 or 0.6, but it is important to know how sure we can be that the elasticity is positive. To the extent that input or factor price endogeneity / exogeneity does not hold perfectly, the coefficient estimates should perhaps not be taken too literally anyway. What matters more is the direction of the effect – do unskilled workers gain or lose – and the confidence one can have in that coefficient sign. Furthermore, the elasticities are functions of coefficients, meaning that the effects of imprecise coefficient estimates could multiply to such an extent that any elasticity estimate could be practically meaningless.
5.2 MOTIVATION OF VALUE-ADDED SPECIFICATION

Chapter 3.6 showed that separability is important for justifying the value-added specification instead of regressing gross output on the factors and raw materials. It also showed what this means in terms of the translog coefficients. The next three pages deal with empirical aspects of separability before using test results to justify the value-added specification in the production function and the use of value added as a regressor in the cost function.

Of the 12 studies listed by Chung (1994), only three test for value-added separability while Hamermesh (1993) does not discuss value-added separability in his comprehensive survey, although a lack of gross output and/or intermediate input data prevent such tests in many cases. In translog production studies, Grant & Hamermesh (1981) express output, taken from annual manufacturing censuses and surveys, in terms of capital and four forms of labour, while Mak (2000) does not have raw materials inputs in her system of share equations. Neither tests the validity of omitting raw materials. In cost studies, Binswanger (1974a) has no output or value added term, while Bergström & Panas (1992) and Teal (2000) have value added in their cost share equations but don’t mention any tests for separability.

Denny & May (1977) generally reject separability in both their production and cost function estimates. They ascribe differences between their elasticity findings and others to invalid value added estimates in other studies. However, Humphrey & Wolkowitz (1976) find no significant difference between cost function estimates that impose separability and estimates that don’t: as an alternative to the process discussed in chapter 3.6, Berndt & Christensen (1973a) show a factor is separable if the Allen elasticities of each of the other factors with respect to that factor are equal. Humphrey and Wolkowitz estimate cost functions with the restriction that the AES between capital and raw materials equals the AES between labour and raw materials and estimate the cost function without this restriction to reach their conclusion.

Full output regressions yield sensible results in this study, and can therefore be used to test separability restrictions. Wald tests of additive separability in the form of (3.31) are rejected, but tests of non-additive separability in the form of (3.32) for each combination of factors do not reject separability. The tests therefore justify the use of the value added specification.

However, two issues remain unresolved. The first is why one would prefer to use such a specification if raw materials are available. The second is why separability motivates the value
added specification rather than regressing gross output on the factors excluding raw material inputs. In aggregated studies, value added avoids issues of double counting (Sato, 1975), but the reasons in a firm-level study are not obvious. We tackle these two issues in order.

This study constructs value added using output, which introduces measurement error to value added and the raw materials inputs. The first argument in favour of omitting raw materials is that having measurement error in the dependent variable is less serious than having it in the dependent variables. In the value added specification, let $\nu$ be measured value added and $\nu^*$ actual value added. The measurement error is therefore $e = \nu - \nu^*$. $\nu^*$ is sales(y) minus raw materials ($rm^*$) whilst $\nu$ uses constructed raw materials $rm = py$, where $p$ is data on raw materials as a percentage of costs$^{14}$. Therefore, the error is the extent to which income differs from costs:

$$e = p(y-c)$$ (5.1)

The production function estimate for actual value added is effectively:

$$\nu^* = a + f(\cdot) + \epsilon + p(y-c)$$ (5.2)

where $f(\cdot)$ is the estimated value added function in terms of the factors (\cdot) and $\epsilon$ is the random error. On the assumption that actual value added and profits are uncorrelated $\text{cov}(\nu,\epsilon) = 0$, the estimates of $f(\cdot)$ are not affected. This draws on the standard result in Wooldridge (2002). Because we expect firms to make profits at least on average, $E > 0$ so the constant will be a biased estimate. There is added noise in the error term, making estimates less precise (ibid.).

While having the error in the dependent variable is usually harmless, having it in one of the explanatory variables can bias coefficients of more interest. Assume the non-separable true output function is$^{15}$:

$$y = f(\cdot, rm^*)$$ (5.3)

In the presence of measurement error, the estimate is of:

$$y = f(\cdot, py) + \omega$$

$$= f(\cdot, pc) + f'(\cdot, p(y-c)) + \omega$$ (5.4)

$\omega$ is the error term and $f'(\cdot, p(y-c))$ is some function of the inputs and measurement error. (5.4) reveals many sources of correlation between the right-hand-side variables and the

---

$^{14}$ This was explained in more detail in chapter 4.2
$^{15}$ While the constant is also affected, it is suppressed in the proceeding analysis.
composite error term (in square brackets) and therefore endogeneity bias. This is an example of the errors in variables problem, which results when one assumes that measurement error is uncorrelated with the unobserved true measure of raw materials. The high multicollinearity between the inputs, shown in appendix 3.1, means the biases on all the estimates could be quite high (Wooldridge, 2002).

However, omitting raw materials can analogously introduce omitted variable bias. Omitting raw materials from (5.3) would result in estimation of

\[ y = f'(\cdot) + [f''(\cdot, pc) + w] \]  

(5.5)

This creates endogeneity bias both through the presence of the six factors in some form in the error term and the correlation between raw materials and the factors estimated. If the function is additively separable as in (3.34), the estimated function is

\[ y = f(\cdot) + [g(pc) + w] \]  

(5.6)

This also creates potential endogeneity bias through the correlations between raw materials and the factors, but one of the consequences of separability is that raw materials can be omitted without influencing the parameters of interest\(^\text{16}\).

The second argument for omitting raw materials is that the constructed raw materials variable is a function of output, which would be the dependent variable. This creates an artificial positive correlation between raw materials and output. It also creates an artificial correlation between output and those terms interacted with raw materials. The high correlation between inputs means all the estimates will be incorrect. These two arguments, and the fact that the number of factors to be estimated falls by seven, motivates the omission of raw materials.

Having decided to exclude raw materials, the matter of whether to have gross output or value added as the dependent variable remains. There is no example of gross output being used in a study that has the choice not to. There certainly are intuitive grounds for this as it is more sensible to fit value added to factors than to fit gross output, even if separability technically validates both.

\(^{16}\) There is no guidance in the literature as to whether omitting a separable variable results in unbiased regression estimates as opposed to biasing them in a way that doesn’t affect marginal rates of technical substitution. This raises the possibility that the validity of omitting a variable, in an econometric sense, justified by separability provides analogous arguments for not being concerned with measurement error. Preliminary investigations suggest the issue is not a simple one and that analysing separability in an econometric framework is an area for future research.
However, one concern for this study is the potential efficiency loss associated with the measurement error in the constructed value added measure. Initial comparisons of value-added and gross output regressions show the latter has no efficiency advantages in terms of root mean square errors and $R^2$ values. More importantly, the coefficients and implied elasticities do not differ materially. We therefore follow intuition (and the literature) and present only results from value-added estimation.

### 5.3 HETEROSEDASTICITY

Given the sample size, the unavailability of more precise systems estimators, and the variance-inflating nature of the highly correlated data, any steps to improve efficiency are worthwhile. This includes zealous attention to heteroskedasticity.\textsuperscript{17} This is especially the case because the elasticities are a function of many coefficients: errors might get grossly multiplied, leading to very wide confidence intervals for the elasticity estimates, so a slight improvement in the efficiency on of each of the coefficient estimates could have a material impact on the elasticities. Besides, insofar as heteroskedasticity with respect to a particular variable is an indicator of specification error, such attention will lead to the reduction of omitted variables bias and produce better-fitting equations.

A battery of tests are implemented, including the Breusch & Pagan LM tests for multiplicative heteroskedasticity (with respect to all explanatory variables as well as fitted values), the less general but more powerful Szroeter test for residuals that are monotonically increasing in each of the variables, and the information matrix decomposition into skewness, kurtosis and heteroskedasticity (Statacorp, 2003a; Greene, 2003).

Tests and diagrams (in appendix 5.1) show the translog approximation does not fit nearly as well for very small firms as for the rest of the sample, causing efficiency reductions and heteroskedasticity. The crude but effective solution is to drop firms with a value added of less than R1 million (about £ 80 000) at the cost of 30 observations. Tests on the slightly restricted sample find overall insignificant heteroskedasticity but, at first, find moderately strong relationships between the errors and specific variables. Cross plots usually identify the source of the problem. For example, appendix 5.1 reveals a high positive mean error when the managerial/professional category has one worker, which indicates there are efficiency gains.

\textsuperscript{17} White’s Robust Standard Errors (see Greene (2003)) are not available with the constrained regression command in Stat. Even if they were calculated, they only work well for moderate heteroskedasticity and, while they may be more suitable for inference, they do not improve efficiency characteristics.
enjoyed by individual- or owner-managed firms. As expected, the dummy created to represent this is highly significant. Steps like these were highly successful at eradicating heteroskedasticity, indicating well-specified models, improving efficiency and meeting a necessary condition for valid inference.

5.4 PRODUCTION FUNCTIONS AND ENDOGENEITY BIAS

One problem with estimating production functions is the potential for endogeneity bias. Specified such that exogenous inputs generate the endogenous output, they ignore the possibility that inputs are chosen by the firm to achieve a given or profit-maximising level of output. If the assumptions of exogenous factor inputs consistent with the production function and elasticity concepts held perfectly, there would be no problem, but deviations from the assumption have the implication that econometric estimates would be biased. While firm-specific technology is also a source of bias in cost and production estimates, there are enough suitable variables to act as controls for these effects reasonably well, so this is not the issue discussed here.

Because of the importance many researchers attach to endogeneity, and their preference for 2SLS, considerable space is devoted to it. The next seven pages motivate why OLS with proxies is the best of all the options available, including 2SLS. They begin with a detailed discussion of the problem and possible methods for addressing it. The conceptual differences between using proxies and instruments are presented before performing estimations with both approaches. The very clear conclusion is that 2SLS does not necessarily improve the bias characteristics of the coefficients but grossly amplifies the standard errors and that formal tests suggest no need for 2SLS in the first place.

5.4.1 The endogeneity problem and responses

The discussion will be restricted to a Cobb Douglas case for simplicity, but applies equally to more general production functions (Levinsohn & Petrin, 2003). Consider a production function:

\[ q_i = \alpha a + \beta_l + \beta_k k + u \]

(5.7)

The variables \( q, k \) and \( l \) are the logs of output, capital and labour for firm \( i \) at time \( t \). \( a \) is a vector of controls for fixed technology or perhaps efficiency variables. \( u \) is the log of the error term with distribution \( e^U \) observed by the econometrician. For the purposes of this discussion, assume that technological differences are satisfactorily captured by control variables. Although this is certainly not a trivial assumption, it would be evidenced by a high \( R^2 \) and low standard errors,
and the dataset has many variables that should control for technological differences satisfactorily.

This error term actually comprises various components unobserved by the econometrician. The first is a factor the producer observes in time to affect his choice of inputs. A common example is the weather. Another, which is more readily applicable to manufacturing, is changes in various market conditions \( \mathbf{m} \) (Griliches & Mairesse, 1995). For simplicity, assume none of these can be observed or controlled for by the econometrician. The error term also comprises errors on the producer’s part. He can make mistakes in predicting the nature of external shocks \( \mathbf{p} \) and be unable to adjust his inputs once \( \mathbf{p} \) is observed. Due to incompetence or institutional inertia, he may not be able achieve the optimal input combination \( \mathbf{e} \). \( \mathbf{e} \) and \( \mathbf{p} \) are therefore unrelated to the observed input quantities (Zellner, Kmenta & Dreze, 1966). Consequently:

\[
q_i = c + \beta \mathbf{m} + \beta \mathbf{p} + e
\]

The producer chooses his factor inputs according to the conditions in \( \mathbf{m} \). \( l_i \) and \( u_i \) are therefore correlated so there is endogeneity bias.

Griliches & Mairesse (1995) observe a number of responses to this problem in the microeconomic context. The first is denial. Grant & Hamermesh (1981), Johnson (1980) and Mak (2000) make no mention of the issue, nor does Hamermesh (1993) in his empirical review. Second, people can estimate cost functions. Given that production functions are being estimated for reasons already documented, this is not an option. The third is to assume \( \mathbf{m} \) can’t be predicted by the producer. If so, \( l \) is not conditioned on any component of the error term and hence OLS estimates are not biased (Zellner et al, 1966). Griliches & Mairesse note that, despite this being published in *Econometrica* by respected authors, people feel “guilty” (pg 5) about using this assumption.

The fourth option is to try to control for \( \mathbf{m} \) through proxies, also known as controls. The proxy approach has regained impetus recently, with the use of lagged investment (or raw materials) inputs as proxies. The reasoning is that investment is a function of \( \mathbf{m} \) (and the capital stock). If this input demand function is invertible, the inclusion of investment therefore captures \( \mathbf{m} \) (Levinsohn & Petrin, 2003). Identifying the parameters on the other inputs is simple, but identifying the capital and/or raw material input parameters requires non-parametric techniques. This approach holds much promise, but seems more appropriate for multiple period data. The reason is that, according to Levinsohn & Petrin, investment is monotonically increasing in \( \mathbf{m} \) only if \( \mathbf{m} \) is stochastically increasing in past values and monotonicity is required for invertibility. The use of investment as a proxy seems aimed at productivity or other shocks to \( \mathbf{m} \) within firms across...
time. This monotonic relationship between investment and \textbf{m} across firms does not hold and is therefore not appropriate in cross-sectional space.

The fifth approach is two-stage-least-squares (2SLS). Before exploring this in more detail, we mention the sixth option, which is to employ panel data methods using firm- and time-specific fixed effects and 2-period-lagged input values as instruments (Levinsohn & Petrin, 2003). Panel methods are not an option for this study so no further attention is paid to this approach.

\textbf{5.4.2 Conceptual comparison of two-stage least squares and OLS with proxies/controls}

Returning to the possibility of using 2SLS, the key issue is whether instruments for the observed endogenous variables are appropriate or whether the data available are more suitable as proxies/controls for the unobservables causing the endogeneity bias. The principal reference for the ensuing discussion is Wooldridge (2002).

As mentioned, the source of endogeneity bias is the correlation between a factor input and the error term. Instrumenting for \( l \) entails regressing \( l \) on the set of relevant exogenous variables \( w \), obtaining \( \hat{l} \) and using \( \hat{l} \) in equation(5.8). \( w \) must include at least one variable \( w_i \) not in the structural equation, which must be partially correlated with \( l \) net of the other variables. In fact, there must be at least one \( w \) for each endogenous variable. However, in this study, there are far too many inputs and interacted inputs for this requirement to be met, so the aim is to find instruments for those thought “most” endogenous or perhaps to instrument for first order variables only, leaving the higher order ones uninstrumented.

The consistency of the coefficient estimates under IV estimation critically relies on the assumption that the \( w_i \) are not correlated with the error (\( u' \)) in the instrumented equation(5.8). At the same time, they must be sufficiently correlated with the instrumented variable for desirable efficiency characteristics. In practice, standard errors tend to be large as good instruments are hard to find. Furthermore, a low correlation between \( w_i \) and \( u' \), if \( w_i \) is a weak instrument can result in biases that are larger than those under OLS, particularly in small samples.

As an alternative to 2SLS, the source of endogeneity can be viewed as an omitted variable problem in the sense that \textbf{m} is not observed. Rather than remove the endogeneity bias by adjusting \( l \), it can be removed from \( u' \) by finding one or more substitutes for each phenomenon in \textbf{m}. Let one substitute be:
\[ z = \phi_0 + \phi m + r \]  
where \( r \) is a random error term uncorrelated with \( z \). The resulting equation is:

\[ q_i = a + \lambda \phi_0 + \beta_1 l + \beta_2 k + \lambda \phi z + (e + p + \lambda r) \]  

(5.10)

\( z \) must be redundant in equation (5.8) in the sense that it would add no explanatory power to \( q \) if \( m \) was hypothetically included. It is in practice comfortably assumed that \( z \) is not correlated with \( e \) or \( p \). However, the need for the other factor inputs to be uncorrelated with \( r \) is a requirement that cannot be verified, so one must either rely on theoretical priors or judgement to conclude this correlation is lower than the original one between the factor inputs and \( m \). Wooldridge (2002) argues this can be justified if \( z \) is a good proxy.

Instruments must be uncorrelated with \( m \) while proxies must be highly correlated with \( m \), so candidate variables can either make good proxies or good instruments, but not both. The issue is therefore whether to estimate

\[ q_i = \alpha \beta + \beta_1 l + \beta_2 k + \mu z + v \]  

(5.11)

where, along with their vectors of coefficients, \( \alpha \) is a vector of variables that are generally accepted technology controls, like location or industry dummies, and \( z \) is a vector of controls for market and other shocks that vary across firms in the time period observed, or

\[ q_i = \alpha \beta + \beta_1 l + \beta_2 k + \mu z + v \]  

(5.12)

where \( z_1 \) is a subset of \( z \). 2SLS estimation of system (5.12) implies any controls in the structural equation would also be in the first-stage equations for the factor inputs.

Among the candidate variables in appendix 4.1 designed to address these endogeneity issues, it is hard to find an example that offers clear-cut priors in favour of either proxying or instrumenting. One can suggest that a variable for the effects of market conditions on hiring would make a good proxy for market conditions but would not remove the endogenous component from the input if included in the input’s instrumenting equation. Industry dummies and variables on export quantities, while controlling for an aspect of the technology, would also account for some differences in market conditions. Valid instruments are harder to find; recruitment difficulty or wages are themselves endogenous to market conditions – being higher
when demand is high – and would at best only partially purge the factor input of its endogeneity.\textsuperscript{18}

### 5.4.3 Empirical comparison of OLS and 2SLS estimates

We turn now to some preliminary results. This exercise is in a Cobb Douglas context\textsuperscript{19}. Table 5.1 is OLS with all included variables acting as proxies.

|            | Coef. | Std. Err. | t    | P>|t| | [80% Conf. Interval] |
|------------|-------|-----------|------|-----|----------------------|
| Capital    | 0.164 | 0.036     | 4.6  | 0.000 | 0.118    | 0.210               |
| Man/Prof   | 0.277 | 0.071     | 3.9  | 0.000 | 0.186    | 0.368               |
| Sale/Cle   | 0.296 | 0.068     | 4.3  | 0.000 | 0.208    | 0.384               |
| Skil/Art   | 0.156 | 0.044     | 3.5  | 0.000 | 0.099    | 0.213               |
| Semi       | 0.037 | 0.049     | 0.8  | 0.453 | -0.026   | 0.100               |
| Un         | 0.051 | 0.035     | 1.5  | 0.148 | 0.006    | 0.096               |
| Ind dummy 1| 0.007 | 0.129     | 0.1  | 0.957 | -0.159   | 0.173               |
| Ind dummy 2| 0.443 | 0.174     | 2.6  | 0.012 | 0.220    | 0.667               |
| Ind dummy 3| 0.055 | 0.121     | 0.5  | 0.648 | -0.101   | 0.212               |
| Ind dummy 4| 0.208 | 0.159     | 1.3  | 0.193 | 0.003    | 0.412               |
| Ind dummy 5| -0.089| 0.146     | -0.6 | 0.543 | -0.276   | 0.098               |
| Loc dummy 1| 0.055 | 0.118     | 0.5  | 0.642 | -0.097   | 0.207               |
| Loc dummy 2| 0.042 | 0.117     | 0.4  | 0.717 | -0.108   | 0.192               |
| Loc dummy 3| 0.048 | 0.109     | 0.4  | 0.659 | -0.092   | 0.188               |
| Exports %  | 0.360 | 0.180     | 2.0  | 0.047 | 0.128    | 0.591               |
| Recruitment MP| -0.001| 0.051     | 0.0  | 0.982 | -0.067   | 0.064               |
| Recruitment SC| 0.137| 0.068     | 2.0  | 0.044 | 0.050    | 0.224               |
| Recruitment Skilart| 0.008| 0.068     | 0.1  | 0.905 | -0.079   | 0.096               |
| Recruitment Semi| 0.014| 0.083     | 0.2  | 0.866 | -0.092   | 0.120               |
| Recruitment Un| -0.121| 0.098     | -1.2 | 0.216 | -0.247   | 0.004               |
| Productivity dissat. | -0.034| 0.039     | -0.9 | 0.390 | -0.084   | 0.017               |
| Training expenditure| 0.000| 0.000     | 4.2  | 0.000 | 0.000    | 0.000               |
| Market conditions| 0.000| 0.011     | 0.0  | 0.998 | -0.014   | 0.014               |
| Size dummy | 0.450 | 0.136     | 3.3  | 0.001 | 0.275    | 0.626               |
| Ownermanaged| 0.424| 0.180     | 2.4  | 0.019 | 0.193    | 0.656               |
| _cons      | -0.176| 0.405     | -0.4 | 0.664 | -0.698   | 0.345               |

Table 5.1: OLS for value-added with robust standard errors. The $R^2$ is 0.87, the root mean square error is 0.59, and the F-statistic is 111. Significant variables are in bold.

All the significant variables seem to be those controlling for technology or efficiency. Those designed to control for cross-firm market or other shocks that co-determine output and input demand, notably market conditions, are not significant. They could be bad proxies or there could simply not be much cross firm variation in this sample. The factor inputs are generally significant and have reasonably-sized and correctly-signed coefficients; four are highly significant.

Table 5.2 shows 2SLS estimates where all six inputs are instrumented for. It keeps generally accepted technology controls as proxies while using the others in table 5.1 as additional

\footnotesize{\textsuperscript{18} The exogeneity of wages was discussed in chapter 2.5.\textsuperscript{19} This is used because it is much easier to compare two sets of parameters and to distinguish plausible coefficients from unlikely ones. The differences in estimates will be so stark that they will overwhelm any distortions that may result, in this context, from not using the fuller translog specification.}
instruments, on the assertion that they are unsuitable candidates for proxies and their inclusion in the second stage regression would bias the results. Factor prices are additional instruments.

| Coef. | Std. Err. | t | P>|t| | 80% Conf. Interval |
|-------|-----------|---|------------|-------------------|
| Capital | 0.245 | 0.196 | 1.25 | 0.214 | -0.008 to 0.497 |
| Man/Prof | 0.545 | 0.464 | 1.17 | 0.242 | -0.052 to 1.142 |
| Sale/Cle | 0.364 | 0.431 | 0.88 | 0.379 | -0.167 to 0.895 |
| Skil/Art | -0.055 | 0.208 | -0.26 | 0.792 | -0.323 to 0.212 |
| Semi | 0.029 | 0.156 | 0.18 | 0.854 | -0.172 to 0.230 |
| Un | 0.019 | 0.147 | 0.13 | 0.898 | -0.170 to 0.208 |
| Ind dummy 1 | -0.169 | 0.343 | -0.49 | 0.622 | -0.061 to 0.272 |
| Ind dummy 2 | 0.275 | 0.321 | 0.86 | 0.392 | -0.137 to 0.687 |
| Ind dummy 3 | 0.130 | 0.156 | 0.84 | 0.403 | -0.070 to 0.330 |
| Ind dummy 4 | 0.377 | 0.220 | 1.71 | 0.088 | 0.094 to 0.661 |
| Ind dummy 5 | -0.106 | 0.217 | -0.49 | 0.625 | -0.386 to 0.173 |
| Loc dummy 1 | -0.069 | 0.158 | -0.44 | 0.662 | -0.273 to 0.134 |
| Loc dummy 2 | 0.091 | 0.149 | 0.61 | 0.544 | -0.101 to 0.282 |
| Loc dummy 3 | 0.063 | 0.124 | 0.51 | 0.613 | -0.097 to 0.222 |
| Exports % | 0.233 | 0.395 | 0.59 | 0.556 | -0.275 to 0.741 |
| Size dummy | 0.145 | 0.318 | 0.46 | 0.648 | -0.263 to 0.554 |
| Ownermanaged | 0.804 | 0.489 | 1.64 | 0.102 | 0.175 to 1.432 |
| cons | -0.289 | 0.400 | -0.72 | 0.471 | -0.803 to 0.225 |

Table 5.2: IV (2SLS) regression for value added with all 6 factors instrumented, with robust standard errors; additional instruments are factor prices, recruitment difficulties, productivity dissatisfaction, training expenditure and market conditions. The R² is 0.83, the root mean square error is 0.66 and the F-statistic is 69. Significant variables are in bold.

The coefficient (factor share) for skilled/artisans is negative and the coefficient for managers/professionals seems implausibly large. This is consistent with the variables being used as instruments themselves being correlated with the error term. No variable except one industry dummy and, marginally, the ownermanaged dummy is significant. The location dummies are not jointly significant and the factor inputs are not significant either.

While the output is not presented, a summary of the first-stage regressions for each input provides an indication of instrument quality. The R² varies from 0.46 to 0.55. Of the few significant variables, only training expenditure and market conditions are genuine instruments as the others are also in the second stage. The significant market conditions variable could mean it is a good instrument and poor proxy, but it was only significant for some inputs, suggesting market conditions are more relevant for some inputs than others. The wage variable for managerial/professional labour was incorrectly signed and the wage variables were insignificant instruments. For reasons to be explained in chapter 7, factor prices can be very weak instruments and thus seriously dent the quality of the instrumentation procedure.²⁰

The result is very wide confidence bands on the inputs. In fact, they are so wide that those in the OLS regression fall well within the bounds of the IV regression for every variable.²¹

²⁰ Wage size is not a candidate instrument because it is by construction a function of output and hence correlated with the error term.
²¹ Using 80% confidence intervals was designed to induce a smaller chance of this happening, making the result even more potent.
importance of efficiency to this study is reason enough to abandon the IV approach. Furthermore, doubts about the validity of some instruments, supported by the nature of the coefficients, provide no reason to believe the IV estimates are less biased than OLS with proxies.

These conclusions are reinforced (and explained) by formal tests. For details of the exact procedures, see Wooldridge (2002) and Statacorp (2003b). The Hausman specification test for endogeneity compares the regressions, finding a p value of 0.90 for the null hypothesis that the variables in equations (5.11) and the first line of (5.12) do not differ systematically. In other words, there is no endogeneity bias and there is no need to consider IV in the first place. The difficulty with this test is the underlying conjecture that the IV regression coefficients are consistent. The possibility for this not being the case is now tested.

A Hausman type test runs an IV regression, stores the residuals and regresses these residuals on all the instruments. There should be no explanatory power for the residual if the instruments are exogenous. The test statistics is based on the $R^2$ from this regression and has a $\chi^2$ distribution. The test for the residuals from table 5.2 does not reject the exogeneity assumption at the 20% level. Whether the instruments are exogenous “enough” given the weakness of the instruments is a matter of judgement, but, because only moderate rejections of exogeneity can result in large biases if instruments are weak (Wooldridge, 2002), 20% is possibly too low for inclusion. If, on the other hand, one is satisfied with the exogeneity assumption, then the first test is valid and there is no need for 2SLS in the first place.

Given the poor efficiency characteristics, the risk of bias, the formal tests, and of course the fact that any instrumentation procedure cannot deal with all the factor inputs, 2SLS is not an effective solution. One must conclude that OLS with correct controls is a better option, even if some variables are better instruments than controls. Furthermore, the constraints imposed on the functional form, especially linear homogeneity restrictions, would pull the coefficients closer to their true values, especially if all inputs are equally endogenous.

In simple settings and and/or assumptions that the explanatory variables are not correlated, it may be possible to suggest the direction of any biases caused by any remaining endogeneity. However, given the use of proxies, the highly collinear nature of the data, and the fact that higher order terms are used, this is practically impossible. Greene (2003:86) notes, “Although expressions can be derived for these biases in a few of these cases, they generally depend on numerous parameters whose signs and magnitudes are unknown and, presumably, unknowable.”
5.5 ECONOMETRIC ESTIMATION METHOD

The data issues in chapter 4 designate that only a single equation production function is estimated (by OLS). The regressions will, by the way the variables are constructed, impose the profit-maximization restriction \( \beta_{ij} = \beta_j \). Technology restrictions will be tested using Wald tests.

The cost share equations will be estimated together with the cost function using the Zellner seemingly unrelated regressions (SUR) model, which exploits correlations between the errors in each of the share equations to improve efficiency. Scope for such gains is limited by the fact that the explanatory variables in each factor share equation are identical or at least highly correlated. However, cross equation restrictions do allow for efficiency improvements (Greene, 2003). As will be elaborated in chapter 7, restrictions exist because the cost shares are derivatives of the cost function, so some coefficients are the same. Symmetry conditions \( b_{ij} = b_{ji} \) also imply cross equation restrictions. In addition, factor share equations incorporate more information associated with producers’ optimising decisions and are therefore potentially more precise (Kim, 1992).

By construction, the sums of the \( a_i \) coefficients across the factor share equations equal unity for each observation. Therefore, the residual cross product and disturbance covariance matrices are singular and prevent estimation (Berndt, 1991). This has been verified for this dataset. A common response is, after validating it, to impose price homogeneity on the cost function and hence across the share equations:

\[
\sum_i a_i = 1, \quad \sum_i b_{ij} = \sum_j b_{ij} = \sum_i b_{ij} = 0.
\]

Using the first of these restrictions, let \( a_i = 1 - \sum a_i \), where \( k \) refers to capital and \( l \) refers to the five labour inputs. This allows the share equation for capital to be dropped and the remaining five factor share equations to be estimated as

\[
s_i = a_i + \sum_j b_{ij} \ln \frac{w_j}{w_k} + b_{ij} \ln y; \quad (i, j = 1, \ldots, 5)
\]

The capital equation is dropped but the choice is arbitrary if the Zellner iterated efficient (IZEF) procedure is used (Berndt, 1991). The (IZEF) procedure is the dominant method in the literature and is the one employed by this study: instead of one or two-step feasible generalised least

\[\text{For convenience, the subscripts } i,j \text{ are retained but now refer to the five labour inputs. } w_k \text{ is the cost of capital. The original share equation was, for each factor including capital:}
\]

\[
s_i = a_i + \sum_j b_{ij} \ln w_j + b_{ij} \ln y; \quad (i, j = 1, \ldots, 6)
\]
squares estimates, the procedure iterates over the disturbance covariance matrix and parameter estimates until they converge (Statacorp, 2003b).

5.6 INFERENCE

Significant regression coefficients neither imply nor are necessary for significant elasticities (Anderson & Thursby, 1986). “Significant” can refer to rejecting a null hypothesis of the elasticity being zero, in which case we can be confident the factors are complements or substitutes or can refer to the Cobb-Douglas elasticity of unity.

The difficulty lies in the fact that the elasticity estimates are highly non-linear combinations of the coefficients and data (Greene, 2003). Anderson & Thursby (1986) present conditions under which Allen Elasticities of Substitution asymptotically follow the normal or ratio-of-normals distribution. They find the normal distribution or ratio-of-normals is appropriate only if the means of the actual factor shares are used. They are not appropriate under many alternative calculations found in the literature, including using the logs of the means of the factor price and output variables or the means of the corresponding logs to calculate shares.

Reviews of empirical work make no mention of significance (Chung, 1994; Hamermesh, 1993). Some studies do not report confidence intervals for the estimators at all (Bergström & Panas, 1992; Chung, 1987; Teal, 2000). Others (Binswanger, 1974b, Mak, 2000) regard the factor shares as fixed and treat the $\beta_{ij}$ coefficient as the only one with a confidence interval, incorrectly inferring the elasticity significance from a t-statistic. Grant & Hamermesh (1981) use both the t-statistic on the regression coefficient and Taylor approximations that treat the factor shares as stochastic.

One can speculate why some studies do not present more appropriate confidence measures. Given the number of parameters in the regressions, the number of parameters used in calculating the elasticities, and that the translog is an approximation to complex underlying technologies with few restrictions, achieving precise elasticity estimates is a heavy burden to place on sub-optimal sample sizes. In an analysis of famous pioneering translog studies, Anderson & Thursby (1986) find that confidence intervals for many of the elasticity measures span both the negative and positive orthants, bringing their forceful conclusions into doubt. Grant & Hamermesh (1981) have some significant results, but have a data set up to the task. Besides, there is likely a sample selection problem: when they are not an accepted requirement, researchers are more likely to report inference when they find significant values.
This study does not have the option to use the Anderson & Thursby result as no actual shares are available and because it is not necessarily applicable in the stochastic framework of a production function. Abusing the terms “inference” and “significance” slightly from now on, this study provides good indications of the confidence in its conclusions in two ways.

The first is to apply the delta method, which calculates Taylor approximations to underlying distributions of functions of parameters (see Greene (2003)). For example,

\[
\varepsilon_{su} = \frac{\beta_u}{\zeta} + \varepsilon_u
\]

\[
= \frac{\beta_u}{\alpha + \sum_j \beta_{uj} \ln x_j} \text{ } + \alpha_u + \sum_j \beta_{uj} \ln x_j
\]

(5.14)

The delta method accounts for confidence intervals on all the parameters in the elasticity calculation. This method is applied to the production estimates but not the cost functions, for two reasons. First, it is not clear that such a procedure can be used after estimating a system, as suggested in Greene (2003). Second, the residuals produced after the cost estimates are not even approximately normally distributed, unlike for the production estimates. Functions of parameters with irregular distributions could have very unreliable confidence bands, especially in a relatively small sample.

Given the additional burdens on the elasticity estimates discussed, and the relatively small number of observations in the final sample, confidence intervals of 70% or approximately one standard deviation are used. While this stands accused of not being wide enough, when combined with the second procedure, much more meaningful inference can be conducted.

Unlike other studies, this thesis gives an indication of how elasticities vary across the sample of firms. Other research, by reporting one summary statistic, discards possibly the most valuable benefit of the translog model over CES models: variation across the sample. Besides, from a policy point of view, it is far more informative to find the elasticity is positive for 75% of the firms in the sample than to know what the average is. Furthermore, elasticity estimates might be precise at the centre of the data, but could be badly behaved at other percentiles.

Therefore, the elasticity value for each firm is calculated using the coefficient estimates and each firm’s input quantity. In most cases, the median and quartiles of the sample are presented for the estimated parameters and for each of the two confidence bands. There are therefore nine estimates of a particular elasticity. Robustness of the measures will be determined according to
how many of these nine have the same sign. For the cost function, indications of how elasticities vary between the 5th and 95th percentiles are also shown.

5.7 SUMMARY

This chapter has tested, validated and motivated value-added separability for the production function. It has shown why and how heteroskedasticity is vigorously attended to and dedicates a lot of space to comparisons between OLS and 2SLS estimates, concluding that the costs and dangers of 2SLS far outweigh those of OLS. The chapter briefly motivates the use of the IZEF systems procedure for estimating the cost share equations. It ends with two proposals for conducting inference. The first is to construct confidence bands using the delta method while the second is to check across the sample of firms for consistency in the sign of the calculated elasticity.
6. ELASTICITIES FROM PRODUCTION FUNCTION ESTIMATES

6.1 INTRODUCTION

Chapter 6 presents a variety of results derived from the translog production function estimates, namely Hicks Elasticities of Complementarity and elasticities of factor price. The key result is in table 6.4, which shows that a rise in the supply of skilled/artisans will lead to a rise in unskilled wages, but that a rise in the supply of semi-skilled workers will lead to a fall in unskilled wages. To the extent that wages may be rigid, this can be interpreted as a corresponding rise/fall in employment, and adjusting the calculations for rigid unskilled wages confirms the finding. The results then account for imperfectly elastic product demand, taking sub-samples of firms in particular industries or exporting firms and recalculating the elasticities of factor price. The key findings still hold, namely that a rise in the number of skilled/artisans will benefit unskilled labour but that a rise in the number of semi-skilled workers will not. The results are robust to the confidence band estimates and apply to a big enough proportion of the sample to be of clear policy relevance.

There are lots of numbers, so the focus is on the effects on unskilled workers. Nonetheless, some other results emerge. Increases in the supply of managerial/professional workers would benefit all production workers but not sales/clerical workers, while increases in the supply of skilled/artisans would benefit all other occupations. Finally, while the implications for unskilled workers are unchanged, there are cross-industry differences in signs for some input-pairs, driven mainly by different product demand elasticities.

Recall that the production function being estimated is:

\[ \ln y = \ln \alpha_0 + \sum_i \alpha_i \ln x_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln x_i \ln x_j \]  

(6.1)

\( y \) is value added, \( x_i \) are capital and five occupations. \( \kappa_{ij} = \kappa_{ji} \) (aleph) is the elasticity of complementarity and measures \( \frac{\partial \ln w_i}{\partial \ln \gamma_j} \). (6.2) is the elasticity of factor price and measures the effect of an exogenous increase in the supply of one factor on the wage of another.

\[ \epsilon_{ij} = \frac{\partial \ln w_i}{\partial \ln x_j} = \zeta \kappa_{ij} + \zeta \eta \]  

(6.2)
Initial results assume perfectly elastic product demand, which omits the first term. Factors are \( q \)-
complements if \( \epsilon_{ij} > 0 \) and \( q \)-substitutes if \( \epsilon_{ij} < 0 \). Elasticities are calculated using the following
formulae:

\[
\xi_{ij} = \frac{\beta_{ij}}{\zeta_j} + 1 \quad (6.3)
\]

\[
\xi_i = \frac{\beta_{ii}}{\zeta_i} + 1 - \zeta_i \quad (6.4)
\]

\[
\varepsilon_{ij} = \frac{\beta_{ij}}{\zeta_i} + \zeta_j \quad (6.5)
\]

\[
\varepsilon_{ii} = \frac{\beta_{ii}}{\zeta_i} + \zeta_i - 1 \quad (6.6)
\]

where:

\[
\zeta_i = \alpha_i + \sum_j \beta_{ij} \ln x_j \quad (6.7)
\]

are the factor shares derived from the coefficients of the production function estimate.

### 6.2 Regression Results and Derived Factor Shares

Because most of the individual regression coefficients are not of direct interest, the full results
are in appendix 6.1. The \( R^2 \) is 0.97 and the root means square error is 0.47. The \( \beta_{ij} \) coefficients
and their p-values are presented in table 6.1: Significant coefficients are in bold.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>coefficient</th>
<th>value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5*Capital^2</td>
<td>b11</td>
<td>0.059</td>
<td>0.12</td>
</tr>
<tr>
<td>Capital*ManProf</td>
<td>b12</td>
<td>-0.039</td>
<td>0.39</td>
</tr>
<tr>
<td>Capital*SaleCle</td>
<td>b13</td>
<td>0.061</td>
<td>0.15</td>
</tr>
<tr>
<td>Capital*SkilArt</td>
<td>b14</td>
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<td>0.55</td>
</tr>
<tr>
<td>Capital*Semi</td>
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<td>-0.035</td>
<td>0.31</td>
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<tr>
<td>Capital*Un</td>
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<td>-0.027</td>
<td>0.32</td>
</tr>
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<td>ManProf*Semi</td>
<td>b25</td>
<td>0.011</td>
<td>0.85</td>
</tr>
<tr>
<td>ManProf*Un</td>
<td>b26</td>
<td><strong>0.080</strong></td>
<td><strong>0.10</strong></td>
</tr>
<tr>
<td>0.5*SaleCle^2</td>
<td>b33</td>
<td>0.089</td>
<td>0.32</td>
</tr>
<tr>
<td>SaleCle*SkilArt</td>
<td>b34</td>
<td>-0.028</td>
<td>0.57</td>
</tr>
<tr>
<td>SaleCle*Semi</td>
<td>b35</td>
<td>-0.005</td>
<td>0.92</td>
</tr>
<tr>
<td>SaleCle*Un</td>
<td>b36</td>
<td>-0.037</td>
<td>0.36</td>
</tr>
<tr>
<td>0.5*SkilArt^2</td>
<td>b44</td>
<td>-0.083</td>
<td>0.13</td>
</tr>
<tr>
<td>SkilArt*Semi</td>
<td>b45</td>
<td>0.057</td>
<td>0.13</td>
</tr>
<tr>
<td>SkilArt*Un</td>
<td>b46</td>
<td><strong>0.054</strong></td>
<td><strong>0.09</strong></td>
</tr>
<tr>
<td>0.5*Semi^2</td>
<td>b55</td>
<td>0.051</td>
<td>0.29</td>
</tr>
<tr>
<td>Semi*Un</td>
<td>b56</td>
<td><strong>-0.080</strong></td>
<td><strong>0.01</strong></td>
</tr>
<tr>
<td>Un^2</td>
<td>b66</td>
<td>0.010</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Table 6.1: \( \beta_{ij} \) values

By (6.3), a significant \( \beta_{ij} \) coefficient is indicative of that elasticity being significantly different
from the Cobb-Douglas elasticity of unity, but the inference discussion (section 5.6) shows why
this is not strictly correct. Coefficient values suggest the higher-order coefficients have some
explanatory power for certain factor combinations. This study has particular interest in the unskilled, where elasticities are of direct interest, and the significant coefficients suggest the elasticities referring to unskilled labour at least are materially different from one. Nonetheless, using a Cobb Douglas approximation to the technology may not be inappropriate for studies where the focus is not elasticities.

From the regression, factor shares are predicted for each firm. Figure 6.1 presents the values of 25th percentile, median and 75th percentile factor shares in the sample.

![Factor Shares](image)

Figure 6.1: Factor shares for (1 to 6) Capital, Managerial/Professional, Sales/Clerical, Skilled/Artisan, Semi-skilled and Unskilled occupations. Values are presented for the median shares and the quartiles of the shares calculated for all the firms in the sample.

The results imply a median capital:labour ratio of about 17%. In Cobb Douglas estimates for the Gauteng Province, Rankin (2001) calculates a ratio of about 21%. Earlier concerns about possible negative factor shares are realised for semi-skilled labour. This is due to the high variance in numbers of that occupation and the number of firms with zero or close to zero employees. Given that these negative values can produce strange elasticities for some firms, which can badly influence average elasticities, the median and quartiles are the preferred statistics of analysis.

There are nine estimates of a parameter: the median and quartiles of the sample are calculated for the estimated coefficients and for each of the two confidence interval estimates. Robustness of the measures will be measured according to how many of these nine have the same sign.

\[23\] Given that these abnormal elasticities will be produced for firms having highly negative or positive true elasticities – because of the small denominators – these statistics are robust and representative.
6.3 OVERALL ELASTICITY RESULTS ASSUMING PERFECTLY ELASTIC PRODUCT DEMAND

Table 6.2 presents the median elasticities of complementarity, assuming perfectly elastic demand. Median and quartile values at the confidence limits are in appendix 6.2.

\[
\begin{array}{ccccccc}
\text{i} & \text{j} & \text{Capital} & \text{Man/Prof} & \text{Sale/Cle} & \text{Skil/Art} & \text{Semi} & \text{Un} \\
\hline
\text{Capital} & -2.36^* & -0.27 & 2.17^* & 0.11 & -0.33 & -0.24 \\
\text{Man/Prof} & -0.27 & -2.79 & -0.11 & 1.08^* & 1.05^* & 1.46^* \\
\text{Sale/Cle} & 2.17^* & -0.11 & -1.20^* & 0.95^* & 0.99^* & 0.89^* \\
\text{Skil/Art} & 0.11 & 1.08^* & 0.95^* & -13.23 & 0.86 & 0.53 \\
\text{Semi} & -0.33 & 1.05^* & 0.99^* & 0.86 & -2.05 & 0.66^* \\
\text{Un} & -0.24 & 1.46^* & 0.89^* & 0.53 & 0.66^* & -4.46^* \\
\end{array}
\]

Table 6.2: Median elasticities of complementarity; * indicates sign consistent for at least 8 of the 9 values calculated. Positive values are blue and negative values are red.

The results suggest that, for example, a rise in the relative amount of capital per worker will cause the unskilled wage to fall relative to the cost of capital, but they don’t provide evidence of capital skill complementarity\(^{24}\). A relative rise in the supply of managers or professionals would have positive effects on the relative wages of production workers, but the effect is opposite on sales/clerical workers, as one might expect given they are more likely to be substitutes. A rise in the relative number of skilled/artisan workers would lead to relative rises in the wages of all other occupations.

The general complementarity of factors is clearer in table 6.3. The statistics are the percentage of firms with \(R > 0\). In particular, all production occupations are complements for 74% to 85% of firms in the sample.

\[
\begin{array}{ccccccc}
\text{i} & \text{j} & \text{Man/Prof} & \text{Sale/Cle} & \text{Skil/Art} & \text{Semi} & \text{Un} \\
\hline
\text{Capital} & 33% & 94% & 54% & 43% & 39% \\
\text{Man/Prof} & 39% & 98% & 98% & 86% \\
\text{Sale/Cle} & 100% & 100% & 99% \\
\text{Skil/Art} & 82% & 74% \\
\text{Semi} & 85% \\
\end{array}
\]

Table 6.3: Percentage of firms that have a given factor pair as complements. Values above 50% are blue while those below 50% are red.

\(^{24}\) See the last paragraph of section 3.5 for a brief description of this term.
Table 6.4 presents the elasticities of factor price which unlike the HEC are not symmetric. Appendix 6.3 contains median and quartile values at the confidence limits. All the own-price elasticities are significantly negative. For example, a 10% rise in the supply of skilled/artisan workers would lead to a (large) 14.3% fall in their own wage. The only other significant negative elasticities are in the column showing the effects of a rise in the supply of semi-skilled workers. This has a negative effect on the prices of capital and unskilled workers, for whom a 10% rise in the supply of semi-skilled workers would lead to a 4.2% fall in wages. Unlike semi-skilled workers, increases in the supply of the skilled/artisan category will have positive effects on both other production occupations. A 10% rise in the supply of this category would raise semi- and unskilled wages by 3.5% and 4.2% respectively.

The results include a few values which cannot be deemed “significant”. To the extent that this is caused by cross-sample variation, the results show it can be misleading to infer factor relationships for the whole sample using one summary statistic like the mean or median. Furthermore, the value of disaggregating the labour force beyond the standard production/non-production worker analysis is demonstrated in the key result of this study: raising the supply of skilled/artisans workers helps the unskilled but raising the supply of semi-skilled workers harms those who remain unskilled.

### 6.4 ADJUSTING ELASTICITIES FOR RIGID UNSKILLED WAGES

Such numbers need not necessarily be taken literally to be of policy relevance. To the extent that market mechanisms don’t work properly and that there is institutional interference in wages, the signs could be the focus of attention. Over the long run, the changes in wages can arise through

---

25 Because it is the standard in the literature, appendix 6.4 has the statistics calculated at the means of the factor values, which are remarkably similar to those in table 6.4.
collective bargaining processes\textsuperscript{26}. If artificially high wages are contributing to unemployment, the positive sign could be seen as a rise in employment instead of wages. This conclusion is supported by adjusting the elasticities for rigid unskilled wages, using the method explained in chapter 2.6. Table 6.5 presents the adjusted elasticities of factor price for all factors except unskilled workers\textsuperscript{27}.

<table>
<thead>
<tr>
<th>(i)</th>
<th>(j)</th>
<th>Capital</th>
<th>Man/Prof</th>
<th>Sale/Cle</th>
<th>Skil/Art</th>
<th>Semi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>.</td>
<td>-0.05</td>
<td>0.73</td>
<td>-0.10</td>
<td>-0.21</td>
<td></td>
</tr>
<tr>
<td>Man/Prof</td>
<td>-0.05</td>
<td>.</td>
<td>0.02</td>
<td>0.41</td>
<td>-0.14</td>
<td></td>
</tr>
<tr>
<td>Sale/Cle</td>
<td>0.34</td>
<td>0.01</td>
<td>.</td>
<td>-0.02</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>Skil/Art</td>
<td>-0.06</td>
<td>1.02</td>
<td>0.12</td>
<td>.</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Semi</td>
<td>-0.04</td>
<td>0.07</td>
<td>0.27</td>
<td>0.23</td>
<td>.</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.5: Elasticities of factor price for the other factors on the assumption that unskilled wages are rigid. Own elasticities not calculated.

There are no important contradictions between tables 6.4 and 6.5, and the few sign switches that occur don’t involve significant coefficients. Moreover, the signs for the effect on the quantity of unskilled labour are completely consistent with those in table 6.4, as shown in table 6.6.

<table>
<thead>
<tr>
<th>(u)</th>
<th>Capital</th>
<th>Man/Prof</th>
<th>Sale/Cle</th>
<th>Skil/Art</th>
<th>Semi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>-0.05</td>
<td>1.00</td>
<td>0.09</td>
<td>0.53</td>
<td>-0.54</td>
</tr>
</tbody>
</table>

Table 6.6: Expected percentage change in unskilled employment after a 1% rise in the supply of the other factors, assuming unskilled wages are fixed.

The numbers suggest that, if unskilled wages are fixed, a 10% rise in the supply of skilled/artisans would lead to a 5.3% rise in unskilled employment while a 10% rise in the supply of semi-skilled workers would lead to a 5.4% fall in unskilled employment. These results are consistent for most of the firms in the sample, as shown in figure 6.2, which indicates the percentage of firms for which a rise in a given factor leads to a rise in unskilled employment. We propose the terms \(r\) complements for when \(\varphi > 0\) and \(r\)-substitutes for when \(\varphi < 0\).

\textsuperscript{26} Using multi-union models to understand the effects of an increase in supply of one factor on the resulting bargains is a fruitful area for research. This is especially interesting when, as the results show, some inputs are complements.

\textsuperscript{27} Because the values are derived using the adjustment explained in chapter 2.6 and not direct estimates from the production function, inference is not conducted.
ALLOWING FOR IMPERFECTLY ELASTIC PRODUCT DEMAND

6.5.1 Exporters

The discussion has implicitly assumed perfectly elastic product demand, as have all the studies consulted. This is quite possible for internationally traded commodities; Behar & Edwards (2004) find elasticities as high as –6 for South African manufactured exports, leaving open the possibility that they are also perfectly elastic. Substituting –6 into (6.2) for the sub-sample of firms having more than 20% exports yields table 6.7\textsuperscript{28}.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
\textbf{\(E_{ij}\)} & \textbf{Cap} & \textbf{MP} & \textbf{SC} & \textbf{Skilart} & \textbf{Semi} & \textbf{Un} \\
\hline
\textbf{Cap} & -0.47 & -0.10 & 0.70 & -0.10 & -0.20 & -0.13 \\
\textbf{MP} & -0.06 & -0.75 & -0.06 & 0.18 & 0.12 & 0.40 \\
\textbf{SC} & 0.31 & -0.05 & -0.44 & -0.01* & 0.05* & -0.05* \\
\textbf{Skilart} & -0.06 & 0.37 & 0.04* & -1.52 & 0.56 & 0.52 \\
\textbf{Semi} & -0.12* & 0.26 & 0.24 & 0.38* & -0.56 & -0.33* \\
\textbf{Un} & -0.12 & 0.84 & -0.03* & 0.49 & -0.60 & -0.81 \\
\hline
\end{tabular}
\caption{Product-elasticity compensated cross elasticities of factor price for exporting firms; * indicates not consistent across the quartiles of the sample}
\end{table}

\textsuperscript{28} To the extent that many firms appear to export only that which they cannot sell on the local market (Behar and Edwards, 2004), the export elasticity is appropriate even at this relatively low percentage.
Because of the additional difficulties with using external parameter estimates, confidence intervals are not estimated. However, the signs on the coefficients are consistent across quartiles for all but a few values (inconsistent ones are marked with a *). The effect on the signs compared to table 6.4 is not big, as expected for a high product-demand elasticity value. The key implications for unskilled labour are the same and the coefficients are in fact moderately higher for exporting firms despite the slight negative effect of adjusting for product demand elasticity.

6.5.2 Clothing and Furniture Industries

For specific industries, we are able to use recent consumer product demand elasticity estimates for manufacturing industries estimated by Selvanathan & Selvanathan (2003). Treating industries separately may be criticised for the implicit assumption that no workers can change industry, as worker movement would mitigate the price effects of a change in supply in just one industry. However, the values still give an indication of how different industries would be affected by a general rise in the supply of a factor. We contrast two sectors: the relatively inelastic clothing sector (–0.423) and the almost unit-elastic furniture sector (–0.947).

Table 6.8 presents the clothing sector estimates. There are far more negative values than in the analysis so far, as expected for inelastic industries ceteris paribus according to (6.2). Again, * indicates values are not consistent across quartiles.

<table>
<thead>
<tr>
<th></th>
<th>Cap</th>
<th>MP</th>
<th>SC</th>
<th>Skilart</th>
<th>Semi</th>
<th>Un</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cap</td>
<td>-0.69</td>
<td>-0.66</td>
<td>0.03*</td>
<td>-0.30</td>
<td>-0.48</td>
<td>-0.28</td>
</tr>
<tr>
<td>MP</td>
<td>-0.27</td>
<td>-1.32</td>
<td>-0.72</td>
<td>-0.06*</td>
<td>-0.12*</td>
<td>0.22</td>
</tr>
<tr>
<td>SC</td>
<td>0.06</td>
<td>-0.62</td>
<td>-1.16</td>
<td>-0.24</td>
<td>-0.19</td>
<td>-0.21</td>
</tr>
<tr>
<td>Skilart</td>
<td>-0.31</td>
<td>-0.18</td>
<td>-0.65</td>
<td>-1.78</td>
<td>0.18</td>
<td>0.36</td>
</tr>
<tr>
<td>Semi</td>
<td>-0.38</td>
<td>-0.23</td>
<td>-0.46</td>
<td>0.23</td>
<td>-0.99</td>
<td>-0.60</td>
</tr>
<tr>
<td>Un</td>
<td>-0.27</td>
<td>0.21*</td>
<td>-0.61</td>
<td>0.27*</td>
<td>-0.67*</td>
<td>-1.06</td>
</tr>
</tbody>
</table>

Table 6.8: Elasticities of factor price for the clothing sector; * indicates not consistent across quartiles

One difference is the effect of sales/clerical workers on skilled/artisans, which is now negative, but the main results are still in place. The effect of a 10% rise in skilled artisans is now expected to lead to a (lower) 2.7% rise in unskilled wages and a rise in the supply of semi-skilled workers is negative for unskilled workers, but no longer consistently across the sample.
The results for the furniture sector in table 6.9 show the same qualitative effects, which are consistently signed across the sample of firms.

\[
\begin{array}{lcccccc}
      & \text{Cap} & \text{MP} & \text{SC} & \text{Skilart} & \text{Semi} & \text{Un} \\
\hline
\text{Cap} & -0.34^* & -0.45 & 0.66 & -0.22 & -0.40 & -0.31 \\
\text{MP} & -0.13 & -0.99 & -0.28 & 0.06 & 0.03 & 0.25 \\
\text{SC} & 0.21^* & -0.30 & -0.70 & -0.10 & -0.02 & -0.14 \\
\text{Skilart} & -0.17 & 0.14^* & -0.24 & -1.67 & 0.46 & 0.43 \\
\text{Semi} & -0.19^* & 0.04 & -0.04 & 0.30^* & -0.74 & -0.43^* \\
\text{Un} & -0.19 & 0.54 & -0.27 & 0.37 & -0.57 & -0.94 \\
\end{array}
\]

Table 6.9: Factor price elasticities for the furniture sector; * indicates not consistent across sample

More generally, the furniture industry has some sign-switches compared to the clothing industry. There are more positive values compared to the clothing industry, but there are more negative values than for the export firms and as expected for the case where perfect product demand elasticity is implicitly assumed. This suggests the elasticity of product demand can have important effects on calculated elasticities. The results also suggest producing artisans may be more beneficial for the unskilled in some industries than in others. Artisans are particularly complementary with unskilled labour in export-oriented firms.

### 6.6 SUMMARY

The broad conclusions are robust – whether estimating for the whole manufacturing sector or for specific industries with imperfectly elastic product demand, whether adjusting for rigid unskilled wages or not, whether using the estimates at the lower or upper confidence bands, and whether looking at the 25th or 75th percentiles of the estimated elasticities: producing more skilled artisans will help the unskilled while producing more semi-skilled workers will harm the unskilled. This result would not have been revealed were it not for the disaggregated nature of the estimates.
7. COST FUNCTION AND COST SHARE ESTIMATIONS

7.1 INTRODUCTION

This chapter describes how a cost function and cost share equations are estimated to infer Allen Partial Elasticities of Substitution and the associated factor demand elasticities. It shows very briefly that the regressions based on the wage data originally collected are unsatisfactory. The cause is diagnosed to be wages that fail to account for firm-size effects. After accounting for firm size, the results change in the manner anticipated in chapter 4, in particular accepting price homogeneity restrictions and yielding far more plausible results. The estimated nature of demand for sales/clerical workers is still non-intuitive and their share equation is persistently badly specified, so it is dropped on the plausible assumption of separability. The final results, with equations for each occupation relatively well specified, yield an extensive list of elasticities.

The main results are in table 7.3, where labour demand is found to be relatively inelastic, with own-price elasticities of demand having absolute values of less than 1. Capital and all forms of labour are substitutes, while most forms of labour are themselves complements, meaning wage restraint on the part of one occupation can have positive employment effects for all occupations. The results also suggest that a fall in semi-skilled wages will lead to a small rise in unskilled employment. In contrast, a fall in skilled/artisan in wages will lead to a fall in unskilled employment.

Recall that the cost function and factor cost share equations to be estimated are:

\[ \ln C = \ln a_0 + a_1 \ln y + \sum_i a_i \ln w_i + \frac{1}{2} b_{ij} \ln^2 y + \frac{1}{2} \sum_i \sum_{j} b_{ij} \ln w_i \ln w_j + \sum_i b_{ij} \ln w_i \ln y \]  
\[ s_i = a_i + \sum_j b_{ij} \ln \frac{w_j}{w_k} + b_{ij} \ln y; \ (i, j = 1, ..., 5) \]  

where \( w_i \) is the factor price and the \( k \) subscript refers to the cost of capital, \( x_i \) is factor quantity, \( y \) is value added, \( C = \sum w_i x_i \) and \( s_i = \frac{w_i}{C} \). Because a raw materials price is not available, value-added separability is assumed.29

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29 While comparisons with production estimates are deliberately avoided to insure “result independence”, there is support for the separability assumption from those results.
Most studies only estimate the system in (7.2) (Berndt, 1991). The reason for including (7.1) in systems estimation is that measures of returns to scale can be estimated and coefficients for capital are more conveniently available (the capital equation is dropped to allow estimation to take place; see section 5.5). Location and industry dummies, together with various industry- and firm-level controls, are used to capture the unobserved technological differences across firms. The equality of coefficients in (7.1) and (7.2), profit maximizing behaviour and features of the technology provide many possible cross-equation restrictions for improved efficiency.

7.2 RESULTS WITHOUT ADJUSTING WAGES FOR FIRM SIZE

The regressions using wage_all or wage_some, which are not presented, are completely unsatisfactory. With the exception of the value-added variables, there are no significant coefficients and the estimates yield many high positive own-price elasticities. Initial estimations, besides the symmetry conditions $b_{ij} = b_{ji}$, do not impose any restrictions on the technology or those implied by profit maximising behaviour, as even the most plausible of these restrictions are usually rejected, or “accepted” at uncomfortably low p-values $^{30}$. The estimated returns to scale, as shown in appendix 7.1, have a mean $^{31}$ value of about 2 while appendix 7.2 shows that three of the six factors have positive own-price elasticities.

The results are poor, but it is not clear whether this is poor wage data or an econometric problem. This section will investigate one likely econometric problem, namely multicollinearity. Although this is usually of minor concern to most researchers, the analysis will show that the problem is severe in this study. Nonetheless, the rest of this section will also show that the poor results are not attributable to multicollinearity.

Coefficients and elasticities are not robust to the choice of control variable or to slight changes in the subsample, which is symptomatic of the twin problems of multicollinearity and micronumerosity (Gujarati, 1995). Generally, the variance on the coefficient of variable $k$ is higher if (Greene, 2003):

i. the overall regression fit is low
ii. the variance of that variable is low
iii. the $R^2$ in the regression of variable $k$ on the other explanatory variables is high ($R^2_k$)

$^{30}$ Running constrained regressions, imposing price homogeneity and a variety of technological restrictions anyway, does not improve the quality of the regressions.

$^{31}$ Because homogeneity is not imposed, each firm has a different value according to its level of output. This is therefore the average of the values calculated for the firms in the sample.
The root mean square error is 0.65 and the $R^2$ is 0.74. Although not ideal, this is not a serious problem. Depending on the wage definition used, there are a maximum of only 36 different wage levels for each occupation, so there may be micronumerosity. This might favour using $\text{wage}_{\text{all}}$ over $\text{wage}_{\text{some}}$, but makes little difference. The variance inflation factor (VIF) ($\sqrt{1-R^2}$) measures the third and most common source: multicollinearity. Multicollinearity is to be expected, given the way the wage variables are constructed by a common set of characteristics. Furthermore, translog estimates, particularly with a high number of factors, are especially vulnerable to multicollinearity. VIF values are not easily derived after systems estimation, but a single equation value is informative, and as shown shortly can be used as a diagnosis tool. The statistics in appendix 7.3 are staggering, with a maximum VIF of 211 473. The average of 18 702 includes the control variables, which generally have values of less than 2. The estimates are in theory consistent in large samples, but, given the erratic coefficient estimates, this sample is evidently not big enough.

One way to mitigate multicollinearity is to drop higher order terms in the factors, which is tantamount to a Cobb Douglas specification, possibly with higher order value added terms (as found in Greene (2003)). While not useful for calculating elasticities, estimating this simpler model provides a useful diagnosis tool: estimates succeed in mitigating multicollinearity, as the average VIF falls to 1.84. However, these estimates yield negative shares for some factors and measures of returns to scale are still implausibly high, having a median value of (also) 1.84 for a quadratic specification and 2.75 for a (significant) cubic specification.

It is clear that multicollinearity, although serious, is not the cause of the poor results, so, unless firms drastically deviate from cost-minimizing behaviour, the wage data are not adequate.

**7.3 IMPACT OF ADJUSTING WAGES FOR FIRM SIZE**

This section demonstrates using $\text{wage}_{\text{size}}$ alters the estimates in the way predicted by chapter 4.3.3 and produces a meaningful improvement in the results once $\text{wage}_{\text{size}}$ is used. For purposes of comparison with prior results, the only adjustment made is to the wage data and associated

---

32 Estimating the system of equations entails regressing the five derived labour shares on a constant, which is restricted to the relevant $\alpha_i$ in the cost function equation. This naturally leads to positive and significant factor share coefficients. This and the ease of computing variance inflation factors motivates comparisons of single-equation estimates.
factor shares. All other aspects, including choice of control variable and restriction, are kept constant.

Consistent with the analysis in chapter 4.3.3, the null hypothesis of homotheticity is now comfortably accepted, with a p value of over 0.95. Imposing homotheticity permits a comparison of returns to scale with the unadjusted wage. Appendix 7.1 shows how the previous average of 2 falls to an average of 1.6. This latter value is still quite high, but figure 7.1 shows the values do approach unity as firms get big.

![Figure 7.1: Estimates of returns to scale using adjusted (wage_{size}) and unadjusted (wage_{wage}) wages converge with each other and approach a value of unity at high output. (Value Added in the Log of Millions of Rands)](image)

Part of the phenomenon could be technological; South Africa is a small market and it is a stylised fact that many industries are not exploiting potential economies of scale. While there may be slight increasing returns to scale, these were not found to differ significantly from unity in the production function estimates. Another explanation for high returns to scale is the falling cost of capital for bigger firms, driven by the risk-adjustments imposed. This uncovers a tension between cost and production estimates. Returns to scale are supposed to be a technological phenomenon in a world of exogenous factor prices, yet bigger firms do enjoy lower costs of capital so they do have lower relative costs. This is of course a comparison across firms with
possibly optimal sizes for their industries, but the implication is that small firms could exploit major economies of scale by getting bigger or merging and enjoying lower costs of capital.

As shown in appendix 7.4, own-price elasticity estimates are now all negative except for the sales/clerical value of 0.41, although this could merely be random sign switch for two variables (three were positive before). However, this controlled experiment has also had the effects predicted in chapter 4 and the data are now more consistent with cost-minimizing behaviour. Having held everything else constant for comparison, the superior wage data are now used in refined systems estimates.

7.4 FINAL ESTIMATES ADJUSTING FOR FIRM SIZE AND DROPPING SALES/CLERICAL

In the forthcoming systems estimation, the restriction that \( a_i \) in the cost equation equals the constant for share equation \( i \) is not imposed, even if it is supposed to be the same by definition (see equations (7.1) and(7.2)). This is because the equations may still suffer from measurement error and other specification issues. Many of the biases of these imperfections are deposited on the constant (Wooldridge, 2002), so restricting these catchments for error spills the biases throughout the system. The constants are therefore left free and the relevant elasticities are calculated from the cost equation.

In addition, the sales/clerical factor is dropped, which naturally assumes separability. Of all the factors, this is the one one should be most comfortable dropping. After all, it is hard to believe that the number of salespeople or clerks a company employs will have any impact on the relationship between other factors, especially the production workers on the factory floor. The treatment of separability is pragmatically different to that under the production function, as the circumstances are different. First, one has enough confidence in the production specification to test separability, while the motivation behind dropping sales/clerical is poor specification and misleading results. Furthermore, in systems estimation, errors in one equation transmit themselves to other parts of the system, despite this being mitigated as explained in the previous paragraph. The own-price elasticity is persistently positive in single equation estimates, and this continues in systems estimations, where the explanatory power of the sales/clerical factor share equation is persistently lower than for the others. Part of the reason for the bad specification is that this is quite a diverse group in terms of skill-level, so wages are more likely to be inaccurate in this occupation. Also, the role of this diverse group varies more than usual across firms, so the
control variables are less able to refine this role. Therefore, the damage to other results from including the sales/clerical occupation is most likely greater than any damage from excluding it.

In the regression, profit maximization is still accepted, but contrary to the single equation, more precise systems estimates reject the assumption of homotheticity, so the constraints are no longer imposed. There are two possible explanations for this. One is that the wage adjustment is not accurate enough and poor data are still causing false rejections of homotheticity. Another is that factor shares are genuinely a function of output. As discussed earlier, bigger firms have cheaper capital and therefore employ more of it, or it could be a genuine technological feature.

Before proceeding to elasticity estimates, one caveat remains. For most of the equations, tests of residual normality are easily rejected, unlike for the production equation. This suggests that there are still some errors in the specification of the model (Hendry, 1995). Also, the delta method could produce highly misleading confidence bands when the individual coefficients are not even close to normally distributed, and the sample is not large enough for asymptotic theory to remedy this problem. Therefore, confidence bands are not constructed and, while not compensating for the loss of statistical inference, a wider range of the sample is analysed for consistency.

The regression results are presented in appendix 7.5 as they are not of direct interest. Overall measures of fit are good, as suggested in table 7.1, although they are not reliable given the non-normal residuals:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Obs</th>
<th>Parms</th>
<th>RMSE</th>
<th>&quot;R-sq&quot;</th>
<th>chi2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Man/Prof</td>
<td>307</td>
<td>24</td>
<td>0.06</td>
<td>0.43</td>
<td>232.62</td>
<td>0</td>
</tr>
<tr>
<td>Skilart</td>
<td>307</td>
<td>24</td>
<td>0.08</td>
<td>0.18</td>
<td>71.78</td>
<td>0</td>
</tr>
<tr>
<td>Semi</td>
<td>307</td>
<td>24</td>
<td>0.13</td>
<td>0.16</td>
<td>61.78</td>
<td>0</td>
</tr>
<tr>
<td>Un</td>
<td>307</td>
<td>23</td>
<td>0.11</td>
<td>0.11</td>
<td>38.65</td>
<td>0.02</td>
</tr>
<tr>
<td>Cost</td>
<td>307</td>
<td>54</td>
<td>0.54</td>
<td>0.85</td>
<td>2021.29</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 7.2: measures of fit for systems regression using wage size. Sales/Clerical omitted and capital equation dropped.

\[ \sigma_y = \sigma_w \] is the constant output Allen elasticity of substitution and measures \( \frac{\partial \ln \gamma_y}{\partial \ln \gamma_w} \), while (7.3) measures the effect of an exogenous increase in the price of one factor on the quantity of another, holding output constant:

---

33 If the systems inference is correct and the single-equation inference is wrong, the measures of returns to scale compared earlier are strictly speaking not valid and should be used for rough comparison only, but the experiments on the effects of the wages on technological assumptions are still valid. Furthermore, residuals tend to be non-normally distributed, making inference less reliable.
\[ \lambda_{ij} = \frac{d \ln x_{i}}{d \ln w_{j}} = \zeta_{ij} \sigma_{ij} \]  

(7.3)

Because we cannot validly impose constant returns to scale, we don’t adjust (7.3) for non-constant output (see chapter 2.3). This means the numbers are best interpreted in a demand-constrained economy. Chapter 2.3 showed that, despite constant returns to scale not holding, we can still exploit duality and use the following expressions:

\[ \sigma_{ij} = \frac{b_{ij}}{s_{j}s_{i}} + 1 \]  

(7.4)

\[ \sigma_{ii} = \frac{b_{ii}}{s_{i}^{2}} + 1 - s_{i} \]  

(7.5)

\[ \lambda_{ij} = \frac{b_{ij}}{s_{i}} + s_{j} \]  

(7.6)

\[ \lambda_{ii} = \frac{b_{ii}}{s_{i}} + s_{i} - 1 \]  

(7.7)

where:

\[ s_{i} = a_{i} + \sum_{j} b_{ij} \ln w_{j} \]  

(7.8)

**7.5 ELASTICITY MEASURES**

The AES are presented in table 7.2. Values marked with an asterisk are consistently signed across at least 95% of the sample; the other values are consistent across 75% of the sample.

<table>
<thead>
<tr>
<th>( i )</th>
<th>Capital</th>
<th>Man/Prof</th>
<th>Skil/Art</th>
<th>Semi</th>
<th>Un</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>-1.62*</td>
<td>2.19*</td>
<td>2.91*</td>
<td>2.73*</td>
<td>1.74*</td>
</tr>
<tr>
<td>Man/Prof</td>
<td>2.19*</td>
<td>-5.96</td>
<td>-5.77</td>
<td>-1.46</td>
<td>-2.04</td>
</tr>
<tr>
<td>Skil/Art</td>
<td>2.91*</td>
<td>-5.77</td>
<td>-7.53</td>
<td>-7.28*</td>
<td>1.79*</td>
</tr>
<tr>
<td>Semi</td>
<td>2.73*</td>
<td>-1.46</td>
<td>-7.28*</td>
<td>-5.48*</td>
<td>-2.44*</td>
</tr>
<tr>
<td>Un</td>
<td>1.74*</td>
<td>-2.04</td>
<td>1.79*</td>
<td>-2.44*</td>
<td>-5.94*</td>
</tr>
</tbody>
</table>

Table 7.2: Allen Elasticities of Substitution; *denotes consistent across 5th and 95th percentiles; all others are consistent across both quartiles

Table 7.2 suggests capital is a \( p \)-substitute with all occupations, as shown in the first row/column, so a relative fall in the cost of capital will lead to a fall in employment. This common result has a bizarre feature ignored by commentators: a fall in interest rates would increase unemployment. Of course, these values hold output constant; while a fall in interest rates is expected to lead to a rise in output, the rise in output needs to be quite big to dominate the negative constant-output effect on employment.
The suggestion that all forms of labour seem roughly equally substitutable for capital implies separability\textsuperscript{34}, which has two methodological implications. First, studies of labour/capital substitution do not incur a great cost by aggregating various forms of heterogeneous labour. Second, should data constraints prevent the use of costs of capital in studies of intra-labour elasticities, omitting capital would not affect the estimates badly. Another result is that there is no support for capital-skill complementarity in these statistics.

Most occupations share a common substitute – capital – but are themselves p-complements. This result is important, and differs from two-factor studies like those of Edwards (2003), which by construction will find skilled and unskilled labour to be substitutes. While the previous paragraph suggested some simplifications to the model need not be overly damaging, only using two factors can be very misleading.

The values imply that wage restraint by one occupation, by allowing relative wages to fall relative to the cost of capital, would increase employment of that occupation and the other occupations. This has interesting implications for collective bargaining in industries or firms where one or two different occupations are represented by their own union. The exception to this finding is the interaction between skilled artisans and labour, a dynamic more usefully explored using the elasticities of factor demand in table 7.3:

\begin{center}
\begin{tabular}{|c|c|c|c|c|}
\hline
& Capital & Man/Prof & Skill/Art & Semi & Un \\
\hline
Capital & -0.96* & 0.18* & 0.18* & 0.40* & 0.19* \\
Man/Prof & 1.28* & -0.56 & -0.32* & -0.20 & -0.20 \\
Skill/Art & 1.77* & -0.42* & -0.56 & -0.99* & 0.19* \\
Semi & 1.60* & -0.12* & -0.43* & -0.80* & -0.26* \\
Un & 1.03* & -0.16* & 0.12* & -0.34* & -0.65* \\
\hline
\end{tabular}
\end{center}

\textbf{Table 7.3: Elasticities of factor demand; * denotes consistent across 95% of firms; all other values are consistent across 75% of firms.}

Taken literally, the implication is that a 10% fall in skilled/artisan wages will lead to a 1.2% fall in unskilled employment, holding output constant, while the same fall in semi-skilled wages would lead to a 3.4% rise in unskilled employment. This again shows the importance of disaggregating for this study.

All own-price elasticities are negative, falling comfortably in the –0.66 to –0.85 range found in most South African studies (Nattrass, 2004). In particular, we can say that, based on firm-level

\textsuperscript{34} See section 5.2
manufacturing evidence, a 10% fall in unskilled wages should lead to a 6.5% rise in unskilled employment. This value is close to the widely-cited unskilled value of –0.71 for manufacturing estimated by Fallon & Lucas (1998) at an industry-level and raises confidence in all the results.

7.6 SUMMARY

This chapter has experienced one of the key difficulties with disaggregated studies, namely a lack of disaggregated wage data. It has shown some very poor initial results and empirically supported the claim of chapter 4.3.3 that not-accounting for firm-size effects on wages can cause false rejections of linear price homogeneity. The final results suggest that, while some forms of aggregation are not necessarily an obstacle for certain studies, disaggregating distinguishes between the effects of skilled/artisan and semi-skilled workers on the unskilled in this study. They also show labour demand is relatively inelastic and that, while capital and all forms of labour are substitutes, most forms of labour are complements with each other. Such complementarity would not be found in CES functions or other estimates using two factors at a time.
8. CONCLUDING COMMENTS

Using predominantly firm-level manufacturing data, this study estimates various elasticities between capital, managers/professionals, salespeople/clerks, skilled workers / artisans, semi-skilled workers and unskilled workers. Translog functions are used because they impose few restrictions on the technologies and hence the derived elasticities.

This thesis documents the evolution of various elasticity concepts, clarifying confusion over when they are equivalent or otherwise, and showing how each is consistent with different endogeneity assumptions. The Hicks Elasticity of Complementarity and elasticity of factor price concepts are appropriate to studying the effects of exogenous labour supply shifts on endogenous wages. HECs are estimated using a production function. Allen Elasticities of Substitution are estimated using a cost function in order to understand the effects of exogenous wage shifts on endogenous factor quantities. After confirming that an AES can be derived from a cost function for general technologies, this study calculates these elasticities despite the cost function not representing a constant returns to scale technology.

In order to obtain wage data for the cost function, household data are used to predict wages for each firm according to characteristics that are common to both the firm and household surveys. We build wages that are heterogeneous enough to proxy each firm’s wages accurately and yet calculated from enough observations to yield precise wage estimates. The impacts of not accounting for firm-size in constructing wages are predicted to be exaggerated returns to scale as well as false rejections of price homogeneity restrictions and homotheticity assumptions. Existing estimates of wage size effects are used to adjust the constructed wages, support these predictions, and estimate cost functions.

This study also improves on prior work by simultaneously constructing confidence intervals for elasticity parameters and showing how elasticities vary across firms in the sample. True to Marshall’s Rules, and unlike any documented study, this work allows for the effects of imperfect product demand elasticity on the elasticity of factor price.

If a rise in the supply of one factor leads to a rise in the price of another, as measured by the elasticity of factor price, the pair are said to be *q*-complements while if the rise in supply leads to a fall in the price of the other factor, the pair are *q*-substitutes. If a rise in the price of one factor
leads to a fall in the quantity of another, as measured by the elasticity of factor demand, the pair are said to be *p-complements*, while if the rise in the price leads to a rise in the quantity of the other factor, the pair are *p-substitutes*. The elasticity estimates produce the following key results:

- Unskilled workers are *p- and q-complements* with managers/professionals; they are *p-substitutes* with skilled/artisan labour but *q-substitutes* with skilled/artisan labour; they are *p-complements* with semi-skilled workers but *q-substitutes* with semi-skilled workers.

Other findings are documented below:

- Capital and all occupations are *p-substitutes*; capital is a *q-substitute* with managers/professionals, semi-skilled workers and unskilled workers; capital is a *q-complement* with the sales/clerical and skilled/artisan occupations.
- Managerial/Professional labour and all other occupations are *p- and q-complements*.
- Skilled/artisan occupations are *p- and q-complements* with managers/professionals and semi-skilled workers; they are *p-substitutes* but *q-complements* with unskilled labour.
- Semi-skilled workers are *p- and q-complements* with managers/professionals and skilled/artisans; they are *p-complements* but *q-substitutes* with unskilled labour.
- Sales/clerical workers are *q-complements* with production workers and *q-substitutes* with managerial/professional employees; cost function estimates involving sales/clerical workers are not performed owing to specification issues.

However these summary statistics belie some cross-sample variations. For example, only 51% of firms exhibit skilled workers / artisans and semi-skilled workers as *q-complements*. In contrast, 96% of firms employ skilled/artisan and unskilled workers as *q-complements*.

Hamermesh (1993) stresses that factor inputs can be complements of one form and substitutes of another. In this study, evidence thereof at the lower skill levels suggests this is the work of imperfectly functioning markets. Furthermore, bargaining institutions play a major role in wage and, to a lesser extent, quantity setting. Given the complementarity between occupation types and the apparent opportunities for co-ordination, there are clear grounds for research into the interactions between different occupations through their unions. The effects of changes in supply of one occupation on the bargaining equilibria, given this complementarity, provide another avenue for research.

Such evidence also emphasises the importance of using the appropriate model. To study the effects of increased artisan supply, one might be tempted to infer a fall in artisan wages and,
because they are p-substitutes, a fall in unskilled employment. The more appropriate measure, which adjusts for rigid wages if necessary, forcefully shows the opposite result. In the key conclusions of this paper, a rise in the supply of skilled workers / artisans will lead to a rise in unskilled wages or employment and a rise in the supply of semi-skilled workers will lead to a fall in unskilled wages or employment. These results are consistent at lower and upper confidence regions, for the large majority of the sample, and in various industries after allowing for imperfect product demand elasticity. Critically, such results would be impossible to find in aggregated studies.

There are a few cautions against taking the numbers literally, as the nature of wage rigidity is complex. Furthermore, this cross section of firms is being used to simulate supply effects that necessarily will only take place over time. The survey year was a year of recession, which perhaps distorts the production relations between the factors. Moreover, much restructuring took place in the early 1990s and has continued since the sample period, meaning the nature of technological relationships may already have changed since then. In addition, the predicted static effects assume no change in technology choice, which is endogenous to the supply of skilled labour. There is theoretical and empirical evidence that increased skill supply produces more technologies that encourage firms to demand more skilled labour, with negative consequences for those remaining unskilled (Acemoglu, 1998, 2003). The further research that is needed in this area can now be done with a South African emphasis, as the key static elasticity parameters used by these sorts of models have been estimated by this study.

Nonetheless, the estimated values have some clear policy implications. For manufacturing as a whole, and assuming perfectly elastic demand, the elasticity of factor price for unskilled wages is 0.42 with respect to the quantity of skilled/artisans and –0.42 with respect to semi-skilled workers. Extrapolating sample factor proportions to the 1.3 million manufacturing workers in the economy in 1999 (Statistics South Africa, 2000), this means training a modest extra 10 000 skilled/artisans instead of 10 000 semi-skilled workers every year would raise unskilled annual wages by approximately R660 (4%) or increase the number of unskilled jobs by almost 10 500 (2%) every year.

There is therefore cause for serious concern because the opposite seems to happening; training is taking place, but artisans make up far too low a proportion (Paton, 2003). These training programs may currently be increasing unskilled unemployment, so policy-makers must redouble efforts to produce more artisans.
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# APPENDICES

## APPENDIX 3.1: CORRELATION COEFFICIENTS BETWEEN INPUTS

### Table 3: Correlation coefficients for the 6 factors and raw materials

<table>
<thead>
<tr>
<th></th>
<th>Capital</th>
<th>Man/Prof</th>
<th>Sale/Cle</th>
<th>Skil/Art</th>
<th>Semi</th>
<th>Un</th>
<th>Raw materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Man/Prof</td>
<td>0.7588</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sale/Cle</td>
<td>0.7635</td>
<td>0.8417</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skil/Art</td>
<td>0.6392</td>
<td>0.6914</td>
<td>0.6848</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semi</td>
<td>0.6992</td>
<td>0.7326</td>
<td>0.7391</td>
<td>0.6549</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Un</td>
<td>0.705</td>
<td>0.6445</td>
<td>0.671</td>
<td>0.5439</td>
<td>0.6292</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Raw materials</td>
<td>0.8088</td>
<td>0.7922</td>
<td>0.8477</td>
<td>0.6288</td>
<td>0.7491</td>
<td>0.7201</td>
<td>1</td>
</tr>
</tbody>
</table>
**APPENDIX 4.1: LIST OF VARIABLES**

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>SYMBOL</th>
<th>OTHER</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>VARIABLES IN PRODUCTION FUNCTION (in logarithms)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>y</td>
<td>R million Turnover</td>
</tr>
<tr>
<td>Value Added</td>
<td>y</td>
<td>Turnover*(1 - raw materials percentage)</td>
</tr>
<tr>
<td>Capital</td>
<td>x1</td>
<td>R million fixed assets - adjusted for shift capacity utilization</td>
</tr>
<tr>
<td>Managerial/Professional Labour</td>
<td>x2</td>
<td>Number of fulltime employees + 0.5*(number of part time employees)</td>
</tr>
<tr>
<td>Sales/Clerical Labour</td>
<td>x3</td>
<td>Number of fulltime employees + 0.5*(number of part time employees)</td>
</tr>
<tr>
<td>Skilled/Artisan Labour</td>
<td>x4</td>
<td>Number of fulltime employees + 0.5*(number of part time employees)</td>
</tr>
<tr>
<td>Semiskilled Labour</td>
<td>x5</td>
<td>Number of fulltime employees + 0.5*(number of part time employees)</td>
</tr>
<tr>
<td>Unskilled Labour</td>
<td>x6</td>
<td>Number of fulltime employees + 0.5*(number of part time employees)</td>
</tr>
<tr>
<td>Raw materials</td>
<td>x7</td>
<td>(Raw materials percentage)*(sales)</td>
</tr>
<tr>
<td>These variables are used to calculate the higher order and interaction terms</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**VARIABLES IN COST FUNCTION AND COST SHARE EQUATIONS (in logarithms)**

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>SYMBOL</th>
<th>OTHER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of Capital</td>
<td>w1</td>
<td>real interest rate + depreciation rate + effective tax rate + risk premium</td>
</tr>
<tr>
<td>Managerial/Professional Labour</td>
<td>w2</td>
<td>4 wages predicted using household survey data</td>
</tr>
<tr>
<td>Sales/Clerical Labour</td>
<td>w3</td>
<td>4 wages predicted using household survey data</td>
</tr>
<tr>
<td>Skilled/Artisan Labour</td>
<td>w4</td>
<td>4 wages predicted using household survey data</td>
</tr>
<tr>
<td>Semiskilled Labour</td>
<td>w5</td>
<td>4 wages predicted using household survey data</td>
</tr>
<tr>
<td>Unskilled Labour</td>
<td>w6</td>
<td>4 wages predicted using household survey data</td>
</tr>
<tr>
<td>Total Cost</td>
<td>C</td>
<td>Sum of factor costs; each factor cost (i) = w(i)x(i)</td>
</tr>
<tr>
<td>Value Added</td>
<td>y</td>
<td>Sales*(1 - raw materials percentage)</td>
</tr>
<tr>
<td>Factor share Man/Prof</td>
<td>ShareManProf</td>
<td>w1*x1/C (not in log form)</td>
</tr>
<tr>
<td>Factor share Sale/Cle</td>
<td>ShareSale/Cle</td>
<td>w2*x2/C (not in log form)</td>
</tr>
<tr>
<td>Factor share Skil/Art</td>
<td>ShareSkilart</td>
<td>w3*x3/C (not in log form)</td>
</tr>
<tr>
<td>Factor share Semiskilled</td>
<td>ShareSemi</td>
<td>w4*x4/C (not in log form)</td>
</tr>
<tr>
<td>Factor share Unskilled</td>
<td>Shareun</td>
<td>w5*x5/C (not in log form)</td>
</tr>
<tr>
<td>Managerial/Professional Labour - Cost of Capital</td>
<td>d2</td>
<td>w2-w1</td>
</tr>
<tr>
<td>Sales/Clerical Labour - Cost of Capital</td>
<td>d3</td>
<td>w3-w1</td>
</tr>
<tr>
<td>Skilled/Artisan Labour - Cost of Capital</td>
<td>d4</td>
<td>w4-w1</td>
</tr>
<tr>
<td>Semi-skilled Labour - Cost of Capital</td>
<td>d5</td>
<td>w5-w1</td>
</tr>
<tr>
<td>Unskilled Labour - Cost of Capital</td>
<td>d6</td>
<td>w6-w1</td>
</tr>
<tr>
<td>These variables are used to calculate the higher order and interaction terms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VARIABLE NAME</td>
<td>SYMBOL</td>
<td>OTHER</td>
</tr>
<tr>
<td>----------------------------------------------------------------</td>
<td>--------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>CANDIDATES FOR CONTROLS/INSTRUMENTS</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 Industry Dummies</td>
<td>ind</td>
<td></td>
</tr>
<tr>
<td>8 Province Dummies</td>
<td>loc</td>
<td></td>
</tr>
<tr>
<td>Exports as a % of sales</td>
<td>q10q9</td>
<td></td>
</tr>
<tr>
<td>Raw materials as % of total costs</td>
<td>q18c</td>
<td></td>
</tr>
<tr>
<td>% raw materials imported</td>
<td>q18d</td>
<td></td>
</tr>
<tr>
<td>Difficulty recruiting Man/Prof workers</td>
<td>q43_a</td>
<td>index; higher value means the firm has less difficulty</td>
</tr>
<tr>
<td>Difficulty recruiting Sale/Cle workers</td>
<td>q43_b</td>
<td>index; higher value means the firm has less difficulty</td>
</tr>
<tr>
<td>Difficulty recruiting Skill/Art workers</td>
<td>q43_c</td>
<td>index; higher value means the firm has less difficulty</td>
</tr>
<tr>
<td>Difficulty recruiting Semi-skilled workers</td>
<td>q43_d</td>
<td>index; higher value means the firm has less difficulty</td>
</tr>
<tr>
<td>Difficulty recruiting Unskilled workers</td>
<td>q43_e</td>
<td>index; higher value means the firm has less difficulty</td>
</tr>
<tr>
<td>Productivity dissatisfaction</td>
<td>q45a</td>
<td>index</td>
</tr>
<tr>
<td>Training expenditure</td>
<td>q47</td>
<td>R million</td>
</tr>
<tr>
<td>Market effects</td>
<td>q50atot</td>
<td>Aggregation of six indices of effects macro-factors have on hiring</td>
</tr>
<tr>
<td></td>
<td></td>
<td>decisions; index ranging from serious obstacle to major benefit</td>
</tr>
<tr>
<td>Unionised</td>
<td>q52q53</td>
<td>Index combining firm's exposure to firm- or plant-level collective</td>
</tr>
<tr>
<td></td>
<td></td>
<td>bargaining and bargaining council agreements</td>
</tr>
<tr>
<td>Size dummy</td>
<td>nf2a</td>
<td>1 if there are more than 50 employees</td>
</tr>
<tr>
<td>Ownermanaged</td>
<td>ownermanaged</td>
<td>1 if there is only 1 managerial/professional worker</td>
</tr>
<tr>
<td>Computer Investment in past year as percentage of assets</td>
<td>q19b1q11</td>
<td>Constructed using information on what percentage of equipment is in</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a certain age range</td>
</tr>
<tr>
<td>Average equipment age</td>
<td>q24_e</td>
<td>Capacity-adjusted capital stock divided by total number of employees</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(weighted to account for part-time workers)</td>
</tr>
<tr>
<td>K:L ratio</td>
<td>klratio</td>
<td></td>
</tr>
</tbody>
</table>
### APPENDIX 4.2: GROSS MONTHLY WAGES ACCORDING TO WAGE\_IND DEFINITION

<table>
<thead>
<tr>
<th>Category by Occupation and Industry</th>
<th>Mean</th>
<th>Std. Err.</th>
<th>Obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Managerial/Professional Food &amp; Beverages</td>
<td>3475</td>
<td>468</td>
<td>38</td>
</tr>
<tr>
<td>Managerial/Professional Wood, Pulp &amp; paper</td>
<td>2844</td>
<td>565</td>
<td>16</td>
</tr>
<tr>
<td>Managerial/Professional Chemicals, Rubber &amp; plastic</td>
<td>3427</td>
<td>523</td>
<td>17</td>
</tr>
<tr>
<td>Managerial/Professional Auto</td>
<td>2603</td>
<td>584</td>
<td>9</td>
</tr>
<tr>
<td>Managerial/Professional Textiles and Clothing</td>
<td>2562</td>
<td>326</td>
<td>19</td>
</tr>
<tr>
<td>Managerial/Professional Fabricated Metal</td>
<td>3527</td>
<td>947</td>
<td>5</td>
</tr>
<tr>
<td>Managerial/Professional Furniture</td>
<td>3699</td>
<td>833</td>
<td>5</td>
</tr>
<tr>
<td>Managerial/Professional Electrical, Electronic and other Machinery</td>
<td>3048</td>
<td>431</td>
<td>11</td>
</tr>
<tr>
<td>Managerial/Professional Printing &amp; Publishing</td>
<td>3051</td>
<td>398</td>
<td>4</td>
</tr>
<tr>
<td>Sales/Clerical Food &amp; Beverages</td>
<td>2479</td>
<td>431</td>
<td>58</td>
</tr>
<tr>
<td>Sales/Clerical Wood, Pulp &amp; paper</td>
<td>1803</td>
<td>242</td>
<td>18</td>
</tr>
<tr>
<td>Sales/Clerical Chemicals, Rubber &amp; plastic</td>
<td>2157</td>
<td>291</td>
<td>20</td>
</tr>
<tr>
<td>Sales/Clerical Auto</td>
<td>2977</td>
<td>629</td>
<td>8</td>
</tr>
<tr>
<td>Sales/Clerical Textiles and Clothing</td>
<td>1535</td>
<td>163</td>
<td>29</td>
</tr>
<tr>
<td>Sales/Clerical Fabricated Metal</td>
<td>2600</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Sales/Clerical Furniture</td>
<td>2755</td>
<td>442</td>
<td>9</td>
</tr>
<tr>
<td>Sales/Clerical Electrical, Electronic and other Machinery</td>
<td>2159</td>
<td>417</td>
<td>19</td>
</tr>
<tr>
<td>Sales/Clerical Printing &amp; Publishing</td>
<td>1571</td>
<td>420</td>
<td>8</td>
</tr>
<tr>
<td>Skilled/Artisans Food &amp; Beverages</td>
<td>1562</td>
<td>161</td>
<td>88</td>
</tr>
<tr>
<td>Skilled/Artisans Wood, Pulp &amp; paper</td>
<td>1689</td>
<td>139</td>
<td>50</td>
</tr>
<tr>
<td>Skilled/Artisans Chemicals, Rubber &amp; plastic</td>
<td>2134</td>
<td>255</td>
<td>45</td>
</tr>
<tr>
<td>Skilled/Artisans Auto</td>
<td>2510</td>
<td>303</td>
<td>34</td>
</tr>
<tr>
<td>Skilled/Artisans Textiles and Clothing</td>
<td>1337</td>
<td>66</td>
<td>187</td>
</tr>
<tr>
<td>Skilled/Artisans Fabricated Metal</td>
<td>2047</td>
<td>437</td>
<td>17</td>
</tr>
<tr>
<td>Skilled/Artisans Furniture</td>
<td>1310</td>
<td>143</td>
<td>18</td>
</tr>
<tr>
<td>Skilled/Artisans Electrical, Electronic and other Machinery</td>
<td>2408</td>
<td>265</td>
<td>38</td>
</tr>
<tr>
<td>Skilled/Artisans Printing &amp; Publishing</td>
<td>2462</td>
<td>317</td>
<td>31</td>
</tr>
<tr>
<td>Semi-skilled Food &amp; Beverages</td>
<td>1720</td>
<td>88</td>
<td>131</td>
</tr>
<tr>
<td>Semi-skilled Wood, Pulp &amp; paper</td>
<td>1604</td>
<td>119</td>
<td>38</td>
</tr>
<tr>
<td>Semi-skilled Chemicals, Rubber &amp; plastic</td>
<td>1706</td>
<td>120</td>
<td>49</td>
</tr>
<tr>
<td>Semi-skilled Auto</td>
<td>1908</td>
<td>124</td>
<td>35</td>
</tr>
<tr>
<td>Semi-skilled Textiles and Clothing</td>
<td>1533</td>
<td>69</td>
<td>120</td>
</tr>
<tr>
<td>Semi-skilled Fabricated Metal</td>
<td>1756</td>
<td>229</td>
<td>17</td>
</tr>
<tr>
<td>Semi-skilled Furniture</td>
<td>1387</td>
<td>197</td>
<td>17</td>
</tr>
<tr>
<td>Semi-skilled Electrical, Electronic and other Machinery</td>
<td>1313</td>
<td>143</td>
<td>20</td>
</tr>
<tr>
<td>Semi-skilled Printing &amp; Publishing</td>
<td>1817</td>
<td>260</td>
<td>10</td>
</tr>
<tr>
<td>Unskilled Food &amp; Beverages</td>
<td>1200</td>
<td>83</td>
<td>177</td>
</tr>
<tr>
<td>Unskilled Wood, Pulp &amp; paper</td>
<td>1047</td>
<td>70</td>
<td>39</td>
</tr>
<tr>
<td>Unskilled Chemicals, Rubber &amp; plastic</td>
<td>1898</td>
<td>223</td>
<td>35</td>
</tr>
<tr>
<td>Unskilled Auto</td>
<td>1925</td>
<td>268</td>
<td>21</td>
</tr>
<tr>
<td>Unskilled Textiles and Clothing</td>
<td>1351</td>
<td>73</td>
<td>86</td>
</tr>
<tr>
<td>Unskilled Fabricated Metal</td>
<td>1401</td>
<td>140</td>
<td>18</td>
</tr>
<tr>
<td>Unskilled Furniture</td>
<td>955</td>
<td>114</td>
<td>16</td>
</tr>
<tr>
<td>Unskilled Electrical, Electronic and other Machinery</td>
<td>1780</td>
<td>232</td>
<td>22</td>
</tr>
<tr>
<td>Unskilled Printing &amp; Publishing</td>
<td>1171</td>
<td>237</td>
<td>12</td>
</tr>
</tbody>
</table>
### APPENDIX 4.4: COMPARISON OF \( \text{WAGE}_{\text{SOME}} \) AND \( \text{WAGE}_{\text{ALL}} \)

<table>
<thead>
<tr>
<th>Wage Series</th>
<th>Categories</th>
<th>ManProf</th>
<th>Salecl</th>
<th>Skilart</th>
<th>Semi</th>
<th>Un</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{WAGE}_{\text{all}} )</td>
<td>Observations</td>
<td>3.9</td>
<td>5.3</td>
<td>14.6</td>
<td>11.8</td>
<td>13.2</td>
</tr>
<tr>
<td>( \text{WAGE}_{\text{some}} )</td>
<td>Observations</td>
<td>17.7</td>
<td>16.9</td>
<td>33.9</td>
<td>43.7</td>
<td>30.2</td>
</tr>
</tbody>
</table>

Number of categories each wage series is divided into, and the average number of observations in the household survey used to calculate the mean wage for each wage category.

### APPENDIX 4.4: COSTS OF CAPITAL

<table>
<thead>
<tr>
<th>Cost of Capital</th>
<th>Frequency (# firms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>1</td>
</tr>
<tr>
<td>41</td>
<td>8</td>
</tr>
<tr>
<td>42</td>
<td>28</td>
</tr>
<tr>
<td>43</td>
<td>62</td>
</tr>
<tr>
<td>44</td>
<td>7</td>
</tr>
<tr>
<td>45</td>
<td>70</td>
</tr>
<tr>
<td>46</td>
<td>89</td>
</tr>
<tr>
<td>47</td>
<td>78</td>
</tr>
<tr>
<td>48</td>
<td>97</td>
</tr>
<tr>
<td>49</td>
<td>20</td>
</tr>
<tr>
<td>50</td>
<td>74</td>
</tr>
<tr>
<td>51</td>
<td>60</td>
</tr>
<tr>
<td>52</td>
<td>14</td>
</tr>
<tr>
<td>53</td>
<td>17</td>
</tr>
</tbody>
</table>

Costs of capital
APPENDIX 5.1: PLOTS OF RESIDUALS

Evidence of Heteroskedasticity

Plot of residuals against Log Man/Prof. Firms with Log(Man/Prof) = 0 are assumed to be owner-managed and hence enjoy efficiency gains suggested by the residuals.

Evidence of Heteroskedasticity

Plot of residuals against Log Value Added; there is clearly misspecification in values below 0, which are not rectified with another size dummy.
### APPENDIX 6.1: REGRESSION RESULTS

#### Dependent Variable: Value Added

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Parameter</th>
<th>Coefficient</th>
<th>p value</th>
<th>Variable name</th>
<th>Coefficient</th>
<th>p value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>a1</td>
<td>0.201</td>
<td>0.001</td>
<td>nd2</td>
<td>0.116</td>
<td>0.480</td>
</tr>
<tr>
<td>ManProf</td>
<td>a2</td>
<td>0.156</td>
<td>0.167</td>
<td>nd3</td>
<td>-0.055</td>
<td>0.762</td>
</tr>
<tr>
<td>SaleCle</td>
<td>a3</td>
<td>0.361</td>
<td>0.000</td>
<td>nd4</td>
<td>0.520</td>
<td>0.002</td>
</tr>
<tr>
<td>SkillArt</td>
<td>a4</td>
<td>-0.020</td>
<td>0.769</td>
<td>nd5</td>
<td>0.163</td>
<td>0.367</td>
</tr>
<tr>
<td>Semi</td>
<td>a5</td>
<td>0.088</td>
<td>0.177</td>
<td>nd6</td>
<td>0.169</td>
<td>0.358</td>
</tr>
<tr>
<td>Un</td>
<td>a6</td>
<td>0.214</td>
<td>0.000</td>
<td>nd7</td>
<td>0.163</td>
<td>0.312</td>
</tr>
<tr>
<td>0.5*Capital^2</td>
<td>b11</td>
<td>0.059</td>
<td>0.119</td>
<td>nd8</td>
<td>-0.078</td>
<td>0.668</td>
</tr>
<tr>
<td>Capital*ManProf</td>
<td>b12</td>
<td>-0.039</td>
<td>0.387</td>
<td>nd9</td>
<td>0.146</td>
<td>0.397</td>
</tr>
<tr>
<td>Capital*SaleCle</td>
<td>b13</td>
<td>0.061</td>
<td>0.148</td>
<td>loc2</td>
<td>0.321</td>
<td>0.237</td>
</tr>
<tr>
<td>Capital*SkilArt</td>
<td>b14</td>
<td>-0.020</td>
<td>0.547</td>
<td>loc3</td>
<td>0.122</td>
<td>0.332</td>
</tr>
<tr>
<td>Capital*Semi</td>
<td>b15</td>
<td>-0.035</td>
<td>0.306</td>
<td>loc4</td>
<td>0.205</td>
<td>0.116</td>
</tr>
<tr>
<td>Capital*Un</td>
<td>b16</td>
<td>-0.027</td>
<td>0.315</td>
<td>loc5</td>
<td>0.405</td>
<td>0.140</td>
</tr>
<tr>
<td>0.5*ManProf^2</td>
<td>b22</td>
<td>0.009</td>
<td>0.931</td>
<td>loc6</td>
<td>-0.326</td>
<td>0.415</td>
</tr>
<tr>
<td>ManProf*SaleCle</td>
<td>b23</td>
<td>-0.080</td>
<td>0.330</td>
<td>loc7</td>
<td>-0.924</td>
<td>0.026</td>
</tr>
<tr>
<td>ManProf*SkillArt</td>
<td>b24</td>
<td>0.020</td>
<td>0.720</td>
<td>loc8</td>
<td>-0.084</td>
<td>0.815</td>
</tr>
<tr>
<td>ManProf*Semni</td>
<td>b25</td>
<td>0.011</td>
<td>0.845</td>
<td>loc9</td>
<td>0.224</td>
<td>0.069</td>
</tr>
<tr>
<td>ManProf*Un</td>
<td>b26</td>
<td>0.080</td>
<td>0.102</td>
<td>exports as % sales</td>
<td>0.233</td>
<td>0.263</td>
</tr>
<tr>
<td>0.5*SaleCle^2</td>
<td>b33</td>
<td>0.089</td>
<td>0.315</td>
<td>raw materials as % costs</td>
<td>-0.016</td>
<td>0.000</td>
</tr>
<tr>
<td>SaleCle*SkillArt</td>
<td>b34</td>
<td>-0.028</td>
<td>0.571</td>
<td>imports as % raw materials</td>
<td>-0.001</td>
<td>0.348</td>
</tr>
<tr>
<td>SaleCle*Semi</td>
<td>b35</td>
<td>-0.005</td>
<td>0.921</td>
<td>Recruitment ease ManProf</td>
<td>0.111</td>
<td>0.122</td>
</tr>
<tr>
<td>SaleCle*Un</td>
<td>b36</td>
<td>-0.037</td>
<td>0.355</td>
<td>Recruitment ease SaleCle</td>
<td>0.023</td>
<td>0.740</td>
</tr>
<tr>
<td>0.5*SkillArt^2</td>
<td>b44</td>
<td>-0.083</td>
<td>0.131</td>
<td>Recruitment ease SkilArt</td>
<td>-0.018</td>
<td>0.765</td>
</tr>
<tr>
<td>SkillArt*Semi</td>
<td>b45</td>
<td>0.057</td>
<td>0.128</td>
<td>Recruitment ease Semi</td>
<td>0.062</td>
<td>0.390</td>
</tr>
<tr>
<td>SkillArt*Un</td>
<td>b46</td>
<td>0.054</td>
<td>0.089</td>
<td>Recruitment ease Un</td>
<td>0.110</td>
<td>0.301</td>
</tr>
<tr>
<td>0.5*Semni^2</td>
<td>b55</td>
<td>0.051</td>
<td>0.292</td>
<td>Productivity dissatisfaction</td>
<td>-0.017</td>
<td>0.643</td>
</tr>
<tr>
<td>Semi*Un</td>
<td>b56</td>
<td>-0.080</td>
<td>0.014</td>
<td>Training expenditure</td>
<td>0.000</td>
<td>0.042</td>
</tr>
<tr>
<td>Un^2</td>
<td>b66</td>
<td>0.010</td>
<td>0.779</td>
<td>Market conditions</td>
<td>0.018</td>
<td>0.114</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.759</td>
<td>0.057</td>
<td></td>
<td>CollectiveBargaining</td>
<td>-0.014</td>
<td>0.839</td>
</tr>
<tr>
<td>Firm size &gt; 50 employees</td>
<td>0.506</td>
<td>0.000</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ownermanaged</td>
<td>0.420</td>
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<td>(3) b13 + b23 + b33 + b34 + b35 + b36</td>
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<td>(4) b14 + b24 + b34 + b44 + b45 + b46 = 0</td>
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<td>(5) b15 + b25 + b35 + b45 + b55 + b56 = 0</td>
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<td>(6) b16 + b26 + b36 + b46 + b56 + b66 = 0</td>
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<td>(7) a1 + a2 + a3 + a4 + a5 + a6 = 1</td>
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bij=bji imposed in variable construction
### APPENDIX 6.2: MEDIAN AND QUARTILES OF ESTIMATES AND CONFIDENCE BANDS

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Table: AES between pairs ij, at median and quartiles, for estimate (ij), lower (ijl) and upper (ijh) confidence band.
## APPENDIX 6.3: DETAILED ELASTICITIES OF FACTOR PRICE

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### APPENDIX 6.4 MEASURES OF ELASTICITY OF FACTOR PRICE AT MEAN OF FACTOR VALUES

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APPENDIX 7.1: RETURNS TO SCALE USING FIRM-SIZE ADJUSTED (WAGE_{SIZE}) AND UNADJUSTED (WAGE_{SOMD}) WAGES

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APPENDIX 7.2: OWN PRICE ELASTICITIES USING WAGE_{ALL}

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<th>(\lambda_{\text{Salecle}})</th>
<th>(\lambda_{\text{Skilart}})</th>
<th>(\lambda_{\text{Semi}})</th>
<th>(\lambda_{\text{Un}})</th>
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APPENDIX 7.3: VARIANCE INFLATION FACTORS FROM SINGLE EQUATION COST ESTIMATE

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<td>B35</td>
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<td>B34</td>
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<td>B56</td>
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APPENDIX 7.4: OWN-PRICE ELASTICITIES USING FIRM-SIZE ADJUSTED WAGES BUT INCLUDING SALES/Clerical

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<th>(\lambda_{\text{Skilart}})</th>
<th>(\lambda_{\text{Semi}})</th>
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APPENDIX 7.5: REGRESSION USED AS BASIS FOR FINAL COST FUNCTION ELASTICITY RESULTS

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